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CEIS Tor Vergata - Research Paper Series, Vol. 5, No. 15 May 2003

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Fiscal Deficits and Currency Crises

by

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February 2003

Abstract

This paper investigates currency and financial crises in an optimizing general equilibrium model. It is shown that a rise in current and expected future budget deficits generates a real exchange rate appreciation and a decumulation of external assets, leading up to a currency crisis when foreign reserves approximate a critical level. Strong empirical support for our model is obtained by a probit estimation for Latin American and Asian countries.

JEL classification: F31; F32; F41; E52; E62
Key words: budget deficits, foreign exchange reserves, currency crises

Acknowledgements:

We wish to thank Barbara Annicchiarico, Fabio C. Bagliano, Marianne Baxter, Leonardo Becchetti, Lorenzo Bini Smaghi, Alberto Dalmazzo, Bassam Fattouh, Laurence Harris, Fabrizio Mattesini, Pedro M. Oviedo, Alberto Petrucci, Cristina M. Rossi, Pasquale Scaramozzino, Pietro Senesi and Salvatore Vinci for helpful comments and suggestions.

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I. INTRODUCTION

The events of the 90's have cast serious doubts on the validity of standard models of currency crises. The economic conditions in the Asian and Latin American countries did not appear to show the kind of macroeconomic and financial distress that typically is at the core of the traditional models of balance of payments crises. Alternative explanations have been provided by a new literature emphasizing moral hazard problems and financial panics. The moral hazard based models stress the role played by the government bailout promise in determining excessive risk taking by financial intermediaries. The channels through which this could have operated in emerging economies are poor banking regulations and the so called “carry trade”, by which banks borrow in international markets at low interest rates and lend at higher rates at home (OECD, 1999, pp. 177-83). This resulted in a lending boom fuelled by large capital inflows, thereby generating overinvestment in risky projects and asset price bubbles. The bubble grew up until an adverse shock burst it, revealing the fragility of the banking system and generating a financial and currency crisis.\footnote{For an interpretation of financial crises in Southeast Asia along these lines see Calvo, et al. (1994), McKinnon and Pill (1996), Dooley (1997), Corsetti et al. (1998, 1999), Krugman (1998), and Chinn et al. (1999), among others.}

Proponents of this view argue that if the bailout promise is at the core of moral hazard problems, then the only policy cure is to abolish the lender of last resort facilities.

The problem with the moral hazard view, as pointed out in Radelet and Sachs (1998, pp. 35-42), is that the data in the early 1990s did not show a dramatic deterioration in either loan quality or investment riskiness for the crisis countries. Furthermore, spreads on Asian bonds fell between 1995 and 1997, and ratings of long term government bonds by Moody's, Standard and Poor's, and Euromoney remained unchanged until the onset of the crisis, revealing that foreign lenders did not perceive an increase in risk. No warning of an asset price bubble was present in the reports of the investment houses, showing that expectations of a financial crash and a subsequent bailout were absent.\footnote{See also BIS (1997), IMF (1998) and for a contrasting view, Sarno and Taylor (1999).}
The financial panic based models, on the other hand, stress self-fulfilling prophecies and herding behavior as the determinants of a crisis. According to this view, the crisis may be triggered by either rumors or fundamentals resulting in a massive withdrawal by investors attempting to avoid capital losses. This is rationalized by using multiple rational expectations equilibrium models where the financial panic represents a self-fulfilling bad equilibrium leading to the collapse, along the lines sketched by Diamond and Dybvig (1983) in the context of banking institutions. Crises are thus unavoidable and can occur even when countries show sound or non-deteriorating fundamentals.3

The key factor behind the sudden shifts in expectations is “the excess volatility “ in international financial markets. Evidence may be found in the large and, to some extent, unanticipated swings of capital flows that played a critical role in pushing the emerging market economies into crises. The sharp reversal of capital flows from Latin American and Asian countries, respectively in 1994 and 1997, was the start of the currency and banking crises through herding behavior and contagion effects (e.g. Radelet and Sachs, 1998; Kaminsky and Schmukler, 1999). Proponents of this view argue that there is a strong rationale for an international lender of last resort, so that the crisis could be stopped and not be allowed to spread.

The problem with the financial panic view is that the data in the 1990s in many emerging countries do show macroeconomic imbalances, and this must have played some role in the subsequent crises (see, for example, OECD, 1999, pp. 183-91; Corsetti et al., 1998).

However, no satisfactory model based on fundamentals has been presented in the literature with the notable exceptions by Burnside, Eichenbaum and Rebelo (2000, 2001) and Daniel (2000) who explain currency crises as the consequence of expected changes about the course of future policy. In particular, Burnside, Eichenbaum and Rebelo examine the role of large prospective budget deficits (associated with implicit bailout guarantees to banking systems) in the 1997 Asian currency crisis. Daniel explores how different combinations of expansionary future policies determine the timing of the exchange rate collapse.

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In both models the collapse of the peg is driven by the increase in the present value of seignorage revenue associated with the change in expected future policies.

In this paper we present, on the other hand, a model of currency crises based on fundamentals, where current account deficits brought about by current and prospective fiscal deficits push foreign reserves to a critical level, where the attack starts. The main result is that crises can occur even when both monetary and fiscal policy are correctly designed. There is no need for prospective seignorage revenue, since the intertemporal budget constraint of the government is always respected and monetary policy obeys the rules of the game. The conclusion is that the sustainability of fixed exchange rate system may require giving up both monetary and fiscal sovereignty.

The paper is organized as follows. Section II presents the theoretical model. Section III describes the dynamics of the model and the time of the speculative attack. Section IV presents the empirical results. Section V contains the summary and conclusions of the paper.

II. THE OPTIMIZING MODEL

Consider a semi-small open economy operating under a fixed exchange rate regime. The economy is populated by households, firms, and the government. Agents have perfect foresight and consume two physical goods, which we denote as $C^H$ (domestically produced good) and $C^F$ (foreign or imported good), so that total real consumption, $C$, can be written as

$$C = C^H + \rho C^F = qC + (1-q)C,$$

where $\rho = \frac{EP^*}{P}$ is the relative price of foreign goods in terms of domestic goods, or real exchange rate, $E$ is the nominal fixed exchange rate, $P^*$ and $P$ the foreign and domestic price of the consumption goods, respectively, and $q$ and $(1-q)$ the proportions of domestic and foreign goods over total consumption. Since this is a semi-small open economy, the price of import goods is exogenous but the price of the export good is domestically set.

The household sector consists of many identical, utility maximizing agents. The utility
function is logarithmic in consumption and real money balances. There is no bequest motive, population is equal to the labor force and normalized to unity, and lifetimes are uncertain as in Yaari (1965) and Blanchard (1985). Household’s non-human wealth is the sum of domestic money, capital stock and traded bonds denominated in foreign currency. Traded bonds pay an exogenously given world real interest rate, \( r^* \). International capital markets are perfect and uncovered interest parity holds at all times.

Each household face the following maximization problem:

\[
\max V_t \equiv \int_0^\infty \log [c_t(s)^\epsilon \ell_t(s)^{1-\epsilon}] e^{-(\beta+\delta)t} dt
\]

s.t. \( \dot{w}_t(s) = (r^*+\delta)w_t(s) + \omega_t(s) - \tau_t(s) - c_t(s) - i_t\ell_t(s) \)

and the transversality condition preventing Ponzi games

\[
\lim_{t \to \infty} w_t(s)e^{-(r^*+\delta)t} = 0,
\]

where \( t \) denotes time, \( s \) the birth date of an individual to identify the generation and \( \dot{x} = dx/dt \). \( c, \ell, \omega, \) and \( \tau \) are real consumption, real money balances, real labor income and real lump-sum taxes, respectively. Real non-human wealth is defined as \( w \equiv \ell + k + \rho b \), where \( k \) denotes the capital stock and \( b \) traded bonds. The nominal interest rate is \( i = r^* + \pi = i^* \) and \( \pi = \dot{p}/p \) is the expected and actual rate of inflation, set for simplicity equal to zero. The weight of consumption in the utility function is \( 0 < \epsilon < 1 \), \( \beta \) is the subjective discount factor and \( \delta \) is the instantaneous probability of death. The effective discount factor thus is \( (\beta+\delta) \) and \( \delta^{-1} \) is the expected lifetime of agents.

