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Essays on Quantification and Disaggregation of Time Series

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Introduction

The availability for researchers and policy makers of timely, comprehensive and disaggregate indicators has become in the last years one of the most important issues for the economic analysis. As a consequence econometrics has experimented a wide diffusion and improvement to more sophisticated methods.

My interest has been focussed on the issue of nowcasting and forecasting macroeconomic data, among which a primary place is taken by the Gross Domestic Product (GDP). Although this series is overall considered as the major stance of the economic activity, the official release of its value is published with huge delay. In order to get a timely measure of the state of the economy, several attempts have been carried out by Institutions, Statistical offices, Universities and many others actors.

The most common approach, already used in the first econometrics applications and still apparently successful, consists in exploiting the information from more timely and disaggregated indicators to estimate the GDP. Taking into consideration available official data for Europe, the timeliest information come from the Business and Consumer Surveys of the European Commission. Faced to a delay of more than 40 days for all macro series, Surveys are published at the end of the month and refer to the same month. This advantage in term of timeliness is the main strength of these data. The use of this information, which is collect according to qualitative categories, describing the feelings of economic actors (firms and consumers), remains to be understood and fully investigated.

Accordingly, the aim of the first Chapter, dedicated to Business Survey data, is to present new methods for the quantification of the qualitative information provided by firms on the state of the economy. The Spectral Envelope, as well as the Cumulative Logit model with a non linear Kalman filter, is applied to get a quantified indicator of the level of production. Due of the availability of micro data the application is carried out only for Italy, as the lack of similar data for the others countries prevent extending the method to all Europe.

The main result is that the quantified indicator is highly coherent with the cycle of the Industrial production.

In the second Chapter we deal with the issue of GDP nowcasting and disaggregation of the quarterly value added series from National Accounts in a monthly base for the Euro area. By using a mixed frequency model (quarterly and monthly) defined in a state space framework, the monthly indicator for GDP is obtained conditionally on a set of monthly and timely series. The estimation is carried out from the output and the expenditure side, hence the final computation of the monthly GDP is obtained by combining the two estimates with weights based on their precision. A procedure to deal with the chain linked nature of the National Account is also provided. The results show that, in contrast with part of the recent literature, Surveys do not play a relevant role in the estimation of GDP.

More in-depth investigations on this direction are provided in the third Chapter, where the basic model in state space form is extended to allow for more than one common factor and low frequency cycles. After some analysis of real time data and revisions, the main conclusion of this section is that the inclusion of Survey data is useful to produce more accurate estimates and forecast in a context where "hard" data are not released yet.

The last Chapter is devoted to the estimation of a mean-variance coincident index for the US economy. There exists a broad consensus that most of the macroeconomic series has become less volatile in the last 20 year in US. To capture this empirical evidence, the framework developed for Europe is extended to allow for time varying volatility of the economy. We propose a Garch-type model with two regimes, which mimic the so-called "great moderation" period. The main findings are that the volatility of the coincident index shows the response of the US economy to negative shocks such as wars, oil crises, terroristic attacks. Furthermore, while the level of the economy is mainly driven by the industrial production, the uncertainty is the reflection of other series, such as income and employment.

Chapter 1

Quantification of qualitative survey data

ABSTRACT [1]**:** In this chapter we deal with several issues related to the quantification of Business Surveys. In particular, we propose and compare new ways of scoring the ordinal responses concerning the qualitative assessment of the state of the economy, such as the spectral envelope and cumulative logit unobserved components models, and investigate the nature of seasonality in the series. We conclude with an evaluation of the type of business cycle fluctuations that is captured by the qualitative surveys.

Keywords: Spectral envelope. Seasonality. Deviation cycles.
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1.1 Introduction

An important set of indicators on current economic conditions arises from the monthly business survey conducted by various national institutions and coordinated by the European Commission. Their relevance stems from the fact that they provide timely information on economic variables that are either difficult to measure, such as expectations or capacity utilisation, or whose measurement on a quantitative scale is more expensive and time consuming (turnover and production in volume).

The data collected are mostly categorical or ordinal and timeliness is achieved by a suitable survey design. Survey questions are kept to a minimum and bear on the direction of the trend in an economic variable, as perceived by the respondent. For instance, with respect to orders and the level of production, the respondent is asked whether they are low, normal, or high, abstracting from seasonal fluctuations. The individual data are finally aggregated into a single time series by subtracting the percentage of responses falling in the below normal category from the percentage of the above normal. These differences are called balances and are often used for the quantification of the survey responses, insofar as qualitative information is translated into a quantitative scale. See Pesaran and Weale (2006) for a general exposition and review of alternative quantification methods.

With reference to the assessment of the current level of production, this Chapter discusses two alternative quantification methods. The first is based on the notion of the spectral envelope and originates a signal extraction filter which has solely cross-sectional dimension, i.e. only contemporaneous values are employed. As a result the quantification suffers from excess roughness. The second is based on a dynamic cumulative logit model for the time series of ordered responses; the signal extraction filter for the underlying latent cycle is nonlinear and has also a time series dimension. The Chapter also addresses explicitly whether a particular quantification adheres to a specific notion of business cycle and discusses the presence of seasonality and calendar components in the business survey indicators.

The individual survey data take the form of a categorical time series, $y_t, t = 1, \dots, T$, with k ordered response categories identified by the labels $c_1, \dots, c_k$. For algebraic manipulation it is often convenient to represent the response categories introducing the $k \times 1$ vectors $\mathbf{e}_j, j = 1, \dots, k,$, where $\mathbf{e}_j$ has the value 1 in the j-th position and zero elsewhere. We thus define a multinomial vector time series, $\mathbf{Y}_t$, taking the value $\mathbf{Y}_t = \mathbf{e}_j$ if $y_t = c_j$, that is if the j-th category is selected. In the sequel we shall denote $\pi_{jt} = P(\mathbf{Y}_t = \mathbf{e}_j) = P(y_t = c_j), \sum_j \pi_{jt} = 1$. The unconditional mean and covariance matrix of $\mathbf{Y}_t$ are $\mathrm{E}(\mathbf{Y}_t) = \boldsymbol{\pi}_t = (\pi_{1t}, \dots, \pi_{kt})'$ and $\mathrm{Var}(\mathbf{Y}_t) = \mathrm{diag}(\boldsymbol{\pi}_t) - \boldsymbol{\pi}_t\boldsymbol{\pi}_t'$, respectively.

Given n_t independent observations, $\mathbf{Y}_{it}, i = 1, \dots, n$, interest often centers on analysing the number of responses in each category, $\mathbf{Y}_{.t} = \sum_i \mathbf{Y}_{it}$. We assume throughout that sampling is such that at any given time t, $\mathbf{Y}_{.t}$ has a multinomial distribution, that is it takes the values $\mathbf{n}_t = (n_{1t}, \dots, n_{jt}, \dots, n_{kt})', \sum_j n_{jt} = n_t, n_{jt} = \sum_i \mathbf{e}_{ij}$, with probability

$$P(\mathbf{Y}_{.t} = \mathbf{n}_t) = \frac{n_t!}{n_{1t}! \cdots n_{jt}! \cdots n_{kt}!} \pi_{1t}^{n_{1t}} \cdots \pi_{jt}^{n_{jt}} \cdots \pi_{kt}^{n_{kt}}, \;\; n_t = \sum_j n_{jt}, 1 = \sum_j \pi_{jt}.$$

Typically, the total sample size n_t does not change with time, although nonresponse affects it. We assume anyway that nonresponse is fully ignorable, that is it only affects the sample through a reduction of the sample size. Further, we define $\mathbf{p}_t = (p_{1t}, \ldots, p_{kt})' = n_t^{-1}\mathbf{Y}_{.t}$, the proportion of responses in category j. The latter is such $\mathbf{i}_k'\mathbf{p}_t = 1$, where $\mathbf{i}_k$ is a $k \times 1$ vector of 1s, and has a scaled multinomial distribution with mean $\boldsymbol{\pi}_t$ and covariance matrix $n_t^{-1}(\mathrm{diag}(\boldsymbol{\pi}_t) - \boldsymbol{\pi}_t\boldsymbol{\pi}_t')$.

Finally, in the contemporaneous aggregation of the individual responses across groups (e.g branches and sectors of economic activity), weights can be used that stands for the relative importance of the group. Often the data are made available to the public in the form $\mathbf{Y}^*_{.t} = \sum_{is} w_{st}\mathbf{Y}_{ist}$ where s denotes the group to which unit i belongs, and $w_{st}, \sum_s w_{st} = 1$, is the group weight (e.g. the share of gross domestic product or employment, or a measure of size). The aggregate series can be written $\mathbf{Y}^*_{.t} = \sum_s w_{st}\mathbf{Y}_{.st} = \sum_s w_{st}n_{st}\mathbf{p}_{st}$, where $\mathbf{p}_{st}$ is the vector containing the proportions of the responses of each category in group s and n_{st} are the number of respondents in the same group. The scaled series is thus $\mathbf{p}^*_t = \sum_s w^*_{st}\mathbf{p}_{st}$, where the group weights are $w_{st}n_{st}\backslash\sum_s w_{st}n_{st}$. Taking advantage of micro data provided by ISAE, the Italian Institute responsible for Surveys, we can use both proportion of responses than counts. Unfortunately the European Commission made available only the series $\sum_s w_{st}\mathbf{p}_{st}$ that prevent extending our method to the other European countries and building an indicator for Europe as whole. In the sequel we will ignore the complications that arise due to the weighted aggregation of the responses and will continue to assume that the observed counts or proportions arise from a multinomial distribution.

A number of methods have been proposed in the literature for converting these proportions into aggregate measures of perceived business conditions and expectations. The study evaluates some novel quantification methods based on the notion of spectral envelope (section 1.3) and cumulative logit unobserved components models (section 1.4). Some issues related to seasonality in survey data are also presented (section 1.2) and finally the evaluation of the type of business cycle fluctuations captured by the qualitative surveys is attempted.

1.2 Seasonality

Although the respondent is explicitly asked to abstract from seasonal movement in forming his/her judgement, a well known common feature of business survey indicators is the presence of seasonality. The seasonal dynamics in the business survey indicators reflect the seasonality in the underlying quantitative indicators (orders, turnover and industrial production) as far as the location of seasonal peaks and troughs within the year is concerned. This evidence has been advocated in support of the notion that seasonal fluctuations are not independent of the trend-cycle, which implies that economic time series are not decomposable (Franses, 1996).

The presence of seasonality can be illustrated from the month-by-month plots of the Industrial production series and the responses about the level of the production from the Survey on firms, which are presented in figure 1.1. While Industrial production displays a very deep seasonal trough in August and a minor one in December, the percentage of low and high or no change (transformed into logits) display seasonal peaks in correspondence.

This descriptive evidence can be supported by formal statistical tests, such as the Canova -

Hansen (1995) and Busetti-Harvey (2003) test, concerning the presence and the nature of the seasonal movements. A related issue is whether the responses are affected by the number of working days in the month and any other calendar effect, such as the length of the month and Easter.

These issues can be addressed in an unobserved components framework by decomposing the univariate time series, y_t, according to the following model:

$$y_t = \mu_t + \gamma_t + \varepsilon_t \qquad t = 1...T, \tag{1.2.1}$$

where μ_t is the level component, $\mu_t = \mathbf{x}_t'\boldsymbol{\delta}$, $\mathbf{x}_t$ is a vector of linearly independent deterministic regressors, e.g. $\mathbf{x}_t = [1, t-(T+1)/2]'$ for a linear trend, γ_t denotes the seasonal component and $\varepsilon_t \sim \text{NID}(0, \sigma_\varepsilon^2)$. Busetti and Harvey (2003) derive the locally best invariant test of the null that there is no seasonality against the alternative of a permanent seasonal component, which can be either deterministic or stochastic. The seasonal pattern could be decomposed in two components: $\gamma_t = \gamma_t^D + \gamma_t^S$, where the first is a deterministic term expressed as linear combination with fixed coefficients of sines and cosines defined at the seasonal frequencies $\omega_j = 2\pi j/s$, $j = 1...[s/2]$, where s is the number of seasons in a year (e.g. 12 for monthly time series), and $[s/2]$ is the nearest integer resulting from the division in the argument. The second term (γ_t^S) is a nonstationary stochastic component, resulting from a linear combination of the same trigonometric elements with random coefficients.

More formally, defining $\mathbf{z}_t = \left[\cos\omega_{1t}, \sin\omega_{1t}, ..., \cos\omega_{jt}, \sin\omega_{jt}, ..., \cos\omega_{[s/2]t}, \sin\omega_{[s/2]t}\right]'$, we can express the deterministic seasonality as $\gamma_t^D = \mathbf{z}_t'\gamma_0$, with γ_0 being fixed coefficients. The stochastic component is $\gamma_t^S = \mathbf{z}_t'\sum_{i=1}^{t}(k_i)$ where k_t is a vector of serially independent disturbances with zero mean and covariance matrix $\sigma_k^2 W$, independent from ε_t.

The null hypothesis of no seasonality is then formulated as $H_0 : \gamma_0 = 0, \sigma_k^2 = 0$, while a permanent seasonal component is supposed under the two alternatives: $H_a : \gamma_0 \neq 0, \sigma_k^2 = 0$ (deterministic seasonality), $H_b : \gamma_0 = 0, \sigma_k^2 > 0$ (stochastic seasonality). The test statistic proposed by Busetti and Harvey (2003) is consistent against both alternative hypotheses, and it features as follows:

$$\varpi = \sum_{j}^{[s/2]} \varpi_j, \quad \varpi_j = \frac{a_j}{T^2\sigma^2} \sum_{t=1}^{T} \left[\sum_{i=1}^{t} (e_i \cos\omega_j i)^2 + \sum_{i=1}^{t} (e_i \sin\omega_j i)^2 \right], \tag{1.2.2}$$

where e_i are the OLS residuals obtained from the regression of y_t on the deterministic regressors $\mathbf{x}_t$. Under the null ϖ is asymptotically distributed according to a Cramér von Mises (CvM) distribution with $(s-1)$ degrees of freedom.

One might be interested in testing the null of a deterministic seasonal component (H_a) against the alternative of a nonstationary seasonal process (H_b), i.e. characterised by the presence of unit roots at the seasonal frequencies. This issue is addressed by the Canova-Hansen test statistic which has a similar formalization of (1.2.2), with the difference being e_i the residuals from the OLS regression including the trigonometric functions $\mathbf{z}_t$ as additional explanatory variables. Under the null of deterministic seasonality, the test has a Cramér von Mises distribution, with $(s-1)$ degrees of freedom.

Both the mentioned tests allow testing for seasonality (nonstationary, if $\mathbf{z}_t$ is included in the set of regressors) at any single frequency by using ϖ_j. In this case, the tests follow under the null a CvM with 2 degrees of freedom if $j = 1, ..., (s-1)/2$ and 1 degree of freedom for $j = s/2$, for an even s. If $j = 0$ the tests corresponds to the usual tests of stationarity at the long–run frequency.

An estimate of σ^2 is required. It can be computed nonparametrically by rescaling of 2π the estimate of the spectrum of the sequence e_t at the frequency ω_j, using a Bartlett window. Canova and Hansen (1995) further allow for seasonal heteroscedasticity.

The same framework can be used to test for the presence of calendar effects in the series. As it is well known calendar effects impact on the underlying quantitative indicators (production, turnover). Therefore, it seems reasonable to check for such component also in the qualitative surveys. For this purpose the test statistic ϖ_j in (1.2.2) is applied at the specific frequencies $.348 \times 2\pi$, $.432 \times 2\pi$, $.304 \times 2\pi$, which Cleveland and Devlin (1982) have shown to be corresponding to the spectral peaks induced by the trading days effect.

In the sequel we discuss the empirical evidence arising from the application of seasonality and stationarity tests on the first differences of the proportions p_{it} emerging from the ISAE survey on firms in Italy. The results, reported in the tables 1.1– 1.3, are invariant to the adoption of a transformation, such as the logit transformation.

The null of no permanent seasonal effects is always rejected for all the three response categories “low”, “normal”, “high”, either for the single frequencies either for the overall test (see tab. 1.1), although the results for survey data do not always mirror those concerning the Industrial production series. As for the nature of the seasonality, for some of the seasonal frequencies the null of a deterministic seasonal pattern cannot be rejected and the overall evidence is against the null for all the series under consideration (see tab. 1.2).

Finally, as far as the trading-days effects are concerned, the overall test is significant for all the response categories as well as for the balance series, although the test statistic takes a much smaller value with respect to the Industrial production series (see tab. 1.3). More generally, the evidence is that calendar effects constitute a much less relevant source of variation for the business survey series.

1.3 The spectral envelope

Stoffer, Tyler and McDougall (1993) proposed a frequency domain approach to scaling categorical time series which can be applied to the quantification of business surveys. They introduced the notion of spectral envelope for a categorical time series $\mathbf{Y}_t$, which can be defined at any angular frequency $\omega \in [0, \pi]$ as the spectral density of the univariate synthetic time series $\mathbf{b}(\omega)'\mathbf{Y}_t$ (or $\mathbf{b}(\omega)'\mathbf{p}_t$ for time series of proportions) that is obtained when the optimal scores $\mathbf{b}(\omega)$ are applied to the categories. The notation $\mathbf{b}(\omega)$ stresses dependence of optimal scores vary with the frequency. The optimality lies on the fact that $\mathbf{b}(\omega)$ provide the greatest evidence for periodicity at frequency ω, in that the scaled time series has the greatest relative power at that frequency.

Since the distribution of the vector time series $\mathbf{Y}_t$ (or $\mathbf{p}_t$) is singular ($\mathbf{i}'\mathbf{Y}_t = 1$ with probability one), one of the series is redundant and can be assigned a score equal to zero. Let $\mathbf{Z}_t = \mathbf{A}'\mathbf{Y}_t$ be a linearly independent subset of series, where $\mathbf{A}$ is a fixed selection matrix, and let $\mathbf{\Gamma}(\tau)$ denote the crosscovariance matrix at lag τ of $\mathbf{Z}_t$; then, $\mathbf{F}(\omega) = (2\pi)^{-1}\sum_{\tau=-\infty}^{\infty}\mathbf{\Gamma}(\tau)\exp(-\imath\omega\tau)$ is then the spectral density at frequency ω and let $\mathbf{F}^r(\omega) == (2\pi)^{-1}\sum_{\tau=-\infty}^{\infty}\mathbf{\Gamma}(\tau)\cos(\omega\tau)$ is its real part. In our case study concerning the assessment of the level of production we can assign a zero score to the central category and choose $\mathbf{A} = (\mathbf{e}_1, \mathbf{e}_3)$.

Defining $\mathbf{b}^*(\omega) = \mathbf{A}'\mathbf{b}(\omega)$ the vector of scores attached to the selected series, the quantification $\mathbf{b}^{*'}(\omega)\mathbf{Z}_t$ takes place by choosing the scores so as to emphasize the periodic features in the series. This corresponds to maximize the power at frequency ω relative to the total power. In particular, the spectral density of the scaled series is $(2\pi)^{-1}\mathbf{b}^{*'}(\omega)\mathbf{F}^r(\omega)\mathbf{b}^*(\omega)$, whereas $\mathbf{b}^{*'}(\omega)\mathbf{\Gamma}_Y(0)\mathbf{b}^*(\omega) = \int_0^{\pi}(2\pi)^{-1}\mathbf{b}^{*'}(\omega)\mathbf{F}^r(z)\mathbf{b}^*(\omega)dz$ is the variance of the scaled series. Thus, $\mathbf{b}^*(\omega)$ is chosen so as to maximize the ratio:

$$\frac{\mathbf{b}^{*'}(\omega)\mathbf{F}^r(\omega)\mathbf{b}^*(\omega)}{\mathbf{b}^{*'}(\omega)\mathbf{\Gamma}_Y(0)\mathbf{b}^*(\omega)}. \tag{1.3.1}$$

Differentiating with respect to $\mathbf{b}^*(\omega)$, and denoting by $\lambda(\omega)$ the supremum of 1.3.1, the first order conditions lead to the system of equations $\mathbf{F}^r(\omega)\mathbf{b}^*(\omega) = \lambda(\omega)\mathbf{\Gamma}(0)\mathbf{b}^*(\omega)$. Hence, $\lambda(\omega)$ is the largest eigenvalue of $\mathbf{\Gamma}(0)^{-1/2}\mathbf{F}^r(\omega)\mathbf{\Gamma}(0)^{-1/2}$, $\mathbf{\Gamma}(0)^{1/2}\mathbf{\Gamma}(0)^{1/2} = \mathbf{\Gamma}(0)$ (in practice, the symmetric square root matrix is constructed from the spectral decomposition of $\mathbf{\Gamma}(0)$) and $\mathbf{b}^*(\omega) = \mathbf{\Gamma}(0)^{-1/2}\mathbf{v}(\omega)$, where $\mathbf{v}(\omega)$ is the corresponding eigenvector. The scalar $\lambda(\omega)$ is the spectral envelope at frequency ω.

It should be noticed that the scores are not a monotonic function of the category index j, which may be regarded as a drawback. Anyway, the estimation of a vector of scores β_j that is monotonic in j is left to future research. It can however be argued that the monotonic solution is bounded from above by the standard solution.

Given a realization of $\mathbf{Y}_t$ or $\mathbf{p}_t$, the spectral envelope and the associated optimal scores can be estimated using a nonparametric estimator of the real part of the cross-spectrum. In the application below we use a Parzen lag window with truncation parameter at 60.

Figure 1.2 plots the spectral envelope for the assessment of the level of production. Its main features are the concentration of power at the long run and business cycle frequencies, along with the presence of spectral peaks at the seasonal frequencies. The estimated optimal scaling corresponding to the spectral envelope is plotted against the angular frequency in the central panel of the former figure. It is very interesting to notice that the scores have different signs, i.e. produce a contrast, only at a subset of the business cycle frequencies, ranging from 2.5 to 5 years (shaded area). Since we have assigned the score 0 to the central category (normal) the optimal scores are a monotonic function of the index j. The bottom panel of figure 1.2 displays the scaled series using the spectral envelope and the balance of opinions (linearly transformed so as to match the mean and standard deviation). Essentially, for our particular application, the spectral envelope validates the use of the balance for quantification.

1.4 Cumulative logit model

The quantification method proposed in this section can be regarded as a dynamic version of the probability approach initiated by Carlson and Parkin (1975), described, among others, in Pesaran and Weale (2006), which postulates the existence of a common latent variable with known distribution function. The method, which is based on a dynamic cumulative logit model, see Fahrmeir (1992) and Fahrmeir and Tutz (1994 sec. 3.3), overcomes the independence assumption for the latent variable. As a consequence, the quantification will be based on a dynamic nonlinear combination of the observed proportions.

The most popular approach to the analysis of ordinal responses is to assume the existence of a latent continuous response variable, ς_t such that $\mathbf{Y}_t = \mathbf{e}_j$, i.e. the individual response is category i if $q_{i-1} < \varsigma_t \leq q_i$, where q_i is a threshold $q_0 = -\infty < q_1 < \cdots < q_k = \infty$. As we shall see shortly, the natural choice for the latent variable is a stochastic cycle plus a seasonal components as suggested by the tests results of section 1.2.

The process ς_t is a linear Gaussian time series process that is parameterised according to the state space model

$$\varsigma_t = \mathbf{z}'\boldsymbol{\alpha}_t + \epsilon_t, \qquad \boldsymbol{\alpha}_{t+1} = \mathbf{T}\boldsymbol{\alpha}_t + \mathbf{H}\boldsymbol{\eta}_t$$

where ϵ_t has logistic distribution function $F(\epsilon) = 1/(1+\exp(-\epsilon))$.

The most popular approach to modelling the series y_{it} uses logits of cumulative probabilities, also termed cumulative logits. These can be related to the latent signal as follows:

$$P(y_t \leq c_i) = \sum_{j=1}^{i} \pi_{jt} = P(\varsigma_t \leq q_i|\boldsymbol{\alpha}_t) = P(\epsilon_t \leq q_i - \mathbf{z}'\boldsymbol{\alpha}_t) = \left(\frac{1}{1+\exp(\mathbf{z}'\alpha_t - q_i)}\right).$$

Consider now the assessment of the level of production and let $i = 1, 2, 3$ label the three response categories. Then,

$$\ln\left[\frac{P(y_t \leq c_i)}{1 - P(y_t \leq c_i)}\right] = \ln\left[\frac{P(\varsigma_t \leq q_i)}{1 - P(\varsigma_t \leq q_i)}\right] = q_i - \mathbf{z}'\boldsymbol{\alpha}_t.$$

Hence, we can express the cumulative probabilities as

$$\pi_{1t} = \frac{\exp(\theta_{1t})}{1+\exp(\theta_{1t})}, \quad \pi_{1t} + \pi_{2t} = \frac{\exp(\theta_{2t})}{1+\exp(\theta_{2t})},$$

and write $\theta_{1t} = \text{logit}(\pi_{1t}), \theta_{2t} = \text{logit}(\pi_{1t} + \pi_{2t})$.

Assuming that, conditionally on $\boldsymbol{\alpha}_t$, the observations are independent, the multinomial density of the counts n_{it} is

$$p(\mathbf{n}_t|\boldsymbol{\pi}_t) = \sum_{t=1} \left\{\ln K_t + n_t \ln \pi_{kt} + \sum_{i=1}^{k-1} n_{it} \left[\ln \pi_{jt} - \ln \pi_{kt}\right]\right\}$$

where K_t denotes the multinomial coefficient $n_t!/\left(\prod_{i=1}^{k} n_{it}!\right)$. The likelihood can be expressed in terms of the cumulative logits $\theta_{it} = q_i - \mathbf{z}'\boldsymbol{\alpha}_t$. In particular, when $k = 3$:

$$p(n_{1t}, n_{2t}|\theta_{1t}, \theta_{2t}) = \begin{array}{l}\sum_{t=1} \{\ln K_t + n_{1t}[\theta_{1t} + \ln(1 + \exp(\theta_{2t}))] + \\ n_{2t}[\exp(\theta_{2t}) - \exp(\theta_{1t})] - (n_{1t} + n_{2t}) \ln(1 + \exp(\theta_{1t}))\} .\end{array} \tag{1.4.1}$$

When the sample size is constant, $n_t = n$, we can reexpress the likelihood in terms of the proportions $p_{jt} = n_{jt}/n_t$ and the parameters $\theta_{it}, i = 1, 2$.

$$p(p_{1t}, p_{2t}|\theta_{1t}, \theta_{2t}) = \begin{array}{l}\sum_{t=1} \{\ln K_t^* + p_{1t}[\theta_{1t} + \ln(1 + \exp(\theta_{2t}))] + \\ p_{2t}[\exp(\theta_{2t}) - \exp(\theta_{1t})] - (p_{1t} + p_{2t}) \ln(1 + \exp(\theta_{1t}))\} .\end{array} \tag{1.4.2}$$

As for as the specification of the linear and Gaussian state space model for the latent component, we have

$$\mathbf{z}'\boldsymbol{\alpha}_t = \psi_t + \gamma_t,$$

where ψ_t is a stochastic cycle, generated by the dynamic equation:

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}$$

where $\kappa_t \sim \mathrm{NID}(0, \sigma_\kappa^2)$ and $\kappa_t^* \sim \mathrm{NID}(0, \sigma_\kappa^2)$ and $\mathrm{E}(\kappa_t\kappa_t^*) = 0$. The seasonal component is parameterised according to the trigonometric seasonal model (see Harvey, 1989), which results from the sum of $[s/2]$ nonstationary stochastic cycles defined at the seasonal frequencies $\omega_t = \sum_{j=1}^{[s/2]} \omega_{jt}$,

$$\begin{bmatrix} \omega_{jt} \\ \omega_{jt}^* \end{bmatrix} = \begin{bmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{bmatrix} \begin{bmatrix} \omega_{j,t-1} \\ \omega_{j,t-1}^* \end{bmatrix} + \begin{bmatrix} \chi_{jt} \\ \chi_{jt}^* \end{bmatrix}, \quad j = 1, ..., [s/2],$$

with χ_{jt} and χ_{jt}^* being a set of serially and mutually uncorrelated WN sequences with common variance σ_χ^2.

The cumulative logit model is a particular instance of dynamic generalised linear model. Inference on the unknown parameters and the underlying cycle can be made using the approach based on importance sampling by Durbin and Koopman (2001), according to which the linear and Gaussian approximating model is derived as follows. Let us suppose that we are able to start from trial values $\tilde{\theta}_{it} = q_i - \mathbf{z}'\tilde{\boldsymbol{\alpha}}_t$, $i = 1, 2$, and set $\tilde{\boldsymbol{\theta}}_t = [\tilde{\theta}_{1t}, \tilde{\theta}_{2t}]'$. Consider the first order Taylor expansion of the gradient of the logarithm of the multinomial density, denoted $\mathbf{g}(\boldsymbol{\theta}_t)$, with respect to the $\theta_{it}, i = 1, 2$.

$$\mathbf{g}(\boldsymbol{\theta}_t) = \mathbf{g}(\tilde{\boldsymbol{\theta}}_t) + \mathbf{D}(\tilde{\boldsymbol{\theta}}_t)(\boldsymbol{\theta}_t - \tilde{\boldsymbol{\theta}}_t), \mathbf{g}(\boldsymbol{\theta}_t) = \frac{\partial p(\mathbf{n}_t|\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}, \quad \mathbf{D}(\boldsymbol{\theta}) = \left[\left.\frac{\partial^2 p(\mathbf{n}_t|\boldsymbol{\theta}_t)}{\partial\boldsymbol{\theta}_t\partial\boldsymbol{\theta}_t'}\right|_{\boldsymbol{\theta}_t=\tilde{\boldsymbol{\theta}}_t} \right]. \tag{1.4.3}$$

Equating (1.4.3) to zero and rearranging yields the pseudo-linear approximating state space model

$$\tilde{\mathbf{y}}_t = \mathbf{q_i} - \mathbf{i}_2\mathbf{z}'\boldsymbol{\alpha}_t + \mathbf{u}_t, \quad \mathbf{u}_t \sim \mathrm{NID}(\mathbf{0}, \mathbf{D}^{-1}(\tilde{\boldsymbol{\theta}}_t)), \qquad \boldsymbol{\alpha}_{t+1} = \mathbf{T}\boldsymbol{\alpha}_t + \mathbf{H}\boldsymbol{\eta}_t$$

where the pseudo-observations are $\tilde{\mathbf{y}}_t = \tilde{\boldsymbol{\theta}}_t - \mathbf{D}^{-1}(\tilde{\boldsymbol{\theta}}_t)\mathbf{g}(\tilde{\boldsymbol{\theta}}_t)$, $\mathbf{q} = [q_1, q_2]'$. The linear Gaussian approximating model that is used for simulation is found by iterating the Kalman filter and Smoother on the linearised model. Starting from $\tilde{\boldsymbol{\alpha}}_t$ (and thus $\tilde{\boldsymbol{\theta}}_t$) the KFS is applied to the pseudo observations to get a new estimate of the state vector $\tilde{\boldsymbol{\alpha}}_t$, and of $\tilde{\boldsymbol{\theta}}_t$, which are in turn replaced into the gradient and the hessian so as to give a new set of pseudo observations. At each run the system matrices of the linear Gaussian approximating model are update and the process, iterated until convergence, yields the estimates of the posterior mode of $\boldsymbol{\theta}_t$ and $\boldsymbol{\alpha}_t$, conditional on the available data. The Gaussian density is used to draw samples and to estimate functions of the state by means of importance sampling techniques, and a Monte Carlo estimate of $\mathrm{E}(\boldsymbol{\alpha}_t|\mathbf{n}_t)$ is available. Importance sampling is used also for Monte Carlo estimation of the likelihood, which can be maximised using a quasi-Newton method.

1.5 Application to the Level of Production

1.5.1 Description of the available data

Business Surveys are a well established source of timely information on the current state of the economy and its future developments. In the European Union the *Joint Harmonised EU Programme of Business and Consumer Surveys* (see European Commission, 1997) has contributed to enhance the comparison and the harmonisation both in terms of sampling design and questionnaire. We present an application for Italy based on micro data from the survey on firms. The *Manufacturing Survey* for Italy, as part of the join harmonized EU program, is carried out on a monthly basis by ISAE from 1962. Several modifications and improvements on the sampling design and survey questions have occurred through time in order to improve the accuracy and to enforce the harmonisation with other EU countries. More recently, particular attention has been devoted to the issue of weighting the individual responses to produce sectorial aggregates. After the last revision which took place in 2003, the current weights are based on economic activity (3-digit l NACE Rev 1.1 classification), 20 administrative regions and firm size.

The questionnaire is divided into three parts, the first concerning the assessment of the situation for the current month and the last relating to the expectations for the following 3 months. The central part is devoted to a specific question formulated in terms of the variation with respect to the previous month. The survey refers to the main aspect of economic activity: level of production and orders, unsold stocks, liquidity, selling prices, employees. In addition, in the last month of each quarter the questionnaire is supplemented with questions on capacity utilization, raw materials, worked hours, export and constraints to growth. For each question the respondent firm should provide a qualitative answer on an ordinal balanced scale, with 3 or 5 response categories, always formulated with a central neutral position.

The number of firms participating to the survey has increased steadily: from about 2,500 in 1991 to almost 4,000 in 2005. The selection of the sample is made by ISAE purposively, according to the representativeness of the firm (an implicit measure of size is used for sample selection), and the dataset takes the form of a panel, even though only 14 firms have a continuous participation record from 1991 to 2005. This number does not increase considerably if we restrict our analysis to the last decade (from 1996 to 2005 only 59 complete records are available). Partial nonresponse is widespread and may be detrimental to the quality of the survey especially in August, which is the traditional holiday period in Italy. As for the assessment of the level of production, which is the focus of this study, the survey question is formulated as follows: "Excluding seasonal patterns, the level of production is: high, normal, low?". No attempt at providing a detailed explanation of the reference, or "normal", value is made. In the following application we use a monthly sample of firms from 1991.1 to 2005.12 weighted by firm size and sector, following the ISAE aggregation scheme (see tab. 5 pag 27 in Malgarini *et al*, 2005). Proportions of responses for the level of production survey question are shown in Figure 1.3.

1.5.2 Main results

In our approach no account is made for the fact that the responses refer to the same units; we will postulate that at each time point an independent sample of manufacturing units is drawn, and that the autocorrelation of the responses is fully explained by a latent variable which expresses the state of the economy. We leave to future research the modelling and the investigation accounting for individual panel time series, while we report in section 1.5.4 an exercise for group of activities. Following the cumulative logit model former discussed we estimate [2] the quantified indicator for the level of production as from ISAE Survey in Italy. The maximum likelihood estimates of the threshold parameters resulted $\hat{\mathbf{q}} = [-1.4, 2.1]$ with asymptotic standard error by the delta method equal to 0.07 for both parameters. We report in Figure 1.4 the estimates of the posterior mean of the latent cyclical factor, $\mathrm{E}(\psi_t|\mathbf{n}_t)$, obtained via importance sampling using 1000 replications and an antithetic variable (see Durbin and Koopman, 2001, p. 205), along with an approximated 95% confidence interval. The estimated cycle shows a period of about 5 years and half (69 months) and appears to be a persistent component with an estimated damping factor equal to 0.97. Moreover, seasonality is a smaller source of variation, with $\hat{\sigma}^2_\kappa = 0.004\hat{\sigma}^2_\chi$.