First-order conditions for maximization imply the following demand functions:

1. \( c_t(s) = \frac{\beta + \delta}{1 + \eta} [w_t(s) + h_t(s)] \)
2. \( \ell_t(s) = \frac{\eta c_t(s)}{i_t} \)

\(^4\) The approach of entering money in the utility function to allow for money holding behavior within a Yaari-Blanchard framework, is common to a number of papers including Spaventa (1987), Marini and van der Ploeg (1988), van der Ploeg (1991), Daniel (1993), and Kawai and Maccini (1990, 1995). Similar results could also be obtained by use of cash-in advance or liquidity cost models (see Feenstra 1986).
where \(0 < \eta \equiv \frac{1 - \epsilon}{\epsilon} < 1\), and \(h_i(s)\) is human wealth defined as

\[
h_i(s) = \int_0^\infty \left[ \omega_i(s) - \tau_i(s) \right] e^{-(r + \delta)\tau} dt.
\]

The individual consumption function (1) is a linear function of total wealth. This is a straightforward implication of the assumed logarithmic utility function. Equation (2) is the portfolio balance condition showing that individuals equalize the marginal rate of substitution between consumption and real money balances to the opportunity cost of holding real cash balances.

Assuming that all households have the same human wealth, and using the following procedure to derive aggregate variables

\[
N_j = \int_{-\infty}^0 x_i(s) \delta e^{\kappa} ds,
\]

where \(N_j\) and \(x_i(s)\) denote any aggregate variable and its individual counterpart, yields:

\[
C = \frac{\beta + \delta}{1 + \eta} [H + W],
\]

\[
m \equiv \frac{M}{p} = \frac{\eta C}{i},
\]

\[
\dot{W} = r^* W + \omega - T - C - im, \quad W \equiv m + K + B,
\]

\[
\dot{H} = (r^* + \delta) H - \omega + T, \quad H = \int_0^\infty \left[ \omega - T \right] e^{-(r + \delta)\tau} dt,
\]

where the time subscript has been dropped for simplicity and \(C, T, K\) and \(B\) indicate aggregate consumption, taxes, capital stock and traded bonds, respectively. \(H\) and \(W\) are aggregate human wealth and non-human wealth, \(m\) is real money balances and \(M\) the nominal quantity of money.

The production sector consists of many identical, profit maximizing firms, each with the same technology described by a two factor neoclassical production function with constant returns to scale. The assumption of constant returns imply that the number of firms is of no consequence in a competitive environment. Thus, for notational convenience, we set the number of firms
equal to 1 and focus on the representative agent. We write the production function of the firm as \( Y = Y(K) \), normalizing labor input to unity, where \( Y \) is domestic output. For simplicity, we assume that capital stock does not depreciate and does not incur adjustment costs. Domestic output equals the sum of private and government consumption, net exports and investment.

The first-order conditions for profit maximization imply:

\[
F'(K) = r^*
\]

\[
F(K) - KF'(K) = \omega(K).
\]

The government spends on goods and finance its expenditures through lump-sum taxes, debt or money creation. The flow budget constraint is

\[
(7) \quad \dot{D} = \frac{1}{\rho} \left[ r^* \rho D + G - \frac{\dot{M}}{p} \right],
\]

where \( D \) is the stock of traded bonds issued by the domestic government and \( G \equiv G'' + \rho G' \) is total government spending on goods.

Subtracting (7) from (5) and applying Euler's theorem to the production function yields

\[
(8) \quad \dot{F} = \frac{1}{\rho} \left\{ X(\rho) - \left[ (1 - q)C + \rho F' \right] + r^* \rho F' \right\},
\]

where \( F \equiv B - D \) is the net stock of traded bonds of the domestic economy and \( X \equiv Y(K) - qC - G'' - \dot{K} \) are exports. This equation describes the dynamics of the current account, which equals the trade balance plus the real interest on foreign assets. Combining eqs. (5), (7) and (8) gives

\[
(9) \quad \dot{K} = Y(K) - (qC + G'') - X(\rho),
\]

which characterizes the evolution of capital stock.

The aggregate behavior of the economy is described by the following relationships:

\[
(10) \quad C = \frac{\delta + \beta}{1 + \eta} \left[ \frac{\omega(K) - T}{r^* + \delta} + K + \rho F' + m + \rho D \right]
\]

\[
(11) \quad \dot{m} = (r^* + \mu)m - \eta C
\]

\[
(12) \quad \dot{K} = Y(K) - (qC + G'') - X(\rho)
\]
\[
\hat{F} = \frac{1}{\rho} \left\{ X(\rho) - \left[ (1-q)C + \rho G^F \right] + r^* \rho F \right\} 
\]

(14) \quad \dot{\rho} = [Y'(K) - r^*] \rho 

(15) \quad \dot{D} = \frac{1}{\rho} \left\{ r^* \rho D + (G'' + \rho G^F) - T - \frac{\dot{M}}{\rho} \right\},

together with the transversality conditions

\[
\lim_{t \to \infty} We^{-rt} = \lim_{t \to \infty} me^{-rt} = \lim_{t \to \infty} Ke^{-rt} = \lim_{t \to \infty} Fe^{-rt} = \lim_{t \to \infty} De^{-rt} = 0,
\]

where \( \mu \equiv \dot{M} / M \) is the growth rate of nominal money stock. Note that in a sustainable fixed exchange rate the capital stock is determined by the exogenously given world interest rate \( r^* \).

Hence, real labour income, \( \omega(K) \), is also given in equation (10). Equation (12) describes the time evolution of real money balances driven by the Yaari-Blanchard consumption dynamics.

It can be obtained combining the portfolio balance condition (4) with the real money growth equation

\[
\hat{m} = (\mu - \pi)m.
\]

Equation (14) shows the dynamics of real exchange rate depreciation.

In a sustainable fixed exchange rate regime no seignorage revenues are available to the government. Setting \( \dot{M} = 0 \) in equation (15), as in Burnside et al. (2001), and integrating it under the constraint implied by the transversality conditions we may write the intertemporal budget constraint of the government as

\[
D = \int^\infty_\tau \frac{1}{\rho} (T_v - G_v) e^{-r(v-\tau)} dv,
\]

which states that the level of government debt is equal to the present discounted value of future surpluses.

Since fiscal policy affects demand through the effects on wealth, relative price and consumption, it is convenient, following Blanchard (1985), to summarize all these effects by the following index

\[
\rho d = \left\{ G - \left( \frac{\delta + \beta}{1 + \eta} \right) \int G_v e^{-(r + \delta)(v-\tau)} dv + \left( \frac{\delta + \beta}{1 + \eta} \right) \rho D + \int (G_v - T_v) e^{-(r + \delta)(v-\tau)} dv \right\},
\]
which reduces to

\begin{equation}
\rho d = \left( \frac{\delta + \beta}{1 + \eta} \right) \rho D \left[ - \int_{v}^{\infty} e^{-1(r^{*} + \delta)(v-t)} dv \right]
\end{equation}

where, for simplicity, government spending has been set equal to zero on the entire path.

The tax policy rule is assumed to be given by:

\begin{equation}
T = \alpha \rho D - Z , \quad \dot{D} = \frac{1}{\rho} (r^{*} \rho D - T) , \quad D_{0} = 0 .
\end{equation}

Taxes are positively linked to the level of debt through the \( \alpha \) parameter, in order to prevent an explosive path for the government debt. \( Z \) is the lump-sum component of the tax policy rule. Solvency requires \( \alpha > r^{*} \geq 0 \).

Solving equation (17) for the time path of \( D \) and \( T \) and substituting in (16) yields

\begin{equation}
\rho d = \frac{\delta(\delta + \beta)}{(\alpha - r^{*})(1 + \eta)} \left[ \frac{1}{r^{*} + \delta} - \frac{e^{-(\alpha - r^{*})(t-t_{0})}}{\alpha + \delta} \right] Z
\end{equation}

from which we get

\( (\rho d)_{t_{0}} = \left[ \frac{\delta(\delta + \beta)}{(r^{*} + \delta)(\alpha + \delta)(1 + \eta)} \right] Z \)

\[
\dot{d}_{x} \equiv \dot{\rho \bar{d}} = \left[ \frac{\delta(\delta + \beta)}{(r^{*} + \delta)(\alpha - r^{*})(1 + \eta)} \right] Z = \left[ \frac{\delta(\delta + \beta)}{(r^{*} + \delta)(1 + \eta)} \right] D_{x} > (\rho d)_{t_{0}} ,
\]

where \( (\rho d)_{t_{0}} \) and \( d_{x} \) denote the initial and steady-state value of \( d \), while \( D_{x} = \frac{Z}{(\alpha - r^{*})} \) is the steady-state value of debt. The initial value of \( d \) depends on the entire sequence of current and anticipated future budget deficits. Differentiating equation (18) with respect to time we obtain

\begin{equation}
\dot{d} = (r^{*} - \alpha - \frac{\dot{\rho}}{\rho}) d + \left[ \frac{\delta(\delta + \beta)}{\rho(r^{*} + \delta)(1 + \eta)} \right] Z ,
\end{equation}

which describes the equation of motion of the fiscal index.

In order to analyze the dynamic effects of fiscal policy we can rewrite the model, making use of (18) and (19), as
\[ C = \frac{\delta + \beta}{1 + \eta} \left[ \frac{\omega(K)}{r^* + \delta} + K + \rho F + m \right] + \rho d \]

\[ \dot{m} = r^* m - \eta C \]

\[ \dot{K} = Y(K) - (q C + G^H) - X(\rho) \]

\[ \ddot{F} = \frac{1}{\rho} \left\{ X(\rho) - \left[ (1 - q) C + \rho G^F \right] + r^* \rho F \right\} \]

\[ \dot{\rho} = [Y'(K) - r^*] \rho \]

\[ \dot{d} = (r^* - \alpha - \frac{\dot{\rho}}{\rho}) d + \left[ \frac{\delta(\delta + \beta)}{\rho(r^* + \delta)(1 + \eta)} \right] Z, \]

where \( \mu \) has been set to 0 for simplicity.

### III. FISCAL DEFICITS AND CURRENCY CRISSES

In this section we examine the dynamic effects of fiscal policy on the macrovariables of the model to derive the links between expected future budget deficits and currency crises in a pegged exchange rate economy. The policy is centered on a lump-sum tax cut, that is a once and for all increase in \( Z \). There is a fiscal deficit at \( t = t_0 \), generated by the tax cut, followed by future surpluses as debt accumulates, so as to always satisfy the intertemporal government budget constraint without recourse to seignorage revenues.

We assume, in fact, that the monetary authorities accommodate any change in money demand in order to keep the relative price of the currency fixed to \( E \), and finance current account imbalances through changes in foreign reserves. For simplicity, we also assume that capital and government bonds are owned entirely by domestic residents.

A unique stable saddle-point equilibrium path characterizes the model if \( \beta \leq r^* < \delta + \beta \) and the transversality conditions are met\(^6\). Solving the model for short-run and steady-state

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equilibrium, we obtain the following set of relationships among the variables of interest, as
shown in detail in the Mathematical Appendix.

**Short-Run Equilibrium**

\[(26)\] 
\[C = C(K, F, r^*, d) \quad C_K > 0, C_F > 0, C_{r^*} < 0, C_d > 0\]

\[(27)\] 
\[\rho = \rho(K, F, r^*, d) \quad \rho_K > 0, \rho_F < 0, \rho_{r^*} > 0, \rho_d < 0\]

\[(28)\] 
\[m = m(K, F, r^*, d) \quad m_K > 0, m_F > 0, m_{r^*} < 0, m_d > 0\]

**Steady-State Equilibrium**

\[(29)\] 
\[C = C(r^*, d) \quad \bar{C}_{r^*} > 0, \bar{C}_d < 0\]

\[(30)\] 
\[\rho = \rho(r^*, d) \quad \bar{\rho}_{r^*} < 0, \bar{\rho}_d > 0\]

\[(31)\] 
\[K = K(r^*, d) \quad \bar{K}_{r^*} < 0, \bar{K}_d = 0\]

\[(32)\] 
\[F = F(r^*, d) \quad \bar{F}_{r^*} > 0, \bar{F}_d < 0\]

\[(33)\] 
\[m = m(r^*, d) \quad \bar{m}_{r^*} > 0, \bar{m}_d < 0\]

where the upper bars indicate long-run effects.

From equations (26) through (33) we can see that an increase in \(d\) implies a rise in consumption and real money balances and an appreciation of the real exchange rate, in the short-run: in the steady-state equilibrium consumption, foreign assets and real money balances are, however, below their original levels. The capital stock is unchanged and there is a real exchange rate appreciation.

The dynamics of this economy can be determined by substituting the short-run solution for \(C, \rho\) and \(m\) into the dynamic equations of the model. The critical equation (23) can be rewritten as

\[\dot{F} = \frac{X[\rho(\bar{W}, d, r^*)]}{\rho(\bar{W}, d, r^*)} - \frac{(1 - q)C(\bar{W}, d, r^*)}{\rho(\bar{W}, d, r^*)} + r^* F,\]

where \(\bar{W} \equiv K + \rho F\).

Linearizing around the steady state, we obtain

\[\dot{F} = \Theta(d_0 - \bar{d}) e^{\lambda t} + r^* (F - \bar{F}),\]

where \(\lambda_i\) is the negative root associated with the stable arm of the saddle path,
\[ \Theta \equiv \frac{1}{\rho} \left[ (\nu \rho_W - \rho C^F_W) \rho + (\nu \rho_d - \rho C^F_d) \right], \quad \rho_W \equiv \rho_K + \rho_F, \quad C^F_W \equiv \frac{(1-q)}{\rho} \left( C_W - \frac{C}{\rho} \rho_W \right), \]

\[ C^F_d \equiv \frac{(1-q)}{\rho} \left( C_d - \frac{C}{\rho} \rho_d \right), \quad C_W \equiv C_K + C_F, \quad \nu \equiv X' - \frac{X}{\rho} > 0 \quad \text{and} \quad b = \frac{(1+\eta)\rho}{\alpha - (\delta + \beta)} < 0 \]

is a parameter linking \( \tilde{W} \) and \( d' \) along the stable path.

The current stock of foreign assets is given by

\[ F_t = \bar{F} + \frac{\Theta}{\lambda_1 - r^*} (d_0 - \bar{d}) e^{\lambda d^t} + \left[ F_0 - \bar{F} - \frac{\Theta}{\lambda_1 - r^*} (d_0 - \bar{d}) \right] e^{\lambda t}. \]

The dynamics towards the steady-state are described by

\[ F_t = \bar{F} + \frac{\Theta}{\lambda_1 - r^*} (d_0 - \bar{d}) e^{\lambda d^t}, \]

or,

\[ F_t = \bar{F} + \frac{\Theta}{\lambda_1 - r^*} (d_0 - \bar{d}) e^{\lambda (t-t_0)}. \]

Equation (34) shows the relationship between the accumulation of foreign assets and the evolution of budget deficits along the path approaching the steady-state equilibrium. There are both direct effects on the real exchange rate \( (\nu \rho_d) \) and consumption \( (\rho C^F_d) \), and indirect effects via changes in real wealth \( (\nu \rho_W - \rho C^F_W) \). A rise in the budget deficit generates a depletion of external assets (or current account deficits) during the transition to the steady state if \( \Theta > 0 \).