In order to asses the accuracy of the model we provide some graphic diagnostics for the residuals, as well as for the importance sampler. The Pearson's residuals, presented in the top left panel of Figure 1.5, feature a not desirable cyclical pattern in the first years of the sample. This might reflect the Survey design revision occurred in the late '90s, which is visible also by the inspection of Figure 1.3. The additional plots in Figure 1.5 are dedicated to importance sampling weights diagnostics and show the weights for a simulation of 1.000 replications, together with the largest 100 weights and the recursive estimation of their standard deviation. This last graph provides evidence on the existence of the variance of the sampling weights and its robustness against outliers, which is a desirable feature for Monte Carlo experiment.

Figure 1.6 compares the estimated latent cycle with the cycle in the corresponding quantitative indicator, the index of Industrial production for the Italian manufacturing sector, and with the balances ($p_{3t} - p_{1t}$), appropriately rescaled and seasonally adjusted. The deviation cycle in Industrial production was estimated by the Harvey and Jaeger (1993) model with seasonality, using the time series produced by ISTAT for the sample period 1990.1–2005.12. Accordingly, the series is decomposed into a local linear trend (see Harvey, 1989), a stochastic cycle with the same

[2]Estimation was carried out using Ssfpack 3.0 beta by Koopman et al. (1999).

representation as ψ_t above and a trigonometric seasonal component including trading days effect correction.

As discussed previously, survey data has become more reliable and homogeneous in the last decade, evidence emerging also by the inspection of Figure 1.3. This motivates the estimation of the same model in a restricted sample: (1996.1-2005.12). Results, shown in Figure 1.7, are very similar to the former: although the Pearson's residuals show a better pattern for the resticted sample (central and bottom panels), the survey data cycle on the whole sample is closer to the balance series (top panel).

Several interesting considerations emerge: first and foremost, our model based quantification can be seen as a smoothed version of the balances. The amount of smoothing is dictated by the parameters of the model; the turning points are more clearly identifiable and the assessment of cyclical stance is made much easier. Secondly, the latent cycle is highly coherent with that in Industrial production which was estimated independently.

1.5.3 Time varying thresholds and sample size

In the model presented above we made two implicit assumptions: first we have assumed thresholds constant over time and secondly, working on percentage of responses, we have considered a fixed sample size (n=100). These two restrictions could be relaxed in order to provide more generality.

As far as the first issue is concerned, we claim that is likely to expect that during periods of high (low) production growth, the thresholds in the indifference interval $\mathbf{q} = [q_1, q_2]'$ would increase (decrease). This evolution in time might be formalized in several ways, among which the random walk, proposed in this study, might be seen as a naive model. In the State Space Form this is straightforward by supplemented the the state equation by: $q_{it} = q_{it-1} + \vartheta_t$ with $\vartheta_t \sim NID(0, \sigma^2_\vartheta)$ and independent from other errors in the model. We present in Figure 1.8 the results as for the application of the model with time varying threshold. The plot suggests that the indifference interval has moved to the right part of the real axis during the last ten years. Diagnostics are very similar to the fixed thresholds model and the coherence with the Industrial production deviation cycle is slightly improved (compare with Figure 1.4).

The log-likelihood for the two model is very similar (respectively -85.055 for the fixed thresholds and -85.096 for the time varying), which suggests not significant difference between the two specifications. As matter of fact, any test of model choice based on the likelihood, like the Likelihood Ratio test (LR), would need some additional considerations. If the order of integration do not change between one specification (fixed threshold) and the other (time varying threshold), the LR is distributed as a mixture of Chi-squared, otherwise it follows a Cramèr-von-Mises distribution [3]. We omit the presentation of the time varying thresholds model on the restricted sample from 1996 since results degenerate into a time constant thresholds model because of the short sample size.

As mentioned before, there is one possible generalization of the model allowing for different sample size in different periods. The model with percentage of responses was the first natural formalization, given that data on Business and Consumers Surveys are diffuse by the European Commission in this form. However, the availability of micro data allows working with count data.

[3] The complexity of this issue discourage any further application that would be included in the future agenda.

This generalization attempts considering a time varying sample size, which should be related to seasonality and/or attrition. Results as from the application of the model with counts data are presented in figure 1.9. There is not relevant difference from the estimated cycle, unless for the fact that the model with counts produce a more volatile cycle in recent years. The core pattern is confirmed and this is also a signal of robustness of our methodology.

1.5.4 Sectorial Analysis

The availability of individual firm data provided by ISAE allows deepening the analysis, up to individual level. However, because of the strong attrition in the sample, a careful treatment of missing data would be required, which is out of the goal of this study. Therefore, we consider more cautious to deal with group of activities of firms instead of single firms. In order to carry the sectorial analysis we group firms in homogeneous activities starting from the 3 digits ATECO classification, which we are reported in table 1.4 along with some descriptive statistics.

The groups of activities differ along several directions and especially in terms of number of observations and dimension of firms, and for internal heterogeneity as well. In particular for “Chemicals” we observe a very small sample size, under 30 observations in several years. As far as sample size of firms is concerned, it is worth to notice that there are groups extremely heterogenous, such as “Motor vehicles and transport equipment”, where the smallest firms has 4 employees and the biggest about 80 thousands.

For every group of activity, after weighting the observations by the dimension of firm, we apply the same model as for the total manufacture sector to extract the quantified indicator for the level of production by using the cumulative logit model. We do not present results for Chemicals because the small number of observations invalidates any statistics. The inspection of figure 1.10 suggests that in all the sub-sectors there were two peaks of production, the first in the 1998 and the second in the late 2000, unless for “Food, beverages and Tobacco”, whose pattern is not well defined. However, the turning points are not synchronized for all of them, with a shift of few months. In particular,“ Other manufacture” seems to anticipate a bit the total cycle, while production in “Textiles” is the latest activity to turn after the peak of 2000. The total cycle appears to be driven mainly by “ Motor vehicles and transport equipment” and this is reasonable if we consider that this activity gathers more than 50% of the total employment in Manufacture.

Following the same approach as for the general case, we extract the cycle from the industrial production -disaggregated at the same ATECO digits level- and we compare the pair of cycles from surveys and hard data. Unless for “Other manufacture” we obtain very good results in term of correlation and coherence (see figure 1.11) between the cumulative logit model indicator and the Industrial production cycle. This confirms the validity of our method for quantification of Survey data also at sub-sectorial level.

1.6 Conclusions

This study has investigated several issues related to the quantification and the analysis of business survey variables. The presence and nature of seasonal fluctuations and trading days variation was addressed and two novel quantification methods were proposed and investigated. The first is based on a purely cross–sectional filter derived from the notion of a spectral envelope. The second is based on a dynamic cumulative logit model, which extracts the latent cycle from the available time series of proportions and yields signal extraction filters that have both time series and cross-sectional dimensions. We present an application for the level of production, in the form of proportion of responses and counts, for two different time periods. The empirical analysis includes also a model with time varying thresholds and a sub-sectorial exercice.

As a result the quantification has a smoother appearance, which is more amenable for the identification of turning points and for the characterization of the perceived cyclical stance.

The underlying cycle is highly coherent with the deviation cycle in the corresponding quantitative indicator, the index of Industrial production. This raises the important issue of understanding what notion of business cycle (deviation or growth rate) the economic agents have in mind when they answer the qualitative survey question. We leave to future research this important issue.

Table 1.1: Harvey-Busetti general tests for seasonality

	overall test ϖ	single frequencies						
		0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
Industrial production	7816.82	0.399	44.341	601.34	2306.1	1302.7	1517.7	2044.3
Low	171.79	0.049	3.531	11.097	50.207	14.804	53.772	38.331
Normal	156.48	0.052	4.530	12.666	48.338	15.956	54.804	20.132
High	134.36	0.043	7.570	6.566	38.069	23.934	23.892	34.287
Balance	165.34	0.046	4.524	9.565	48.200	16.779	45.580	40.650

Note: Critical values (see tab. 1 in Nyblom (1989)) are respectively:
CvM(11)=9.03 for overall test, CvM(1)=1.65 for 0 and π and CvM(2)=2.63 for other frequencies

Table 1.2: Canova-Hansen test for stationary seasonality with correction for heteroschedasticity

	overall test ϖ	single frequencies						
		0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
Industrial production	3.496	0.093	0.596	0.714	0.785	0.353	0.879	0.076
Low	3.90	0.045	0.937	0.681	0.820	0.472	0.444	0.499
Normal	4.04	0.041	1.096	0.538	1.157	0.307	0.661	0.241
High	5.14	0.048	0.611	0.940	1.020	0.483	1.425	0.609
Balance	4.53	0.047	0.748	0.941	0.665	0.544	0.743	0.838

Note: Critical values (see tab. I(a) in Harvey (2001)) are respectively:
CvM(11)=2.739 for overall test, CvM(1)=.461 for 0 and π and CvM(2)=.748 for other frequencies

Table 1.3: Harvey-Busetti test for calendar effects

	overall test ϖ	single frequencies		
		$.348 * 2\pi$	$.432 * 2\pi$	$304 * 2\pi$
Industrial production	367.39	222.469	128.584	16.335
Low	11.23	8.931	2.020	0.283
Normal	11.36	3.672	6.504	1.179
High	18.76	16.090	2.227	0.441
Balance	13.58	12.123	1.165	0.290

Note: Critical values (see tab. 1 in Nyblom (1989)) are respectively:
CvM(6)=5.68 for overall test and CvM(1)=.461 for single frequencies

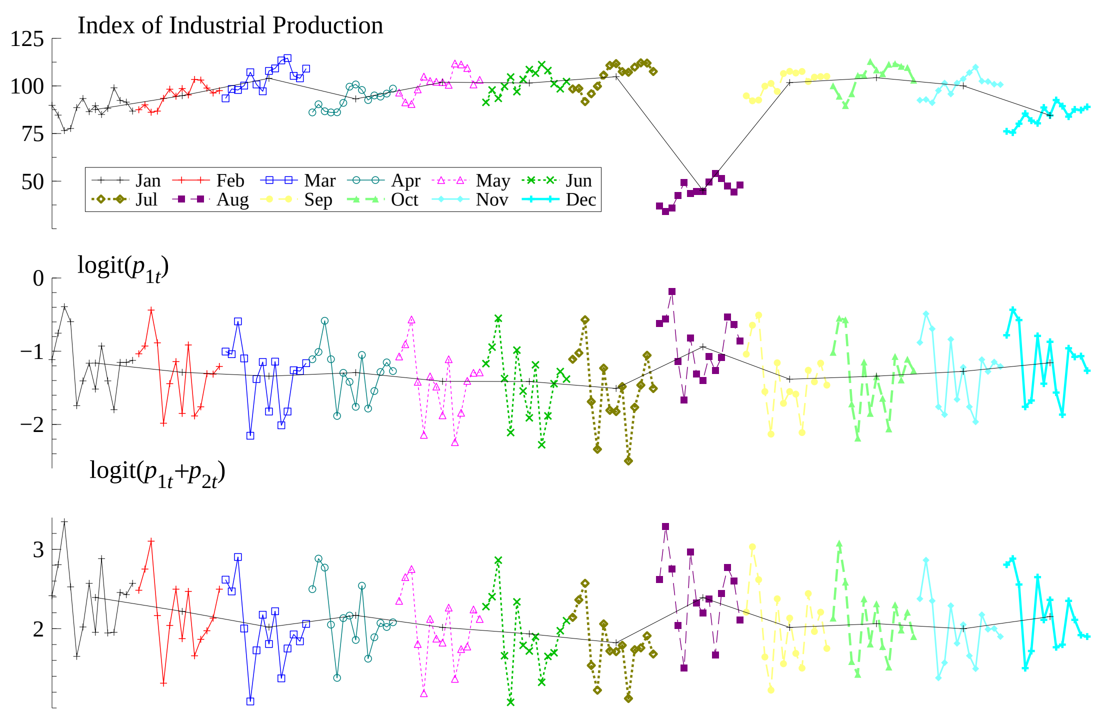
Table 1.4: Groups of activity in the Manufacture Sector and descriptive statistics (1996M1-2005M12)

Group of activity	firms on average	Employees		
		mean	std	max
Food, beverages and tobacco	93	107	486	7,302
Textile	123	76	136	1,202
Chemicals	39	185	319	6,764
Metal manufacture and metal products	202	90	197	7,912
Motor vehicles and transport equipment	136	534	3,408	80,542
Other manufacture	209	94	166	1,836

Note: The 3-digits ATECO classes are grouped as follow: Food, beverages, tobacco=DA; Textile=DB+DC; Chemicals=DG;Metal manufacture=DI+DJ;Motor vehicles=DK+DL+DM; Other manufacture=DD+DE+DF+DH+DN.

The minimum size of firms is for all groups of 4 or 5 employees.

Figure 1.1: Monthplots of Industrial production and Survey data on the assessment of the production for Italy (1991-2005).



Note: p_{it} is the proportion of response i="low", "normal","high" from ISAE Survey on firms in Italy.

Figure 1.2: Spectral envelope, scores and quantification of survey questions on the level of production for Italy.

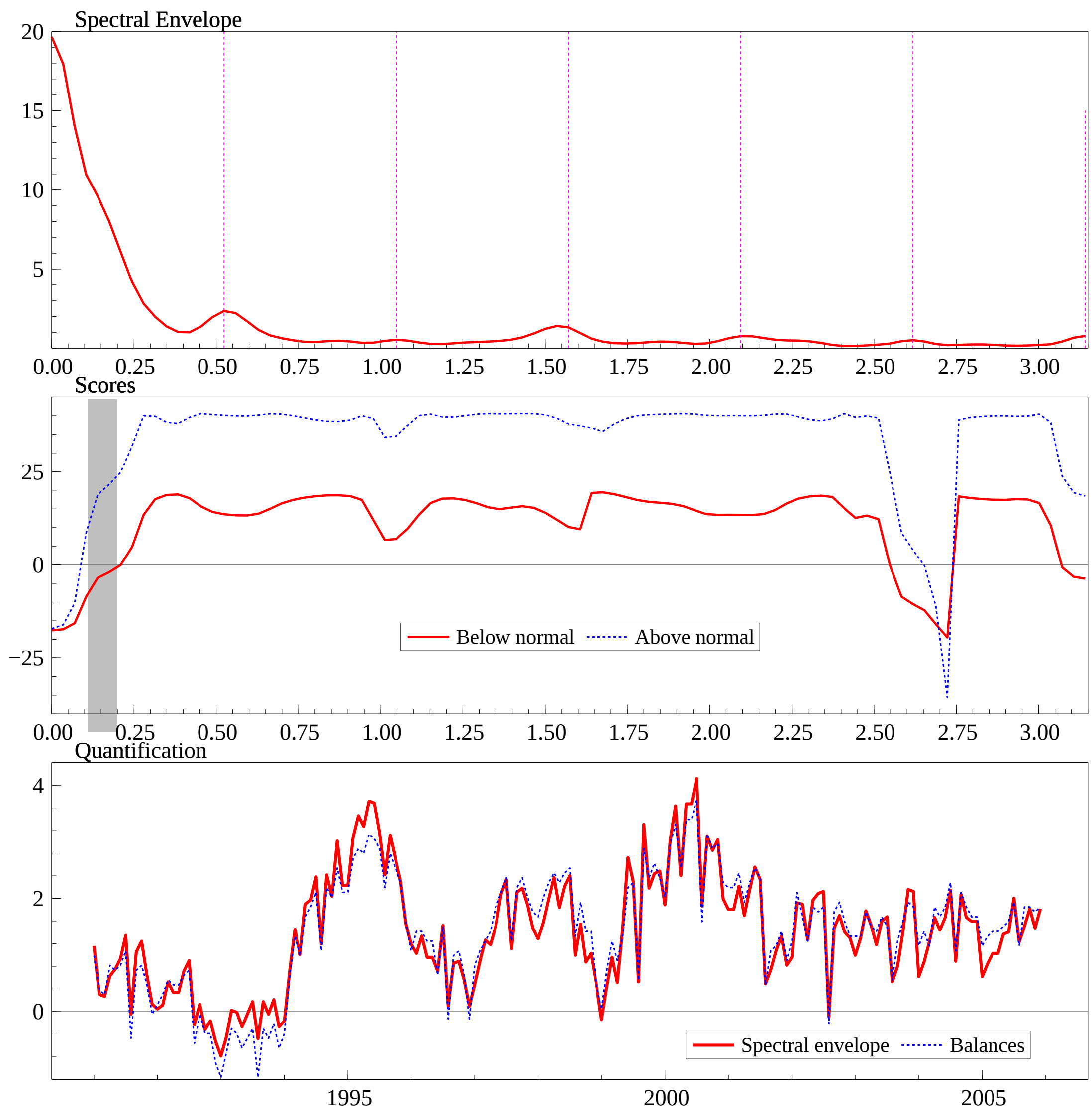


Figure 1.3: Proportion of responses for the assessment of the level of production, ISAE Business Survey for Italy

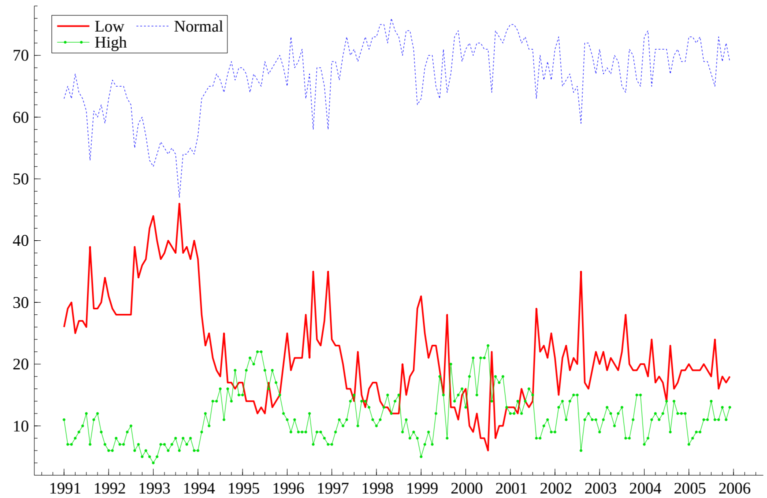


Figure 1.4: Dynamic cumulative logit model: estimates of the conditional mean of the latent cyclical factor, $\mathrm{E}(\psi_t|\mathbf{n}_t)$ obtained via importance sampling using 1000 replications.

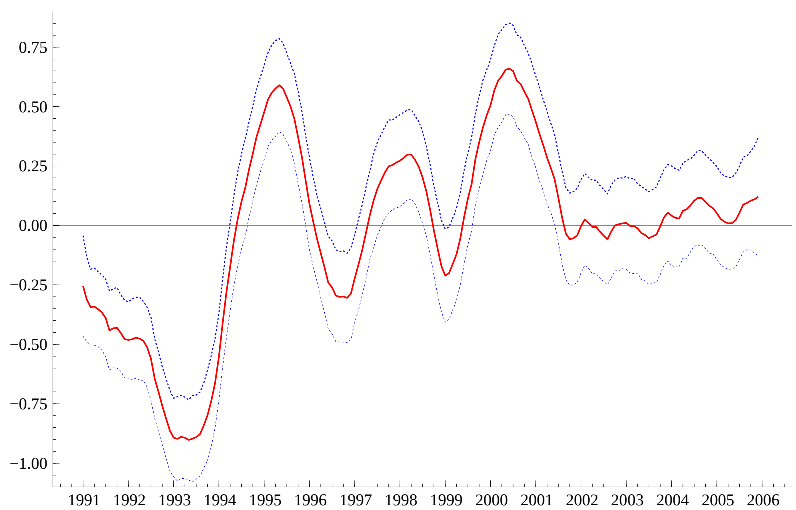


Figure 1.5: Pearson's residuals and importance sampling diagnostics for the cumulative logit model

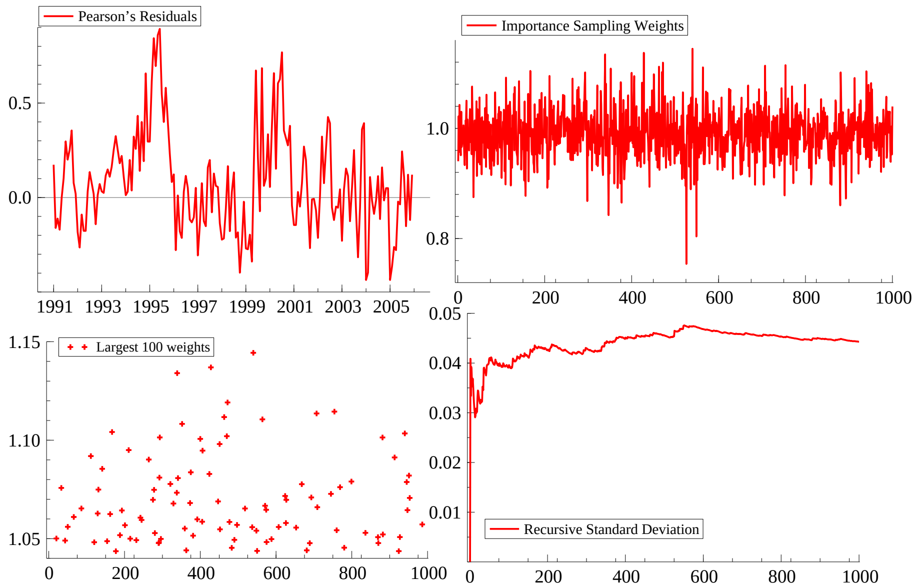


Figure 1.6: Comparison of Industrial production deviation cycle, survey data latent cycle with constant thresholds, quantification through balances

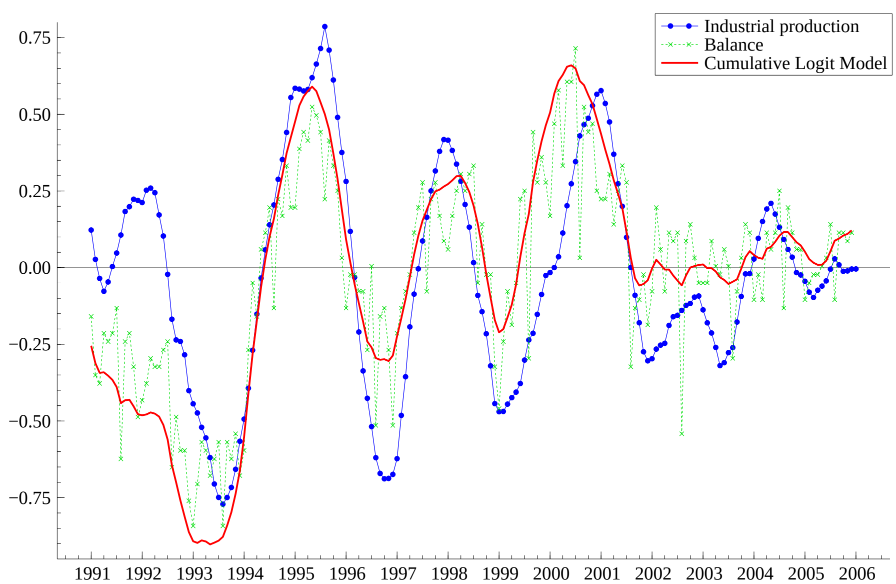


Figure 1.7: Dynamic cumulative logit model on the sample (1996-2005): estimates of the conditional mean of the latent cyclical factor, comparison with the Industrial production deviation cycle and diagnostics.

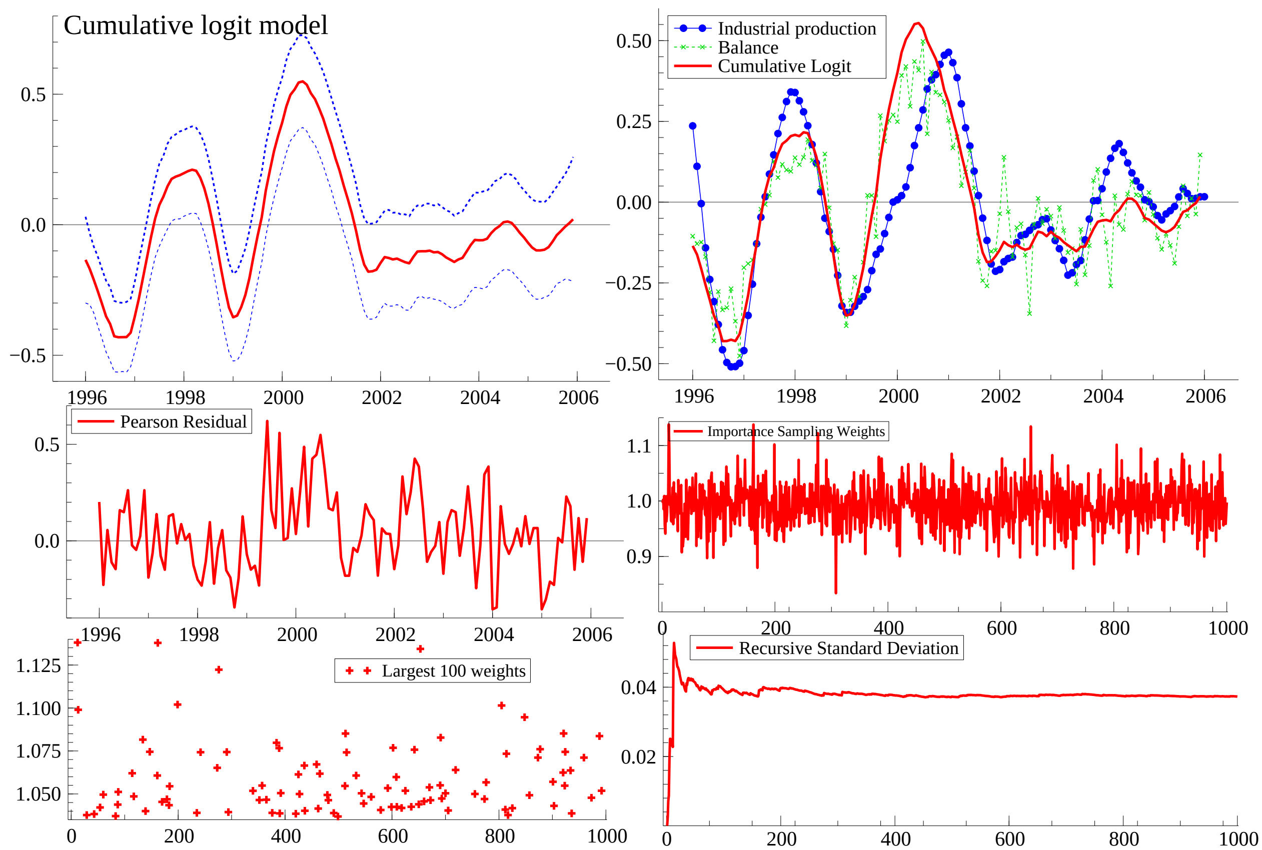


Figure 1.8: Dynamic cumulative logit model with time varying thresholds: estimates of the conditional mean of the latent cyclical factor, comparison with the Industrial production deviation cycle and diagnostics.

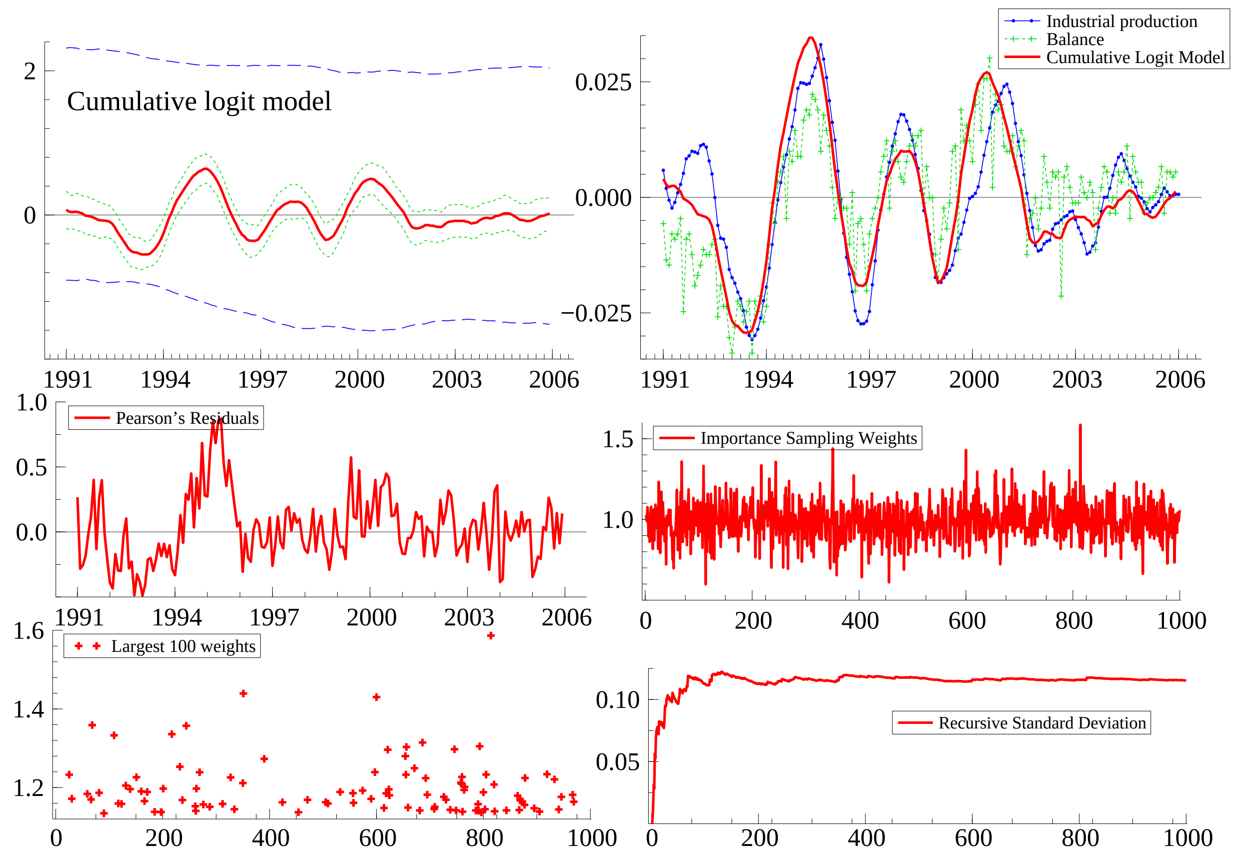


Figure 1.9: Latent cycle from the model applied to the counts

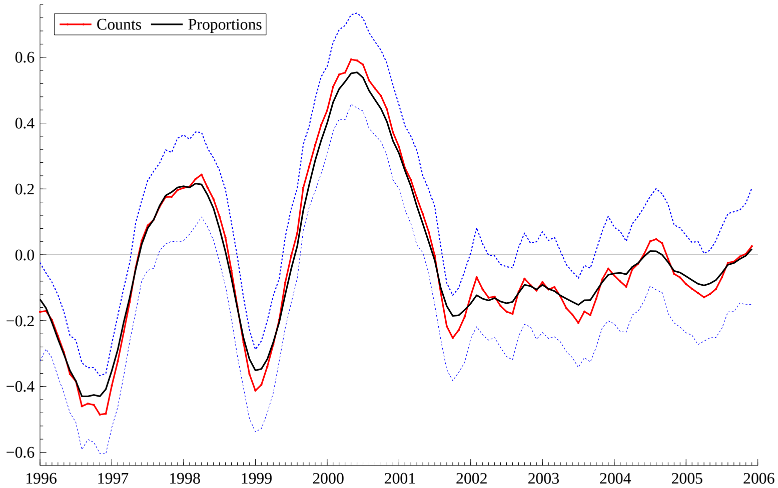


Figure 1.10: Cumulative logit model for groups of activity and the total manufacture sector

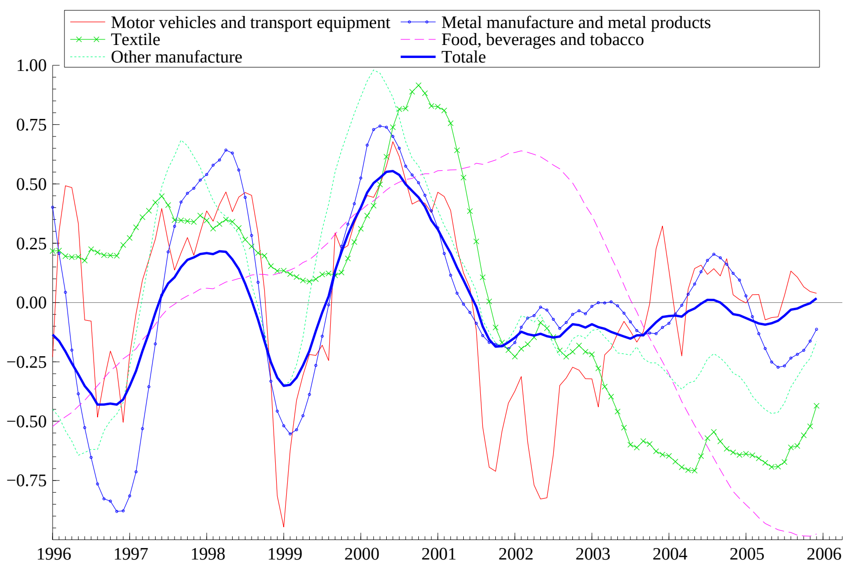
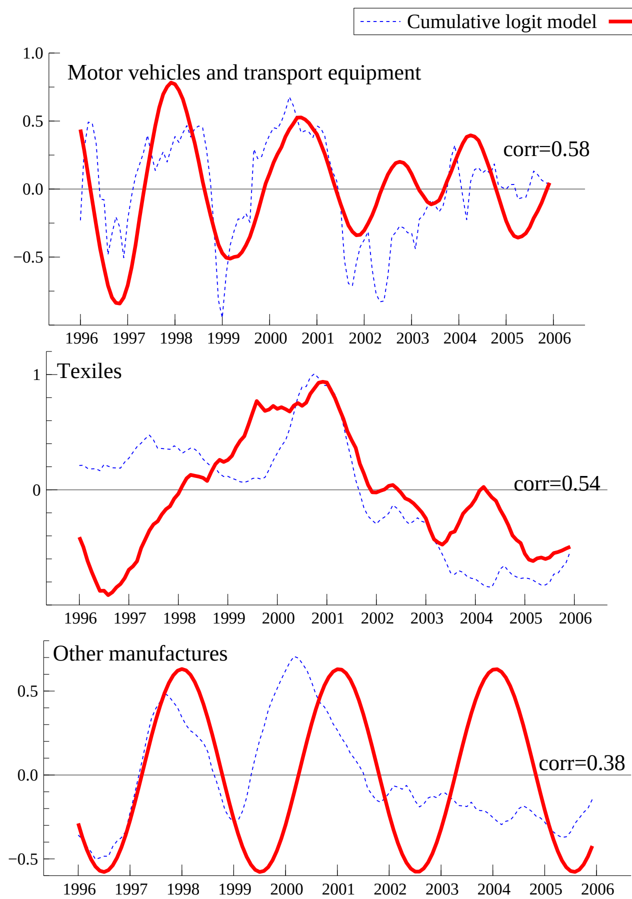


Figure 1.11: Cumulative logit model and industrial production cycle for groups of activity



Chapter 2

A Monthly Indicator for the Euro Area GDP

ABSTRACT [1]: A continuous monitoring of the evolution of the economy is fundamental for the decisions of public and private decision makers. This paper proposes a new monthly indicator of the euro area real Gross Domestic Product (GDP), with several original features. First, it considers both the output side (six branches of the NACE classification) and the expenditure side (the main GDP components) and combines the two estimates with optimal weights reflecting their relative precision. Second, the indicator is based on information at both the monthly and quarterly level, modelled with a dynamic factor specification cast in state-space form. Third, since estimation of the multivariate dynamic factor model can be numerically complex, computational efficiency is achieved by implementing univariate filtering and smoothing procedures. Finally, special attention is paid to chain-linking and its implications, via a multistep procedure that exploits the additivity of the volume measures expressed at the prices of the previous year.