The remaining dynamic equations of the model are:

\[ K_t = \bar{K} - \frac{\Psi}{\lambda_1 - Y^*(K)} (d_0 - \bar{d}) e^{\lambda (t-t_0)}, \]

\[ \rho_t = \bar{\rho} - \frac{\Phi}{\lambda_1 - [Y^*(k) - r^*]} (d_0 - \bar{d}) e^{\lambda (t-t_0)}, \]

\[ m_t = \bar{m} - \frac{\Gamma}{\lambda_1 - r^*} (d_0 - \bar{d}) e^{\lambda (t-t_0)} \]

\[ ^7 \text{Strong empirical support for a positive relationship between the current account deficit and current and expected future budget deficits, as implied by equation (34), is found in Piersanti (2000). See also Baxter (1995) for a more general discussion on this issue.} \]
\begin{align*}
(38) \quad C_t &= \bar{C} + a \left( W_t - \bar{W} \right) e^{\gamma (r - \rho)} \\
(39) \quad W_t &= \bar{W} + b \left( d_t - \bar{d} \right) e^{\lambda (t - t_0)},
\end{align*}

where \( \Psi \equiv \left[ (q C_w + X^\prime \rho_w)b + q C_d + X^\prime \rho_d \right], \quad \Phi \equiv \left[ (Y'')(K) \rho \Psi / \left( \lambda_i - r^* \right) \right], \quad \Pi \equiv \eta (C_w b + C_d), \quad a \equiv \frac{\delta + \beta}{1 + \eta} > 0.

Equations (35), (36) and (37) describe the transitional dynamics towards the long-run equilibrium for the capital stock, the real exchange rate and real money balances. Equations (38) and (39) express the dynamical relationship between consumption and non-human wealth along the adjustment path, as shown in the Mathematical Appendix. During the transition to the steady state, the paths for \( K \) and \( \rho \) are positively sloped if \( \Psi, \Phi > 0 \), while that of \( m \) is negatively sloped if \( \Pi < 0 \).

The adjustment process can be better understood by making use of figs. 1(a) - 1(e), where \( d_{t_0} \) and \( \bar{d} \) denote the initial and steady state value of the fiscal index, respectively. Assume, for example, a tax cut at \( t = t_0 \). There is, on impact, an appreciation of the real exchange rate, an increase in consumption and real money balances and a deterioration in the current account. These effects are visualized in figs 1(a), 1(d) and 1(e) as jumps from the initial equilibrium points \( \bar{C}_0, \bar{\rho}_0 \) and \( \bar{m}_0 \) to \( C_{t_0}, \rho_{t_0} \) and \( m_{t_0} \). The capital stock and foreign assets also move to \( K_{t_0} \) and \( F_{t_0} \), leaving their sum unchanged (see figs. 1(b) and 1(c)). The total amount of \( K \) and \( F \) is predetermined, but not its two components: agents can substitute foreign assets for capital stock instantaneously and exchange them for money. Hence, the domestic real interest rate jumps upwards.

As the economy moves towards its new long-run equilibrium point \((\bar{C}, \bar{F}, \bar{K}, \bar{\rho}, \bar{m})\) and the government budget goes from deficit to surplus, non-human wealth, consumption and real money balances decline, foreign assets are run down, while the real exchange rate and the capital stock rise. Since the current account is in deficit during the period of adjustment, foreign assets end up at a lower level in the new steady-state. The capital stock returns back to its original level, \( \bar{K} \), in the long-run, while foreign assets and human wealth decline.
Total real wealth decreases and consumption and real money balances fall below their initial levels. The real exchange rate overshoots and the domestic real interest rate is higher than the world rate during the transition to the steady state.

Currency crises occur along the stable path to the new equilibrium if foreign reserves decline below the threshold level, $F^c$, stirring up a speculative attack and the collapse of the peg. Substituting $F^c$ for $\bar{F}$ into equation (34), we can determine the time of the attack, $t^*$, when the government abandons the peg. Solving equation (34) for $t^*$, we find:

$$t^* = t_0 + \frac{1}{\lambda_1} \ln \left( \frac{F_t - F^c}{\Omega(d_0 - \bar{d})} \right),$$

where $\Omega = \frac{\Theta}{\lambda_1 - r^*}$.

The value of $t^*$ depends on both the level of foreign reserves and the magnitude of the deficit being financed. Given $d_0$, the larger the foreign reserves holdings $F_t$ the longer the fixed exchange rate regime will last. On the other hand, given $F_t$, the larger $d_0$ the smaller $t^*$ will be.

The dynamics of foreign reserves until the time of the attack, $t^*$, is depicted in fig. 2. Following the tax cut, the economy starts (at $t_0$) reducing its holding of foreign assets (hence reserves fall) to finance the higher consumption level along the transitional path. If, during the adjusting process, reserves come to a threshold level ($F^c$), a speculative attack takes place, forcing the government to give up the peg. Thereafter the economy shifts to a flexible exchange rate regime and the money supply becomes exogenous.

The main result of our model of currency crises thus appeals to intuition and can be restated as follows. A rise in current and expected future budget deficits generates both an appreciation of the real exchange rate and current account deficits. Hence, there is a decumulation of foreign reserves along the transitional path to the new steady state. If reserves approach a critical level in the adjusting process, a speculative attack occurs causing the collapse of the fixed exchange rate regime. The next section tests the predictions of the model.
FIG. 1 The transitional dynamics to the steady state
III. THE EMPIRICAL ANALYSIS

In this section we test the power of our model in predicting financial and currency crises by using a simple probit model linking the onset of a crisis to the relevant macroeconomic variables of the theoretical model. Our empirical investigation is focused on all the Latin American and Asian countries for which we could find reliable data. The data are annual from 1990 through 2000. The countries are: Argentina, Brazil, Mexico, Venezuela, Chile, Colombia, Peru, Uruguay, Bolivia, Honduras, Indonesia, Korea, Malaysia, Philippines, Thailand, Turkey, Singapore, China (P. R. Mainland), India, Pakistan and Sri Lanka.

Our probit framework implies that the left-hand-side variable takes on a value of one if the country fell into a crisis during the year and zero otherwise. For this purpose, we define a crisis as a drastic depreciation of the currency (and/or the collapse of the peg) or a significant balance of payment disruption. Ten cases out of two hundred and thirty-one are set equal to one: Turkey and Venezuela in 1994; Argentina and Mexico in 1995; Indonesia, Korea, Malaysia, Philippines and Thailand in 1997; Brazil in 1999.

On the right-hand-side, as suggested by our model, we use the following variables as determinants of crises: i) current and expected future government budget deficits or surpluses as a percentage of GDP ($EFGB$); ii) current account balance as a percentage of GDP ($CA$); iii) accumulated real exchange rate appreciation$^9$ ($REXA$); iv) total reserves as a percentage of imports ($RESR$), which we use as a proxy for the stock of reserves; v) the domestic real interest rate ($RRATE$); and vi) domestic credit as a percentage of GDP ($DCR$), modeling the increase in the domestic component of the money stock along the transitional path to the steady state.

The specification used in the estimation of our probit model may thus be written as:

$$P(CS_i) = \mathcal{N}(EFGB_{i-1}, CA_{i-1}, REXA_{i-1}, RESR_{i-1}, RRATE_{i-1}, DCR_{i-1}), \quad i = 0, 1, \ldots, n,$$

where $CS$ is the binary variable ($Crises$) and the lagged values for the independent variables encompass the dynamics implied by equation (34).