Keywords: Temporal Disaggregation. Multivariate State Space Models. Dynamic factor Models. Kalman filter and smoother. Chain-linking.
JEL Classification: E32, E37, C53

[1] I am sincerely grateful to Tommaso Proietti for nominating me research assistant in the project for the estimation of the Monthly GDP for the Euro area in the context of which this chapter has been produced. My thanks are also to Gianluca Cubadda for his great comments. This chapter is mainly based on the homonymous paper written with Tommaso Proietti, Massimiliano Marcellino and Gianluigi Mazzi.

2.1 Introduction

The availability of a representative, reliable and timely set of high frequency macroeconomic indicators is quintessential for the assessment of the state of the Eurozone economy and the conduct of monetary policy.

With the purpose of satisfying the information requirements of policy makers, economic analysts, researchers and business cycle experts, Eurostat has organised a very comprehensive and representative number of monthly and quarterly time series in the Euro-IND database, accessible through the Euro-Indicators website. The latter contains time series observations on 80 macroeconomic variables for the Euro-zone, the European Union, as well as for Member States and EFTA countries, concerning the following domains: balance of payments; business and consumer surveys; external trade; industry, commerce and services; labour market; monetary and financial indicators; national accounts; consumer prices. Among this set, 19 indicators have been selected by the ECB and the Commission's Economic and Financial Affairs Directorate-General with the qualification of Principal European Economic Indicators (PEEI).

In recent years there have been substantial advances in the methodology and the quality of infra-annual statistical information for the Eurozone, well accounted in the report "Towards improved methodologies for Eurozone statistics and indicators" by the Commission of the European Communities (2002). In particular, the statistical methodology has made it possible to increase the length, coverage, and timeliness of short-term statistics for the Eurozone. Nevertheless, some of the PEEI are available at the quarterly frequency, whereas it would be desirable to have monthly estimates of the corresponding aggregates. The leading example is Gross Domestic Product (GDP), which is usually considered as a comprehensive measure of the level of economic activity of an economy.

The relevance of GDP and the need to make it available at higher (monthly) frequency provides the motivation for this Chapter. Using recent advances in statistical methodology and the availability of timely and reliable statistical information on related indicators at the monthly frequency, we can produce indirect estimates of monthly GDP that are informative for short run analysis.

As the monthly indicators represent measures of sectorial output (industrial production, retail turnover, number of passengers, etc.) or of sectorial input (employment, hours worked), we consider the breakdown of GDP into the value added of six branches of the NACE-Clio rev. 1 classification and, for each branch, we proceed to the estimation of the monthly value added. The observed quarterly value added series will be distributed over the three months composing the quarters so as to preserve the quarterly aggregation constraint, that is, ensuring that the sum of the three distributed values is consistent with the quarterly figure. The same approach is followed to estimate gross domestic product at market prices from the expenditure side, by using monthly indicators of the final demand. Finally the estimates of total GDP are reconciliated by combining the supply side and expenditure side estimates using optimal weights, which reflect the relative precision.

Of course, several alternative monthly indicators of the economic conditions in the euro area are available. A first type of indicators relies on the non model based methodology adopted by the Conference Board for the US. In this context, a composite coincident index (CCI) is constructed as a simple weighted average of selected standardized single indicators. Examples are provided in Carriero and Marcellino (2007a).

A second type of indicators are model based. Within this approach, two main methodologies have emerged: dynamic factor models and Markov switching models. In both cases there is a single unobservable force underlying the current status of the economy, but in the former approach this is a continuous variable, while in the latter it is a discrete variable that evolves according to a Markov chain. While Markov switching models do not perform particularly well in this context for European countries, likely because of the availability of rather short and noisy time series (see e.g. Carriero and Marcellino (2007b)), factor models have been more successfully used. Examples include Carriero and Marcellino (2007b) for the UK, and Charpin (2005) and Altissimo et al. (2001, 2007) for the euro area. The latter reference underlies the eurocoin indicator, published by the CEPR, and is based on the use of a very large information set.

A third type of indicators are based on survey data. The European Commission (more specifically, DG-ECFIN) computes a variety of survey based CCIs, using mostly a non-model based procedure. Gayer and Genet (2006) and Carriero and Marcellino (2007c) propose to summarize the data in the business and consumer survey into a CCI with a large scale dynamic factor model, comparing the static principal component approach of Stock and Watson (2002a,b) and the dynamic principal component approach of Forni et al. (2000, 2003).

A fourth type of monthly indicator of economic activity is more closely related to the method we propose in this study, since the goal is to provide a montly estimate of GDP. A leading example is Mitchell et al. (2005) for the UK.

With respect to the existing literature on monthly indicators of economic activity in the euro area, the main original features of this study are the following. First, it considers both the output side (six branches of the NACE classification) and the expenditure side (the main GDP components). Second, for each disaggregate GDP component, a set of monthly indicators are carefully selected, including both macroeconomic variables and survey answers. Third, our indicator is based on information at both the monthly and quarterly level, rather than monthly only, modelled with a dynamic factor specification cast in state-space form. Fourth, we provide an explicit measure of uncertainty around the indicator, which is particularly relevant in a decision making context. Fifth, since estimation of the multivariate dynamic factor model can be numerically complex, computational efficiency is achieved by implementing univariate filtering and smoothing procedures. Sixth, special attention is paid to chain-linking and its implications for the construction of a monthly indicator of GDP, via a multistep procedure that exploits the additivity of the volume measures expressed at the prices of the previous year. Finally, the estimate of the monthly euro area GDP is obtained by combining the estimates from the output and expenditure sides, with optimal weights reflecting their relative precision. The resulting pooled estimator is more precise than each of its two components, paralleling the results on the usefulness of pooling in the forecasting literature (see e.g. Stock and Watson (1999)).

The Chapter is structured as follows. Section 2.2 discusses the information available. Section 2.3 presents the multivariate disaggregation methods, focusing in particular on the dynamic factor model for the estimation of an index of coincident indicators proposed by Stock and Watson (1991) as a special case of the dynamic factor model introduced by Geweke (1977) and Sargent and Sims

(1977). Section 2.4 discusses the aggregation of the monthly estimates of sectorial value added into GDP at basic and market prices, and how chained-linked volume measures using 1995 as the reference year are obtained. Section 2.5 reports the main empirical results obtained from the output side, from the demand side, and from an optimal combination of these two approaches to the disaggregation of quarterly value added. At the end of this section, some diagnostics and issues related to the revisions of the indicators and hence of the estimates are presented. Section 2.6 summarizes the main findings of the Chapter and puts forward the agenda for future research. Finally, several Appendixes provide additional details on the state space form and the statistical treatment of the multivariate model.

2.2 The information set

The construction of a monthly indicator of the euro area GDP is carried out indirectly through the temporal disaggregation of the value added of the six branches of the NACE Rev. 1 - Level A6 classification and at the same time through the temporal disaggregation of the main components of the demand from the expenditure side. As mentioned before the two monthly estimates, from the supply an demand approach, are at the end combined with appropriate weights reflecting their precision.

Quarterly observations on each branch of activity and expenditure components are available at moment of writing from the national accounts compiled by Eurostat for the sample 1995Q1-2006Q3. All the series are in seasonally adjusted form and refer to the Eurozone12.

Unfortunately a major structural break in the variable concerning the statistical allocation of Financial Intermediation Services Indirectly Measured (FISIM) makes the series relatively short.

Secondly, in 2005 and 2006, most Eurozone member states have introduced chain-linking into their quarterly and annual national accounts to measure the development of economic aggregates in volume terms. This innovation, introduced in the 3rd quarter of 2005, bears important consequences for the estimation of a monthly indicator of the Euro area gross domestic product since, as a result of chaining, additivity is lost. The issue of aggregation of chain-linked volume measures is the topic of section 2.4.

The monthly indicators available for each branch are listed in table 2.1 along with the delay of publication. A remarkable fact is that no indicator is available for the primary sector (AB). For Industry (CDE) and Construction (F), a core indicator is represented by the index of industrial production. For the remaining branches (services), the monthly variables tend to be less directly related to the economic content of value added.

From the expenditure side the monthly indicators suitable for the disaggregation of GDP are listed in table 2.2. In particular, for Final consumption expenditure some indicators of demand are available together with the production of consumer goods. For Gross capital formation a core indicator is the production index (both for industry and constructions), in addition to some specific variables for constructions. As far as the External Balance is concerned, the monthly volume index of Imports and Exports is provided by Eurostat, although with more than 3 month of delay. In order to catch sentiments and expectations of economic agents we complete this set of variables with the Business and Consumers Surveys data published by the European Commission.

2.3 Methodology

The construction of an indicator of monthly GDP, that is consistent with Eurostat's quarterly estimates is an exercise in temporal disaggregation. The aggregate series, concerning the quarterly totals of value added and other economic flows, such as taxes less subsidies, have to be distributed across the months, using related time series that are available monthly and timely. In this section we provide an overview of the statistical methods that we adopt in our empirical analysis, and illustrate how univariate filtering and smoothing procedures can be used to analyze multivariate models in order to increase the computational efficiency of the disaggregation procedure.

For the primary sector and taxes less subsidies, due to the lack of reliable related monthly time series, we use univariate disaggregation methods. The procedure for handling temporal aggregation/disaggregation of univariate models in a state space framework is based on Harvey (1989) and Proietti (2004)(for more detail see also the technical report by Proietti and Frale (2007)).

There are two main related sources of criticism that arise with respect to the univariate disaggregation methods. The first deals with the exogeneity assumption, according to which the indicator is considered as an explanatory variable in a regression model. In general there is no causal relationship between, say, the monthly (deflated) turnover of the retail sector and its value added. Rather, the two phenomena share a common environment and are related measures of the level of economic activity of the branch. The second is that the regression based methods assume that the indicators are measured without error. The consequence is that the information on the indicators is transmitted to the disaggregated series by a single regression coefficient and thus any outlying and purely idiosyncratic feature, such as trading day variation, is automatically attributed to the estimated series. This problem can be also better tackled in a multivariate set-up. Therefore, for the disaggregation of the other production sectors and of the demand components, our methodology is based on a multivariate method.

Multivariate disaggregation methods move away from the criticisms that affects the regression based methods. There are however several degrees of freedom as far as the specification of the model is concerned, and there are relatively few examples in the literature of applications of these models for temporal disaggregation. Harvey and Chung (2000) use a bivariate unobserved components model. Moauro and Savio (2005) have proposed multivariate disaggregation methods based on the class of Sutse models. Proietti and Moauro (2006) estimated monthly GDP for the U.S. and the Euro area using a direct approach by formulating a dynamic factor model proposed by Stock and Watson (1989), specified in the logarithms of the original variables and at the monthly frequency. This poses a problem of temporal aggregation with a nonlinear observational constraint when quarterly time series are included (see also Proietti, 2006).

Stock and Watson (1991, SW henceforth) developed an explicit probability model for the composite index of coincident economic indicators. They proposed a dynamic factor model featuring a common difference-stationary factor that defines the composite index. The reference cycle is assumed to be the value of a single unobservable variable, "the state of the economy", that by assumption represents the only source of the co-movements of four time series: industrial production, sales, employment, and real incomes.

On the other hand, GDP is perhaps the most important coincident indicator, although it is available only quarterly and it is subject to greater revisions than the four coincident series in the original SW model.

This consideration motivated Mariano and Murasawa (2003) to extend the SW model with the inclusion of quarterly real GDP growth, proposing a linear state space model at the monthly observation frequency that entertains the presence of an aggregated flow. Although their model is formulated explicitly in terms of the logarithmic changes in the variables, the nonlinear nature of the temporal aggregation constraint is not taken into account. This is done in Proietti and Moauro (2006).

In this study we apply the SW dynamic factor model to obtain estimates of the monthly GDP components from the output and expenditure sides, to be later aggregated into an indicator of monthly GDP. Since this requires to apply several times the SW model, we also want to improve the computational efficiency of the procedure, by casting the multivariate SW model into an extended univariate framework.

We continue this Section with a review of the SW model, cast it into state space framework, show how it can be used for temporal disaggregation, transform it into an extended univariate model to achieve computational efficiency, and finally discuss diagnostic checking and provide formulae for the estimation of the disaggregate variables of interest.

2.3.1 The Stock and Watson dynamic factor model

Let $\mathbf{y}_t$ denote an $N \times 1$ vector of time series, that we assume to be integrated of order one, or $I(1)$, so that $\Delta y_{it}, i = 1, \dots, N$, has a stationary and invertible representation. The model is of course generalisable to higher orders of integration, but our applications concerns only the $I(1)$ case. The dynamic factor model decomposes $\mathbf{y}_t$ into a common nonstationary component and an idiosyncratic one, which is specific to each series.

Although SW formulate their model in terms of $\Delta \mathbf{y}_t$ we prefer to set up the model in the level of the variables. The advantages of this formulation are twofold: in the first place the mean square error of the estimated coincident index are immediately available both in real time (filtering) and after processing the full available sample (smoothing). Moreover, the treatment of the aggregation constraint in the levels is more transparent and efficient from the computational standpoint, in that it leads to a reduced state vector dimension.

The level specification of the SW model expresses $\mathbf{y}_t$ as the linear combination of a common cyclical trend, that will be denoted by μ_t, and an idiosyncratic component, $\boldsymbol{\mu}_t^*$. Letting $\boldsymbol{\vartheta_0}$ and $\boldsymbol{\vartheta_1}$ denote $N \times 1$ vectors of loadings, and assuming that both components are difference stationary and subject to autoregressive dynamics, we can write:

$$\begin{array}{lcll} \mathbf{y}_t & = & \boldsymbol{\vartheta}_0\mu_t + \boldsymbol{\vartheta}_1\mu_{t-1} + \boldsymbol{\mu}_t^* + \mathbf{X}_t\boldsymbol{\beta}, & t = 1, ..., n, \\ \phi(L)\Delta\mu_t & = & \eta_t, & \eta_t \sim \text{NID}(0, \sigma_\eta^2), \\ \mathbf{D}(L)\Delta\boldsymbol{\mu}_t^* & = & \boldsymbol{\delta} + \boldsymbol{\eta}_t^*, & \boldsymbol{\eta}_t^* \sim \text{NID}(\mathbf{0}, \boldsymbol{\Sigma}_{\eta^*}), \end{array} \qquad (2.3.1)$$

where $\phi(L)$ is an autoregressive polynomial of order p with stationary roots:

$$\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$$

and the matrix polynomial $\mathbf{D}(L)$ is diagonal:

$$\mathbf{D}(L) = \text{diag}\left[d_1(L), d_2(L), \ldots, d_N(L)\right],$$

with $d_i(L) = 1 - d_{i1}L - \cdots - d_{ip_i}L^{p_i}$ and $\mathbf{\Sigma}_{\eta^*} = \text{diag}(\sigma_1^2, \ldots, \sigma_N^2)$. The disturbances η_t and $\boldsymbol{\eta}_t^*$ are mutually uncorrelated at all leads and lags.

The lag polynomial $\boldsymbol{\vartheta}_0 + \boldsymbol{\vartheta}_1 L$ can also be rewritten as $\boldsymbol{\theta}_0 + \boldsymbol{\theta}_1 \Delta$, where $\boldsymbol{\theta}_0 = \boldsymbol{\vartheta}_0 + \boldsymbol{\vartheta}_1$ and $\boldsymbol{\theta}_1 = -\boldsymbol{\vartheta}_1$. The measurement equation can thus be reparameterised as

$$\mathbf{y}_t = \boldsymbol{\theta}_0 \mu_t + \boldsymbol{\theta}_1 \Delta\mu_t + \boldsymbol{\mu}_t^* + \mathbf{X}_t \boldsymbol{\beta}.$$

The model postulates that each series, in differences, Δy_{it}, is composed of a mean term δ_i, an individual AR(p^*) process, $d_i(L)^{-1}\eta_{it}^*$, and a common AR(p) process, $\phi(L)^{-1}\eta_t$. Both μ_t and $\boldsymbol{\mu}_t^*$ are difference stationary processes and the common dynamics are the results of the accumulation of the same underlying shock η_t; moreover, the process generating the index of coincident indicators is usually more persistent than a random walk and in the accumulation of the shocks produces cyclical swings.

Notice that (2.3.1) assumes a zero drift for the single index and a unit variance for its disturbances($\sigma_\eta^2 = 1$). These identification restrictions (we may alternatively restrict to unity one of the loadings in $\boldsymbol{\theta}_0$ and include a nonzero drift in the common index equation, provided we impose one linear constraint on $\boldsymbol{\beta}$) can be removed at a later stage to enhance the interpretability of the estimated common index.

2.3.2 The advantages of the dynamic factor approach

The dynamic factor model framework has several appealing features. First and foremost, it rationalises the practice of statistical offices which amounts to summarise the available indicators in a unique summary (a weighted average, if a priori weights are available, or a statistical summary achieved through the use of static principal components analysis, or a simple combination with weights that are inversely proportional to the volatility of each indicator). The common indicator would then be smoothed and corrected for outliers and structural breaks. In our approach all these operations are carried out simultaneously in a model based framework, and the common factor extracts the dynamics that are common to the indicators and that are relevant for the disaggregation of the quarterly flows.

Finally, it has the flexibility of handling seasonal effects, calendar components and other effects affecting the level of the series (additive outliers, level shifts, etc.) simultaneously; in the regression approach typically adopted by statistical institutes these operations are carried out as preliminary corrections, which makes the disaggregation exercise more elaborate and less internally consistent.

It should be noticed that the dynamic factor model formulated in the previous section is such that each of the component series is integrated of order 1, or $I(1)$, and there is no cointegration among the series, unless more than one of the idiosyncratic variances were equal to zero. Under such circumstances, there exists a dynamic linear combination that is stationary. On the contrary, static cointegration is ruled out by the formulation of the model. Another way in which dynamic cointegration may arise is when the representation adopted for the idiosyncratic component is a stationary ARMA process. The original formulation of the model of coincident indicators by Stock and Watson specified an $I(1)$ idiosyncratic component; the presence of cointegration was explicitly ruled out by pretesting. In our case, there are no theoretical and empirical arguments for assuming that say retail turnover, new car registrations and value added of the branch *Trade, transport and communication services* are cointegrated. In general, the dangers of assuming cointegration when it is not present are greater compared to the dangers of assuming it away when it is present. Roughly speaking, overdifferencing is a less costly form of dynamic misspecification compared to underdifferencing. Additional evidence in favour of our no cointegration assumption is reported in Appendix 2.C.

It should be stressed, perhaps, that the dynamic factor model with idiosyncratic ARIMA(1,1,0) dynamics is an unobserved components version of the Litterman model, where the common index μ_t summarises the information that is common to a set of indicators. The ARIMA(1,1,0) specification is too rich for the quarterly temporally aggregated GDP series; essentially, this is so because the AR parameter is difficult to identify (see Proietti, 2006) on this point. Therefore, we will restrain the AR parameter to be equal to 0, so that effectively we use a random walk specification for the idiosyncratic component of the temporally aggregated series. It must nevertheless be kept in mind that the monthly indicators are endogenous, which is another desirable feature of our approach.

2.4 Chain-linking and temporal disaggregation

The disaggregation methods exposed in the previous section are applied to the quarterly chained volume measures of sectorial output and expenditure components produced by Eurostat. Currently the available series feature the year 1995 as the common reference year.

The Eurozone member states chain-link the quarterly data on an annual basis, i.e. the quarterly volume measures are expressed at the average prices of the previous years. Two alternative techniques are applied for annual chain-linking of quarterly data by the member countries: one quarter overlaps (Austria) and annual overlaps (other states). These are described in Bloem, et al. (2001, Chapter IX); the annual overlap technique, which implies compiling estimates for each quarter at the weighted annual average prices of the previous year, has the advantage of producing quarterly volume estimates that add up exactly to the corresponding annual aggregate. The annual overlap technique is also the method used by Eurostat in the imputation of the chain-linked volume measures of those countries for which no quarterly data at previous year's prices are available.

As it is well known, chain-linking results in the loss of cross-sectional additivity (if the one quarter overlap is used also temporal additivity is lost and benchmarking techniques have to be employed in order to restore it). However, for the annual overlap, the disaggregated (monthly and quarterly) volume measures expressed at the prices of the previous year preserve both the temporal and cross-sectional additivity.

These facts motivate the choice of a multistep procedure for the estimation of monthly GDP at basic and market prices, which is advocated, e.g., also by the IMF (see Bloem *et al.*, 2001). It is described in the sequel.

Let us index the month of the year by $j, j = 1, \ldots, 12$, and the year by m, $m = 1, \ldots, M = [n/12]$, so that the time index is written $t = j + 12m, t = 1, \ldots, n$. We are interested in estimating the monthly values of GDP at basic and market prices, which are, respectively, the sum of the value added of the six branches, and this sum plus taxes less subsidies (or the sum of expenditure components), by aggregating the monthly estimates of sectorial value added and expenditure components. If the estimates were expressed at current prices, then no consistency problem would arise, as the monthly disaggregated estimates would be perfectly additive.

For a particular component of GDP let us denote by y_{jm} the value at current prices of month j in year m, $y_{.m} = \sum_j y_{jm}$ the annual total, $\bar{y}_m = y_{.m}/12$ the annual average. The chain-linked volume estimate with reference year b (the year 1995 in our case) will be denoted $y_{jm}^{(b)}$. The temporal disaggregation methods described in the previous section are applyed to the quarterly chained-linked volume series with reference year b and yield estimates that add up to the quarterly and annual totals (temporal consistency), but are not additive in a horizontal (that is cross-sectional) sense.

The following multistep procedure enables the computation of volume measures expressed at the prices of the previous year that are additive.

1. Transform the monthly estimates into Laspayres type quantity indices with reference year b (volumes are evaluated at year b average prices), by computing

$$I_{jm}^{(b)} = \frac{y_{jm}}{y_{.b}}, j = 1, \ldots, 12, m = 1, \ldots, M,$$

where the denominator is the annual total of year b at current prices. In our case $b = 1$ (year 1 is the calendar year 1995).

2. Change the reference year to $m = 2$, the second year of the series (1996 in our case), by computing:

$$I_{jm}^{(2)} = \frac{I_{jm}^{(b)}}{\bar{I}_2^{(b)}}, j = 1, \ldots, 12, m = 1, \ldots, M,$$

where $\bar{I}_2^{(b)} = \sum_j I_{j2}^{(b)}/12$ is the average quantity index for year 2.

3. Transform the quantity indices for year $m = 2, 3 \ldots, M$, into indices with reference year $m - 1$ (the previous year), by rescaling $I_{jm}^{(2)}$ as follows:

$$I_{jm}^{(m-1)} = \frac{I_{jm}^{(2)}}{\bar{I}_{m-1}^{(2)}}, j = 1, \ldots, 12, m = 2, \ldots, M,$$

where

$$\bar{I}^{(2)}_{m-1} = \begin{cases} \frac{1}{12}\sum_j I^{(2)}_{j,m-1}, & m = 3, \ldots, M \\ \frac{y^{(b)}_{.2}}{y_{.b}}, & m = 2 \end{cases}$$

4. Compute the series at the average prices of the previous year as:

$$y^{(m-1)}_{jm} = I^{(m-1)}_{jm}\bar{y}_{m-1}, \;\; j = 1, \ldots, 12, m = 2, \ldots, M,$$

5. *Aggregation step*: the values $y^{(m-1)}_{i,jm}$ for the i-th component series (the index $i = 1, \ldots, N$ was omitted in the previous steps for notation simplicity) are additive and can be summed up to produce the aggregate GDP measure,

$$Y^{(m-1)}_{jm} = \sum_{i=1}^{N} y^{(m-1)}_{i,jm}, \;\; j = 1, \ldots, 12, m = 2, \ldots, M.$$

6. *Chain-linking* (annual overlap):

   (a) Convert the aggregated volume measures into Laspeyres-type quantity indices with respect to the previous year:

   $$\mathcal{I}^{(m-1)}_{jm} = \frac{Y^{(m-1)}_{jm}}{\bar{Y}_{m-1}}, \;\; j = 1, \ldots, 12, m = 2, \ldots, M,$$

   where $\bar{Y}_{m-1} = \sum_j Y_{j,m-1}/12$ is the average GDP of the previous year at current prices.

   (b) Chain-link the indices using the recursive formula (the first year is the reference year):

   $$\mathcal{I}^{(1)}_{jm} = \mathcal{I}^{(m-1)}_{jm}\bar{\mathcal{I}}^{(b)}_{m-1}, \quad j = 1, \ldots, 12, m = 2, \ldots, M,$$

   where

   $$\bar{\mathcal{I}}^{(1)}_{m-1} = \frac{1}{12}\sum_j \mathcal{I}^{(b)}_{j,m-1}.$$

   (c) If $b > 1$ then change the reference year to year b:

   $$\mathcal{I}^{(b)}_{jm} = \frac{\mathcal{I}^{(1)}_{jm}}{\bar{\mathcal{I}}^{(1)}_{b}} \quad j = 1, \ldots, 12, m = 2, \ldots, M.$$

   (This is not needed in our case, since $b = 1$).

   (d) Compute the chain-linked volume series with reference year b:

   $$Y^{(b)}_{jm} = \mathcal{I}^{(b)}_{jm}\bar{Y}_b \quad j = 1, \ldots, 12, m = 2, \ldots, M,$$

where $\bar{Y}_b = \frac{1}{12}\sum_j Y_{jb}$ is the value of GDP (at basic or market prices) at current prices of the reference year.

The multistep procedure just described enables to obtain estimates of monthly GDP in volume such that the values $Y_{jm}^{(m-1)}$ expressed at the average prices of the previous year add up to their quarterly and annual totals published by Eurostat and to sum of the values of the component series. Moreover, the chain-linked volumes $Y_{jm}^{(b)}$ with reference year b are temporally consistent (they add up to the quarterly and monthly totals published by Eurostat for the GDP), but are not horizontally consistent (cross-sectional additivity cannot be retained).

2.5 Empirical results: temporal disaggregation of GDP

The estimates of monthly value added and GDP presented in this Section cover the sample period January 1995 – December 2006. The last three monthly estimates, concerning the fourth quarter of 2006 can be considered as genuine out of sample forecasts, whereas the estimates up to September 2006 can be considered as " nowcasts", as they exploit the preliminary Eurostat estimate of quarterly value added for the third quarter of GDP and the timely monthly indicators (industrial production, turnover, and so forth).

Two important model specification issues concern whether or not we should assume cointegration between the temporally aggregated flow and the indicator variables, and whether or not we should apply the logarithmic transformation to GDP. Some preliminary analysis, reported in Appendix 2.C, provides evidence in favour of not imposing cointegration and working with the raw data rather than logs. Maintaining these two assumptions, the estimation of GDP at market prices is carried out both from the output side (first subsection) and the expenditure side (second subsection). Regressors accounting for calendar effects (trading days, Easter and length of the month) where included in the equations to provide working day adjustement.

We also report results for each of the output sectors and demand components, which can be of interest by itself. The results from the output and demand sides are later balanced by combining the estimates using optimal weights (final subsection).[2]

2.5.1 The output side

The smoothed estimates of the coincident index, μ_t, and of monthly value added are presented in figure 2.1, along with their 95% confidence interval. In the same plot we report also the original quarterly value added series while the maximum likelihood estimates of the parameters of the model are presented in table 2.3.

In this paragraph we summarize the main results for the disaggregation on the supply side, while the analysis of the estimates from the demand side is the topic of section 2.5.2.

According to the NACE classification the GDP at basic price is obtained by summing up the following branch of activities:A–B: Agriculture, hunting, forestry and fishing; C–D–E: Industry,

[2]All the algorithms and procedures used in the Chapter are implemented in Ox, the matrix programming language by Doornik (2001), version 3.3

incl. Energy; F: Construction ; G–H–I: Trade, transport and communication services; J–K: Financial services and business activities; L–P: Other services.

As mentioned before the branch Agriculture is characterised by the lack of coincident indicators available at the monthly frequency. We thus proceeded to the temporal disaggregation of the value added at constant prices according to the Fernàndez method, i.e. assuming a random walk with constant drift for the unobserved underlying monthly series.

As far as Industry is concerned six monthly indicators are selected. Among them three are quantitative indicators - the index of industrial production, employment and hours worked- and the remaining three are business survey indicators compiled in the form of balances of opinions by the European Commission- the industrial confidence indicator, the production trend observed in recent months and the assessment of order book levels.

For the quantification of surveys and their role in econometric analysis see Pesaran and Weale (2006). We found their inclusion in the dynamic factor model and thus in the disaggregation of value added problematic as we argument below and we propose to investigate the issue further in future research.

Business cycle indicators are supposed to be stationary at the long run frequency (see also the evidence arising from stationarity tests in Proietti and Frale, 2007), so that we can postulate a relationship only with the changes in the coincident index, $\Delta\mu_t$, plus a further idiosyncratic stationary component. As a consequence, survey variables have been included in our models in integrated form so as to preserve the level specification of the regression and the dynamic factor models.

The SW dynamic factor model was estimated for the seven series, the six monthly indicators plus quarterly value added, by specifying an AR(2) process for the common component $\Delta\mu_t$ and the idiosyncratic components of the monthly indicators. For value added, the idiosyncratic component is formulated as a random walk with drift. This restricted specification is motivated by the fact that there are identification problems of the kind that have been discussed by Proietti (2004) with reference to the Litterman model, which affect the estimation of autoregressive effects.

The estimation results are such that the common factor μ_t is driven mostly by the business survey variables, which dominate in variation the other quantitative variables. Moreover the factor loading of industry's value added are not significant.

Be that as it may, when the business survey indicators are removed from the analysis, the estimation results are much more satisfactory as the common factor is strongly related to the dynamics of industrial production and value added. After some experimentation we focussed on a trivariate model with two monthly indicators - Industrial production and hours worked- and the quarterly value added.

For hours worked we consider the possibility of a lagged relationship with the common factor, which however did not result significant.

As well as for Industry, for the Constructions sector six candidate monthly indicators were selected (see table 2.1) and two business survey indicators (Construction Confidence Indicator and Trend of activity over recent months). However, Survey data were dismissed after a preliminary analysis, for reasons that are similar to those exposed about Industry: essentially when they are included in the SW factor model, they drive the common factor so that value added does not load significantly on the common factor and it is fully idiosyncratic.

The main evidence is that the index of production in construction is highly significant.

Value added presents sharp drops at the beginning of the sample, in correspondence to January and February. These are well reflected in the indicators, in particular the index of production and hours worked and thus there is no need for particular interventions. The SW dynamic factor model was estimate for a five variable system consisting of production in constructions, building permits, employment, hours worked and value added.

It is interesting to notice (see table 2.3) that all the variables, including value added, load significantly on the common factor, except for building permits.

The third branch of activity- Trade, transport and communication services- account for about 22% of total value added at constant prices, includes wholesale and retail trade, repair of motor vehicles, motorcycles and personal and household goods; hotels and restaurants; transport, storage and communication. While for industry and construction it is possible to find a very good indicator of value added, the production index, the relationship with the monthly indicators becomes more blurred for this sector.

Seven indicators were considered, see table 2.1, and after preliminary analysis based among others on Fernàndez univariate method, the SW dynamic factor model was formulated with respect to a trivariate system including the industrial production index for consumption goods, the number of registered cars (both available at the monthly frequency) and value added (quarterly).

Value added loads significantly on the coincident single index. The coincident index, plotted in figure 2.1, is highly coherent with the same index estimated for the industry sector.

For the branch of financial intermediation, real estate, renting and business activities, we select two monthly indicators, which are provided by the European Central Bank, and that measure the liabilities and the loans of the monetary and financial institution. Both series were deflated using the harmonised consumer price index. Two intervention variables were included so as to account for a level shift in the January 2001, presumably due to the fact that the previous data referred to 11 countries excluding Greece.

The estimation results for the trivariate dynamic factor model are reported in table 2.4. The loading of value added on the common factor is not significant and most variation is captured by the idiosyncratic random walk.

Nevertheless, the monthly disaggregated estimates of monthly value added appear to be very reliable (see Figure 2.1).

Finally the last branch of NACE classification (labelled L-P) gathers a variety of economic activities (public administration and defence, compulsory social security; education; health and social work; other community, social and personal service activities; private households with employed persons) for which it is not easy to find reliable and timely monthly indicators of value added. For our disaggregation exercise we tried several macroeconomic aggregates related to the state of the economy, such as the unemployment rate, the index of industrial production. We ended up selecting a single monthly indicator, the total amount of debt securities issued by central government, deflated by the harmonised consumer price index.

2.5.2 The Expenditure side

So far we have deal with the disaggregation of GDP into branch of activity, nevertheless the quarterly value added might be obtained from the main National account identity: GDP (market prices) = Final consumption expenditure + Gross capital formation + External balance. Final consumption expenditure is made up of households expenditure and government expenditure. While for the latter no monthly indicator is available, for private consumption the most plausible indicators are those referring to the final demand, among which we select retail trade and new cars registration. The index of industrial production for consumer goods may also provide useful information. Furthermore, we include in the set of indicators some soft variables from Consumer Survey, such as the confidence indicator, the assessment of the financial situation, and price trend, to capture economic agents expectations and feelings. The specification adopted for the coincident indicator and the idiosyncratic components is AR(1), rather than AR(2), which produces a more smooth estimates. Both indicators and the national accounts aggregate load positively and significantly on the coincident index. On the contrary, the loading coefficient of the survey variables was not significant. This motivated the use of only car registration and retail trade as regressors in the final model, whose estimation results are presented in table 2.5 and in Figure 2.2.

Gross capital formation is mainly the result of investments in the industry and construction sectors. The monthly indicators preliminary selected featured industrial production, also for capital goods, building permits and the survey variables listed in Table 2.2.

As well as in former exercises we tentatively conclude that survey data do not play a significant role, whereas the general industrial production index resulted strongly significant. The coincident index is specified as an AR(1) process as in the case of final consumption.

As far as the external balance is concerned, we first point out that quarterly imports and exports have to be disaggregated separately since the chain-linking mechanism cannot be performed directly on variables that can take both negative and positive values.

As indicators we use the monthly volume indices of Imports and Exports produced by Eurostat; these are published with a delay of about 90 days. We also include the real exchange rate of the euro, and survey variables concerning orders (internal and external demand). From a preliminary Fernàndez model we obtained that volume monthly indexes, Exchange Rate and industrial production for intermediate goods are significant, while in the SW dynamic model, Exchange Rate looses its explanatory power. Survey data are not relevant in both univariate and multivariate models. We also consider some quarterly information from the Business Survey questionnaire, in particular the questions about production capacity and export expectations. Unfortunately neither helped in estimating the coincident index. We ended up to a model with only two indicators- volume index and industrial production of intermediate goods- whose results are listed in table 2.5 and shown in Figure 2.2.