According to our model, we expect that the probability of a crisis be negatively correlated to $EFGB, CA, RESR$ and $RRATE$, and positively linked to $REXA$ and $DCR$, so that increases in expected future budget deficits and domestic credit, reductions in the current account, foreign reserves and the domestic real interest rate, or a real exchange rate appreciation raise the probability that a crisis will eventually break up.

Equation (34) predicts that the budget and current account deficits should be causally linked. The hypothesis of no causality between the two deficits is strongly rejected by our tests reported in table 2, where it is shown that budget deficits do cause current account deficits. Preliminary tests investigating the stationarity of the time series employed show that we may accept the hypothesis of stationarity (see table 1).

The specification of equation (41) raises the important issue of modeling and generating data on future expectations of government budget balance. Our model developed in section II implies that we may define $EFGB$ as:

---

$^9$ This variable is an index that starts at 100 in 1990 and then reflects the accumulated real appreciation of the national currency. It may be found in Veiga (1999), who analyzes the causes of failure of stabilization plans in chronic inflation countries.
\[ EFGB \equiv \sum_{i=0}^{n} \phi^i GB_{t+i}^{e}, \]

where \( \phi \) is the discounting factor, \( GB_{t+i}^{e} \) is the expected government budget balance as a percentage of GDP for the year \( t+i \), and \( n \) is the planning horizon of private agents. We have performed GMM estimations of \( EFGB \) under the hypothesis of rational expectations in order to generate data for future expected government budget balances to be used in the estimation of (41). We have considered values of \( \phi \) in the range \([0.9, 0.99]\) and \( n = 5 \), using the Newey-West (1987a) consistent estimator of the variance-covariance matrix to deal with the presence of MA (4) in the errors. Table 3 reports only the estimates for \( \phi = 0.9 \), since both coefficients and statistics were almost identical for alternative values of \( \phi \) within the chosen range.

We have generated data for \( EFGB \) that were used as a proxy for market’s expectations of current and future government budget deficit in the estimation of the probit model formulated in (41), by carrying out the static forecast of the econometric equation described in Table 3.

Probit estimates are presented in table 4. We also report the marginal effects of the explanatory variables on the conditional probability of a crisis evaluated at the mean of the data.

We performed all the model estimates with country dummies in order to control for fixed effects.

It can immediately be seen from table 4 that our theoretical model fits the Asian and Latin American crises extremely well. The most striking result is the high statistical significance of the key variables \( EFGB^e \) and \( CA \), suggesting that expected future budget deficits and current account deficits do play a critical role in determining crises.

Since the traditional reserves to imports ratio is not regarded as the best measure of the reserves adequacy, we also tried other indicators suggested in the literature, such as the M1 to reserves ratio and the M2 to reserves ratio\(^\text{11}\). We found no statistical significance for the M1 to reserves ratio and the M2 to reserves ratio\(^\text{11}\). We found no statistical significance for

\(^{10}\) This value for the planning horizon of agents, originally suggested by Feldstein (1986), emerged from robustness checks

\(^{11}\) These measures of reserves adequacy have been suggested by Krugman (1979) and Calvo and Mendoza (1996a, b).
these variables. Tests for the presence of interactive effects among the independent variables were equally negative.

In conclusion, the estimates give strong empirical support to the main prediction of our theoretical model, according to which current and expected future budget deficits, current account deficits, foreign reserves, real exchange rate appreciation, domestic credit and domestic real interest rate are the key variables in predicting the onset of currency crises.

We have assessed the power of the above probit model in predicting the likelihood of a crisis. We forecasted the in-sample probability of a crisis for each country and appraised the resulting probability values for the cutoff levels of 0.5 and 0.25. The results of the goodness of fit estimation are reported in table 5.

We also evaluated the in-sample forecasts by three measures of accuracy known as quadratic probability score (QPS), log probability score (LPS) and global squared bias (GSB). Both the QPS and GSB range from 0 to 2, with zero corresponding to perfect accuracy and perfect global calibration, while LPS ranges from 0 to infinity, with zero corresponding to perfect accuracy.

We can see that our model shows both excellent scores and accurate goodness of fit measures from table 5. It correctly calls more than 99% of total observations both at the 0.5 and 0.25 cutoff levels. Nine out of ten country crises are correctly predicted with probability values falling in the range $(0.71, 0.99)$\(^{12}\).

Based on this evidence, we may then conclude that the main cause of the financial and currency turmoil of 1990's in Latin America and Asia has been prospective budget deficits. The empirical results, obtained from estimating and forecasting a probit-based model give strong support to the main implication of our theoretical model, according to which a rise in current and expected future budget deficits generates a real exchange rate appreciation and current account deficits leading up to a depletion of foreign reserves. A currency crisis occurs when foreign reserves approach a critical level. The evidence thus seems to suggest a simple explanation of the crises entirely based on fundamentals, according to the theoretical results of our optimizing model.

\(^{12}\) A probability below the cutoff levels was found only for Malaysia.
IV SUMMARY AND CONCLUSIONS

In this paper we have used an optimizing general equilibrium model to investigate the currency crises of 1990's in emerging markets. It is shown that a rise in current and expected future budget deficits generates, during the transition to the steady state, a real exchange rate appreciation and a depletion of foreign reserves, leading up to a currency crisis when reserves decline below a critical level.

The implications of the model are strongly confirmed by probit estimates for a panel of 21 Latin American and Asian countries in the 1990’s.

Crises can thus occur even when policies are correctly designed. The sustainability of fixed exchange rate system may thus require giving up both monetary and fiscal sovereignty.
<table>
<thead>
<tr>
<th></th>
<th>ADF</th>
<th></th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GB</td>
<td>CA</td>
<td>GB</td>
</tr>
<tr>
<td>-3.673</td>
<td>-4.524</td>
<td>-4.245</td>
<td>-6.057</td>
</tr>
</tbody>
</table>

Critical values for ADF and PP: -3.460 (1%); -2.874 (5%); -2.574(10%)

Legend:

ADF: Augmented Dickey-Fuller unit root test based on OLS regression of the first difference of the dependent variable (budget deficit, or current account) on a constant, the one period lagged level of the depended variable and four lagged difference terms. Similar results are obtained for lagged differences in the range [2, 6].

PP: Phillips-Perron unit root test based on OLS regression of the first difference of the dependent variable on a constant and its one period lagged level, using the Newey-West (1987a) adjusted variance-covariance matrix of the parameter estimates. The figures reported for this test have been obtained using a window size (or truncation point) of 4, but similar results are obtained in the range [2, 6].

GB and CA denote the ratio of budget deficit to GDP and of current account balance to GDP, respectively.

More detailed definitions and sources of the variables employed are found in the Data Appendix.
### Table 2: Causality tests

<table>
<thead>
<tr>
<th>GB → CA</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SIMS</td>
<td>GRANGER</td>
<td>GRANGER INST.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>LM</td>
<td>LMF</td>
<td>LM</td>
<td>LMF</td>
<td>LM</td>
</tr>
<tr>
<td>CA → GB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SIMS</td>
<td>GRANGER</td>
<td>GRANGER INST.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>LM</td>
<td>LMF</td>
<td>LM</td>
<td>LMF</td>
<td>LM</td>
</tr>
<tr>
<td></td>
<td>4.41(2)</td>
<td>2.16(2,224)</td>
<td>4.40(2)</td>
<td>2.16(2,225)</td>
<td>4.62(3)</td>
</tr>
</tbody>
</table>

**Legend:**

- **GB → CA** denotes the causal relationship running from GB to CA.
- **SIMS, GRANGER and GRANGER INST.** denote Sims, Granger and Granger instantaneous causality test, respectively. Under **GB → CA**, the Sims’ test has been performed by regressing the budget deficit on its lagged values together with past and future values of the current account balance. The Granger causality test have been carried out by regressing the current account balance on its lagged values together with past values of the budget deficit. Current values of the budget deficit where also included in the regressors for the Granger instantaneous causality test. An analogous procedure have been applied for the case **CA → GB**.
- **LM** is the Lagrange multiplier statistics used to test the null hypothesis of no causality, asymptotically distributed as $\chi^2(k)$ under the null hypothesis, where $k$ is the maximum lag (lead) term chosen to perform the test.
- **LMF** is the modified LM statistics, asymptotically distributed as $F(k, T-h)$, where $T$ is the sample size and $h$ the number of regressors.
- The single and double asterisks denote a statistical level of significance better than 5 and 1%, respectively.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>T-statistic</th>
<th>Sample size</th>
<th>$\bar{R}^2$</th>
<th>SE</th>
<th>J-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const.</td>
<td>4.735</td>
<td>0.979</td>
<td>231</td>
<td>0.924</td>
<td>5.364</td>
<td>0.029(1)</td>
</tr>
<tr>
<td>EFGB(-1)</td>
<td>0.379</td>
<td>4.472</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>REXA(-1)</td>
<td>-0.056</td>
<td>1.371</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CA(-1)</td>
<td>-0.378</td>
<td>2.554</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>-0.983</td>
<td>1.226</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T^2$</td>
<td>0.078</td>
<td>1.369</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Instrument list: Const., EFGB(-1), CA(-1), REXA(-1), RESR(-1), T, $T^2$ and 20 country dummies.

Legend:
- $\bar{R}^2$: Adjusted R-squared.
- SE: Standard error of regression.
- J-statistic: Newey and West(1987b) test for the validity of the $(s-h)$ overidentifying restrictions, where $s$ is the number of instruments and $h$ the number of regressors. Asymptotically distributed as $\chi^2(s-h)$ under the null hypothesis that the restrictions are not binding.
- Equation estimated with twenty country dummies. $T$ and $T^2$ are, respectively, a time trend and a squared time trend.
Table 4                              Probit estimates: 1990-2000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>z-statistic</th>
<th>slope derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFGB^</td>
<td>-0.909</td>
<td>2.730</td>
<td>1.509</td>
</tr>
<tr>
<td>CA(-1)</td>
<td>-3.591</td>
<td>2.950</td>
<td>5.962</td>
</tr>
<tr>
<td>REXA(-1)</td>
<td>0.204</td>
<td>4.204</td>
<td>0.338</td>
</tr>
<tr>
<td>RESR(-1)</td>
<td>-0.245</td>
<td>2.801</td>
<td>0.407</td>
</tr>
<tr>
<td>DCR</td>
<td>0.997</td>
<td>3.163</td>
<td>1.655</td>
</tr>
<tr>
<td>RRATE</td>
<td>-0.134</td>
<td>2.465</td>
<td>0.222</td>
</tr>
</tbody>
</table>

LF          | -7.006      |
LR          | 52.807(6)   |
\textit{McF.} R^2 | 0.830      |

Legend:
- EFGB^: Estimated value of EFGB obtained by static forecast of the econometric equation shown in table 3.
- LF: Maximized value of the log likelihood function.
- LR: Likelihood ratio statistic to test the null hypothesis that all slope coefficients except the constant and the country dummies are zero, asymptotically distributed as $\chi^2(n)$, where $n$ is the number of the variables tested.
- \textit{McF.} R^2: McFadden R-squared.

Probit slope derivatives are expressed in percentage values. The model is estimated by maximum likelihood with a constant and twenty country dummies. The z-statistics uses the robust standard errors estimated by quasi-maximum likelihood method.
<table>
<thead>
<tr>
<th>Table 5</th>
<th>In-sample prediction evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accuracy and calibration of forecasts</strong></td>
<td></td>
</tr>
<tr>
<td>Quadratic probability score (QPS)</td>
<td>0.017328</td>
</tr>
<tr>
<td>Log probability score (LPS)</td>
<td>0.030462</td>
</tr>
<tr>
<td>Global squared bias (GSB)</td>
<td>0.00000002</td>
</tr>
<tr>
<td><strong>Goodness-of-fit (cut-off probability of 0.5)</strong></td>
<td></td>
</tr>
<tr>
<td>Dependent variable = 0</td>
<td></td>
</tr>
<tr>
<td>% of correct observations $(\hat{P} \leq 0.5)$</td>
<td>99.55</td>
</tr>
<tr>
<td>% of incorrect observations $(\hat{P} &gt; 0.5)$</td>
<td>0.45</td>
</tr>
<tr>
<td>Dependent variable = 1</td>
<td></td>
</tr>
<tr>
<td>% of correct observations $(\hat{P} &gt; 0.5)$</td>
<td>90.00</td>
</tr>
<tr>
<td>% of incorrect observations $(\hat{P} \leq 0.5)$</td>
<td>10.00</td>
</tr>
<tr>
<td>% of total correct observations</td>
<td>99.13</td>
</tr>
<tr>
<td>% of total incorrect observations</td>
<td>0.87</td>
</tr>
<tr>
<td><strong>Goodness-of-fit (cut-off probability of 0.25)</strong></td>
<td></td>
</tr>
<tr>
<td>Dependent variable = 0</td>
<td></td>
</tr>
<tr>
<td>% of correct observations $(\hat{P} \leq 0.25)$</td>
<td>99.55</td>
</tr>
<tr>
<td>% of incorrect observations $(\hat{P} &gt; 0.25)$</td>
<td>0.45</td>
</tr>
<tr>
<td>Dependent variable = 1</td>
<td></td>
</tr>
<tr>
<td>% of correct observations $(\hat{P} &gt; 0.25)$</td>
<td>90.00</td>
</tr>
<tr>
<td>% of incorrect observations $(\hat{P} \leq 0.25)$</td>
<td>10.00</td>
</tr>
<tr>
<td>% of total correct observations</td>
<td>99.13</td>
</tr>
<tr>
<td>% of total incorrect observations</td>
<td>0.87</td>
</tr>
</tbody>
</table>
QPS : Quadratic probability scores, defined as \( QPS \equiv (1/T) \sum_{i=1}^{T} 2(\hat{P}_i - P_i)^2 \), where \( \hat{P}_i \) is the probability forecast generated by the model shown in table 4 for the year \( t \) and \( P_i \) is our binary variable (Crisis) which is equal to 1 if a crisis occurs in the year \( t \) and zero otherwise.

LPS : Log probability score, defined as \( LPS \equiv (1/T) \sum_{i=1}^{T} \left[ (1 - P_i) \ln(1 - \hat{P}_i) + P_i \ln(\hat{P}_i) \right] \).

GBS : Global squared bias, defined as \( GBS \equiv 2(\overline{P} - \overline{P})^2 \), where \( \overline{P} \equiv (1/T) \sum_{i=1}^{T} \hat{P}_i \) and \( \overline{P} \equiv (1/T) \sum_{i=1}^{T} P_i \). For a more detailed discussion see Diebold and Lopez (1996).

An observation is classified as “correct” when the predicted probability is less than or equal to the cut-off value and the observed Crisis = 0, or when the predicted probability is greater than the cut-off value and the observed Crisis = 1. Analogously, an observation is classified as “incorrect” when the predicted probability is greater than the cut-off level and the observed Crisis = 0, or when the predicted probability is less than or equal to the cut-off level and the observed Crisis = 1.

A · DATA APPENDIX ·


1 Government budget deficit or surplus/GDP. The ratio of government budget deficit (-) or surplus (+) (IFS line 80) to GDP (IFS line 99b).

2 Current Account/GDP. The ratio of current account (IFS line 78ald) to GDP (IFS line 99b) converted into dollars (using IFS line rf).