Before coming into next paragraph, that deal with the issue of the combining supply and demand side to get the final estimate of monthly GDP, it is worth to concerns about "Taxes less subsidies on products". This aggregate is the gap between the value added at market price, obtained by the expenditure side, and the value added at basic price, computed from the output side. The temporal disaggregation of Taxes less subsidies at chained 1995 prices was carried out using a trivariate dynamic factor model for monthly industrial production, deflated turnover and quarterly Taxes less subsidies. The latter does not load significantly on the monthly indicators and thus the disaggregation method is not different from the Fernàndez univariate method with a constant drift.

2.5.3 Monthly gross domestic product

The estimation of the monthly indicator of the Euro area GDP at basic and market prices was carried out using the methodology outlined in section 2.4. The components series (the estimated monthly sectorial value added and taxes less subsidies, the estimated expenditure components), expressed as chain-linked volume with reference year 1995, were de-chained and expressed at the average prices of the previous year, and then contemporaneously aggregated. The corresponding GDP measures are fully additive and are later chain-linked to express the volume measure with a common reference year, which is 1995.

The estimates and their standard errors are reproduced in table 2.6 along with monthly and yearly growth rates.

As it is well known, as a result of chain-linking the GDP estimates fail to be additive in a horizontal sense. Thus, the sum of components (for the six branches, or expenditure components) differs from GDP at basic prices and market prices, respectively. However, the discrepancy is very small.

As far as the standard errors are concerned, these are obtained as the square root of the sum of the estimation error variances of the individual components time series, made available by the Kalman filter and smoother. Strictly speaking they do not represent the estimation standard errors for GDP at basic and market prices, as the latter arise from the elaborate procedure described in section 2.4. The latter involves a sequence of multiplicative transformations, which makes the computation of the standard errors prohibitive. Nevertheless, the statistical discrepancy is negligible because it never overcomes 0.1%.

Figure 2.3 plots the percent coefficient of variation of the estimates (100 times the standard error relative to the GDP estimate) both from the output side and from the demand side for the last year. This increases rapidly for the last three estimates, which concern the last quarter of 2006 and constitute out of sample predictions. The right top graph of each panel is a fan plot of the level of GDP at market prices for the last year, and the two subsequent plots show the point estimates and the 95% interval estimates of the monthly and yearly growth rates.

The estimates of GDP at market prices from the expenditure side are slightly more volatile and are characterised by an higher estimation error variance. Their quarterly sum is nevertheless equal to that obtained from the disaggregated estimates from the output side.

The two estimates, obtained respectively from the output side, here denoted Y_t^o, and from the expenditure side, Y_t^e, are combined with time-invariant weights $w_o = 0.88$ and $w_e = 0.12$, $0 < w_o < 1$ and $w_e = 1 - w_o$, so as to form the estimate

$$Y_t^c = w_o Y_t^o + w_e Y_t^e.$$

If S_t^{2o} and S_t^{2e} denote respectively the estimation error variance of the output and expenditure estimates, then w_o is the sample average of the relative precision of the output estimates, that is the average of

$$\frac{1/S_t^{2o}}{1/S_t^{2o} + 1/S_t^{2e}}.$$

The combined estimates, along with their standard error, which is $(w_o^2 S_t^{2o} + w_e^2 S_t^{2e})^{1/2}$ are presented in table 2.6. Obviously, the combined estimate is more precise than Y_t^o and Y_t^e. The percent reduction in variance with respect to Y_t^o is about 12%. Finally, the combined estimates of the level of GDP and its monthly and annual growth are displayed in figure 2.4. It is worth noticing that the last three predictions, concerning the last quarter of 2006 arising from the output approach are more optimistic than those obtained from the expenditure approach.

2.5.4 Diagnostic checking and disaggregated estimates

Diagnostics and goodness of fit are based on the innovations, which are given by $\tilde{v}_{t,i} = v_{t,i} - \mathbf{V}'_{t,i}\mathbf{S}_{t,i}^{-1}\mathbf{s}_{t,i}$, with variance $\tilde{f}_{t,i} = f_{t,i} + \mathbf{V}'_{t,i}\mathbf{S}_{t,i}^{-1}\mathbf{V}_{t,i}$. The standardised innovations, $\tilde{v}_{t,i}/\sqrt{\tilde{f}_{t,i}}$ can be used to check for residual autocorrelation and departure from the normality assumption. The innovations have the following interpretation:

$$\tilde{v}_{t,i} = y_{t,i}^{\dagger} - \mathrm{E}(y_{t,i}^{\dagger}|\mathbf{Y}_{t-1}^{\dagger}, y_{t,j}^{\dagger}, j < i), \tag{2.5.1}$$

where $\mathbf{Y}_t^{\dagger}$ denotes the information set $\{\mathbf{y}_1^{\dagger}, \ldots, \mathbf{y}_t^{\dagger}\}$.

It is worth to notice that the elements in the innovation vector $\tilde{v}_{t,i}$ are not the same as in the univariate treatment, unless for the first element. In the case of multivariate series the innovations should be considered as the one step ahead forecast error conditional on the current values of the indicators. We report in Figure 2.5 and 2.6 the innovations for GDP by sectors and components. We observed a better pattern from the expenditure side than from the output side. Several improvements for future applications are suggested by the inspection of these plots. For instance, it seems that there still a trend component not explained by the model for Construction and a shift in the level for Financial services, which could be modelled by using an intervention variable.

The filtered, or real time, estimates of the state vector and the estimation error matrix are computed as follows:

$$\tilde{\boldsymbol{\alpha}}_{t,i}^* = \mathbf{a}_{t,i}^* - \mathbf{A}_{t,i}^*\mathbf{S}_{t,i}^{-1}\mathbf{s}_{t,i} + \mathbf{P}_{t,i}^*\mathbf{z}_i^*\tilde{v}_{t,i}/f_{t,i}, \;\; \tilde{\mathbf{P}}_{t,i}^* = \mathbf{P}_{t,i}^* + \mathbf{A}_{t,i}^*\mathbf{S}_{t,i}^{-1}\mathbf{A}_{t,i}^{*'} - \mathbf{P}_{t,i}^*\mathbf{z}_i^*\mathbf{z}_i'\mathbf{P}_{t,i}^*/f_{t,i},$$

where $\tilde{\boldsymbol{\alpha}}_{t,i}^* = \mathrm{E}(\boldsymbol{\alpha}_t^*|\mathbf{Y}_{t-1}^{\dagger}, y_{t,j}^{\dagger}, j \leq i)$, $\tilde{\mathbf{P}}_{t,i}^* = \mathrm{Var}(\boldsymbol{\alpha}_t^*|\mathbf{Y}_{t-1}^{\dagger}, y_{t,j}^{\dagger}, j \leq i)$.

The smoothed estimates are obtained from the augmented smoothing algorithm proposed by de Jong (1988), appropriately adapted to handle missing values. Defining $\mathbf{r}_{n,N} = 0, \mathbf{R}_{n,N} = 0, \mathbf{N}_{n,N} = 0$, for $t = n, \ldots, 1$, and $i = N, \ldots, 1$ if $y_{t,i}^{\dagger}$ is available:

$$\begin{aligned}
\mathbf{L}_{t,i} &= \mathbf{I}_m - \mathbf{K}_{t,i} z_i^{*'} \\
\mathbf{r}_{t,i-1} &= \mathbf{z}_i^* v_{t,i}/f_{t,i} + \mathbf{L}_{t,i}\mathbf{r}_{t,i}, \;\; \mathbf{R}_{t,i-1} = \mathbf{z}_i^*\mathbf{V}'_{t,i}/f_{t,i} + \mathbf{L}_{t,i}\mathbf{R}_{t,i}, \\
\mathbf{N}_{t,i-1} &= \mathbf{z}_i^*\mathbf{z}_i^{*'}/f_{t,i} + \mathbf{L}_{t,i}\mathbf{N}_{t,i}\mathbf{L}'_{t,i}.
\end{aligned}$$

Else, if $y^{\dagger}_{t,i}$ is missing,

$$\mathbf{r}_{t,i-1} = \mathbf{r}_{t,i}, \quad \mathbf{R}_{t,i-1} = \mathbf{R}_{t,i}, \quad \mathbf{N}_{t,i-1} = \mathbf{N}_{t,i}.$$

$$\mathbf{r}_{t-1,N} = \mathbf{T}^{*'}_{t+1}\mathbf{r}_{t,i}, \quad \mathbf{R}_{t,i-1} = \mathbf{T}^{*'}_{t+1}\mathbf{R}_{t,i}, \quad \mathbf{N}_{t,i-1} = \mathbf{T}^{*'}_{t+1}\mathbf{N}_{t,i}\mathbf{T}^{*}_{t+1}.$$

The smoothed estimates are obtained as

$$\begin{aligned}
\tilde{\boldsymbol{\alpha}}^{*}_{t|n} &= \mathbf{a}^{*}_{t,1} + \mathbf{A}^{*}_{t,1}\tilde{\boldsymbol{\beta}} + \mathbf{P}^{*}_{t,1}(\mathbf{r}_{t-1,N} + \mathbf{R}_{t-1,N}\tilde{\boldsymbol{\beta}}) \\
\mathbf{P}^{*}_{t|n} &= \mathbf{P}^{*}_{t,1} + \mathbf{A}^{*}_{t,1}\mathbf{S}^{-1}_{n+1}\mathbf{A}^{*'}_{t,1} - \mathbf{P}^{*}_{t,1}\mathbf{N}_{t-1,N}\mathbf{P}^{*}_{t,1}.
\end{aligned}$$

The reliability of the estimated disaggregate series and its changes can be assessed by augmenting the state space representation of the model through the inclusion of $\mathbf{y}_{2,t-j}, j = 0, \ldots, s$, in the state vector. The mean square error of the growth rate

$$\frac{y_{2,t} - y_{2,t-s}}{y_{2,t-s}}$$

can be estimated by the delta method, as

$$\mathbf{g}'\mathbf{V}\mathbf{g}, \mathbf{g}' = \left[\frac{1}{\tilde{y}_{2,t-s}}, \quad -\frac{\tilde{y}_{2,t}}{\tilde{y}^{2}_{2,t-s}}\right],$$

and $\mathbf{V}$ is the conditional mean square error matrix of $(y_{2,t}, y_{2,t-s})$.

2.5.5 Revisions

Macroeconomic data published by Eurostat are revised every time a new observation is release. As a consequence also our estimates are subject to the revision process. In figure 2.7 we report the estimates of monthly GDP as obtained running the model for all months in the year 2007. It is quite visible that the more relevant change in the estimates occurs when a new observation for the quarterly GDP is published, or in the third month of the quarter.

As matter of fact, there are two different source of variability affecting our results. First, as we said we pay the revision of the monthly indicators and of the GDP as well, which is a source of uncertainty totally out of our control. Secondly, every time the model is run to produce an additional estimate the parameters are re-estimated according to the new information set. A rough attempt to split those two effects is considering the results produced by running the model taking parameter fixed and using real time data. In Figure 2.8 we show the relative differences between the estimates with fixed and varying parameters, as obtained month by month in the last year. Unless for February, whose pattern is not in line with the others, we obtained that variability in our results is mainly driven by the revisions of the information set rather than by the estimation process.

The total monthly GDP is obtained combining the estimates from the expenditure and output side, according to their relative precision. It is worth to analyze the contribution of each sectors/components to the final uncertainty, as plotted in figure 2.10 and 2.9. It is evident that standard errors are basically stable month by month, and therefore the period in which the model is run do not affect the composition of uncertainty of the estimates. Among sectors, the more volatility appears the services sector, which is also broadly considered one of the most difficult to estimate. Among components of expenditure, the biggest contribution to the final GDP uncertainty is due to Investments. When nowcast observations are added, the feature does not change, the total level of uncertainty increase, but the relative position of sectors and components of expenditure is still the same.

2.6 Conclusions

In this Chapter we present a monthly indicator of Euro area gross domestic product based prominently on the Stock and Watson (1991) dynamic factor model of coincident indicators. We propose a multivariate approach that encompass the drawbacks of univariate treatments. The model is cast in state space form and a convenient statistical treatment is carried out to handle with mixed frequencies -monthly and quarterly- and with the temporal constraint -the quarterly GDP is the sum over three consecutive monthly values. In addition, a suitable procedure to compute the chained link values for the total GDP at market price is presented.

The application of the model to the sectorial data is satisfactory and as a byproduct we obtain estimates of monthly sectorial value added, which can in turn be aggregated into an estimate of gross domestic product. The disaggregation exercise is also conducted on the expenditure side. The estimates from this approach are less reliable, due to the higher volatility of national accounts aggregate such as gross capital formation and exports and imports. The greater sectional disaggregation and the relative stability of output of industry and services provides an explanation for the greater precision of the output side estimates. The combination of the estimates obtained from the two approaches, with weights reflecting their relative precision, yielded a more accurate combined precision. Accordingly, we present a set of post-estimation diagnostics, focussing on the contribution of sectors and components to the total precision of the estimates and on the impact of revisions of the indicators on the GDP disaggregation.

One of the benefits of the approach is that measures of reliability concerning the estimated levels and growth rates of the indicator of monthly GDP are available. Furthermore, by using the Kalman filter we solve endogenously the problem of the unbalanced sample due to different delay of released data.

Table 2.1: Monthly indicators employed for the disaggregation of sectorial value added

Label	*Quarterly Aggregate*	*Label*	*Monthly Indicators*	*Delay*
A–B	Agriculture, hunting and fishing			
C–D–E	Industry, incl. Energy			
		prod	Monthly production index (CDE)	45
		empl	Number of persons employed	70
		howk	Volume of work done (hours worked)	60
		EA99	Industrial Confidence Indicator (Dg Ecfin)	15
		EA.1	Production trend observed in recent months (Dg Ecfin)	15
		EA.2	Assessment of order-book levels (Dg Ecfin)	15
F	Construction			
		prod_F	Monthly production index (F)	70
		b4610	Building permits	70
		empl	Number of persons employed	70
		howk	Volume of work done (hours worked)	70
		EA99	Construction Confidence Indicator (Dg Ecfin)	15
		EA.1	Trend of activity over recent months (Dg Ecfin)	15
G–H–I	Trade, transport and communication services			
		prod_cons	Monthly production index for consumption goods	45
		tovv	Index of deflated turnover	35
		empl	Number of persons employed	90
		car_reg	Car registrations	15
		EA99	Retail trade Confidence Indicator (Dg Ecfin)	15
		EA.1	Business activity over recent months (Dg Ecfin)	15
		EA.2	Assessment of stocks (Dg Ecfin)	15
J–K	Financial services and business activities			
		M3	Monetary aggregate M3 (deflated)	27
		Loans	Loans of MFI (deflated)	27
L–P	Other services			
		Debt	Debt securities issued by central government (deflated)	27
	Total Gross Value Added (A6)			
	Taxes less subsidies on products			
		prod	Monthly production index (CDE)	45
		tovv	Index of deflated turnover	35

Table 2.2: Monthly indicators employed for the disaggregation of GDP from expenditure side

Label	*Quarterly Aggregate*	*Label*	*Monthly Indicators*	*Delay*
CONS	Final consumption expenditure			
		prod_cons	Monthly production index for consumption goods	45
		car_reg	Car registrations	15
		tovv	Index of deflated turnover retail	35
		EA99	Consumer Confidence Indicator (Dg Ecfin)	15
		EA.1	Financial situation (Dg Ecfin)	15
		EA.3	General Economic situation(Dg Ecfin)	15
		EA.5	Price trends (Dg Ecfin)	15
INV	Gross capital formation			
		prod	Monthly production index (CDE)	45
		prod_F	Monthly production index (F)	70
		prod_cap	Monthly production index for capital goods	45
		b4610	Building permits	70
		EA99_F	Construction Confidence Indicator (Dg Ecfin)	15
		EA.1_F	Assessment of order in construction (Dg Ecfin)	15
		EA99	Industrial Confidence Indicator (Dg Ecfin)	15
		EA.1	Production trend observed in recent months (Dg Ecfin)	15
		EA.2	Assessment of order-book levels (Dg Ecfin)	15
EXP	Exports of goods and services			
		Mexp	Monthly Export volume index	90
		prod_int	Monthly production index for intermediate goods	45
		Er	Real Effective Exchange Rate (deflator: producer price indices)	30
		EA.3	Assessment of export order-book levels (CDE) (Dg Ecfin)	15
IMP	Imports of goods and services			
		Mimp	Monthly Import volume index	90
		prod_int	Monthly production index for intermediate goods	45
		rex	Real Effective Exchange Rate (deflator: producer price indices)	30
		EA.3	Assessment of export order-book levels (CDE) (Dg Ecfin)	15

Table 2.3: Output side: parameter estimates and asymptotic standard errors, when relevant

C-D-E-INDUSTRY

Parameters	*prod*	*howk*	*Value added*
θ_{i0}	0.576	0.191	0.708
	(0.119)	(0.064)	(0.191)
θ_{i1}		0.000	
		(0.054)	
δ_i	0.317	-0.153	0.186
	(0.073)	(0.028)	(0.040)
d_{i1}	-0.664	-0.295	
d_{i2}	-0.305	-0.078	
σ_{η^*}	0.160	0.077	0.001
$1+0.394L+0.104L^2 \quad \Delta\mu_t = \eta_t, \quad \eta_t \sim N(0,1)$			

F-CONSTRUCTIONS

Parameters	*c_pro_m*	*Building permits*	*empl*	*howk*	*Value added*
θ_{i0}	2.371	-0.168	0.207	1.290	0.436
	(0.315)	(0.607)	(0.051)	(0.277)	(0.071)
θ_{i1}			-0.095	-0.080	
			(0.028)	(0.158)	
δ_i	0.149	-0.086	0.015	-0.188	0.022
	(0.313)	(0.308)	(0.014)	(0.103)	(0.023)
d_{i1}	-0.831	-0.224	0.453	-0.313	
d_{i2}	-0.770	-0.341	0.256	0.069	
σ_{η^*}	0.540	3.607	0.162	0.760	0.097
$1+0.496L+0.191L^2 \quad \Delta\mu_t = \eta_t, \quad \eta_t \sim N(0,1)$					

GHI-TRADE AND COMMUNICATIONS

Parameters	*prod_cons*	*car_reg*	*Value added*
θ_{i0}	0.286	1.014	0.536
	(0.140)	(0.596)	(0.114)
δ_i	0.104	0.173	0.211
	(0.064)	(0.263)	(0.031)
d_{i1}	-0.462	-0.430	
σ_{η^*}	0.700	2.939	0.001
$(1+0.462L)\,\Delta\mu_t = \eta_t, \quad \eta_t \sim N(0,1)$			

Table 2.4: Output side: parameter estimates and asymptotic standard errors, when relevant (cont.)

JKI-FINANCIAL SERVICES				**LP-OTHER SERVICES**		
Parameters	*M3*	*Loans*	*Value added*	*Parameters*	*Debt*	*Value added*
θ_{i0}	0.182	0.198	0.059	θ_{i0}	0.137	0.023
	(0.022)	(0.033)	(0.068)		(0.009)	(0.024)
θ_{i1}			θ_{i1}		0.022	
						(0.045)
δ_i	0.348	0.064	0.275	δ_i	0.077	0.125
	(0.047)	(0.012)	(0.035)		(0.012)	(0.011)
d_{i1}	-0.357	0.441		d_{i1}	0.970	
d_{i2}	-0.457	0.324		d_{i2}	-0.987	
σ_{η^*}	0.093	0.123	0.399	σ_{η^*}	0.008	0.123
$1-0.301L-0.101L^2 \quad \Delta\mu_t = \eta_t, \quad \eta_t \sim N(0,1)$				$1+0.005L-0.031L^2 \quad \Delta\mu_t = \eta_t, \quad \eta_t \sim N(0,1)$		

Note: standard errors in parenthesis.

Table 2.5: Expenditure side: parameter estimates and asymptotic standard errors, when relevant

CONSUMPTION				**INVESTMENTS**			
Parameters	*prod_cons*	*tovv*	*Value added*	*Parameters*	*prod*	*c_prod_m*	*Value added*
θ_{i0}	1.526	0.481	1.155	θ_{i0}	0.362	0.869	1.850
	(0.540)	(0.162)	(0.219)		(0.134)	(0.432)	(0.331)
δ_i	0.188	0.179	0.631	δ_i	0.215	0.064	0.244
	(0.256)	(0.067)	(0.067)		(0.060)	(0.208)	(0.108)
d_{i1}	-0.414	-0.490		d_{i1}	-0.382	-0.485	
σ_{η^*}	2.661	0.626	0.0004	σ_{η^*}	0.618	2.301	8.74e-005
$(1-0.461L)\,\Delta\mu_t = \eta_t, \quad \eta_t \sim N(0,1)$				$(1-0.454L)\,\Delta\mu_t = \eta_t, \quad \eta_t \sim N(0,1)$			
IMPORTS				**EXPORTS**			
Parameters	*Mimp*	*prod_int*	*Value added*	*Parameters*	*Mexp*	*prod_int*	*Value added*
θ_{i0}	1.185	0.681	1.923	θ_{i0}	0.915	0.806	1.434
	(0.296)	(0.173)	(0.616)		(0.189)	(0.177)	(0.546)
δ_i	0.512	0.220	0.863	δ_i	0.380	0.216	0.874
	(0.163)	(0.087)	(0.130)		(0.084)	(0.090)	(0.153)
d_{i1}	-0.507	-0.375		d_{i1}	-0.078	-0.348	
σ_{η^*}	1.448	0.786	0.686	σ_{η^*}	0.647	0.676	1.443
$(1-0.404L)\,\Delta\mu_t = \eta_t, \quad \eta_t \sim N(0,1)$				$(1-0.318L)\,\Delta\mu_t = \eta_t, \quad \eta_t \sim N(0,1)$			

Note: standard errors in parenthesis.

Table 2.6: Monthly gross domestic product at market prices: combined estimates, combined standard errors and growth rates (monthly and yearly)

	GDP	SE	Mgr	Ygr		GDP	SE	Mgr	Ygr		GDP	SE	Mgr	Ygr
Jan-96	462042	490			Sep-99	509592	488	0.3	3.25	May-03	542289	452	-0.2	0.4
Feb-96	461376	454	-0.14		Oct-99	512494	491	0.57	3.81	Jun-03	542737	490	0.08	0.27
Mar-96	463475	491	0.46		Nov-99	514437	451	0.38	3.95	Jul-03	545937	489	0.59	0.83
Apr-96	464514	493	0.22		Dec-99	516385	489	0.38	4.47	Aug-03	545288	453	-0.12	0.52
May-96	466152	453	0.35		Jan-00	517566	491	0.23	3.74	Sep-03	546027	489	0.14	0.5
Jun-96	466947	494	0.17		Feb-00	521936	453	0.84	5.11	Oct-03	547648	488	0.3	0.9
Jul-96	466689	488	-0.06		Mar-00	521947	490	0.00	4.29	Nov-03	548368	456	0.13	0.93
Aug-96	468499	449	0.39		Apr-00	524164	499	0.42	4.7	Dec-03	548767	488	0.07	1.28
Sep-96	469425	489	0.2		May-00	526403	453	0.43	5.04	Jan-04	550319	488	0.28	1.3
Oct-96	468648	488	-0.17		Jun-00	524773	490	-0.31	4.14	Feb-04	553032	454	0.49	1.75
Nov-96	468961	451	0.07		Jul-00	526620	490	0.35	3.91	Mar-04	553522	490	0.09	1.84
Dec-96	470051	488	0.23		Aug-00	527702	452	0.21	3.87	Apr-04	553769	497	0.04	1.91
Jan-97	469106	493	-0.2	1.53	Sep-00	528439	488	0.14	3.7	May-04	553860	456	0.02	2.13
Feb-97	472142	461	0.65	2.33	Oct-00	528930	487	0.09	3.21	Jun-04	554580	490	0.13	2.18
Mar-97	473732	500	0.34	2.21	Nov-00	531399	452	0.47	3.3	Jul-04	555662	487	0.2	1.78
Apr-97	476664	488	0.62	2.62	Dec-00	533704	491	0.43	3.35	Aug-04	554199	449	-0.26	1.63
May-97	475521	450	-0.24	2.01	Jan-01	534638	502	0.18	3.3	Sep-04	556159	489	0.35	1.86
Jun-97	478471	488	0.62	2.47	Feb-01	535759	464	0.21	2.65	Oct-04	556095	491	-0.01	1.54
Jul-97	479605	488	0.24	2.77	Mar-01	535869	491	0.02	2.67	Nov-04	556355	451	0.05	1.46
Aug-97	480148	453	0.11	2.49	Apr-01	535063	495	-0.15	2.08	Dec-04	557079	489	0.13	1.51
Sep-97	481192	489	0.22	2.51	May-01	536198	454	0.21	1.86	Jan-05	559080	494	0.36	1.59
Oct-97	484684	488	0.73	3.42	Jun-01	536900	490	0.13	2.31	Feb-05	558685	461	-0.07	1.02
Nov-97	484169	456	-0.11	3.24	Jul-01	535214	487	-0.31	1.63	Mar-05	558526	501	-0.03	0.9
Dec-97	486338	489	0.45	3.46	Aug-01	537218	453	0.37	1.8	Apr-05	560646	487	0.38	1.24
Jan-98	487765	489	0.29	3.98	Sep-01	536548	491	-0.12	1.53	May-05	559604	450	-0.19	1.04
Feb-98	488418	459	0.13	3.45	Oct-01	536548	488	0.00	1.44	Jun-05	562182	488	0.46	1.37
Mar-98	488877	489	0.09	3.2	Nov-01	536498	451	-0.01	0.96	Jul-05	563382	491	0.21	1.39
Apr-98	489473	497	0.12	2.69	Dec-01	537540	491	0.19	0.72	Aug-05	564426	451	0.19	1.85
May-98	490837	456	0.28	3.22	Jan-02	537282	493	-0.05	0.49	Sep-05	565345	489	0.16	1.65
Jun-98	490242	490	-0.12	2.46	Feb-02	537739	461	0.09	0.37	Oct-05	564662	489	-0.12	1.54
Jul-98	493005	489	0.56	2.79	Mar-02	539182	499	0.27	0.62	Nov-05	566442	452	0.32	1.81
Aug-98	492795	453	-0.04	2.63	Apr-02	539287	488	0.02	0.79	Dec-05	568144	487	0.3	1.99
Sep-98	493564	488	0.16	2.57	May-02	540155	453	0.16	0.74	Jan-06	569648	490	0.26	1.89
Oct-98	493670	487	0.02	1.85	Jun-02	541294	492	0.21	0.82	Feb-06	571146	459	0.26	2.23
Nov-98	494902	451	0.25	2.22	Jul-02	541465	488	0.03	1.17	Mar-06	572552	490	0.25	2.51
Dec-98	494276	489	-0.13	1.63	Aug-02	542472	449	0.19	0.98	Apr-06	574550	500	0.35	2.48
Jan-99	498890	493	0.93	2.28	Sep-02	543324	489	0.16	1.26	May-06	577670	454	0.54	3.23
Feb-99	496546	460	-0.47	1.66	Oct-02	542756	488	-0.1	1.16	Jun-06	578133	492	0.08	2.84
Mar-99	500465	495	0.79	2.37	Nov-02	543309	451	0.1	1.27	Jul-06	578399	519	0.05	2.67
Apr-99	500641	490	0.04	2.28	Dec-02	541809	487	-0.28	0.79	Aug-06	580611	461	0.38	2.87
May-99	501131	455	0.1	2.1	Jan-03	543237	491	0.26	1.11	Sep-06	580313	540	-0.05	2.65
Jun-99	503894	488	0.55	2.78	Feb-03	543494	459	0.05	1.07	*Oct-06*	*581579*	*1145*	*0.22*	*3.00*
Jul-99	506824	487	0.58	2.8	Mar-03	543538	492	0.01	0.81	*Nov-06*	*582541*	*1416*	*0.17*	*2.84*
Aug-99	508064	449	0.24	3.1	Apr-03	543374	496	-0.03	0.76	*Dec-06*	*583415*	*1660*	*0.15*	*2.69*

Figure 2.1: Quarterly National Account, Monthly estimates with standard errors and Coincident Index- Output approach.

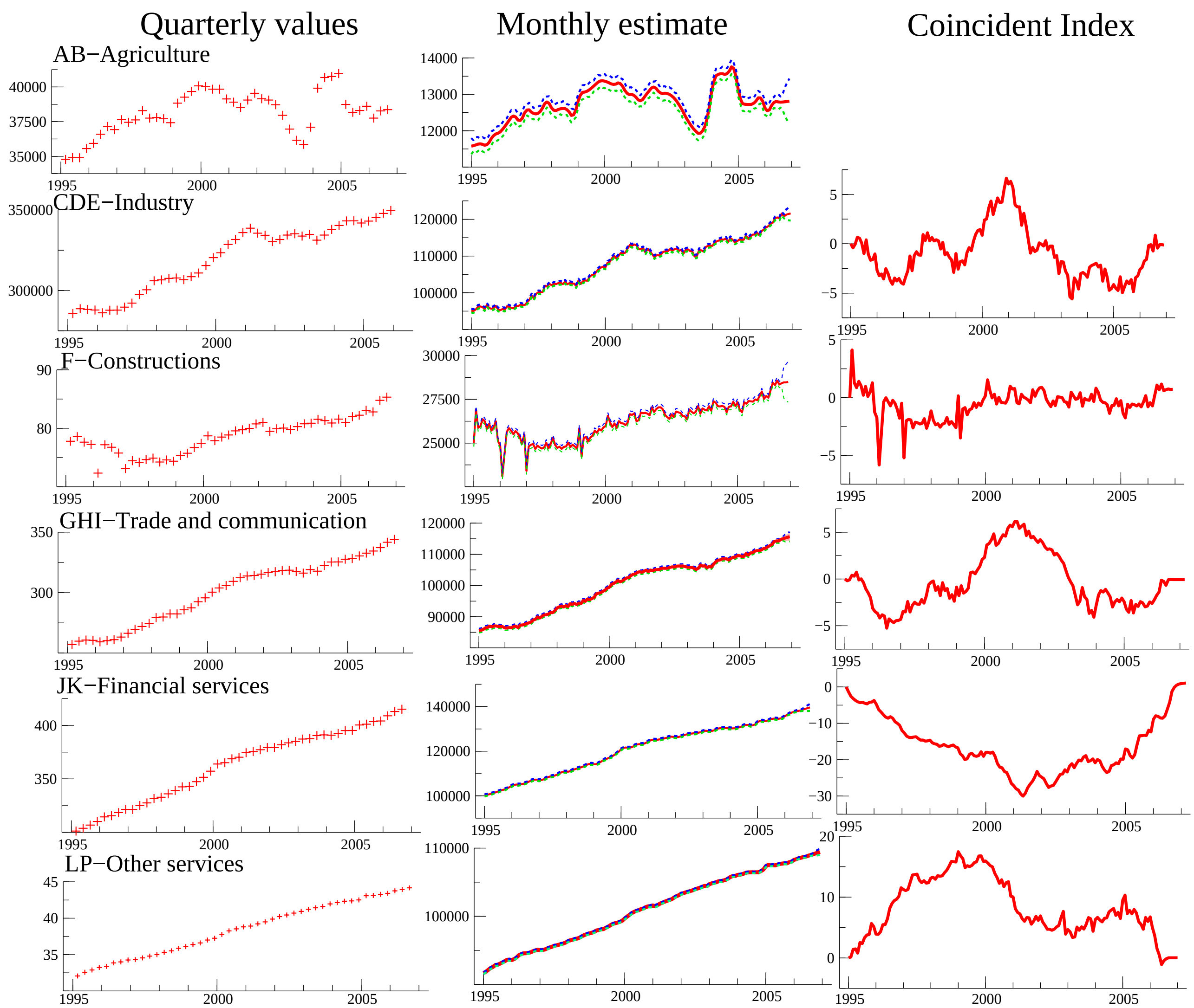


Figure 2.2: Quarterly National Account, Monthly estimates with standard errors and Coincident Index- Expenditure approach.