3 Real exchange rate. The real exchange rate is the nominal exchange rate (IFS line rf) adjusted for the relative consumer prices (IFS line 64). The measure is defined as the price foreign goods (using United States as the foreign country) to the price of domestic goods.

4 Reserves/Imports. The ratio of total reserves (IFS line 11.d) to imports (IFS line 98c) converted into dollars (using IFS line rf).
Total reserves: IFS line 11.d.

Domestic credit/GDP: IFS line 32 divided by IFS line 99b.

Nominal rate of interest: Money market rate (IFS line 60b) for Argentina, Brazil, Mexico, Indonesia, Korea, Malaysia, Thailand, Turkey, Singapore; discount rate (IFS line 60) for Venezuela, Philippines, Pakistan, India, Uruguay, Colombia, Peru; deposit rate (IFS line 60l) for Bolivia, Honduras, Sri Lanka, Chile; lending rate (IFS line 60p) for China.

Real interest rate: Nominal rate minus annual inflation rate, using consumer prices (IFS line 64).

M1/Reserves: IFS line 34 converted into dollars divided by IFS line 11.d.

M2/Reserves: IFS line 35 converted into dollars divided by IFS line 11.d

B - MATHEMATICAL APPENDIX -

B.I) Short-run and long-run equilibrium

The short-run macroeconomic equilibrium is obtained by combining the aggregate consumption equation (10) together with equation (11) and the product market equilibrium condition

\[ Y(K) = qC + G^H + \dot{K} + X(\rho) \]

These equation can be solved for \( C, \rho \) and \( m \) obtaining equations (18)-(20) in the text. The partial derivatives are

\[
\frac{\partial C}{\partial K} = X \left[ \frac{\omega'(K)}{r^* + \delta} + 1 \right] + Y(K) \left( F + \frac{1 + \eta}{\delta + \beta} d \right) \frac{\Lambda}{\Lambda} > 0; \quad \frac{\partial C}{\partial F} = \frac{\rho X'}{\Lambda} > 0; \quad \frac{\partial C}{\partial d} = \frac{\rho X'}{\Lambda} \left( \frac{\delta + \beta}{1 + \eta} \right) > 0;\]

\[
\frac{\partial C}{\partial r^*} = -X \left[ \frac{\omega(K)}{(r^* + \delta)^2} + \frac{m}{r^*} \right] \frac{\Lambda}{\Lambda} < 0; \quad \frac{\partial \rho}{\partial K} = Y(K) \left( \frac{1 + \eta}{\delta + \beta} - \frac{\eta}{r^*} \right) - q \left[ \frac{\omega'(K)}{r^* + \delta} + 1 \right] \frac{\Lambda}{\Lambda} > 0;
\]
\[ \frac{\partial \rho}{\partial F} = -\frac{q \rho}{\Lambda} < 0; \quad \frac{\partial \rho}{\partial r^*} = \frac{q}{\Lambda} \left( \frac{\omega(K)}{(r^* + \delta)^2} + \frac{m}{r^*} \right) > 0; \quad \frac{\partial \rho}{\partial d} = -\frac{q \rho}{(\delta + \beta)(1 + \eta)} < 0; \]

\[ \frac{\partial m}{\partial K} = \frac{\eta}{r^*} Y'(K) \left[ \frac{F + (1 + \eta)}{\delta + \beta} d + X' \left( \frac{\omega(K)}{r^* + \delta} + 1 \right) \right] > 0; \quad \frac{\partial m}{\partial F} = \frac{\eta \rho X'}{\Lambda} > 0; \]

\[ \frac{\partial m}{\partial r^*} = \frac{m}{r^*} \left[ q F + \frac{(1 + \eta)}{\delta + \beta} \left( q d + X' \right) \right] + \eta \frac{\omega(K)}{r^*} \frac{\omega(K)}{(r^* + \delta)^2} X' \Lambda < 0; \quad \frac{\partial m}{\partial d} = \frac{\eta}{r^*} \frac{\rho X}{(\delta + \beta)(1 + \eta)} \Lambda > 0; \]

where: \( \Lambda \equiv X' \left( \frac{1 + \eta}{\delta + \beta} - \frac{\eta}{r^*} \right) + q \left( F + \frac{1 + \eta}{\delta + \beta} d \right) > 0 \) if \( \frac{1 + \eta}{\delta + \beta} \geq \frac{\eta}{r^*} \).

The long-run equilibrium is reached when \( \dot{K} = \dot{r} = \dot{m} = \dot{\rho} = \dot{d} = 0 \) in the model given by equations (20)-(25). The partial derivatives reported in (29)-(33) are obtained as

\[ \frac{\partial C}{\partial r^*} = \frac{\left( \frac{\delta + \beta}{1 + \eta} \right) X' \left( Y'(K) \frac{\omega(K) r^*}{(r^* + \delta)^2} + \rho F + m \right) - r^* \left( \frac{1 + \eta}{\delta + \beta} \frac{r^* d}{X'} + \frac{\omega(K)}{r^* + \delta} \right)}{Y'(K)\Delta} > 0; \]

\[ \frac{\partial C}{\partial d} = \frac{r^* \rho X'}{\Delta} < 0; \quad \frac{\partial K}{\partial r^*} = \frac{1}{Y'(K)} < 0; \quad \frac{\partial K}{\partial d} = 0; \quad \frac{\partial \rho}{\partial d} = \frac{r^* q \rho}{\Delta} > 0; \]

\[ \frac{\partial \rho}{\partial r^*} = \frac{\left( \frac{\delta + \beta}{1 + \eta} \right) \left( \frac{\omega(K)}{r^* + \delta} + (1 + \eta) \left( 1 - \frac{r^*}{\delta + \beta} \right) \right) - Y'(K) q \left( \frac{\omega(K) r^*}{(r^* + \delta)^2} + \rho F + m \right)}{Y'(K)\Delta} < 0; \]

\[ \frac{\partial F}{\partial r^*} = \frac{1}{Y'(K)\Delta \rho} \left( \frac{\delta + \beta}{1 + \eta} \right) \left[ Y'(K) \left( \frac{m + \omega(K)}{r^* + \delta} \right) - \left( \frac{\omega(K)}{r^* + \delta} + 1 \right) \right] \left( X' + qr^* F \right) + \]

\[ + Y'(K) F \rho \left[ q \left( F + \frac{1 + \eta}{\delta + \beta} d \right) + X' \left( \frac{1 + \eta}{\delta + \beta} - \frac{\eta}{r^*} \right) \right] + \]

\[ - r^* \left( X' + qr^* F \left( \frac{\eta}{r^*} - \frac{1 + \eta}{\delta + \beta} \right) + (1 - q) \left( F + \frac{1 + \eta}{\delta + \beta} d \right) \right) > 0; \]
\[
\frac{\partial F}{\partial d} = \frac{X' + qr^* F}{\Delta} < 0; \quad \frac{\partial m}{\partial d} = -\eta p X' < 0;
\]

\[
\frac{\partial m}{\partial r^*} = \frac{\left( \frac{\delta + \beta}{1 + \eta} \right) X'}{Y'(K)\Delta} \left[ Y'(K) m \left[ \left( 1 + q \frac{d}{X'} \right) \left( \frac{1 + \eta}{r^* + \delta} \right) r^* - 1 \right] + \eta \left[ Y'(K) \left( \frac{p F}{r^*} + \frac{\omega(K)}{r^* + \delta} \right) \right] - \left( \frac{r^* d}{X'} \left( \frac{1 + \eta}{\delta + \beta} \right) + \frac{\omega'(K)}{r^* + \delta} \right) \right] > 0;
\]

where \( \Delta \equiv X'\left[ (\delta + \beta) - r^* \right] - qr^* d > 0 \) if \( r^* < (\delta + \beta) \) and \( \frac{(\delta + \beta) - r^*}{r^* q d} > 1. \)

B.II) Stability conditions and transitional dynamics

We can demonstrate that our model has a saddle-point equilibrium path by analyzing the stability conditions when \( d = 0 \) and \( \dot{d} \neq 0 \). We assume, for simplicity, \( Y'(K) = r^* \), so that \( \dot{\rho} = 0 \) and \( \omega \) is constant. Denoting the sum of \( K, F \) and \( m \) by \( W \), the dynamics are given by the two linear differential equations

\[
\dot{C} = (r^* - \beta) C - \delta \left( \frac{\delta + \beta}{1 + \eta} \right) W
\]

\[
\dot{W} = r^* W + \omega - (1 + \eta) C,
\]

when \( d = 0 \).