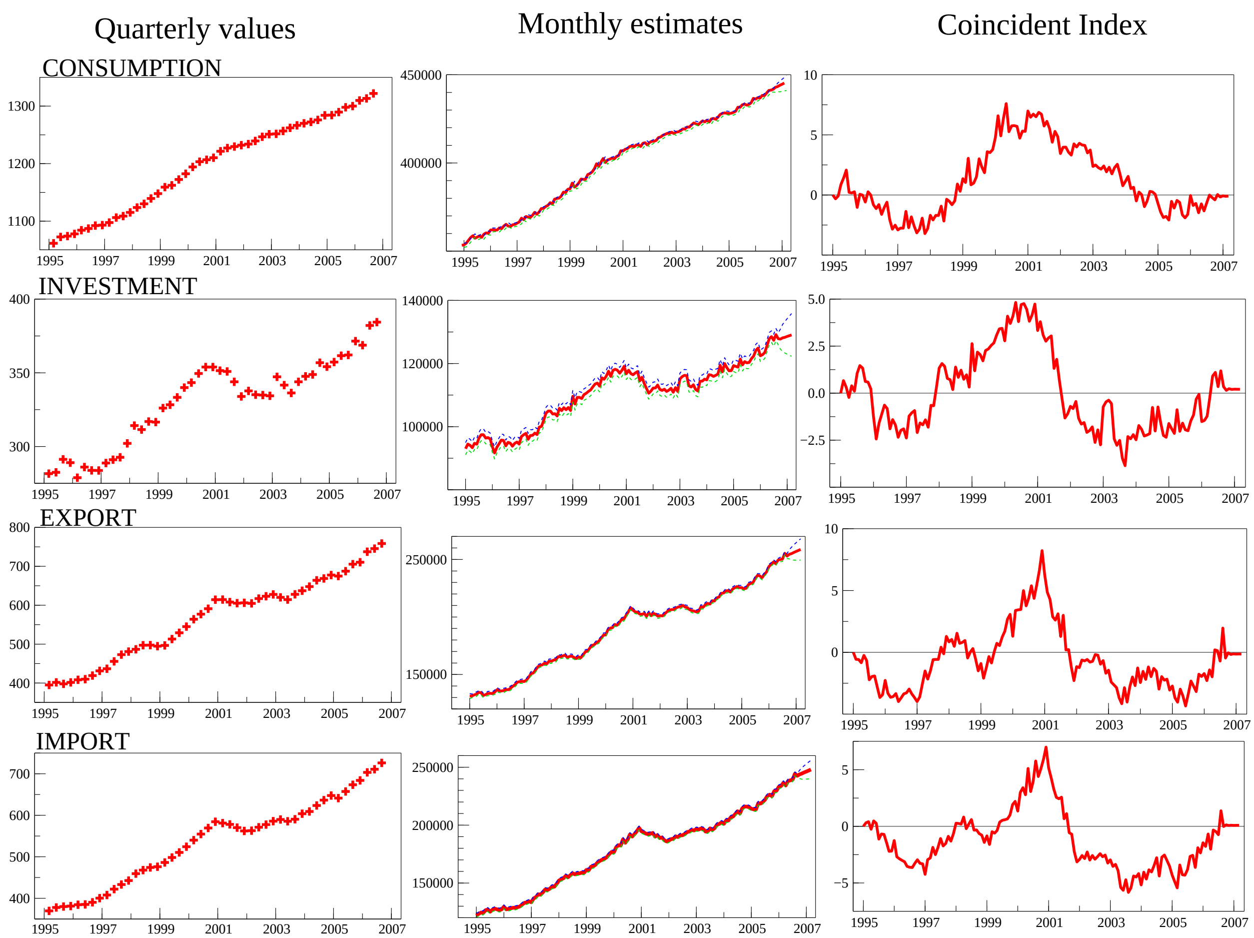


Figure 2.3: Monthly gross domestic product estimates for the Euro Area (eurozone12)

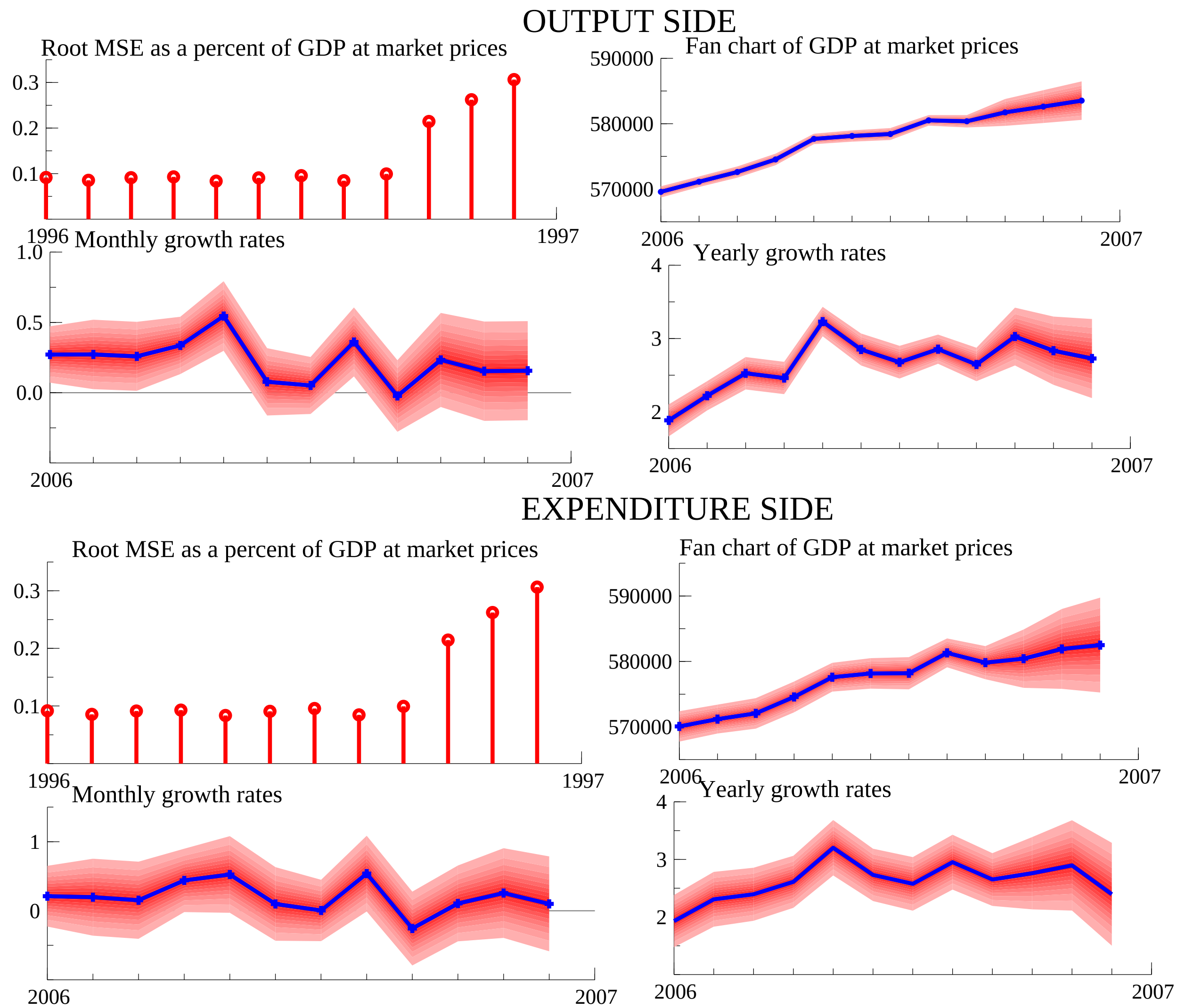


Figure 2.4: Combined monthly estimates of gross domestic product at market prices.

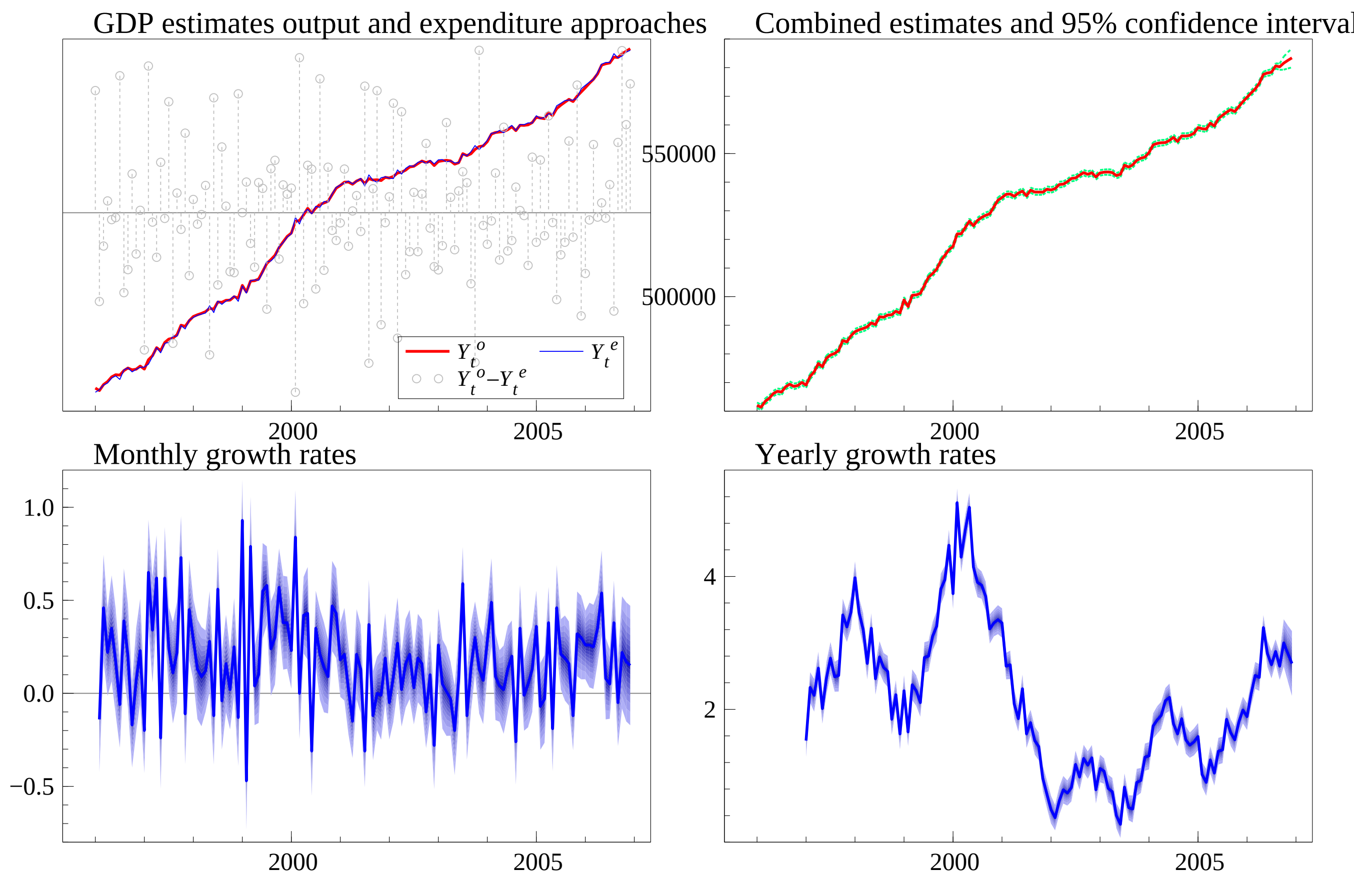


Figure 2.5: Multivariate Innovations for GDP by sector

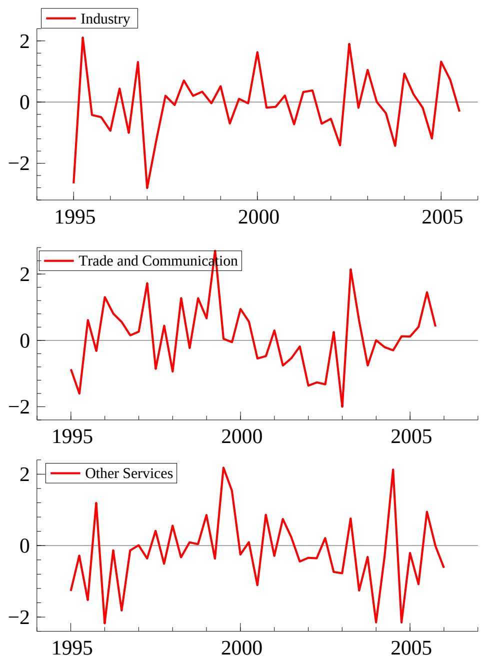


Figure 2.6: Multivariate Innovations for GDP by components of expenditure

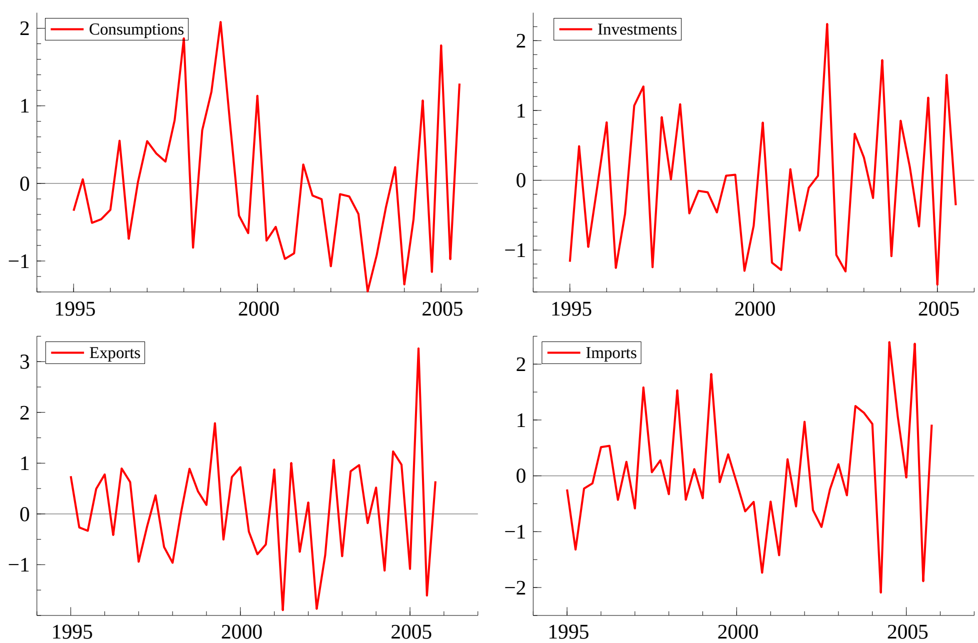


Figure 2.7: GDP estimates revisions

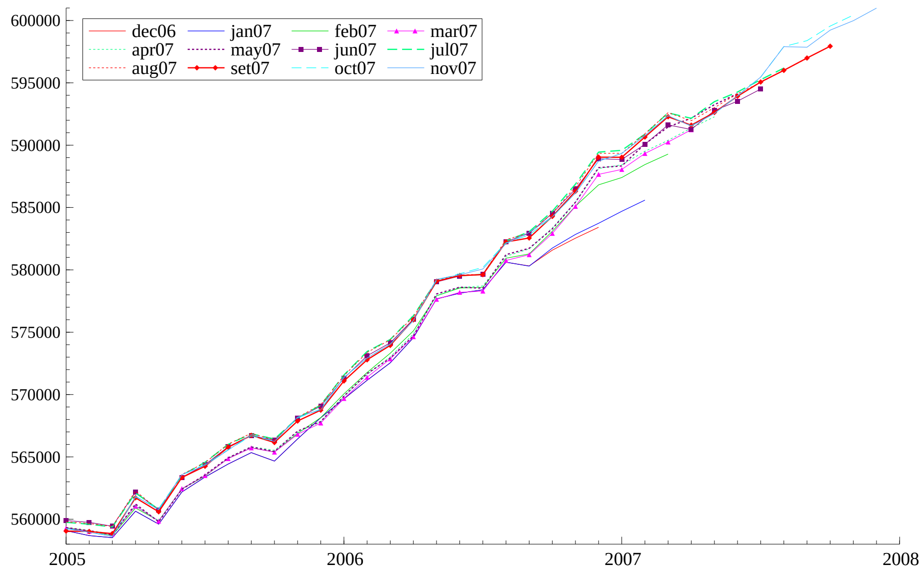
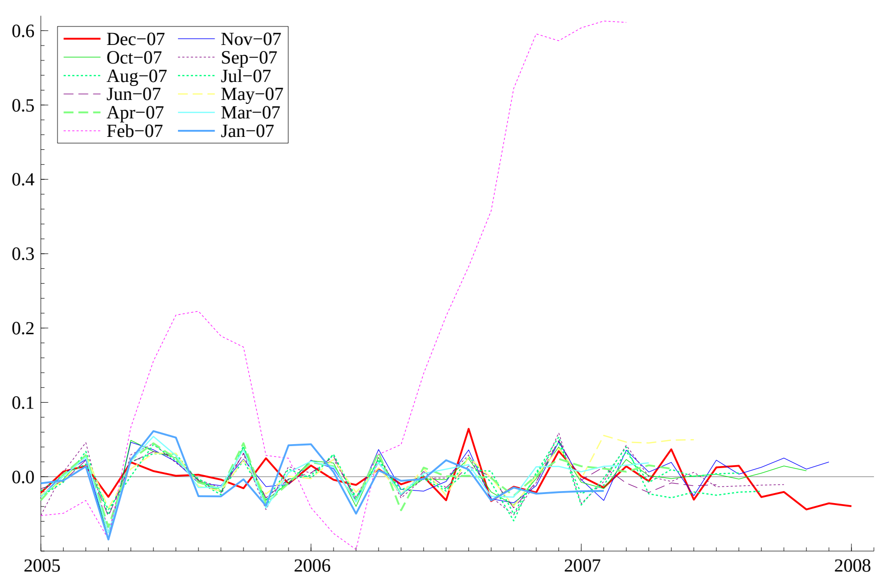


Figure 2.8: Net impact of data revisions in GDP estimates



Note: The plot shows the percentage relative difference between the estimated and constant parameter(as estimated in December 2006) estimates of Monthly GDP.

Figure 2.9: Standard errors by sectors, output side

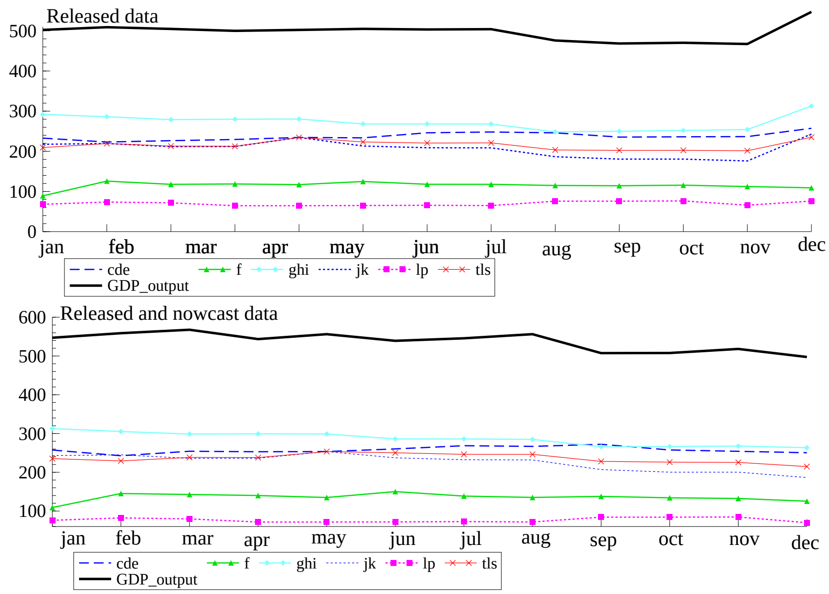
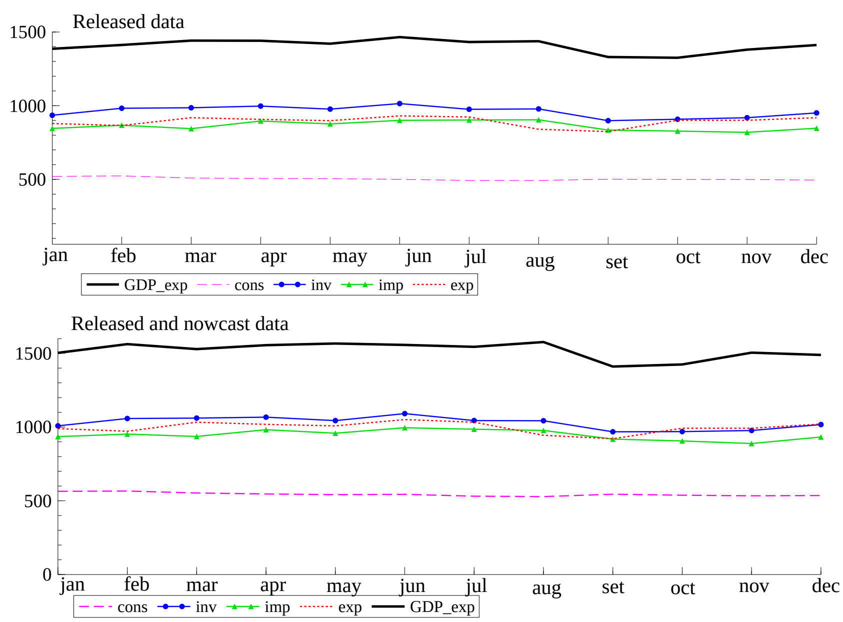


Figure 2.10: Standard errors by components, expenditure side



2.A Appendix-State space representation

In this section we cast model (2.3.1) in the state space form (SSF). We start from the single index, $\phi(L)\Delta\mu_t = \eta_t$, considering the SSF of the stationary AR(p) model for the $\Delta\mu_t$, for which:

$$\begin{aligned} \Delta\mu_t &= \mathbf{e}'_{1p}\mathbf{g}_t, \\ \mathbf{g}_t &= \mathbf{T}_{\Delta\mu}\mathbf{g}_{t-1} + \mathbf{e}_{1p}\eta_t, \end{aligned}$$

where $\mathbf{e}_{1p} = [1, 0, \ldots, 0]'$ and

$$\mathbf{T}_{\Delta\mu} = \left[\begin{array}{cc} \phi_1 & \\ \vdots & \mathbf{I}_{p-1} \\ \phi_{p-1} & \\ \phi_p & \mathbf{0}' \end{array} \right].$$

Hence, $\mu_t = \mu_{t-1} + \mathbf{e}'_{1p}\mathbf{g}_t = \mu_{t-1} + \mathbf{e}'_{1p}\mathbf{T}_{\Delta\mu}\mathbf{g}_{t-1} + \eta_t$, and defining

$$\boldsymbol{\alpha}_{\mu,t} = \left[\begin{array}{c} \mu_t \\ \mathbf{g}_t \end{array} \right], \quad \mathbf{T}_\mu = \left[\begin{array}{cc} 1 & \mathbf{e}'_{1p}\mathbf{T}_{\Delta\mu} \\ 0 & \mathbf{T}_{\Delta\mu} \end{array} \right],$$

the Markovian representation of the model for μ_t becomes

$$\mu_t = \mathbf{e}'_{1,p+1}\boldsymbol{\alpha}_{\mu,t}, \quad \boldsymbol{\alpha}_{\mu,t} = \mathbf{T}_\mu\boldsymbol{\alpha}_{\mu,t-1} + \mathbf{H}_\mu\eta_t,$$

where $\mathbf{H}_\mu = [1, \mathbf{e}'_{1,p}]'$.

A similar representation holds for each individual μ^*_{it}, with ϕ_j replaced by d_{ij}, so that, if we let p_i denote the order of the i-th lag polynomial $d_i(L)$, we can write:

$$\mu^*_{it} = \mathbf{e}'_{1,p_i+1}\boldsymbol{\alpha}_{\mu_i,t}, \quad \boldsymbol{\alpha}_{\mu_i,t} = \mathbf{T}_i\boldsymbol{\alpha}_{\mu_i,t-1} + \mathbf{c}_i + \mathbf{H}_i\eta^*_{it},$$

where $\mathbf{H}_i = [1, \mathbf{e}'_{1,p_i}]'$, $\mathbf{c}_i = \delta_i\mathbf{H}_i$ and δ_i is the drift of the $i-th$ idiosyncratic component, and thus of the series, since we have assumed a zero drift for the common factor.

Combining all the blocks, we obtain the SSF of the complete model by defining the state vector $\boldsymbol{\alpha}_t$, with dimension $\sum_i (p_i + 1) + p + 1$, as follows:

$$\boldsymbol{\alpha}_t = [\boldsymbol{\alpha}'_{\mu,t}, \boldsymbol{\alpha}'_{\mu_1,t}, \ldots, \boldsymbol{\alpha}'_{\mu_N,t}]'. \tag{2.A.1}$$

Consequently, the measurement and the transition equation of SW model in levels is:

$$\mathbf{y}_t = \mathbf{Z}\boldsymbol{\alpha}_t + \mathbf{X}_t\boldsymbol{\beta}, \quad \boldsymbol{\alpha}_t = \mathbf{T}\boldsymbol{\alpha}_{t-1} + \mathbf{W}\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_t, \tag{2.A.2}$$

where $\boldsymbol{\epsilon}_t = [\eta_t, \eta^*_{1,t}, \ldots, \eta^*_{N,t}]'$ and the system matrices are given below:

$$\begin{aligned} &\mathbf{Z} = \left[\boldsymbol{\theta}_0, \,\vdots\, \boldsymbol{\theta}_1 \;\;\vdots\, \mathbf{0} \,\vdots\, \text{diag}(\mathbf{e}'_{p_1}, \ldots, \mathbf{e}'_{p_N})\right], \qquad \mathbf{T} = \text{diag}(\mathbf{T}_\mu, \mathbf{T}_1, \ldots, \mathbf{T}_N), \\ &\mathbf{H} = \text{diag}(\mathbf{H}_\mu, \mathbf{H}_1, \ldots, \mathbf{H}_N). \end{aligned} \tag{2.A.3}$$

The vector of initial values is written as $\boldsymbol{\alpha}_1 = \mathbf{W}_1\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_1$, so that $\boldsymbol{\alpha}_1 \sim \mathrm{N}(\mathbf{0}, \mathbf{W}_1\mathbf{V}\mathbf{W}'_1 + \mathbf{H}\text{Var}(\boldsymbol{\epsilon}_1)\mathbf{H}')$, $\text{Var}(\boldsymbol{\epsilon}_1) = \text{diag}(1, \sigma^2_1, \ldots, \sigma^2_N)$.

The first $2N$ elements of the vector $\boldsymbol{\beta}$ are the pairs $\{(\mu_{01}, \delta_i, i = 1, \ldots, N\}$, the starting values at time $t = 0$ of the idiosyncratic components and the constant drifts δ_i.

The regression matrix $\mathbf{X}_t = [\mathbf{0}, \quad \mathbf{X}^*_t]$ where $\mathbf{X}^*_t$ is a $N \times k$ matrix containing the values of exogenous variables that are used to incorporate calendar effects (trading day regressors, Easter, length of the month) and intervention variables (level shifts, additive outliers, etc.), and the zero block has dimension $N \times 2N$ and corresponds to the elements of $\boldsymbol{\beta}$ that are used for the initialisation and other fixed effects.

The $2N + k$ elements of $\boldsymbol{\beta}$ are taken as diffuse.

For $t = 2, \ldots, n$ the matrix $\mathbf{W}$ is time invariant and selects the drift δ_i for the appropriate state element:

$$\mathbf{W} = \begin{bmatrix} \mathbf{0} \\ \text{diag}(\mathbf{C}_1, \ldots, \mathbf{C}_N) \end{bmatrix}, \mathbf{C}_i = [\mathbf{0}_{p_i+1,1} \vdots \mathbf{c}_i],$$

whereas $\mathbf{W}_1$

$$\mathbf{W}_1 = \begin{bmatrix} \mathbf{0} \\ \text{diag}(\mathbf{C}^*_1, \ldots, \mathbf{C}^*_N) \end{bmatrix}, \mathbf{C}^*_i = \left[\mathbf{e}_{1,p_i+1} \vdots \mathbf{c}_i\right].$$

2.B Appendix- Temporal aggregation and the Univariate treatment of multivariate models

Suppose that the set of coincident indicators, $\mathbf{y}_t$, can be partitioned into two groups, $\mathbf{y}_t = [\mathbf{y}'_{1,t}, \mathbf{y}'_{2,t}]'$, where the second block gathers the flows that are subject to temporal aggregation, so that

$$\mathbf{y}^*_{2\tau} = \sum_{i=0}^{\delta-1} \mathbf{y}_{2,\tau\delta-i}, \quad \tau = 1, 2, \ldots, [T/\delta],$$

where δ denote the aggregation interval: for instance, if the model is specified at the monthly frequency and $\mathbf{y}^\dagger_{2,t}$ is quarterly, then $\delta = 3$.

The strategy proposed by Harvey (1989) consists of operating a suitable augmentation of the state vector (4.A.1) using an appropriately defined cumulator variable. In particular, the SSF (4.A)-(4.A) need to be augmented by the $N_2 \times 1$ vector $\mathbf{y}^c_{2,t}$, generated as follows

$$\begin{aligned} \mathbf{y}^c_{2,t} &= \psi_t \mathbf{y}^c_{2,t-1} + \mathbf{y}_{2,t} \\ &= \psi_t \mathbf{y}^c_{2,t-1} + \mathbf{Z}_2\mathbf{T}\boldsymbol{\alpha}_{t-1} + [\mathbf{X}_{2,t} + \mathbf{Z}_2\mathbf{W}_t]\boldsymbol{\beta} + \mathbf{Z}_2\mathbf{H}\boldsymbol{\epsilon}_t \end{aligned}$$

where ψ_t is the cumulator variable, defined as follows:

$$\psi_t = \begin{cases} 0 & t = \delta(\tau - 1) + 1, \quad \tau = 1, \dots, [n/\delta] \\ 1 & \text{otherwise} , \end{cases}$$

and $\mathbf{Z}_2$ is the $N_2 \times m$ block of the measurement matrix $\mathbf{Z}$ corresponding to the second set of variables, $\mathbf{Z} = [\mathbf{Z}_1', \ \ \mathbf{Z}_2']'$ and $\mathbf{y}_{2,t} = \mathbf{Z}_2\boldsymbol{\alpha}_t + \mathbf{X}_2\boldsymbol{\beta}$, where we have partitioned $\mathbf{X}_t = [\mathbf{X}_1' \quad \mathbf{X}_2']'$. Notice that at times $t = \delta\tau$ the cumulator coincides with the (observed) aggregated series, otherwise it contains the partial cumulative value of the aggregate in the seasons (e.g. months) making up the larger interval (e.g. quarter) up to and including the current one.

The augmented SSF is defined in terms of the new state and observation vectors:

$$\boldsymbol{\alpha}_t^* = \begin{bmatrix} \boldsymbol{\alpha}_t \\ \mathbf{y}_{2,t}^c \end{bmatrix}, \quad \mathbf{y}_t^\dagger = \begin{bmatrix} \mathbf{y}_{1,t} \\ \mathbf{y}_{2,t}^c \end{bmatrix}$$

where the former has dimension $m^* = m + N_2$, and the unavailable second block of observations, $\mathbf{y}_{2,t}$, is replaced by $\mathbf{y}_{2,t}^c$, which is observed at times $t = \delta\tau, \tau = 1, 2, \dots, [n/\delta]$, and is missing at intermediate times. The measurement and transition equation are therefore:

$$\mathbf{y}_t^\dagger = \mathbf{Z}^*\boldsymbol{\alpha}_t^* + \mathbf{X}_t\boldsymbol{\beta}, \quad \boldsymbol{\alpha}_t^* = \mathbf{T}^*\boldsymbol{\alpha}_{t-1}^* + \mathbf{W}^*\boldsymbol{\beta} + \mathbf{H}^*\boldsymbol{\epsilon}_t, \tag{2.B.1}$$

with starting values $\boldsymbol{\alpha}_1^* = \mathbf{W}_1^*\boldsymbol{\beta} + \mathbf{H}^*\boldsymbol{\epsilon}_1$, and system matrices:

$$\mathbf{Z}^* = \begin{bmatrix} \mathbf{Z}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_2} \end{bmatrix}, \quad \mathbf{T}^* = \begin{bmatrix} \mathbf{T} & \mathbf{0} \\ \mathbf{Z}_2\mathbf{T} & \psi_t\mathbf{I} \end{bmatrix}, \quad \mathbf{W}^* = \begin{bmatrix} \mathbf{W} \\ \mathbf{Z}_2\mathbf{W} + \mathbf{X}_2 \end{bmatrix}, \quad \mathbf{H}^* = \begin{bmatrix} \mathbf{I} \\ \mathbf{Z}_2 \end{bmatrix}\mathbf{H}. \tag{2.B.2}$$

The state space model (4.A)-(4.A) is linear and, assuming that the disturbances have a Gaussian distribution, the unknown parameters can be estimated by maximum likelihood, using the prediction error decomposition, performed by the Kalman filter; given the parameter values, the Kalman filter and smoother will provide the minimum mean square estimates of the states $\boldsymbol{\alpha}_t^*$ (see Harvey, 1989, and Shumway and Stoffer, 2000) and thus of the missing observations on $\mathbf{y}_{2,t}^c$ can be estimated, which need to be "decumulated", using $\mathbf{y}_{2,t} = \mathbf{y}_{2,t}^c - \psi_t\mathbf{y}_{2,t-1}^c$, so as to be converted into estimates of $\mathbf{y}_{2,t}$. In order to provide the estimation standard error, however, the state vector must be augmented of $\mathbf{y}_{2,t} = \mathbf{Z}_2\boldsymbol{\alpha}_t + \mathbf{X}_2\boldsymbol{\beta} = \mathbf{Z}_2\mathbf{T}\boldsymbol{\alpha}_{t-1} + [\mathbf{X}_2 + \mathbf{Z}_2\mathbf{W}]\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_t$.

The estimation of multivariate dynamic factor model of this sort can be numerically complex. We solve this issue by using a univariate statistical treatment. This was first considered by Anderson and Moore (1979) and provides a very flexible and convenient device for filtering and smoothing and for handling missing values. Our treatment is prevalently based on Koopman and Durbin (2000). However, for the treatment of regression effects and initial conditions we adopt the augmentation approach by de Jong (1990).

The multivariate vectors $\mathbf{y}_t^\dagger$, $t = 1, \dots, n$, where some elements can be missing, are stacked one on top of the other to yield a univariate time series $\{y_{t,i}^\dagger, i = 1, \dots, N, t = 1, \dots, n\}$, whose elements are processed sequentially.

The state space model for the univariate time series $\{y^{\dagger}_{t,i}\}$ is constructed as follows. the measurement equation for the i-th element of the vector $\mathbf{y}^{\dagger}_t$ is:

$$y^{\dagger}_{t,i} = \mathbf{z}^{*'}_i \boldsymbol{\alpha}^*_{t,i} + \mathbf{x}'_{t,i}\boldsymbol{\beta}, \quad t = 1, \ldots, n, \ \ i = 1, \ldots, N, \tag{2.B.3}$$

where $\mathbf{z}^{*'}_i$ and $\mathbf{x}'_{t,i}$ denote the i-th rows of $\mathbf{Z}^*$ and $\mathbf{X}_t$, respectively. When the time index is kept fixed the transition equation is the identity: $\boldsymbol{\alpha}^*_{t,i} = \boldsymbol{\alpha}^*_{t,i-1}, i = 2, \ldots, N$, whereas, for $i = 1$, $\boldsymbol{\alpha}^*_{t,1} = \mathbf{T}^*_t \boldsymbol{\alpha}^*_{t-1,N} + \mathbf{W}^*\boldsymbol{\beta} + \mathbf{H}^*\boldsymbol{\epsilon}_{t,1}$.

The state space form is completed by the initial state vector which is $\boldsymbol{\alpha}^*_{1,1} = \mathbf{W}^*_1\boldsymbol{\beta} + \mathbf{H}^*\boldsymbol{\epsilon}_{1,1}$, where $\text{Var}(\boldsymbol{\epsilon}_{1,1}) = \text{Var}(\boldsymbol{\epsilon}_{t,1}) = \text{diag}(1, \sigma^2_1, \ldots, \sigma^2_N) = \boldsymbol{\Sigma}_\epsilon$.

The augmented Kalman filter, taking into account the presence of missing values and the computer programs are presented extensively in the technical report by Proietti and Frale (2006).

Under the fixed effects model maximising the likelihood with respect to $\boldsymbol{\beta}$ and σ^2 yields:

$$\hat{\boldsymbol{\beta}} = -\mathbf{S}^{-1}_{n+1,1}\mathbf{s}_{n+1,1}, \text{Var}(\hat{\boldsymbol{\beta}}) = \mathbf{S}^{-1}_{n+1,1}, \quad \hat{\sigma}^2 = \frac{q_{n+1,1} - \mathbf{s}'_{n+1,1}\mathbf{S}^{-1}_{n+1,1}\mathbf{s}_{n+1,1}}{cn}.$$

The profile likelihood is $\mathcal{L}_c = -0.5\left[d_{n+1,1} + cn\left(\ln\hat{\sigma}^2 + \ln(2\pi) + 1\right)\right]$.

When $\boldsymbol{\beta}$ is diffuse (de Jong, 1991), the maximum likelihood estimate of the scale parameter is

$$\hat{\sigma}^2 = \frac{q_{n+1,1} - \mathbf{s}'_{n+1,1}\mathbf{S}^{-1}_{n+1,1}\mathbf{s}_{n+1,1}}{cn - 2N - k},$$

and the diffuse profile likelihood, denoted $\mathcal{L}_\infty$, takes the expression:

$$\mathcal{L}_\infty = -0.5\left[d_{n+1,1} + (cn - 2N - k)\left(\ln\hat{\sigma}^2 + \ln(2\pi) + 1\right) + \ln|\mathbf{S}_{n+1,1}|\right]. \tag{2.B.4}$$

2.C Appendix - The role of cointegration and the logarithmic transformation

In this Appendix we report some empirical evidence concerning a few important model specification issues. The first concerns whether we should assume cointegration between the temporally aggregated flow and the indicator variables. The multivariate dynamic factor model with $I(1)$ idiosyncratic factors, does not assume cointegration - see the discussion in section 2.3.2. The second is whether the linear Gaussian models considered in the previous sections can be assumed to hold only after all the variables are transformed into logarithms.

Our previous experience, dealing with the temporal disaggregation of the Italian national accounts and with the the dynamic factor model for the U.S. and Euro area economy (reported in Istat, 2005, Proietti, 2006, and Proietti and Moauro, 2006), is that it is usually safer to assume that cointegration is not present. In particular, the univariate disaggregation of the Italian value added series with the Fernàndez method were more satisfactory with respect to those obtained by the Chow-Lin methodology, as the results of out-of sample rolling forecast exercises and in sample diagnostics indicated. The Litterman model was ruled out instead due to a fundamental identifiability problem.

Secondly, the logarithmic transformation was found to be most suitable when a long time series is available and the growth rate of the series is sustained and homoscedastic, as it occurs in the U.S. case. For the Euro area the time series are short and growth is not sustained, so that disaggregating the time series on the original scale is usually appropriate.

These a priori information is reinforced by the empirical evidence originating from a rolling forecast experiment for the Industrial sector that we report below. The experiment is based on the comparison of the revision histories that characterise four alternative univariate methods of disaggregating the total value added of the branches C-D-E. For brevity, we do not report the results for the other branches, that confirm anyway our findings.

The four methods are the following:

1. Chow-Lin with regression effects represented by a constant and the indicators (see table 2.1 for a list of the indicators).

2. Chow-Lin with a linear trend and the indicator.

3. The double-logarithmic Chow-Lin model, featuring both value added and the indicators in logarithms. This poses a non-linear temporal disaggregation problem.

4. The Fernández model with a constant and the indicator.

The revision histories are generated as follows: starting from 2001 we perform a rolling forecast experiment such that at the beginning of each subsequent quarter we make predictions for the three months using the information available up to the beginning of the quarter and revise the estimates concerning the three months of the previous quarter. This assumes that the quarterly aggregate at time τ accrues between the end of the month 3τ and the beginning of month $3\tau + 1$. At the end of the experiment 23 sets of predictions are available for three horizons (one month to three months); these are compared with the revised estimates, which incorporate the quarterly aggregate information. The models are re-estimated as a new quarterly observation becomes available.

The decision between alternative methods should be based on a careful assessment of the revision of the estimates as the new total, sometimes referred to as the quarterly benchmark, becomes available. Hence, revision histories are a diagnostic tool, referring to the discrepancy between the estimates not using the last aggregate data and those incorporating it, that complies with the criterion proposed by the European System of National and Regional Accounts (par. 12.04).

> *The choice between the different indirect procedures must above all take into account the minimisation of the forecast error for the current year, in order that the provisional annual estimates correspond as closely as possible to the final figures.*

The following table presents summary statistics pertaining to the revision histories at the three horizons considered: the mean revision error, also as a percentage of the final estimate, the mean absolute and square revision errors. Obviously the performance of the methods deteriorates with the horizons. More importantly, the random walk model (Fernández) outperforms the three CL specifications according to all the measures presented, including the specification in logarithms.

As far as the latter is concerned, the profile likelihood with respect to the Box-Cox tranformation parameter λ for the Fernández model

$$y_t(\lambda) = \mathbf{x}_t(\lambda)'\boldsymbol{\beta} + u_t, \quad \Delta u_t = \epsilon_t.$$

where

$$y_t(\lambda) = \begin{cases} \frac{y_t^\lambda - 1}{\lambda}, & \lambda \neq 0, \\ \ln y_t, & \lambda = 0. \end{cases}$$

takes the following values:

λ	0.0	0.2	0.4	0.6	0.8	1.0
Likelihood	-1.6316	-1.6207	-1.6104	-1.6008	-1.5919	-1.5836

Hence, the likelihood ratio test of the hypothesis that $\lambda = 1$ (no transformation) against the alternative $\lambda = 0$ is not significant.

Table 2.7: Revision history for Industrial value added (years 2002-2006).

Model	*Mean percentage revision error*		
	1 step	2 steps	3 steps
Chow-Lin (constant)	0.18	0.24	0.24
Chow-Lin (trend)	0.08	0.09	0.09
CL Logarithms	0.06	0.07	0.06
Fernández	-0.00	-0.00	-0.00
	Mean revision error		
	1 step	2 steps	3 steps
Chow-Lin (constant)	204.67	268.71	266.52
Chow-Lin (trend)	91.86	106.20	99.38
CL Logarithms	71.95	79.99	63.86
Fernández	-2.18	-2.87	-1.11
	Mean absolute revision error		
	1 step	2 steps	3 steps
Chow-Lin (constant)	284.75	382.66	389.93
Chow-Lin (trend)	248.86	320.96	316.22
CL Logarithms	245.09	312.42	305.40
Fernández	217.20	314.22	367.10
	Mean square revision error		
	1 step	2 steps	3 steps
Chow-Lin (constant)	149255.27	253934.56	246612.17
Chow-Lin (trend)	117564.20	186504.38	176205.58
CL Logarithms	115590.47	183408.05	169103.50
Fernández	68264.69	142244.25	194418.59

Chapter 3

Do Surveys Help in Macroeconomic variables Disaggregation and Estimation?

ABSTRACT [1]: In this chapter we explore the potential of Business Survey data for the estimation and disaggregation of macroeconomic variables at higher frequency. We propose a multivariate approach which is an extension of the Stock and Watson (1991) dynamic factor model, considering more than one common factor and low-frequency cycles. The multivariate model is cast in State Space Form and the temporal aggregation constraint is converted into a problem of missing values. An application in real time for the value added of the Industry sector in the Euro area is presented.

Keywords: Temporal Disaggregation. Multivariate State Space Models. Dynamic factor Models. Kalman filter and smoother. Survey data
JEL Classification: E32, E37, C53

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3.1 Introduction

This chapter deals with the well known topic of forecasting and nowcasting macroeconomic variables. The requirement of reliable and timely indicators, available at high frequency, is the first step for the assessment of the state of the business cycle and the conduct of the economic policy. Using the statistical methodology and the recent advances in the literature on temporal disaggregation we can indirectly disaggregate macroeconomic variables (e.g. GDP and National Accounts) by using indicators available at higher frequency (monthly indicators of economic activity). In the previous chapter a procedure to estimate monthly GDP based on Stock and Watson(1991)(SW) was presented. In that application we found survey data being not relevant for the estimation. However, these data differ from hard data along several direction, either for the economic meaning of the variables concerned, either for the statistical technique of collection. Therefore, some additional investigation in that direction might be worth and a feasible extension of the standard SW model might be of such interest. In particular, we extend the standard framework allowing for more than one common coincident index, and for a re-parametrization of the standard autoregressive model(AR), suitable for low frequency cycle (Morton and Tunnicliffe-Wilson (2004)). The model is cast in State Space Form (SSF) and the statistical treatment, as well as the disaggregation technique is the same as in the previous chapter.

We claim that our method as many appealing advantages. First of all a model based approach allows figuring out an interpretation of the coincident index and idiosyncratic components in terms of the original variables, preserving the economic meaning of the series and of their relationship. Secondly, we deal with mixed frequencies, including information about past values of the GDP in addition to the monthly indicators. Third, the Kalman filter and smoother is an efficient way to solve the unbalanced sample issue induced by different delays in the released series. Last, the inclusion of a low-frequency cycle process allow capturing the features of survey data.

An application for Euro Area value added of Industry is provided and the model is evaluated in term of forecast ability and estimation accuracy trough real time experiments. As a benchmark we estimate the monthly value added for Industry by univariate ADL models. Particular attention is devoted to understand the information and the news content of survey data. Via similar models it is possible to compute the value added of each sector and obtain the monthly GDP by summing up the sectorial estimates. This procedure is in general preferable to a direct estimate of GDP, because allows the use of specific indicators for each sector. However we present in details results as for the estimation of Industry, putting in the future agenda a similar application for the others sectors of activity.

The Chapter is structured as follows. After a review of the univariate treatment (section 3.2), Section 3.3 introduces the Stock and Watson dynamic factor model with the mentioned extension, for which we present the State Space parametrization in section 3.3.1. For the temporal aggregation of the monthly estimates and the univariate treatment we refer to the procedure presented in Chapter 2 and listed in Appendix 2.B.

Section 4 summarizes the main estimation results as applied to the disaggregation of quarterly Euro Area Value Added of Industry, with particular focus on news content and timeliness of Survey data trough real time experiments. Finally, some conclusion ends.

3.2 Autoregressive Distributed Lags Models

The delay of official National Accounts data has led business cycle analyst to find an alternative way to produce nowcasts and forecasts. The most common approach is based on the idea of building "bridge" equations from high frequency to quarterly GDP (or his components) trough monthly indicators (survey and/or hard data). Models of this sort, known as Bridge Models, generally outperform traditional models, such as ARIMA, VAR or BVAR. Typically they are derived from an initial unrestricted Autoregressive Distributed Lag (ADL(p,s)) equation, estimated using aggregated data. For instance, real GDP growth on a quarterly basis is regressed on monthly indicators aggregated to a quarterly frequency.

In this study, following Proietti (2004), we cast the ADL models in State Space Form (SSF) and we disaggregate endogenously the National Account components at monthly level, by using the Kalman filter and Smoother in a mixed frequency univariate model.

Let us start from a simple Autoregressive Distributed Lag first order model, ADL(1,1), and suppose for simplicity to use only one indicator to disaggregate at higher frequency the series y_t. The model takes the form:

$$y_t = \phi y_{t-1} + m + gt + x_t'\beta_0 + x_{t-1}'\beta_1 + \epsilon_t \qquad \epsilon_t \sim NID(0;\sigma_t^2),$$

where x_t is the indicator at time t. It is possible to find a corresponding state space representation, which features:

$$\begin{aligned} y_t &= \quad \alpha_t \\ \alpha_t &= \phi \quad \alpha_{t-1} + \mathbf{W}_t \boldsymbol{\beta} + \epsilon_t \end{aligned}$$

where the matrix $\mathbf{W}_t = [1, t, x_t', x_{t-1}']$ includes the drift, the trend and the exogenous variable x_t. To start the system some initial conditions are needed and several initializations are possible. Among them one can assume that the process started in the indefinite past or consider y_1 as a fixed value or assume that y_1 is random and the process is supposed to have started at time t = 0 with a fixed, but unknown value. The hypothesis of stationarity might be relaxed (see Proietti (2004)) and the ADL model could be estimated in first differences:

$$\Delta y_t = \psi \Delta y_{t-1} + x_t'\beta_0 + x_{t-1}'\beta_1 + \epsilon_t \quad \epsilon_t \sim NID(0;\sigma_t^2).$$

The transition equation is $\boldsymbol{\alpha}_t = \mathbf{T}_{t-1}\boldsymbol{\alpha}_{t-1} + \mathbf{W}_t\boldsymbol{\beta} + \boldsymbol{\epsilon}_t$, with state element $\boldsymbol{\alpha}_t = [y_{t-1}, \Delta y_t]'$,the transition matrix is $\mathbf{T} = [1, 1 \vdots 0, \psi]$, and regression effects are included in the matrix $\mathbf{W}_t$. The measurement equation $y_t = [1,1]\boldsymbol{\alpha}_t$ complete the SSF.

The model is formulated at the frequency level of the indicators x_t (e.g. monthly), therefore due to temporal aggregation, y_t (e.g. GDP) is not observed. The data arise, instead, as the sum of s (equal to 3 in our case) consecutive values, $\sum_{j=0}^{s-1} y_{\tau s-j}$, and are available at times $\tau = 1, 2, \ldots [n/s]$ (e.g. representing the quarters), where $[n/s]$ denotes the integral part of n/s.

In order to handle temporal aggregation, a new state space representation is derived, by augmenting the state vector of the original state space representation with a cumulator variable that is only partially observed:

$$y_t^c = \psi_t y_{t-1}^c + y_t, \quad \psi_t = \begin{cases} 0, & t = s(\tau - 1) + 1, \tau = 1, \ldots, [n/s] \\ 1, & \text{otherwise} \end{cases}$$

Extensions to higher order ADL(p,q)D could be derived in a similar way.

The statistical treatment is based upon the augmented Kalman filter due to de Jong (1991), suitably modified to take into account the presence of missing values, which is easily accomplished by skipping certain updating operations. For a comprehensive treatment of the statistical univariate treatment see Proietti (2004).

There are two main related sources of criticism that arise with respect to the univariate disaggregation methods. The first concerns the exogeneity assumption, according to which the indicator is considered as an explanatory variable in a regression model. Actually there is no a priori reason to say that a monthly indicator Granger cause the GDP, more generally they represent different aspects of the same phenomenon, the state of the economy. The second is that the regression based methods assume that the indicators are measured without error. Considering how much macroeconomic data, such as Industrial production, are revised by Statistical Offices is hard to support this hypothesis.

A multivariate framework is in general more realistic.

3.3 The dynamic factor model

There are relatively few examples of multivariate disaggregation methods in the literature. Harvey and Chung (2000) use a bivariate unobserved components model, while Moauro and Savio (2005) propose multivariate disaggregation methods based on the class of Sutse models. Starting from the original work of Stock and Watson (1991) several papers develop an explicit probability model for the composite index of coincident economic indicators. They consider a dynamic factor model to figure out a common difference-stationary factor which is assumed to be the value of a single unobservable variable, the state of the economy. This represents by assumption the only source of the co-movements of few relevant time series: industrial production, sales, employment, and real incomes. Although it is available only quarterly, GDP is perhaps the most important coincident indicator. This consideration motivate Mariano and Murasawa (2003) to extend the SW model with the inclusion of quarterly real GDP growth, proposing a linear state space model defined at the monthly observation frequency, with a time aggregation constraint. The model is formulated in terms of the logarithmic changes in the variables, and the nonlinear nature of the temporal aggregation constraint is addressed considering a geometric mean relation between monthly and quarterly observations. A more technical solution of the nonlinear constraint is presented in Proietti and Moauro (2006).

The recent interest in Survey data and the evidence of their relevance in macroeconomic forecast (Giannone *et al*. 2005, Altissimo *et al*. 2007) suggests a possible extension of the information set on which is based the SW model to include this sort of indicators. Results from the previous Chapter have provided evidence on the inadequacy of the standard formulation of the model to include soft data. Therefore a modification of the SW standard formalization that considers the specific nature of survey data is achieved.

We propose to address this issue in two directions: first considering more than one common factor, secondly including in the common index a predefined Moving Average (MA) part, suitable for processes with peaks in the spectral density at low frequencies. Morton and Tunnicliffe-Wilson have showed how forecast ability of an AR(p) might improve by using the above modification:

$$\phi(L)X_t = (1-\theta L)^p \eta_t,$$

where $\phi(L)$ is a lag polynomial of the form $(1+\phi_1 L+\phi_2 L^2+...+\phi_p L^p)$ and θ is a specified parameter in the interval [0.4-0.7] (mostly $\theta = 0.5$). This re-parametrization for the AR(p), called ZAR(p), squeezes the spectrum in the fraction $(1-\theta)/(1+\theta)$ of frequences at the lower end of the range and therefore accounts for low frequency cycles. In the sequel we present how to extend the SW model in these two directions.

Let $\mathbf{y}_t$ denote a $N \times 1$ vector of time series, that we assume to be integrated of order one, or $I(1)$, so that $\Delta y_{it}, i = 1,\dots,N$, has a stationary and invertible representation. The extended SW dynamic factor model expresses $\mathbf{y}_t$ as the linear combination of two common cyclical trends, denoted by μ_t and $\widetilde{\mu}_t$ respectively, and idiosyncratic components, $\boldsymbol{\gamma}_t$, specific for each series. Letting $\boldsymbol{\vartheta}$ and $\widetilde{\boldsymbol{\vartheta}}$ denote the two $N \times 1$ vectors of loadings, and assuming that both common and idiosyncratic components are difference stationary and subject to autoregressive dynamics, we can write the specification in level:

$$\begin{array}{lcll}
\mathbf{y}_t & = & \boldsymbol{\vartheta}_0\mu_t + \boldsymbol{\vartheta}_1\mu_{t-1} + \widetilde{\boldsymbol{\vartheta}}_0\widetilde{\mu}_t + \widetilde{\boldsymbol{\vartheta}}_1\widetilde{\mu}_{t-1} + \boldsymbol{\gamma}_t + \mathbf{X}_t\boldsymbol{\beta}, & t = 1,...,n, \\
\phi(L)\Delta\mu_t & = & (1-\theta L)^p\eta_t, & \eta_t \sim \text{NID}(0,\sigma^2_\eta), \\
\widetilde{\phi}(L)\Delta\widetilde{\mu}_t & = & \widetilde{\eta}_t, & \widetilde{\eta}_t \sim \text{NID}(0,\sigma^2_{\widetilde{\eta}}), \\
\mathbf{D}(L)\Delta\boldsymbol{\gamma}_t & = & \boldsymbol{\delta} + \boldsymbol{\xi}_t, & \boldsymbol{\xi}_t \sim \text{NID}(\mathbf{0},\boldsymbol{\Sigma}_\xi),
\end{array} \tag{3.3.1}$$

where $\phi(L)$ and $\widetilde{\phi}(L)$ are autoregressive polynomials of order p and $\widetilde{p}$ with stationary roots:

$$\phi(L) = 1-\phi_1 L - \cdots - \phi_p L^p, \widetilde{\phi}(L) = 1 - \widetilde{\phi}_1 L - \cdots - \widetilde{\phi}_{\tilde{p}} L^{\tilde{p}}$$

and $(1-\theta L)^p\eta_t$ is the pre-specified MA(p) term allowing for low-frequency cycles. The matrix polynomial $\mathbf{D}(L)$ is diagonal:

$$\mathbf{D}(L) = \text{diag}\left[d_1(L), d_2(L), \dots, d_N(L)\right],$$

with $d_i(L) = 1 - d_{i1}L - \cdots - d_{ip_i}L^{p_i}$ and $\boldsymbol{\Sigma}_\xi = \text{diag}(\sigma^2_1,\dots,\sigma^2_N)$.
The disturbances η_t ,$\widetilde{\eta}_t$ and $\boldsymbol{\xi}_t$ are mutually uncorrelated at all leads and lags.

3.3.1 State space representation

In this section we cast model (3.3.1) in the state space form (SSF). To make exposition clear we present the state space of every component separately, the two coincident indexes and the idiosyncratic components, and finally we combine all blocks to get the complete form.

Let us start from the single index, $\phi(L)\Delta\mu_t = (1-\theta L)^p \eta_t$, that is an autoregressive process of order (p), AR(p) with the mentioned Morton and Tunnicliffe Wilson (2004) modification, or a ZAR(p). It is possible to write the stationary ZAR(p) model $\Delta\mu_t$ using the following SSF:

$$\begin{aligned}\Delta\mu_t &= \mathbf{e}'_{1p+1}\mathbf{g}_t,\\ \mathbf{g}_t &= \mathbf{T}_{\Delta\mu}\mathbf{g}_{t-1} + \mathbf{h}\eta_t,\end{aligned}$$

where $\mathbf{h} = \sigma_\eta[1, -p\theta, \binom{p}{2}(-\theta)^2, \binom{p}{3}(-\theta)^3, \ldots, (-\theta)^p]'$ and

$$\mathbf{T}_{\Delta\mu} = \left[\begin{array}{cc} \phi_1 & \\ \vdots & \mathbf{I}_p \\ \phi_p & \\ \phi_{p+1} & \mathbf{0}' \end{array}\right].$$

Nevertheless, the model 3.3.1 is express in level, therefore we need to derive the corresponding SSF for μ_t. Hence, considering that $\mu_t = \mu_{t-1} + \mathbf{e}'_{1p+1}\mathbf{g}_t = \mu_{t-1} + \mathbf{e}'_{1p+1}\mathbf{T}_{\Delta\mu}\mathbf{g}_{t-1} + \mathbf{h}\eta_t$, and defining

$$\boldsymbol{\alpha}_{\mu,t} = \left[\begin{array}{c} \mu_t \\ \mathbf{g}_t \end{array}\right], \quad \mathbf{T}_\mu = \left[\begin{array}{cc} 1 & \mathbf{e}'_{1p+1}\mathbf{T}_{\Delta\mu} \\ 0 & \mathbf{T}_{\Delta\mu} \end{array}\right],$$

the SSF representation of the model for μ_t becomes

$$\mu_t = \mathbf{e}'_{1,p+2}\boldsymbol{\alpha}_{\mu,t}, \quad \boldsymbol{\alpha}_{\mu,t} = \mathbf{T}_\mu\boldsymbol{\alpha}_{\mu,t-1} + \mathbf{H}_\mu\eta_t,$$

where $\mathbf{H}_\mu = [1, \mathbf{h}']'$.

A similar approach could be followed to derive the SSF of the second coincident index, that is a standard AR($\tilde{p}$) process. The index in difference $\Delta\widetilde{\mu}_t$ is expressed by:

$$\begin{aligned}\Delta\widetilde{\mu}_t &= \mathbf{e}'_{1\tilde{p}}\widetilde{\mathbf{g}}_t,\\ \widetilde{\mathbf{g}}_t &= \mathbf{T}_{\Delta\tilde{\mu}}\widetilde{\mathbf{g}}_{t-1} + \mathbf{e}_{1\tilde{p}}\widetilde{\eta}_t,\end{aligned}$$

where $\mathbf{e}_{1\tilde{p}} = [1, 0, \ldots, 0]'$ and

$$\mathbf{T}_{\Delta\tilde{\mu}} = \left[\begin{array}{cc} \tilde{\phi}_1 & \\ \vdots & \mathbf{I}_{\tilde{p}-1} \\ \tilde{\phi}_{\tilde{p}-1} & \\ \tilde{\phi}_{\tilde{p}} & \mathbf{0}' \end{array}\right].$$

Hence, as before, we derive the SSF for the level considering that $\widetilde{\mu}_t = \widetilde{\mu}_{t-1} + \mathbf{e}'_{1\tilde{p}}\widetilde{\mathbf{g}}_t = \widetilde{\mu}_{t-1} + \mathbf{e}'_{1\tilde{p}}\mathbf{T}_{\Delta\tilde{\mu}}\widetilde{\mathbf{g}}_{t-1} + \widetilde{\eta}_t$, and defining

$$\boldsymbol{\alpha}_{\tilde{\mu},t} = \left[\begin{array}{c} \widetilde{\mu}_t \\ \widetilde{\mathbf{g}}_t \end{array}\right], \quad \mathbf{T}_{\tilde{\mu}} = \left[\begin{array}{cc} 1 & \mathbf{e}'_{1\tilde{p}}\mathbf{T}_{\Delta\tilde{\mu}} \\ 0 & \mathbf{T}_{\Delta\tilde{\mu}} \end{array}\right].$$

The final SSF of the model for $\widetilde{\mu}_t$ becomes

$$\mu_t = \mathbf{e}'_{1,\tilde{p}+1}\boldsymbol{\alpha}_{\tilde{\mu},t}, \quad \boldsymbol{\alpha}_{\tilde{\mu},t} = \mathbf{T}_{\tilde{\mu}}\boldsymbol{\alpha}_{\tilde{\mu},t-1} + \mathbf{H}_{\tilde{\mu}}\eta_t,$$

where $\mathbf{H}_{\tilde{\mu}} = [1, \mathbf{e}'_{1,\tilde{p}}]'$.

A similar representation holds for each individual γ_{it}, with $\widetilde{\phi}_j$ replaced by d_{ij}, so that, if we let p_i denote the order of the i-th lag polynomial $d_i(L)$, we can write:

$$\gamma_{it} = \mathbf{e}'_{1,p_i+1}\boldsymbol{\alpha}_{\mu_i,t}, \quad \boldsymbol{\alpha}_{\mu_i,t} = \mathbf{T}_i\boldsymbol{\alpha}_{\mu_i,t-1} + \mathbf{c}_i + \mathbf{H}_i\xi_{it},$$

where $\mathbf{H}_i = [1, \mathbf{e}'_{1,p_i}]'$, $\mathbf{c}_i = \delta_i\mathbf{H}_i$ and δ_i is the drift of the $i-th$ idiosyncratic component, and thus of the series, since we have assumed a zero drift for the common factor.

Combining all the blocks, we obtain the SSF of the complete model by defining the state vector $\boldsymbol{\alpha}_t$, with dimension $\sum_i (p_i+1) + (p+2) + (\widetilde{p}+1)$, as follows:

$$\boldsymbol{\alpha}_t = [\boldsymbol{\alpha}'_{\mu,t}, \boldsymbol{\alpha}'_{\tilde{\mu},t}, \boldsymbol{\alpha}'_{\mu_1,t}, \ldots, \boldsymbol{\alpha}'_{\mu_N,t}]'. \tag{3.3.2}$$

Consequently, the measurement and the transition equation of the SW model in levels are:

$$\mathbf{y}_t = \mathbf{Z}\boldsymbol{\alpha}_t + \mathbf{X}_t\boldsymbol{\beta}, \quad \boldsymbol{\alpha}_t = \mathbf{T}\boldsymbol{\alpha}_{t-1} + \mathbf{W}\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_t, \tag{3.3.3}$$

where $\boldsymbol{\epsilon}_t = [\eta_t, \widetilde{\eta}_t, \xi_{1,t}, \ldots, \xi_{N,t}]'$ and the system matrices are given below:

$$\begin{aligned}
\mathbf{Z} &= \left[\boldsymbol{\theta}_0, \vdots\, \boldsymbol{\theta}_1 \;\; \vdots\, \mathbf{0} \,\vdots\, \tilde{\boldsymbol{\theta}}_0, \vdots\, \tilde{\boldsymbol{\theta}}_1 \;\; \vdots\, \mathbf{0} \,\vdots\, \text{diag}(\mathbf{e}'_{p_1+1}, \ldots, \mathbf{e}'_{p_N+1})\right], \\
\mathbf{T} &= \text{diag}(\mathbf{T}_\mu, \mathbf{T}_{\tilde{\mu}}, \mathbf{T}_1, \ldots, \mathbf{T}_N), \\
\mathbf{H} &= \text{diag}(\mathbf{H}_\mu, \mathbf{H}_{\tilde{\mu}}, \mathbf{H}_1, \ldots, \mathbf{H}_N).
\end{aligned} \tag{3.3.4}$$

The vector of initial values is $\boldsymbol{\alpha}_1 = \mathbf{W}_1\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_1$, so that $\boldsymbol{\alpha}_1 \sim \text{N}(\mathbf{0}, \mathbf{W}_1\mathbf{V}\mathbf{W}'_1 + \mathbf{H}\text{Var}(\boldsymbol{\epsilon}_1)\mathbf{H}')$, $\text{Var}(\boldsymbol{\epsilon}_1) = \text{diag}(1, \sigma_1^2, \ldots, \sigma_N^2)$.

The treatment of the initial conditions and of the regression effect follow the procedure explained in the previous Chapter- see discussion in Appendix 2.A.

As for the model in Chapter 2 the state space form (3.3.3)-(3.3.4) is linear and, assuming that the disturbances have a Gaussian distribution, the unknown parameters can be estimated by maximum likelihood, using the prediction error decomposition, performed by the Kalman filter. Given the parameter values, the Kalman filter and smoother will provide the minimum mean square estimates of the states and thus of the missing observations in $\mathbf{y}^c_{2,t}$. Hence, by using $\mathbf{y}_{2,t} = \mathbf{y}^c_{2,t} - \psi_t\mathbf{y}^c_{2,t-1}$, it is possible to derive the estimates of $\mathbf{y}_{2,t}$. In order to provide the estimation standard error, however, the state vector must be augmented of $\mathbf{y}_{2,t} = \mathbf{Z}_2\boldsymbol{\alpha}_t + \mathbf{X}_2\boldsymbol{\beta} = \mathbf{Z}_2\mathbf{T}\boldsymbol{\alpha}_{t-1} + [\mathbf{X}_2 + \mathbf{Z}_2\mathbf{W}]\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_t$.

As far as the the multivariate smoother and filter is concerned we refer to the procedure developed in the previous Chapter, as well as for the temporal aggregation constraint (see Appendix 2.A and Appendix 2.B).

3.4 Empirical Application

3.4.1 Estimation of the Monthly Value Added for Industry

We present an application on the estimation of the Value Added for Industry carried out using the methodology outlined in section 3.3 for the temporal disaggregation of the quarterly values by using monthly indicators. We consider preferable to figure out the total GDP estimation summing up sectorial estimates in order to exploit specific indicators for each sector, although we present in detail the estimation only for the Industry sector leaving to future work the treatment of all the other sectors.

At the time of writing the series of quarterly Value Added are available by Eurostat from the begin of 1995 to the third quarter of 2006. Observations are seasonally adjusted and working day adjusted and refer to the Euro Area. The series are relatively short because of a major structural break concerning the statistical allocation of Financial Intermediation Services Indirectly Measured (FISIM).

After a set of preliminary analysis for variable selection, we end up to consider as monthly indicators five series, shown in the top left panel of figure 3.1. Two are quantitative indicators: the index of industrial production (prod) and hours worked (howk). The remaining three are business survey indicators compiled in the form of balances of opinions by the European Commission: climate confidence indicator (EA.clim), the production trend observed in recent months (EA.prod) and the assessment of order–book levels (EA.ord). As matter of fact, any variable selection is arbitrary. There are literally hundreds of papers on variable selection methods. After a period in which the general tendence was in favor of huge datasets, some recent studies show that often a smaller set of indicators are yet satisfactory or even better than large dataset (se Boivin and Ng (2006) and Bańbura M. and R*ü*nstler (2007)).

However, the aim of this study is to investigate whether the inclusion of survey data improve the performance of the model, producing more accurate estimates and forecasts, not to address the issue of variable selection. Therefore we start from the same information set for all the competitor models, that includes the most widely used hard data for Industry (industrial production, employment, hours work) and all survey data coming form the Business Survey. Hence we proceed from the general to the particular model eliminating variables that result not significant. We consider also Likelihood based criteria, BIC and Akaike lag selection procedures, to discriminate among different models.

As far as survey data are concerned, see Pesaran and Weale (2006) for a discussion on the quantification of surveys and their role in econometric analysis. Business cycle indicators are supposed to be stationary at the long run frequency (see also stationarity tests in Proietti and Frale, 2006), therefore we include survey variables in our models in integrated form so as to preserve the level specification of the regression and the dynamic factor models. We leave to future research the investigation of alternative specifications and quantifications for survey data. In particular, it would be worth to include the new quantified indicator based on the cumulative logit model, as derived in Chapter 1, and investigate whether it represents a better synthesis of the content of the survey. Unfortunately, the construction of such a index for the Euro Area is still under inspection because the lack of micro data.

We estimate three benchmark models: the univariate ADLD, the SW single index and finally the double index SW with modification (SW2-ZAR henceforth). We first present estimation results for each of them, then, in the next paragraph, we compare their forecast ability.

The ADLD model is estimated according to the framework presented in section 3.2. Among alternative specifications in terms of components (drift, trend), in terms of lags and in terms of initialization options, we found that the ADL(1,1)D with trend is the best model, as also suggested by BIC and Akaike lag selection procedures. The estimated regression coefficients, along with their standard error and the t-statistic are reproduced in table 3.1. Although industrial production remains the most relevant indicator, survey data matter, both contemporaneously than with lag. On the contrary the series of hours worked does not enter in the model at any lags. Figure 3.1 shows the official quarterly series along with smoothed and filtered estimation.

As mentioned before multivariate models are a more appropriate solution to the estimation problem, therefore we estimate a dynamic factor model following the standard SW formalization with one common factor. The maximum likelihood estimates of the parameters of the model along with asymptotic standard errors are presented in table 4.2. The coincident index, which is an AR(2), seems to be strongly related to both industrial production and hours worked. Nevertheless indicators do not enter with lags. Survey data appears not significant, neither contemporaneously neither with lag, therefore we took them out presenting results as for the SW single index without survey data. The smoothed estimates of the common factor, μ_t, and of monthly value added are presented in the left column of figure 4.3.

Finally we estimate a SW model with two common factors and correction for low frequency cycles whose results are reported in table 3.3. For the first coincident index we propose a ZAR(2) specification, meanwhile for the second one we use an AR(2). This is the best model in term of significativeness of coefficients and Likelihood, in a set of alternative parametrization, accounting for: numbers of common factors, combination of indicators and combination of lags.

It is relevant to notice that firstly survey data enter in this model and secondly that there is a clear separation between indicators: hard data load in the first coincident index, survey data in the second one. This confirms our a priori that allowing for more than one coincident indicator might point out the relevance of soft data, although the loading of GDP in the second common index is not significant. We consider that variables could enter in the model with lag, nevertheless we have not found evidence on it. The right column of Figure 4.3 shows the estimated monthly value added and the two coincident indexes, along with their first difference. The inclusion of a second coincident index has an evident effect on the first common component (see the central left and right panels of figure 1), which appears more volatile and dampened in the SW2-ZAR model. The second coincident index in differences seems to reproduce the cyclical behavior of the survey data with a positive shift for stocks and negative for the others indicators (compare with the pattern of Indicators in figure 2).

Some diagnostics and goodness of fit measures for the SSF might be based on the one step ahead forecast errors, that are given by $\tilde{v}_{t,i} = v_{t,i} - \mathbf{V}'_{t,i}\mathbf{S}^{-1}_{t,i}\mathbf{s}_{t,i}$, with variance $\tilde{f}_{t,i} = f_{t,i} + \mathbf{V}'_{t,i}\mathbf{S}^{-1}_{t,i}\mathbf{V}_{t,i}$. The standardised innovations, $\tilde{v}_{t,i}/\sqrt{\tilde{f}_{t,i}}$ can be used to check for residual autocorrelation and departure from the normality assumption. However, on the goal of the study we consider more interesting to compare the competitor models in terms of nowcasting and forecasting ability, which is done in the next section.

3.4.2 Comparative performance of rolling forecasts

Bridge models and in general model for monthly GDP has been widely used to produce forecasts, which are an important requirement for the economic analysis and the conduct of the economic policy. As a consequence, it might be worth to evaluate the three competitor models under consideration, the ADLD, the SW single index and the SW2-ZAR, in terms of forecast accuracy. As common in the literature we use a rolling experiment as an out-of-sample exercise. This corresponds to split the sample period in two parts, the first of which is used for the estimation and the second for evaluation, considering a measure of distance between forecasts and realized observations.

In this context a well known issue is how to split the series between the pre-forecast and the test period. There is not a fixed rule, but considering that the sample starts from 1995 and that we are interested in short term forecast, we run the rolling experiment over 54 consecutive observations in the sample 2001M1-2005M6. Hence, starting from January 2001, the three models are estimated at monthly level and quarterly forecasts of the value added are computed up to 3 step-ahead summing up the monthly data. Then, the forecast origin is moved one step forward and the process is repeated until the end of sample is reached, or 54 times. The model is re-estimated each time the forecast origin is updated, and so parameter estimation will contribute as an additional source of forecast variability. As a benchmark, we run the same exercise taking the parameters constant, as estimated using the information set available at the end of the sample.