The first equation is obtained by differentiating equation (20) in the text, while the second is obtained combining equations (21), (22) and (23), setting \( G = 0 \). If \( \beta < r^* < \delta + \beta \) the determinant of the coefficient matrix

\[
\begin{bmatrix}
(r^* - \beta) & -\delta \left( \frac{\delta + \beta}{1 + \eta} \right) \\
-\left( 1 + \eta \right) & r^*
\end{bmatrix}
\]

is negative and the steady-state equilibrium \((C, W)\) is a saddle point with eigenvalues

\( \gamma_1 = r^* - (\delta + \beta) < 0 \) and \( \gamma_2 = r^* + \delta > 0 \).

The stable locus associated with the negative root is

\[
C - \overline{C} = a(W - \overline{W}) e^{\gamma_1(t-t_0)},
\]
where \( a = \frac{\delta + \beta}{1 + \eta} > 0 \). The stable path is positively sloped and consumption and wealth move in the same direction.

The stability conditions, however, do not change if we drop the restriction \( Y'(K) = r^* \). In the more general case, the linear approximation of the five dimensional system in \( \dot{C}, \dot{m}, \dot{K}, \dot{F} \) and \( \dot{\rho} \) in the vicinity of the steady state has the following matrix

\[
\begin{bmatrix}
    (r^* - \beta) & (\sigma C_K + A) & \sigma C_F & \sigma B & \sigma \left( \frac{\delta + \beta}{1 + \eta} \right) \\
    -\eta & 0 & 0 & -\eta B & H \\
    -q & 0 & 0 & -E & -q \left( \frac{\delta + \beta}{1 + \eta} \right) \\
    -\frac{(1-q)}{\rho} \circ N & \circ P & \circ \Sigma & \circ \left( \frac{(1-q)}{\rho} \right) \left( \frac{\delta + \beta}{1 + \eta} \right) \\
    0 & Y''(K)\rho & 0 & 0 & 0
\end{bmatrix}
\]

and

\[
\sigma \equiv [r^* - (\delta + \beta)] < 0, \quad A \equiv \delta \left( \frac{\delta + \beta}{1 + \eta} \right) \frac{\omega'(K)}{r^* + \delta} > 0, \quad B \equiv \left( \frac{\delta + \beta}{1 + \eta} \right) F > 0,
\]

\[
H \equiv \left[ r^* - \eta \left( \frac{\delta + \beta}{1 + \eta} \right) \right] \geq 0, \quad E \equiv qB + X^* > 0, \quad N \equiv \left( \frac{\nu}{\rho} \rho_F - C^F \right) > 0,
\]

\[
P = \left( \frac{\nu}{\rho} \rho_F - C^F + r^* \right) < 0, \quad \Sigma = \left[ \frac{\nu}{\rho} + \frac{(1-q)}{\rho^2} \left( \frac{\delta + \beta}{1 + \eta} \right) \left( \frac{\omega(K)}{r^* + \delta} + K + m \right) \right] > 0.
\]

Denoting by \( \gamma_i \) the roots of the system, the sign restrictions we have imposed ensure that

\[
\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 = (r^* - \beta) + \Sigma > 0,
\]

\[
\gamma_1 \gamma_2 \gamma_3 + \gamma_1 \gamma_2 \gamma_4 + \gamma_1 \gamma_2 \gamma_5 + \gamma_1 \gamma_3 \gamma_4 + \gamma_1 \gamma_3 \gamma_5 + \gamma_1 \gamma_4 \gamma_5 + \gamma_2 \gamma_3 \gamma_4 + \gamma_2 \gamma_3 \gamma_5 + \gamma_2 \gamma_4 \gamma_5 + \gamma_3 \gamma_4 \gamma_5 = \]

\[
(\sigma C_K + A) \left[ \eta \left( \frac{\nu}{\rho} + \frac{(1-q)}{\rho^2} \right) \right] + Y''(K)\rho \left[ \eta \left( \frac{\delta + \beta}{1 + \eta} \right) \right] - r^* (r^* - \beta) + \Sigma
\]

\[
+ \sigma C_F \left[ q \frac{\nu}{\rho} + \frac{(1-q)}{\rho} \left( q \frac{C}{\rho} + X^* \right) \right] + P \left[ \delta \eta B + (r^* - \beta) X^* \right] + \delta \eta BN < 0,
\]
There are four positive and one negative roots, involving a saddle-point equilibrium path in the neighborhood of the steady state.

The dynamics of the system when \( d \neq 0 \) and \( Y'(K) = r^* \) are given by

\[
\dot{C} = \delta + \beta \left( \frac{\omega}{r^* + \delta} + W \right) + \rho d
\]

\[
\dot{W} = r^* W + \omega - (1 + \eta)C
\]

\[
\dot{d} = (r^* - \alpha)d + \frac{\delta(\delta + \beta)}{\rho(r^* + \delta)(1 + \eta)} Z
\]

which reduces to a system of two linear differential equations by substituting the first equation into the second. The coefficient matrix is

\[
\begin{bmatrix}
  r^* - \alpha & 0 \\
  -(1 + \eta)\rho & r^* - (\delta + \beta)
\end{bmatrix}
\]

and the two roots are both negative, with \( \lambda_1 = r^* - \alpha \) and \( \lambda_2 = r^* - (\delta + \beta) \). The linear system, thus, is globally stable when \( \alpha > r^* < (\delta + \beta) \).

The path describing the transition to steady state is given by the equation

\[
W - \overline{W} = b(d_{\tau_0} - \overline{d}) e^{\lambda_1 (\tau - \tau_0)},
\]

where

\[
b \equiv \frac{(1 + \eta)\rho}{\alpha - (\delta + \beta)} < 0 \text{ if } \alpha < (\delta + \beta). \quad \text{Non-human wealth and expected future budget deficits are negatively correlated along the adjustment path.}
\]
The long-run equilibrium, however, will be a saddle point if we take the following system of dimension three

\[
\dot{d} = (r^* - \alpha) d + \left[ \frac{\delta(\delta + \beta)}{\rho(r^* + \delta)(1 + \eta)} \right] Z
\]

\[
\dot{C} = (r^* - \beta) C - \delta \left( \frac{\delta + \beta}{1 + \eta} \right) W - (\delta + \alpha) \rho d + \left[ \frac{\delta(\delta + \beta)}{(r^* + \delta)(1 + \eta)} \right] Z
\]

\[
\dot{W} = r^* W + \omega - (1 + \eta) C,
\]

where the second equation is obtained by differentiating equation (20) in the text under \( d \neq 0 \). The coefficient matrix

\[
\begin{bmatrix}
(r^* - \alpha) & 0 & 0 \\
-(\delta + \alpha) \rho & (r^* - \beta) & -\delta \left( \frac{\delta + \beta}{1 + \eta} \right) \\
0 & -(1 + \eta) & r^*
\end{bmatrix}
\]

has two stable roots (\( \lambda_1 = r^* - \alpha \), \( \lambda_2 = r^* - (\delta + \beta) \)) and one unstable root (\( \lambda_3 = r^* + \delta \)) when \( \alpha > r^* < (\delta + \beta) \). The path towards the steady state is a saddle.
REFERENCES