All forecast experiments are made in "pseudo" real-time, so as to consider at each observation in time the last release for monthly and quarterly indicators. Therefore distinction is made regarding the position of the month inside the quarter, to account for different delays in the indicators releases. In particular, for the third month in the quarter, we should incorporate in the forecast the anticipated release of the quarterly value added. No account is made at this step for data revisions which is the main topic of the next section.

In table 3.4 and 3.5 we report a few basic statistics upon which forecasting accuracy will be addressed, taking parameters constant and re-estimated them each time. Monthly estimates are aggregated at quarterly frequency before computing any measure of errors, being our benchmark the national account value added. Denoting the l-step ahead forecast by $\widehat{y}_{t+l|t}$ and the true realized value by y_t, we compute for the three competitor models: the average of the forecast mean error (ME), $(\widehat{y}_{t+l|t} - y_{t+l})$; the symmetric mean absolute percentage error (sMAPE), given by the average of $100|y_{t+l} - \widehat{y}_{t+l|t}| \setminus [0.5(y_{t+l} + \widehat{y}_{t+l|t})]$, which treats symmetrically underforecasts and overforecasts; the median relative absolute error (mRAE) a robust comparative measure of performance, obtained by computing the median of the distribution of the ratios $|y_{t+l} - \widehat{y}^{(M)}_{t+l|t}| \setminus$

$|y_{t+l} - \widehat{y}^{(ADLD)}_{t+l|t}|$, where M is the model under consideration. Finally, we add the mean square forecast error (MSFE).

For the ADL(1,1)D, the SW2-ZAR and the SW model, these statistics are reported in table 3.4 and 3.5 with distinction of the month in the quarter, and the forecast horizon as resulted from the rolling experiment.

The ADLD model is almost always encompassed by the multivariate models, between which the SW2-ZAR model makes the lowest forecast error, unless in few exceptions and in terms of ME. This evidence is stronger as the forecast horizon goes forward and the information set goes smaller (1st month). In the re-estimated results, this evidence is even stronger and the SW2-ZAR model gains in term of performance especially for 2 and 3 step ahead forecast.

In addition to those statistics, the forecast accuracy of pairwise models could be tested formally by using the Diebold-Mariano test. It is worth to clarify that although the SW and SW2-ZAR models are nested, the real time nature of the rolling experiment validates the applicability of the Diebold-Mariano test (see Giacomini and White 2003). In table 3.6 we report the p-values test for the three models, with distinction of the month in the quarter and the horizon forecast, which intend to be compared with the usual threshold of 5%. There is strong evidence of significant different forecasts between multivariate SW-type and univariate ADLD model. Nevertheless the hypothesis of equal forecast accuracy of the single and the double SW model is not overall rejected, with particular evidence for the re-estimated version. In line with the previous forecast error analysis, the SW2-ZAR model seems to be preferable for 3 step ahead forecast. Although this could not be consider as a general result, for this empirical application the evidence is in favor of multivariate models, especially the SW2-ZAR.

3.4.3 Revisions and Contribution to the estimation

In this section we attempt to isolating the news content of each block of series used in the estimation of GDP, namely survey data and hard data. For this task we present some forecast exercises using real time data from the Euro area Real Time database, providing vintages of time series of several macroeconomic variables. The revision process is supposed to incorporate the more recent information available and therefore could not be neglected in our purpose. In particular, in order to address the issue of timeliness and news of content of data, we consider how much estimates change when a new block of series is released. We wish to figure out whether survey data matter for the estimation of GDP because their timeliness and/or because their content.

As for the forecast exercise, we consider 54 rolling forecasts staring from 2001M1, so that the last estimated quarter is 2005Q2. At each period in time the input in the model are the quarterly revised value added along with the revised indicators, unless for the series of hours worked because of the lack of vintages. The model is run more than once per month, and in particular every time a block of indicators is made available. Because we consider only two blocks of variables, hard and soft data, twice per month a new estimate of the value added is calculated and compared with the previous one.

In table 3.7 are displayed the results for the models with both constat and re-estimated parameters. As expected the most relevant change in the estimate occurs when Industrial Production is released, and this evidence is amplified for the SW2-ZAR model (0.38% on average). Nevertheless contribution of survey data seems to play a role, the more the horizon goes ahead and the more the information set is small. As expected the impact is higher in the first month of the quarter, because of the lack of hard data information. The results are even stronger in the re-estimated model. In particular the impact of survey on the prevision of GDP 3-step ahead made in the 1th month of the quarter (0.39%) is higher than the corresponding of Industrial production (0.38%). The evidence suggests that the more the forecast horizon increase the more timeliness of data is relevant. This is in line with the findings of Giannone *et al.* (2005).

To conclude we claim that survey data contribution to the estimation is not negligible, and this is probably because of their timeliness.

3.5 Conclusions and directions for future research

This Chapter mainly deals with the issue of macroeconomic variables disaggregation and estimation. The aim is to explore whether the inclusion of high frequency data might improve estimation accuracy and forecast ability.

The methodology proposed for the estimation at monthly level is based prominently on the Stock and Watson (1991) dynamic factor model accordingly to the procedure presented in Chapter 2. The extension to a model with more than one common factor and a correction for low frequency cycle is presented.

We propose an application to the valued added for Industry in the Euro Area and we compare the extended model versus the original SW formulation in term of the forecast ability. The issue of data revisions and content of news in each block of series, survey and hard data, is also analyzed.

In conclusion we found evidence for better performance of a model including hard and survey data, especially in term of forecast errors. As far as the news content of data is concerned, information from survey is related to the lack of hard data. This evidence is more persistent as the information set is small (first month in the quarter) and as the horizon forecast increase (three step ahead).

Table 3.1: Autoregressive Distributed Lag model for Industry (ADL(1,1)D) with trend: parameter estimates and asymptotic standard errors, when relevant

Variables	*coef.*	*StDev*	*t-stat*
Drift	22.81	9.35	2.44
Trend	-0.02	0.02	-1.18
production	1.01	0.16	6.40
hours worked	0.20	0.36	0.55
EA.climate	-2.31	0.92	-2.51
EA.production	1.78	0.68	2.62
EA.orders	0.67	0.33	2.01
production(1)	-1.00	0.15	-6.48
hours worked (1)	-0.41	0.35	-1.17
EA.climate(1)	2.28	0.96	2.36
EA.production(1)	-1.72	0.70	-2.45
EA.orders(1)	-0.68	0.34	-1.97

Note: The label EA indicates that the variable comes from the Business Survey on firms. The script (1) stands for one lag of the variable.

Table 3.2: Dynamic factor model for Industry (SW): parameter estimates and asymptotic standard errors, when relevant

Parameters	*prod*	*howk*	*Value added*
θ_{i0}	0.603	0.218	0.745
	(0.087)	(0.053)	(0.121)
δ_i	0.297	-0.164	0.187
	(0.066)	(0.032)	(0.039)
d_{i1}	-0.587	-0.357	
d_{i2}	-0.231	-0.089	
$\sigma^{\mathbf{2}}{}_{\eta}$	0.140	0.099	3.45E-07
$\left(1-0.44L-0.196L^2\right)\Delta\mu_t=\eta_t, \quad \eta_t\sim N\left(0,1\right)$			

Note: standard errors in parenthesis. The label EA indicates that the variable comes from the Business Survey on firms.

Table 3.3: Dynamic factor model with 2 factors and modification for low frequency cycles (SW2-ZAR): parameter estimates and asymptotic standard errors, when relevant

Parameters	*prod*	*howk*	*EA.clim*	*EA.prod*	*EA.ord*	*Value added*
θ_{i0}	0.660	0.196	0.001	-0.005	0.005	0.675
	(0.113)	(0.075)	(0.019)	(0.016)	(0.016)	(0.136)
$\widetilde{\theta_{i0}}$	0.021	0.012	0.175	0.175	0.157	0.708
	(0.018)	(0.009)	(0.017)	(0.02)	(0.028)	(0.191)
δ_i	0.017	-0.156	0.141	0.237	0.025	0.225
	(0.005)	(0.075)	(0.220)	(0.372)	(0.037)	(0.04)
d_{i1}	0.484	-0.644	0.093	0.922	0.691	
d_{i2}	0.434	-0.138	0.495	-0.621	0.234	
$\sigma^{\mathbf{2}}{}_{\eta}$	0.053	0.101	0.006	0.002	0.008	0.103

$$\left(1+0.56L+0.38L^2\right)\Delta\mu_t=(1-0.5L)^2\eta_t,\quad \eta_t\sim N\left(0,1\right)$$
$$\left(1+1.571L-0.597L^2\right)\Delta\widetilde{\mu}_t=\widetilde{\eta}_t,\quad \widetilde{\eta}_t\sim N\left(0,1\right)$$

Note: standard errors in parenthesis. The label EA indicates that the variable comes from the Business Survey on firms.

Table 3.4: Statistics on forecast performance with constant parameters for 54 rolling estimates (2001M1-2005M6).

		ADL(1,1)D Model			**SW Model**			**SW2-ZAR Model**		
		1-step	*2-step*	*3-step*	*1-step*	*2-step*	*3-step*	*1-step*	*2-step*	*3-step*
ME	1^{st} Month	175	$\underline{-483}$	$\underline{-930}$	137	1,265	2,408	$\underline{-126}$	503	1,235
	2^{nd}	620	$\underline{201}$	$\underline{-352}$	$\underline{-45}$	933	2,055	-232	349	1,156
	3^{thd}	$\underline{-246}$	$\underline{-774}$	$\underline{-1{,}574}$	661	1,926	2,727	303	1,179	1,634
MAE	1^{st} Month	1,508	2,738	3,372	$\underline{836}$	2,488	3,894	878	$\underline{2{,}223}$	$\underline{3{,}546}$
	2^{nd}	1,746	3,211	4,255	$\underline{726}$	2,221	3,966	765	$\underline{1{,}999}$	$\underline{3{,}497}$
	3^{thd}	2,024	3,239	4,116	1,323	3,103	4,124	$\underline{1{,}246}$	$\underline{2{,}478}$	$\underline{3{,}569}$
MAPE	1^{st} Month	0.45	0.81	1.00	$\underline{0.25}$	0.74	1.15	0.26	$\underline{0.66}$	$\underline{1.05}$
	2^{nd}	0.52	0.95	1.26	$\underline{0.22}$	0.66	1.17	0.23	$\underline{0.59}$	$\underline{1.03}$
	3^{thd}	0.60	0.96	1.22	0.39	0.92	1.22	$\underline{0.37}$	$\underline{0.73}$	$\underline{1.05}$
RMSFE	1^{st} Month	1,226	2,260	2,809	$\underline{737}$	2,048	3,381	810	$\underline{1{,}728}$	$\underline{3{,}325}$
	2^{nd}	1,384	2,300	$\underline{2{,}912}$	595	1,881	3,665	$\underline{580}$	$\underline{1{,}764}$	3,507
	3^{thd}	1,987	2,680	4,573	868	3,390	3,556	$\underline{872}$	$\underline{2{,}291}$	$\underline{3{,}095}$
mRAE	1^{st} Month				$\underline{0.5}$	$\underline{0.9}$	1.3	$\underline{0.5}$	$\underline{0.9}$	1.2
	2^{nd}				$\underline{0.4}$	$\underline{0.6}$	$\underline{0.8}$	$\underline{0.4}$	$\underline{0.5}$	$\underline{0.8}$
	3^{thd}				$\underline{0.7}$	1.1	1.3	$\underline{0.7}$	$\underline{0.6}$	$\underline{0.8}$

The smallest values for each measure are underlined, unless for the mRAE where the benchmark is 1.

Table 3.5: Statistics on forecast performance with estimated parameters for 54 rolling estimates (2001M1-2005M6).

		ADL(1,1)D Model			**SW Model**			**SW2-ZAR Model**		
		1-step	*2-step*	*3-step*	*1-step*	*2-step*	*3-step*	*1-step*	*2-step*	*3-step*
ME	1^{st} Month	133	-507	-954	131	1,002	2,100	-407	-26	836
	2^{nd}	121	-450	-1,000	-9	701	1,774	-454	-200	622
	3^{thd}	-593	-1,038	-1,886	524	1,628	2,398	206	1,059	1,739
MAE	1^{st} Month	1,827	3,258	3,871	780	2,486	4,009	1,259	2,297	3,295
	2^{nd}	2,199	3,859	4,772	700	2,446	4,112	1,179	2,156	3,551
	3^{thd}	2,349	3,105	4,260	1,351	2,911	3,883	1,552	2,717	3,689
MAPE	1^{st} Month	0.54	0.97	1.15	0.23	0.74	1.19	0.38	0.68	0.98
	2^{nd}	0.65	1.15	1.42	0.21	0.73	1.22	0.35	0.64	1.05
	3^{thd}	0.70	0.92	1.26	0.40	0.86	1.15	0.46	0.81	1.09
RMSFE	1^{st} Month	1,771	2,947	3,239	480	2,137	3,715	876	2,113	2,545
	2^{nd}	1,963	4,283	4,246	442	2,107	3,694	1,042	1,837	3,710
	3^{thd}	2,282	2,719	4,323	837	3,101	3,213	1,370	2,244	2,850
mRAE	1^{st} Month				0.4	0.8	1.5	0.9	0.6	1.0
	2^{nd}				0.5	0.6	0.8	0.8	0.5	0.6
	3^{thd}				0.4	0.9	1.1	0.6	1.0	0.0

The smallest values for each measure are underlined, unless for the mRAE where the benchmark is 1.

Table 3.6: Diebold-Mariano test (p-values) of equal forecast accuracy by horizon of forecast (1,2,3 quarters) and month of the prevision (1st, 2nd, 3td of the quarter).

Constant parameters			
	1-step	*2-step*	*3-step*
SW *vs* ADLD	0.001	0.023	0.470
SW2-ZAR *vs* ADLD	0.001	0.000	0.026
SW2-ZAR *vs* SW	0.688	0.001	0.005
	1^{st} *Month*	2^{nd} *Month*	3^{thd} *Month*
SW *vs* ADLD	0.590	0.001	0.414
SW2-ZAR *vs* ADLD	0.066	0.000	0.064
SW2-ZAR *vs* SW	0.059	0.066	0.043
Estimated parameters			
	1-step	*2-step*	*3-step*
SW *vs* ADLD	0.000	0.000	0.176
SW2-ZAR *vs* ADLD	0.001	0.000	0.008
SW2-ZAR *vs* SW	1.000	0.163	0.039
	1^{st} *Month*	2^{nd} *Month*	3^{thd} *Month*
SW *vs* ADLD	0.188	0.002	0.258
SW2-ZAR *vs* ADLD	0.009	0.000	0.151
SW2-ZAR *vs* SW	0.203	0.219	0.223

Table 3.7: Averaged size of the news in the estimation, real time data for 54 rolling forecasts (2001M1-2005M6).

Constant parameters

		SW2-ZAR Model			SW Model		
News in Ω		1-step	2-step	3-step	1-step	2-step	3-step
Surveys	1^{st} Month	0.03	0.14	0.27			
	2^{nd}	0.02	0.11	0.24			
	3^{thd}	0.00	0.06	0.14			
IP	1^{st} Month	0.40	0.44	0.41	0.35	0.40	0.41
	2^{nd}	0.27	0.43	0.41	0.27	0.41	0.43
	3^{thd}	0.12	0.51	0.44	0.09	0.43	0.40

Estimated parameters

		SW2-ZAR Model			SW Model		
News in Ω		1-step	2-step	3-step	1-step	2-step	3-step
Surveys	1^{st} Month	0.15	0.29	0.39			
	2^{nd}	0.10	0.33	0.41			
	3^{thd}	0.04	0.30	0.32			
IP	1^{st} Month	0.31	0.38	0.38	0.31	0.38	0.39
	2^{nd}	0.27	0.47	0.47	0.23	0.43	0.44
	3^{thd}	0.15	0.43	0.45	0.09	0.45	0.43

The news is measured by the Mean Absolute Relative difference between two consecutive vintages : $100 * abs[(Y1 - Y0)/Y0]$

Figure 3.1: Temporal disaggregation of value added of Industry: Eurozone12, 1995M1-2006M9. ADL(1,1)D with trend.

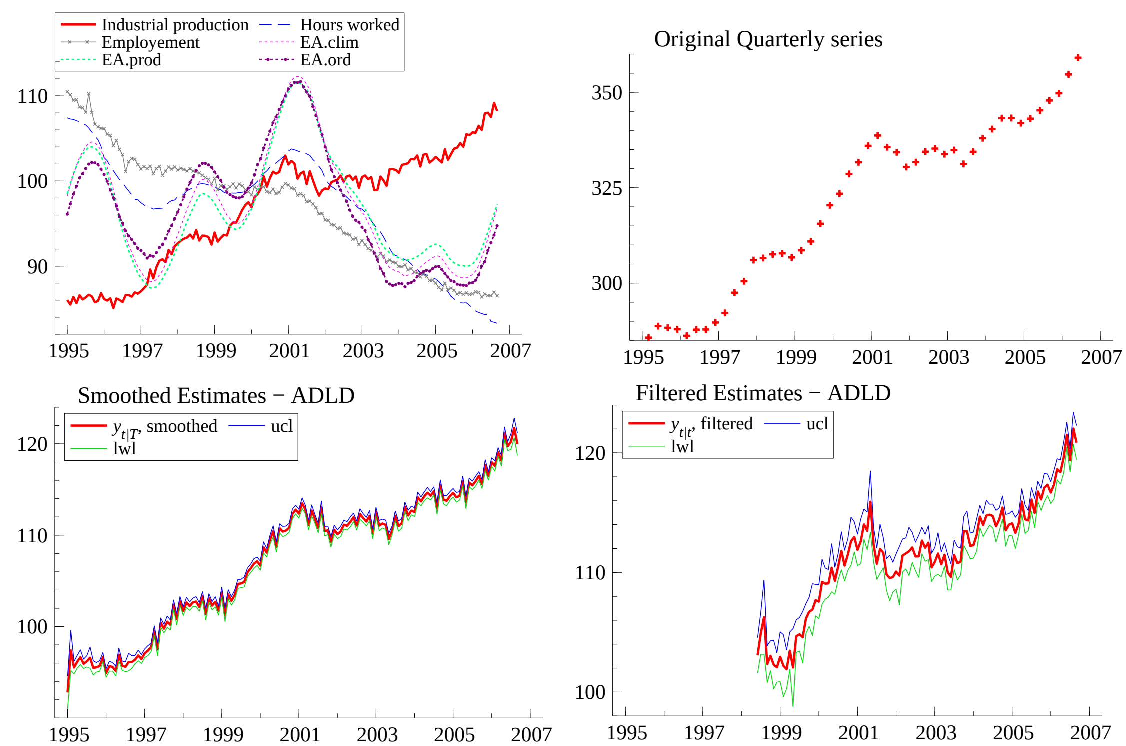
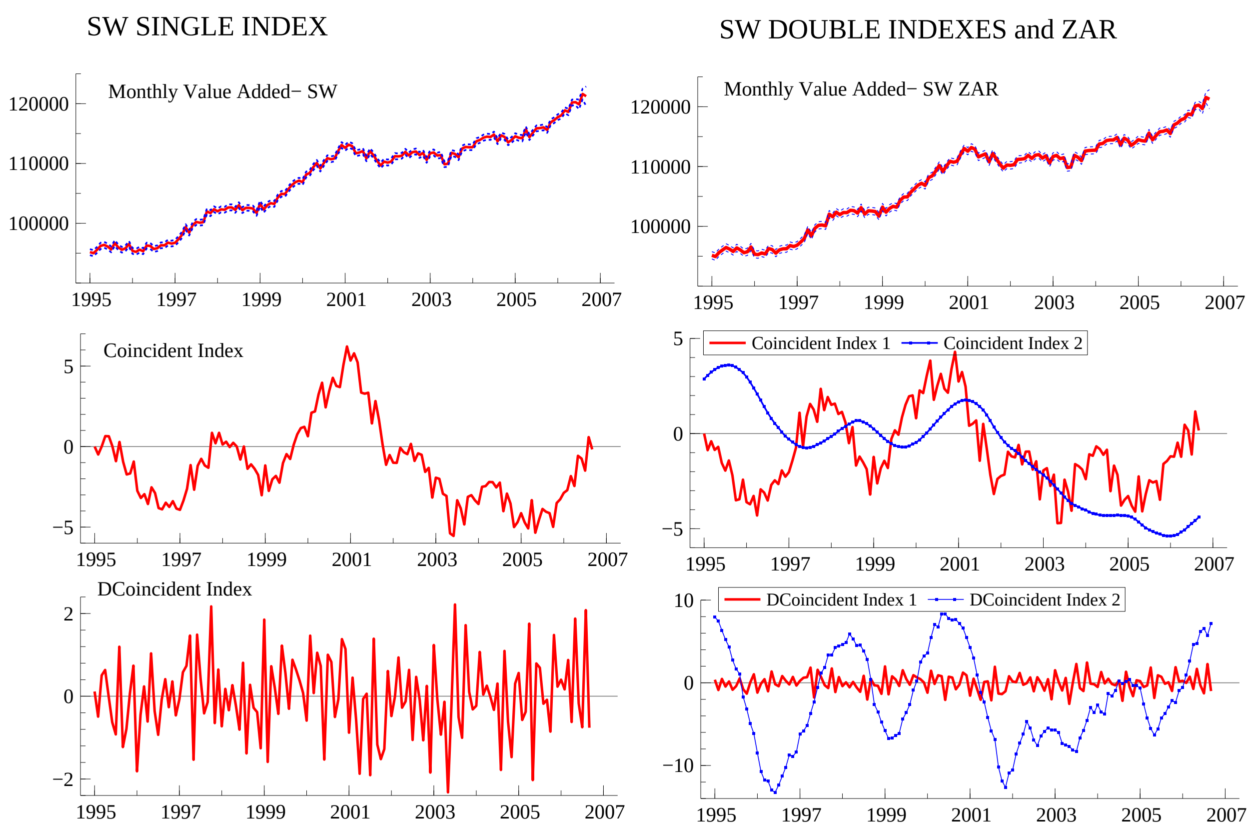


Figure 3.2: Temporal disaggregation of value added of Industry: Eurozone12, 1995M1-2006M9. Dynamic SW factor model.



Chapter 4

A Coincident Index for the US Economy with time Varying Variance

ABSTRACT[1]**:** In this paper we estimate a coincident index for the US economy and its fluctuations, as well as the monthly GDP, extending the model of Mariano and Murasawa (2003). Monthly GDP is highly volatility and there is a clear cut in its behaviour prior and posterior to the great moderation. Moreover, we show that the determinants of growth, mainly industrial production, differ to the determinants of fluctuations in the economy, which are driven by employment and income. The estimated volatility shows that the US economy suffered regular episodes of stress up to the great moderation, while September 11 is the most relevant shock after the mid 80s. Last, we quantify the changes in forecasting accuracy by using VOLINX in contrast with the assumption of constant uncertainty.

Keywords: Temporal Disaggregation. Multivariate State Space Models. Dynamic factor Models. Kalman filter and smoother. Grach model.
JEL Classification: C32, C51, E32, E37

[1]This Chapter has been realized under the advise of David Veredas to whom are directed my sincere thanks for patient and careful supervision during the period I spent in ECARES. I wish to thank also Tommaso Proietti for additional feedbacks and supervision on my results. I also benefitted from the discussion with Charles Bos, Kevin Shepard, Gabriele Fiorentini, Timo Terasvirta, Franco Peracchi and Gianluca Cubadda. Routines are coded in Ox 3.3 by Doornik (2001) and provide an extension allowing for volatility of the programs realized by Proietti T. for the Eurostat project on the Monthly GDP estimation.

4.1 Introduction

Timely information, say monthly, on the state of the economy is of paramount importance for economic policy decision making. This information is typically summarized by the level of the economy and its fluctuations. In this Chapter we provide estimates of these two measures considering a monthly coincident index for GDP and its variance. Furthermore we disaggregate the quarterly GDP into monthly values. From a policy view point, monthly GDP, generally considered as the major indicator of the stance of the economy, can be used to help in the intra quarterly decision making, surveillance of the economy and for GDP nowcast and forecast. As the uncertainty in the economy is concerned, the estimated volatility, expressed as a variance conditional on past square errors of several key economic indicators, allows investigating the determinants of the fluctuations of the economy.

We extent the traditional Mariano Murasawa model and estimate on equal footing on the monthly frequency, the GDP, the coincident indicator and its variance, that we call VOLINX. Using the information contents of the most relevant macroeconomic indicators, as common in the literature [2], we estimates the monthly GDP by disaggregation of the quarterly value according to the technique for state space models developed by Harvey (1989). In a nutshell, this technique is based on augmenting the state vector to include monthly GDP as a latent process that can be estimated with the Kalman filter. A similar approach has been used by Mariano and Murasawa (2003, 2004). However, they express the model in difference and they make use of the EM algorithm as preliminary step for the smoothing and filtering procedure. Our model is formulated in levels so as to provide a measure of the estimation errors for the level of GDP and we use the multivariate treatment of the univariate series by Koopman and Durbin (2000). In particular, we use the initialization by de Jong (1990), which is a very convenient tool for handing with missing observations and intervention variables (such as outliers and calendar effects). This is an other feature that distinguish our model from previous works. For additional details on the statistical treatment and the disaggregation see Proietti and Frale (2006).

To estimate VOLINX, we rely on GARCH type of models, however, the GARCH model that we propose is not standard for two reasons. First, because it is a conditional volatility model for an unobserved component, and therefore the errors are unobserved as well.[3] Second, because we replace the autoregressive component of the GARCH model, the ARCH part, by a linear combination of the standardized squared errors of the idiosyncratic components of the economic indicators. The weights of the linear combination, which are estimated endogenously, have a similar interpretation to the factor loadings for the conditional mean. We can therefore infer which are the most relevant variables that explain the volatility of the economy. Moreover, VOLINX is the common volatility for all the indicators up to an idiosyncratic scale. This is again similar to the conditional mean where the coincident indicator is common to all the indicators up to an idiosyncratic term. In addition, we introduce a multiple regimes mechanism that mimic the well know topic of the ”great moderation”.

It is worthwhile to note that these two issues, disaggregation and dynamic conditional volatility,

[2] We use the same indicators used by Stock and Watson (1991) and Mariano and Murasawa (2003), among others.

[3] This was first noted by Harvey, Ruiz and Sentana (1991).

are strongly related. It is known that the lower the frequency the more homoscedastic and the less dependent the time series become [4].

Our results show that monthly GDP presents heteroscedasticity and volatility clustering with a clear cut in the fluctuations prior and posterior to the great moderation in the mid 80s. The monthly coincident indicator is highly volatile as well and its variance, proportional to VOLINX, and the determinants of volatility in the US economy differ to the determinants of the level of growth. Furthermore VOLINX allows surveillance of the uncertainty detecting periods of stress for the US economy. In particular, our results are in line with the empirical evidence of high fluctuations corresponding to wars (Vietnam 1971), oil crises (OPEC embargo 1974) and instability sentiments (September 11 attacks).

VOLINX has relevance also for forward looking tasks, namely nowcasting and forecasting. Comparing the GARCH model with the standard constant variance model, we show that the later is clearly misspecified as it underestimates the fluctuations prior to the great moderation and exacerbates them posterior to mid 80s.

The structure of the Chapter is as follows. After the revision of the background literature and some stylized facts (section 4.2), Section 4.3 shows the model that allow to disaggregate GDP and deal with volatilities. The model is explained in simple terms and all the technicalities, which are substantial, are relegated to the Appendix. Section 4.4 shows the empirical results and the forecasting exercise on the monthly GDP considering several benchmark models for the volatility. Some preliminary insight about the date of the changing regime for the variance are discussed. Some conclusion ends.

4.2 Stylized fact and background literature

This work is a by-product of two different research directions. On one hand, it relates with the estimation of high frequency GDP and synthetic coincident indexes. On the other hand, it relates with the literature of the causes of the great moderation and its analysis in terms of conditional volatility models.

There is an increasing number of articles on monitoring the evolution of the economy through coincident indexes and high frequency measures of the state of the economy. We can mainly identify two approaches: studies that use monthly information related to GDP to construct a composite coincident indicator, and studies that disaggregate quarterly GDP. The first group of works tracks back to the seminal paper of Stock and Watson (1991)(SW henceforth). They develop a probability parametric model for the coincident index by using a reduced set of well chosen economic variables that are believed to contain the most relevant information about the state of the economy. In a more recent work Stock and Watson (2002a), instead of choosing a small number of indicators, opt for a large scale dynamic factor model, which is also the spirit of Forni *et al* (2000). Working with a monthly observation frequency, this approach does not consider GDP, as it is available quarterly,

[4] In an ARMA-GARCH setting, Drost and Nijman (1993) theoretically show the link between the parameters of ARMA-GARCH models at different frequencies. The parameters that capture the dynamics are a positive function of the aggregation frequency, i.e. the lower the frequency the closer the parameters are to zero and therefore the more memoryless the process becomes. A survey on temporal aggregation techniques is Silvestrini and Veredas (2007).

although it is overall considered as the main measure of the state of the economy. This consideration motivates a second direction of research, which has focused on disaggregation of quarterly GDP into higher frequencies. Chow and Lin (1971) first showed how GDP monthly series could be constructed from regression estimates using monthly data related to GDP. Several authors improved this idea following different multivariate disaggregation methods (see Harvey and Chung, 2000, Moauro and Savio, 2005 and Proietti and Moauro, 2006). Evans (2005) presents an innovative model for estimates of daily GDP. Finally, Mariano and Murasawa (MM)(2003) combines the two approaches -disaggregation and coincident index- casting the initial Stock and Watson model in a linear state space set up defined at monthly frequency and including the quarterly GDP as additional source of information to the monthly indicators.

The other research direction that this Chapter relies on is the analysis of the great moderation in the US economy and its explanation in terms of volatility models. The literature above mentioned neglects the stylized evidence that most of the US macroeconomic series have shown a change in the volatility pattern in the last 30 years. As document by Stock and Watson (2002b), most of the 168 US economic variables they analyze experiment a decline in the volatility, which is characterized by a break in the fluctuations in the mid 80s. Figure 4.1 evidence this fact. It shows the square of the first differences of demeaned quarterly GDP and monthly employment, sales, industrial production an income, all in logs. The time period spans from 1968 to 2006. Since first differences are demeaned, their squares are a good proxy for volatility. The straight lines are the sample variances, which are also shown in Table 4.1. The change in the volatility pattern is clear except, probably, for sales. The great moderation induced a significant decrease in growth fluctuations. For instance, the sample variance of GDP prior to 1985 was 1.26 while for the period 1985-2006 it declined to 0.23, that is a decrease of more than 5 times.

On these grounds, Stock and Watson (2002b) estimate univariate dynamic conditional variances for the 168 series in the form of Stochastic Volatility (SV henceforth) models. Middle panel of Table 4.1 suggests this approach also in our sample. It shows the GARCH(1,1) estimates of the same variables as in former figure. All variances present persistence, measured by α_0, and they are all affected by shocks, measured by α_1. To support this evidence, we test formally the hypothesis of ARCH structure of the residuals in the MM model by using the unilateral Demos and Sentana (1998) test. Results are reported in the bottom panel of Table 4.1. For all monthly series the hypothesis of conditional homoscedasticity could not be accepted at 5% level. For the quarterly GDP the null of homoscedasticity is rejected only at 10% level although the value is very close to the threshold for 5%.

Stock and Watson (2006) develop further the SV model for inflation. Based on the fact that an ARIMA(0,1,1) model can be expressed like a local level model (i.e. random walk plus noise), they disentangle volatility into permanent and transitory components, and consider time-varying moving average parameters. This flexibility allows them to explain a variety of recent univariate inflation forecasting puzzles. Several recent studies have undertaken a similar avenue considering the fluctuation of the US economy with particular focus on inflation after the 80s and monetary policy. By using different models and theoretical approaches, stochastic volatility rather than Bayesian or VAR models, they all end up finding evidence of a decrease in volatility for inflation, which is the reflection of a change in the monetary regime (Cecchetti et al. (2006) and Primiceri

(2005)), or a change in the government policy makers models (Cogley and Sargent (2005))or a change in policy rule coefficients (Sims and Zha (2005)).

4.3 The model

We present in this section the model that we propose for the disaggregation of GDP and for the estimation of a coincident index for the economy and its variance.

Let $\mathbf{y}_t$ denote an $N \times 1$ vector of time series, that we assume to be integrated of order one, or $I(1)$, so that $\Delta y_{it}, i = 1, \ldots, N$, has a stationary and invertible representation. This vector contains both the monthly indicators and the quarterly GDP. We will show later how to deal with the mixed frequency issue. Following the standard literature, $\mathbf{y}_t$ is expressed as the linear combination with loadings $\boldsymbol{\vartheta}_0$ and $\boldsymbol{\vartheta}_1$ of a common cyclical trend (denoted by μ_t) and idiosyncratic components ($\boldsymbol{\mu}_t^*$) specific for each series :

$$\mathbf{y}_t = \boldsymbol{\vartheta_0}\mu_t + \boldsymbol{\vartheta_1}\mu_{t-1} + \boldsymbol{\mu}_t^* + \mathbf{X}_t\boldsymbol{\beta}, \qquad t = 1, ..., T$$

that could be written also as

$$\mathbf{y}_t = \boldsymbol{\theta_0}\mu_t + \boldsymbol{\theta_1}\Delta\mu_{t-1} + \boldsymbol{\mu}_t^* + \mathbf{X}_t\boldsymbol{\beta}, \qquad t = 1, ..., T \tag{4.3.1}$$

with $\theta_0 = \vartheta_0 + \vartheta_1$ and $\theta_1 = -\vartheta_1$. [5]

According to the background literature (SW and MM already mentioned papers) the common index follows an autoregressive difference stationary process, which is an AR(1) in our application:

$$(1 - \phi L)\Delta\mu_t = \eta_t. \tag{4.3.2}$$

The idiosyncratic components are defined as AR(1) difference stationary as well:

$$\mathbf{D}(L)\Delta\boldsymbol{\mu}_t^* = \boldsymbol{\delta} + \eta_t^*, \tag{4.3.3}$$

where $\mathbf{D}(L)$ is a diagonal polynomial matrix:

$$\mathbf{D}(L) = diag\left[(1 - d_1 L), (1 - d_2 L), \ldots, (1 - d_N L)\right].$$

The disturbances $\eta_t \sim \text{NID}(0, \sigma_t^2)$ and $\boldsymbol{\eta^*}_t \sim \text{NID}(\mathbf{0}, \sigma_t^2\boldsymbol{\Sigma}_{\eta^*})$ are mutually uncorrelated at all leads and lags and $\boldsymbol{\Sigma}_{\eta^*} = \text{diag}(\sigma^2_{\eta_1^*}, \ldots, \sigma^2_{\eta_N^*})$ is the time constant matrix of variances of the idiosyncratic specific components.

The matrix $\mathbf{X}_t$, is a $N \times k$ matrix containing the values of exogenous variables that are used to incorporate k calendar effects (trading day regressors, Easter, length of the month) and intervention variables (level shifts, additive outliers, etc.).

[5] We prefer the specification of the model in level, instead of in difference, because this is a very convenient formalization to deal with missing observations and the disaggregation temporal constraint. We wrote the measurement equation according to equation (4.3.1) in the light of the SSF reported in the Appendix.

The fundamental innovation of our model is the inclusion of the time varying factor σ_t^2, which is responsible for the fluctuations of the coincident index and of idiosyncratic components. This factor is what we call VOLINX and represents the volatility of the economy. There are several possibilities for the specification of VOLINX.

In the light of the empirical evidence presented in the introduction, one may think about a variance mechanism that considers two regimes, prior and after the beginning of the great moderation, in line with Stock and Watson (2004) and the top panel of Table 4.1:

$$\sigma_t^2 = \varpi_1 I_{[1]} + \varpi_2 I_{[2]} \tag{4.3.4}$$

where $I_{[1]}$ ($I_{[2]}$) is an indicator function that takes value 1 for t prior (posterior) to the great moderation. This is our first model, to be compared with MM (the benchmark), and can be considered as naive in the sense that it does not capture the conditional heteroskedasticity found in the data (c.f. middle panel of Table 4.1). This apparent dynamic behavior of the volatility suggests a more complex structure.

A finer parametrization is a GARCH type of model. These models are quite cumbersome in this context of unobserved components as the residuals is not the difference between the observed and the prediction but, rather, between the unobserved and its prediction.[6] Harvey, Ruiz and Sentana (1991) introduce a GARCH model for unobserved components defined in a state space framework. Adapted to our case:

$$\sigma_t^2 = (1 - \alpha_0 - \alpha_1) + \alpha_0 \sigma_{t-1}^2 + \alpha_1 \eta_{t-1}^2, \tag{4.3.5}$$

where $0 \leq \alpha_0 < 1$, $0 \leq \alpha_1 < 1$, $\alpha_0 + \alpha_1 < 1$. The authors suggest to replace the unobserved error by its conditional expectation $E(\eta_{t-1}^2 / Y_{t-1})$ where Y_{t-1} is the information set at time (t-1). The intercept term $(1 - \alpha_0 - \alpha_1)$ is such that the unconditional variance of the coincident index is 1, in accordance with SW and MM. Even if appealing from the econometric point of view, this avenue lacks of economic meaning. As indicator of the underlying volatility that drives the economy, it should be caused, in some sense, by the fluctuations of the macroeconomic indicators. Note however, that this model, or any volatility model presented in this section, does explain the contemporaneous volatility of the indicators as their conditional volatility is $\sigma_t^2 \mathbf{\Sigma}_{\eta^*}$.

In order to established the link between past fluctuations of the indicators and the volatility of the coincident index, we consider instead the following model

$$\sigma_t^2 = (1 - \alpha_0 - \alpha_1) + \alpha_0 \sigma_{t-1}^2 + \alpha_1 \sum_{i=1}^{N} \omega_i \nu_{i,t-1}^2, \tag{4.3.6}$$

where $0 \leq \alpha_0 < 1$, $\alpha_1 \geq 0$, $\omega_i > 0$, and $\sum_{i=1}^{N} \omega_i = 1$. The first component is standard in traditional GARCH literature and captures the persistence: the expected variance is expressed as a function of its past values. The second term, accounting for shocks, is expressed as linear combination of past square errors of the idiosyncratic components of each economic indicator. The vector ν_{it} contains the so-called standardized innovations of the Kalman filter: $\nu_{i,t} = (y_{i,t} -$

[6] Another possibility is to opt for SV models, where the aforementioned drawback is not present. But estimation of this models is even more cumbersome.

$\mathrm{E}(y_{i,t}|\mathbf{Y}_{t-1}))/Std(\nu_{i,t})$, where $\mathbf{Y}_{t-1}$ is the information set at time t. The weights of the linear combination, estimated endogenously, have a similar interpretation to the factor loadings for the conditional mean. We can therefore infer which are the most relevant variables that explain the volatility of the economy. Note that the unconditional variance of this model is still 1 as the sum of the weights is 1 and the standardized innovations have second moment equal to 1 as well. This is our second model, to be compared with MM.

The construction of these models have the constraint in the intercept, so they are comparable with MM. If α_0 and α_1 are set to zero, the models boil down to MM. But this is not, in this context, a technical constraint. Rather, it may be a pernicious constraint and a source of misspecification. For instance, while they do account for the stylized fact of the middle part of Table 4.1, they are unable to explain the apparent change in the level of the volatility around 1984 that top part of Table 4.1 and Figure 4.1 suggest. The literature on the great moderation discusses extensively, without widely accepted conclusion, about the starting date and the type of the change, whereas it was a break rather than a declining patter. Stock and Watson (2002b) date the great moderation using different methods (univariate and multivariate). They conclude a break date for most of the components series of GDP in 1983Q2 with a 67% confidence interval of 1982Q4 to 1985Q3, almost three years. Their conclusions are in the direction of a break rather than a trend decline.

We follow their suggestion and taking into account the stylized facts, we consider a feasible and suitable formalization through a 2 regimes GARCH-type model with two regimes for the unconditional variance:

$$\sigma_t^2 = \varpi_1 I_{[1]} + \varpi_2 I_{[2]} + \alpha_0 \sigma_{t-1}^2 + \alpha_1 \sum_{i=1}^{N} \omega_i \nu_{it-1}^2 \tag{4.3.7}$$

where $\varpi_1 \geq 0$, $\varpi_2 \geq 0$, $0 \leq \alpha_0 < 1$, $\alpha_1 \geq 0$, $\omega_i > 0$, and $\sum_{i=1}^{N} \omega_i = 1$. As earlier, $I_{[1]}$ ($I_{[2]}$) is an indicator function taking value 1 for t prior (posterior) to the great moderation. We consider the breaking date to January 1984, roughly the middle date of the confidence interval of Stock and Watson (2002b). Later we check how robust are the results and VOLINX to this choice. This model combines (4.3.4) and (4.3.6), which capture the stylized facts of change in the level of volatility and conditional heteroskedasticity. A decrease in volatility after the great moderation occurs if $\varpi_2 > \varpi_1$.

Regardless of the choice for σ_t^2, the model (4.3.1)-(4.3.3) has a linear state space form (SSF). The unknown parameters, which for (4.3.7) they are $(\vartheta_{0,i}, \vartheta_{1,i}, d_i, \delta_i, \omega_i, \sigma^2_{\eta_i^*}, i = 1, \ldots, N, \beta_k, k = 1, \ldots K, \phi, \varpi_1, \varpi_2, \alpha_0, \alpha_1)$, can be estimated by maximum likelihood by using the prediction error decomposition, which is estimated with the Kalman filter. Given the parameter values, the Kalman filter and smoother will provide the minimum mean square estimates of the coincicent index and the idiosyncratic components. In the Appendix we show all the technicalities. Proietti and Frale (2006) present a deep statistical treatment. Yet, there are two estimation issues that are worth emphasizing.

First, the model involves mixed frequency data, e.g. monthly indicators and quarterly GDP. Following the strategy proposed by Harvey (1989), the state vector in the SSF is suitably augmented by using an appropriately defined cumulator variable. In particular, the cumulator variable, at times $t = 3\tau, \tau = 1, \ldots, [T/3]$ coincides with the (observed) aggregated series, otherwise it contains the partial cumulative value of the aggregate in the seasons (e.g. months) making up the larger interval (e.g. quarter) up to and including the current one. In our analysis the series in the model are expressed in logarithms, therefore to deal with a linear constraint me make the approximation suggested by Mariano and Murasawa (2003, 2004) to disaggregate the quarterly ln(GDP) into three unobserved monthly values by the geometric mean (see Appendix for more details). Although there are finer solution of this problem, as Proietti (2006) shown, we make preference to simplicity, leaving in the future agenda additional investigations.

The resulting state space model is linear and, assuming that the disturbances have a Gaussian distribution, the unknown parameters can be estimated by maximum likelihood, using the prediction error decomposition, performed by the Kalman filter. Given the parameter values, the Kalman filter and smoother will provide the minimum mean square estimates of the states and thus of the missing observations on the cumulator. The monthly estimates of the GDP are obtained by ”decumulation”.

An other relevant aspect of the procedure adopted is the statistical treatments of the multivariate model. Anderson and Moore (1979) first consider the univariate treatment of multivariate models and Koopman and Durbin (2000) shown that it is a very flexible and convenient device for filtering and smoothing and for handling missing values. The main idea is that the multivariate vectors of indicators, where some elements can be missing, are stacked one on top of the other to yield a univariate time series, whose elements are processed sequentially. This facilitate the inclusion of some intervention variable in the model, especially useful for handing with outliers. The inspection of Figure 4.2 and also the results of the GARCH estimates in Table 4.1 suggest the presence of an outlier for the series of employment in August 1983, that cannot be ignored.

4.4 Main results

In this section we present the results as for the application of model (4.3.1) in combination with the four formalization for the volatility discussed in section 4.3- MM (constant volatility), MM-2regimes (4.3.4), GARCH (4.3.5), GARCH-2regimes (4.3.7)- to the US economy in the sample period 1968M1-2006M12. The competitors model are estimated considering the same information set: Employees on non agricultural payrolls, Index of Industrial Production, Manufacturing and trade sales, Personal Income less transfer payments; as well as considering the Real quarterly GDP (see footnote of Table 4.2 for additional details).

The estimated maximum likelihood parameters are reported in Tables 4.2-4.5, while the coincident indexes and the volatility corresponding to the four models are plotted in Figure 4.3.

As expected there is not relevant difference for the autoregressive parameters and idiosyncratic components in the volatility models respect to the standard MM model (confront Table 4.5 and 4.2 with Table 4.4). Industrial production presents the biggest loading[7] and this confirms the general consensus that it is one, if not the most, important determinant of the growth level of the

[7]Indicators are expressed in different unit of measures. For the comparison we consider the standardized loadings, obtained dividing by their standard deviation as suggested by Mariano and Murasawa (2003) . We conclude that their relative importance if not affected by the unit of measure.

economy. One refinement is the inclusion in our formalization of the outlier for employment in August 1983, which is suggested by the inspection of Figure 4.2 and appears to be significantly different from zero in the GARCH formalization (with or without switching). The effect of the outliers is visible in the plot of the monthly estimates of GDP in figure 4.4. Coincident index obtained by the GARCH models shows a similar pattern to the standard MM index (see top panel of figure 4.3), even if the long run values are slightly different because of the small differences in the estimated autoregressive parameters.

Relevant difference appears indeed from the analysis of the fluctuations of the economy.

The monthly coincident indicator is highly volatile. In fact, its variance, which is VOLINX, unveils the most interesting finding of the article. The determinants of volatility in the US economy differ to the determinants of the level of growth: the most important determinant for the volatility is employment, followed by income. And industrial production becomes the less important (compare w_{it} in table 4.4 and 4.5).

The two regimes variance parameters are not significant in the standard MM model, but they do are in the multiple regimes GARCH model (see ϖ_1 and ϖ_2 in table 4.3 and 4.5). This might be an indication of the fact the only a rich volatility formalization is able to capture the change in the level of the fluctuations.

Furthermore, the estimated parameters for the GARCH model with two regimes (presented in table 4.5) confirm the a priori of a higher unconditional variance before 1984. This is clearly visible in the change of the intercept on the GARCH specification, that equals 0.092 before the great moderation and decreases to 0.001 after 1984. We will come back to discuss the issue of dating the "great moderation", providing some sensitivity analysis experiments, at the end of the section.

As already mentioned, VOLINX has multiple practical applications. Surveillance of the uncertainty is one. VOLINX shows (bottom panel of figure 4.3) that, since late 60s, the US economy has suffered regular periods of stress. These have been particularly important prior to the great moderation. In late 1970 and early 1971, under the Nixon administration, the US social agenda was marked by the conflict in Vietnam and the economic agenda by the stagflation.

Late 1974 and 1975 is another period of stress that is detected by VOLINX. The peak of the uncertainty is around these dates, but the stress remained over the next three years. This is the effect of the oil crisis. Although the oil embargo by the OPEC countries to the US only lasted from October 1973 to May 1974, the increases in oil prices led to sudden inflation and economic recession. Another period of stress detected by VOLINX is late 1984 and beginning 1985 which was followed by a fast transition to tranquility. This is the so-called great moderation, one of the most studied economic phenomena over the last years but, still, one of the less understood. There are three recognized explanations. The first is the structural change in economic institutions, technology and business practices that have improved the ability of the economy to absorb shocks. The second is a better monetary policy. Finally, the last explanation is that nothing changed internally in the US economy, nor the monetary policy was more effective. Simply, during the late 80s and 90s a smaller number of shocks, and of less intensity, have hit the economy. In fact, the most stressful period detected by VOLINX after the mid 80s are the months posterior to the September 11 attacks. This attack, followed by the response of the US government of invading Afghanistan, undermined consumer, business confidence and payroll employment fell, which translated into an increasing state of stress of the economy.

While surveillance of the economic uncertainty is a backward looking task, we also use VOLINX for forward looking purpose, namely nowcasting and forecasting (see figure 4.4). The estimated monthly GDP presents heteroscedasticity and volatility clustering. This is expected, as quarterly GDP has time varying variance, see Table 4.1. However, the monthly estimates present a higher degree of clustering and a simple visual inspection of its plot reveals a clear cut in the fluctuations prior and posterior to the great moderation in the mid 80s .

We compares the casting capabilities of our GARCH set up with the standard model that assumes constant variance with some in sample and out of sample exercises (right top and bottom panel of figure 4.4). We show that the later is clearly misspecified as it underestimates the fluctuations prior to the great moderation and exacerbates them posterior to mid 80s. In fact, our results show that the differences between VOLINX and a constant level of fluctuations are up to 300% in the periods where the economy is under stress, like the oil crisis period and the months after the September 11 terrorist attacks. This results, already present in the simple Garch model, are emphasized by the GARCH 2 regimes model.

To support our empirical findings we test formally the GARCH model against the standard MM with constant variance. The two model are nested and the MM could be obtained from imposing the restriction on the GARCH parameters: $\alpha_0 = 0$ and $\alpha_1 = 0$. The LR test statistics $-2 * (lnL(\hat{\theta}_{SW}) - lnL(\hat{\theta}_{SWG}))$ is equal to 44 that exceed considerably the critical value for $\chi^2_{(2)}$ [8]. Therefore we can conclude that, conditional on the data, a time varying volatility framework is more credible.

Once the statistical properties of the model have been analyzed, we might wonder what could be the practical relevance of this sort of extension allowing for volatility.

We claim that the flexibility given by a time varying volatility model improves accuracy of forecast in terms of confidence bands. In periods of high (small) volatility forecast confidence bands should be wider (tighter) and neglecting this fact could bring economic policy makers to underestimate (overestimate) the uncertainty of their estimation. To show the relevance of this issue, we run a rolling forecast exercise for 36 months in two sub-sample of our data, characterized by interesting fluctuations. In the two bottom panel of figure 4.4 we report the ratio between the standard errors of the forecast of Monthly GDP growth for the GARCH-2regimes respect to the standard SW model. The difference between the two standard errors, and of the corresponding confidence bands, is appreciable. In particular, the ratio strongly overtake the unit in the two periods of changing of uncertainty, the begin of the great moderation (1984) and the September 11 attacks. Across this dates the forecast confidence bands resulting from a constant variance model are strongly misspecified.

Few words should be spend about the date of the break. There is no clear agreement in the literature about what is the exact point in time when the great moderation starts, unless everybody agree about a period around 1984.

We run several benchmark analysis for a break date in a period of 2 year: 1984.m1-1985m12,estimating

[8] We leave to future analysis the test of the GARCH model with switching.

rolling the model moving the break date every time of one period ahead. Results (presented in figure 4.5) are encouraging and show that the main findings of the 2 regimes Garch model are robust to the break date shifting in the interval. The position of the switch inside the interval is only responsible for the smoothness of the change of regime of VOLINX, while the pattern outside the interval is not depending on the date point.

Looking at the parameters estimates, shown in the bottom panel of the former figure, we find that the intercept ϖ_2 is always smaller than ϖ_1, confirming that the volatility has become smaller after the mid 80s. Also the relative position of the weights in VOLINX (top right panel) is maintained: Employment has the biggest load followed by Income. Industrial production does not play a relevant role in the determination of the fluctuations of the economy.

The bottom line of this robustness check is that the degree of uncertainty over the 37 years is robust to the choice of the breaking date.

However, only a model with estimated break would get conclusive results about the date of the "great moderation". Unfortunately, multiple regimes GARCH model with estimated threshold are not yet fully explored, which is the reason why we leave this issue to further analysis.

4.5 Conclusions

In this article, starting from Mariano and Murasawa (2003), we provide estimates of a monthly coincident index for the US economy and its variance, that we call VOLINX, and of the monthly GDP. The later is estimated by using the information contents of the most relevant macroeconomic indicators in order to disaggregate the quarterly value. We follow the procedure presented by Proietti and Frale (2006) that relies in a state space model and make use of the disaggregation technique advocated by Harvey(1989).

From a policy view point, monthly GDP, can be used to help in the intra quarterly decision making, surveillance of the economy and for quarterly GDP nowcast and forecast.

The estimated volatility, VOLINX, expressed as a variance conditional on past square errors of several key economic indicators, allows investigating the determinants of incertainty in the economy. We rely on GARCH type of models, following a not standard formalization where the ARCH part is replaced by a linear combination of the standardized squared errors of the idiosyncratic components of the economic indicators. We also consider a multiple regimes mechanism for the unconditional variance that mimic the well know argument of the "great moderation".

Our results show that monthly GDP growth presents heteroscedasticity and volatility clustering with a clear cut in the fluctuations prior and posterior to the great moderation in the mid 80s. We also show that the US economy suffered regular episodes of stress prior to the great moderation, for instance during the Vietnam war and the oil crisis. After mid 80s, the degree of volatility decreased substantially. During the Clinton administration, the economy had a long period of steady growth without significant stress. Only two peaks of uncertainty show up after the great moderation. The first is from mid 1986 to mid 1987. This is probably due to uncertainty that the tax reform act of 1986 produced in income and corporate taxes. The second is September 11 and the subsequent months, though the sudden and violent increase in the uncertainty disappeared as quickly as it appeared. Although the choice of the beginning of a the great moderation is exogenous, we compute, as a robustness check, VOLINX for an array of starting dates, from January 1984 to December 1985. VOLINX is robust to this choice, resembling prior and posterior to the great moderation regardless of the chosen break.

Another usefulness of VOLINX is that it helps us to understand the determinants of uncertainty in the US economy. We use the same set of macroeconomic variables as in SW and MM. By constraining ourselves to this set of macro indicators, we can investigate if they have the same impact in the volatility of the economy as in the level. And the answer is no. We find that, consistently with the literature, industrial production is the most important determinant for the level of the US economy. However, the most important determinants of the volatility are employment and income, relegating industrial production to less important component.

VOLINX has relevance also for forward looking tasks, namely nowcasting and forecasting. By permitting time varying volatility, the in and out of sample forecasts have confidence intervals that shrink and expand with the degree of uncertainty in the economy. This is a useful tool for economic policy decision makers, which may adapt their decisions depending on how wide are the confidence intervals. We show that, in sample, prior to the great moderation the confidence intervals are much wider (up to 50%) with respect to the case of constant variance and, moreover, they change substantially over time. Likewise after the great moderation, with reductions of the confidence intervals of the order of 40%.

We show that, assuming constant volatility, the model with constant variance underestimates the fluctuations prior to the great moderation and exacerbates them posterior to mid 80s.

Additionally, we show that out of sample the confidence intervals do also substantially differ from those of the constant volatility scenario. We perform a one-month ahead rolling forecast experiment for the periods January 1983 - December 1986 (around the great moderation) and January 2000 - December 2003 (around September 11). In both cases the forecasted confidence intervals vary over time, with differences up to 50% to the constant volatility case.

Table 4.1: Descriptive Statistics

	Employment	Industrial Production	Sales	Income	GDP
		Sample Variances			
1968-2006	0.043	0.664	0.133	0.282	0.676
1968-1984	0.075	1.109	0.159	0.283	1.263
1985-2006	0.019	0.321	0.114	0.267	0.231
		Univariate GARCH estimates			
Constant	6.75E-07	2.23E-05	2.72E-06	1.48E-06	0.005
	(2.14E-07)	(5.71E-06)	(1.65E-06)	(7.27E-07)	(0.010)
ARCH	0.464	0.332	0.142	0.116	0.096
	(0.129)	(0.105)	(0.052)	(0.044)	(0.034)
GARCH	0.331	0.281	0.617	0.831	0.893
	(0.067)	(0.141)	(0.177)	(0.061)	(0.038)
		Demos-Sentana ARCH test			
Test	9.238	8.392	14.641	9.194	7.020

Employment: Employees on non agricultural payrolls (thousand, SA). Industrial Production: Index of Industrial Production (2002=100,SA). Sales: Manufacturing and trade sales (Millions of chained (2000) dollars,SA). Income: Personal Income less transfer payments (Billions of dollars, SA,AR). GDP: Real GDP (Billions of chained (2000) dollars, SA, AR). Constant, ARCH and GARCH in the middle panel stand for the ω, α_0 and α_1 of the model $\sigma_t^2 = \omega + \alpha_0\sigma_t^2 + \alpha_1\varepsilon_{t-1}$. Bollerslev-Wooldrige robust standard errors are reported in parenthesis. The test in the bottom panel is the Demos-Sentana ARCH LM test on the residuals of the SW standard model. The test considers 5 lags and is distributed according to a mixture of χ^2 with critical value equal to 7.480 at 5% level and 5.835 at 10%.

Table 4.2: Mariano and Murasawa model

	Employment	Industrial Production	Sales	Income	GDP	CI
$\theta_{0,i}$	0.082**	0.317**	0.070**	0.043**	0.095**	
$\theta_{1,i}$		0.426**				
δ_i	0.194**	0.018	0.194**	0.744**	0.251**	
d_i	-0.259**	0.925**	0.223**	-0.201**		
$\sigma_{\eta_i^*}$	0.112**	0.130**	0.322**	0.519**	0.454**	
ϕ						0.883**

Maximum likelihood estimates. The parameters ($\theta_{0,i}$,$\theta_{1,i}$,δ_i,$\sigma_{\eta_i^*}$) are multiply by 100 for better legibility. CI stands for coincident indicator. The superscript "**" means that parameters are significant at 5% level, while "*" refers to 10% level.

Table 4.3: Mariano and Murasawa model with two regimes variances

	Employment	Industrial Production	Sales	Income	GDP	CI
$\theta_{0,i}$	0.069**	0.245**	0.074**	0.038*	0.072**	
$\theta_{1,i}$		0.484**				
δ_i	0.171**	0.020**	0.194**	0.711**	0.246**	
d_i	-0.211**	0.914**	0.175	-0.267**		
$\sigma_{\eta_i^*}$	0.109**	0.133**	0.338**	0.556**	0.445**	
ϕ						0.909**
Coincident Indicator variance parameters						
ϖ_1						1.463
ϖ_2						0.622

Maximum likelihood estimates The parameters ($\theta_{0,i}$,$\theta_{1,i}$,δ_i,$\sigma_{\eta_i^*}$) are multiply by 100 for better legibility. CI stands for coincident indicator. The superscript "**" means that parameters are significant at 5% level, while "*" refers to 10% level.

Table 4.4: GARCH model

	Employment	Industrial Production	Sales	Income	GDP	CI
$\theta_{0,i}$	0.069**	0.253**	0.071**	0.042**	0.074**	
$\theta_{1,i}$		0.477**				
β_k	-1.799**					
δ_i	0.169**	0.014**	0.209**	0.705**	0.235**	
d_i	-0.159	0.929**	0.142	-0.222**		
$\sigma_{\eta_i^*}$	0.106**	0.111**	0.322**	0.516**	0.454**	
ϕ						0.881
Coincident Indicator variance parameters						
α_0						0.812**
α_1						0.177**
ω_i	0.599**	0.003**	0.112**	0.276		

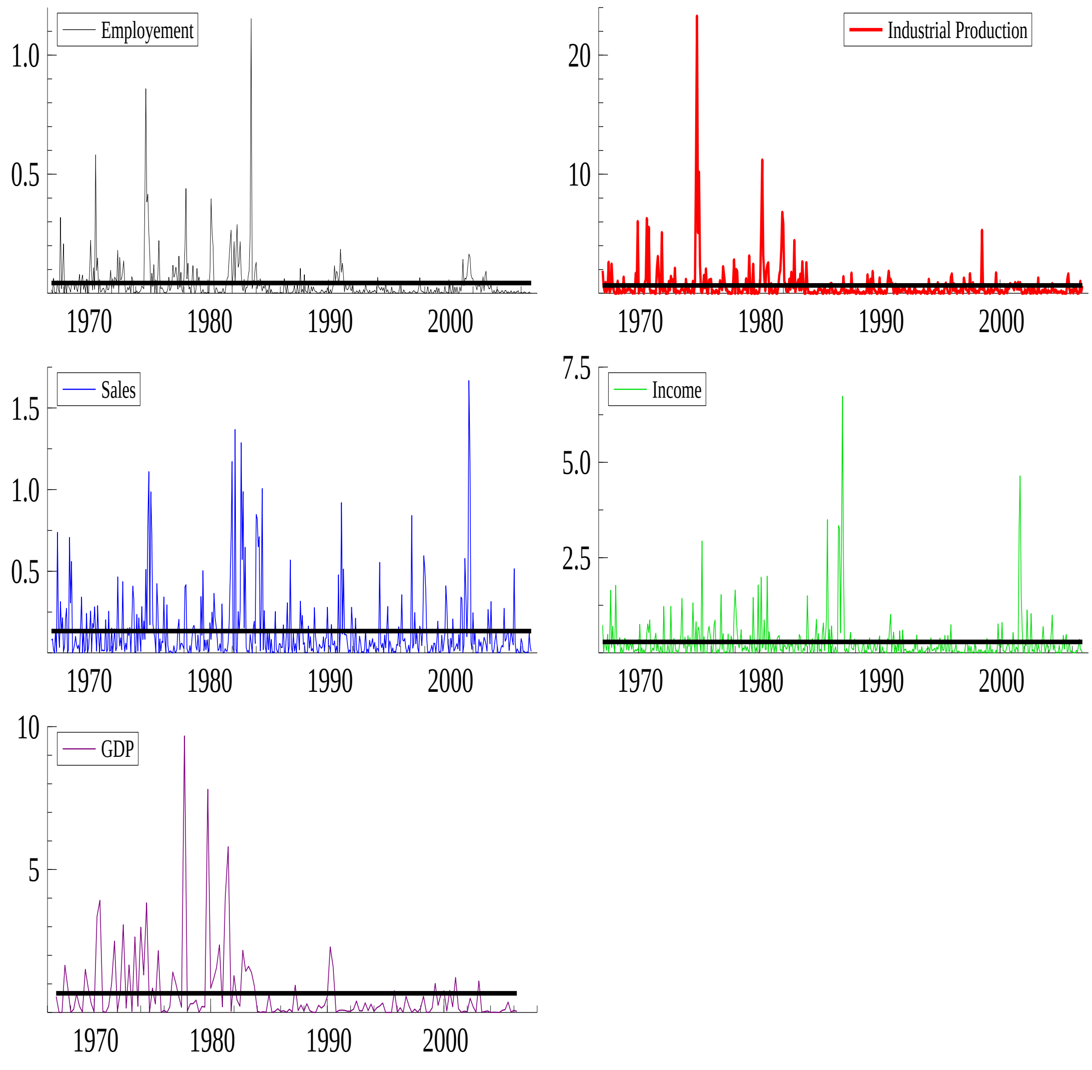
Maximum likelihood parameter estimates. The parameters ($\theta_{0,i}$,$\theta_{1,i}$,δ_i,$\sigma_{\eta_i^*}$) are multiply by 100 for better legibility. CI stands for coincident indicator. The superscript "**" means that parameters are significant at 5% level, while "*" refers to 10% level.

Table 4.5: GARCH 2 regimes model

	Employment	Industrial Production	Sales	Income	GDP	CI
$\theta_{0,i}$	0.090**	0.317**	0.096**	0.048*	0.094**	
$\theta_{1,i}$		0.676**				
β_k	-1.851**					
δ_i	0.165**	0.018**	0.201**	0.685**	0.242**	
d_i	-0.158	0.916**	0.132	-0.221		
$\sigma_{\eta_i^*}$	0.149**	0.158**	0.444**	0.700**	0.600**	
ϕ						0.900
Coincident Indicator variance parameters						
ϖ_1						0.095**
ϖ_2						0.001**
α_0						0.773**
α_1						0.081**
ω_i	0.374**	0.002**	0.290**	0.333**		

Maximum likelihood parameter estimates.The parameters ($\theta_{0,i}$,$\theta_{1,i}$,δ_i,$\sigma_{\eta_i^*}$) are multiply by 100 for better legibility. CI stands for coincident indicator. The superscript “**” means that parameters are significant at 5% level, while “*” refers to 10% level.

Figure 4.1: Indicators squared differences and sample variances



Note: The indicators are expressed in logarithms. The straight lines are the sample variances.

Figure 4.2: Monthly Indicators and Quarterly GDP in US (1968-2006)

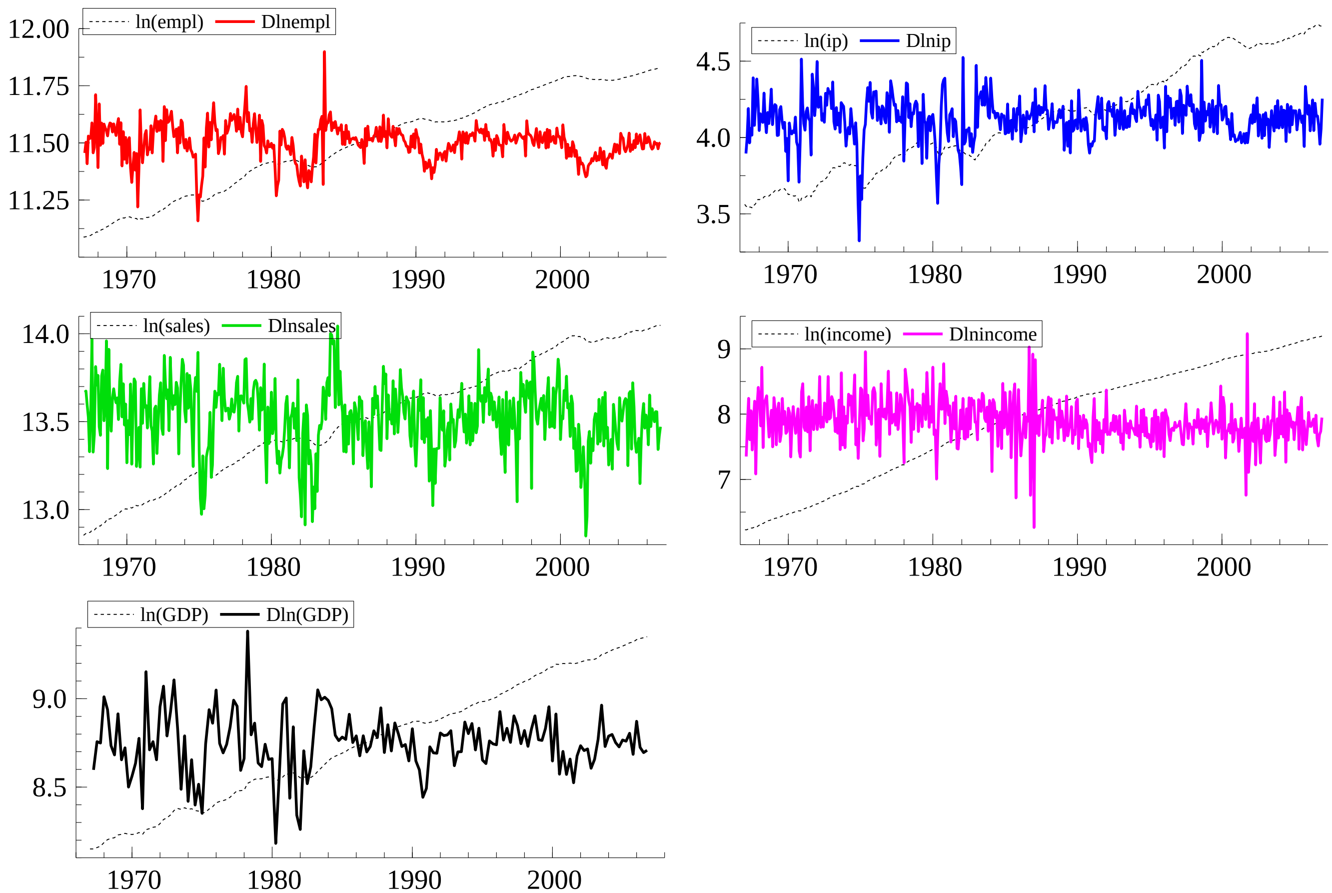
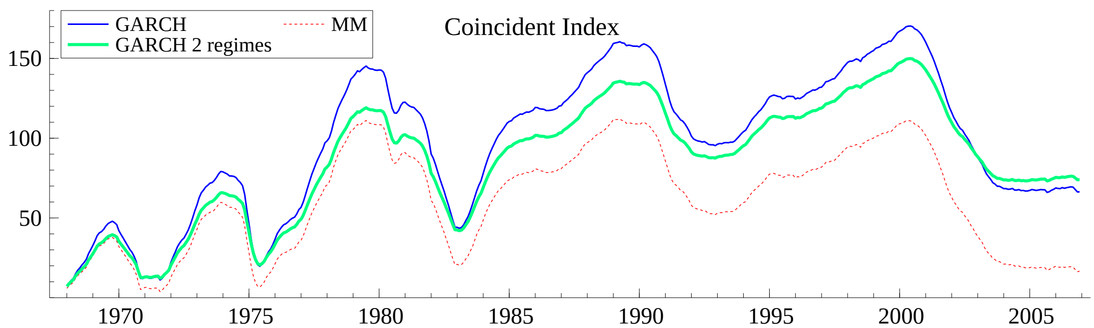


Figure 4.3: Coincident Index and volatility in US, benchmark of different models (1968-2006).



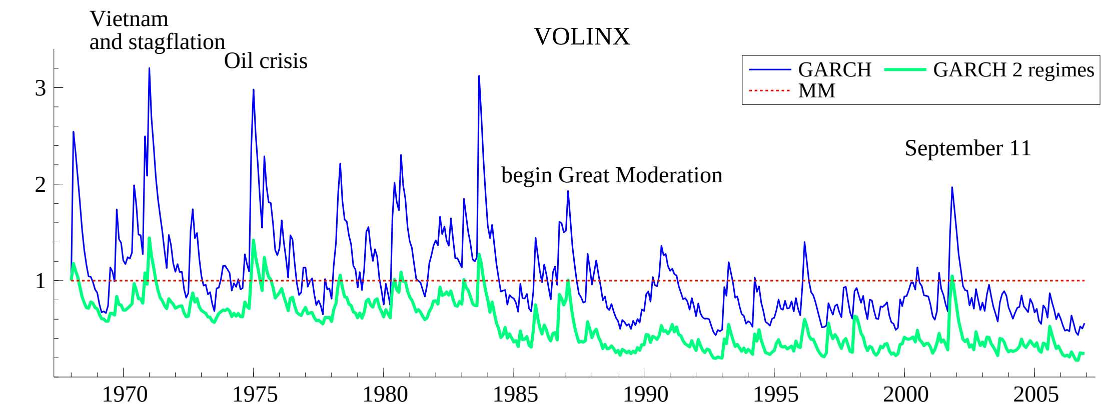
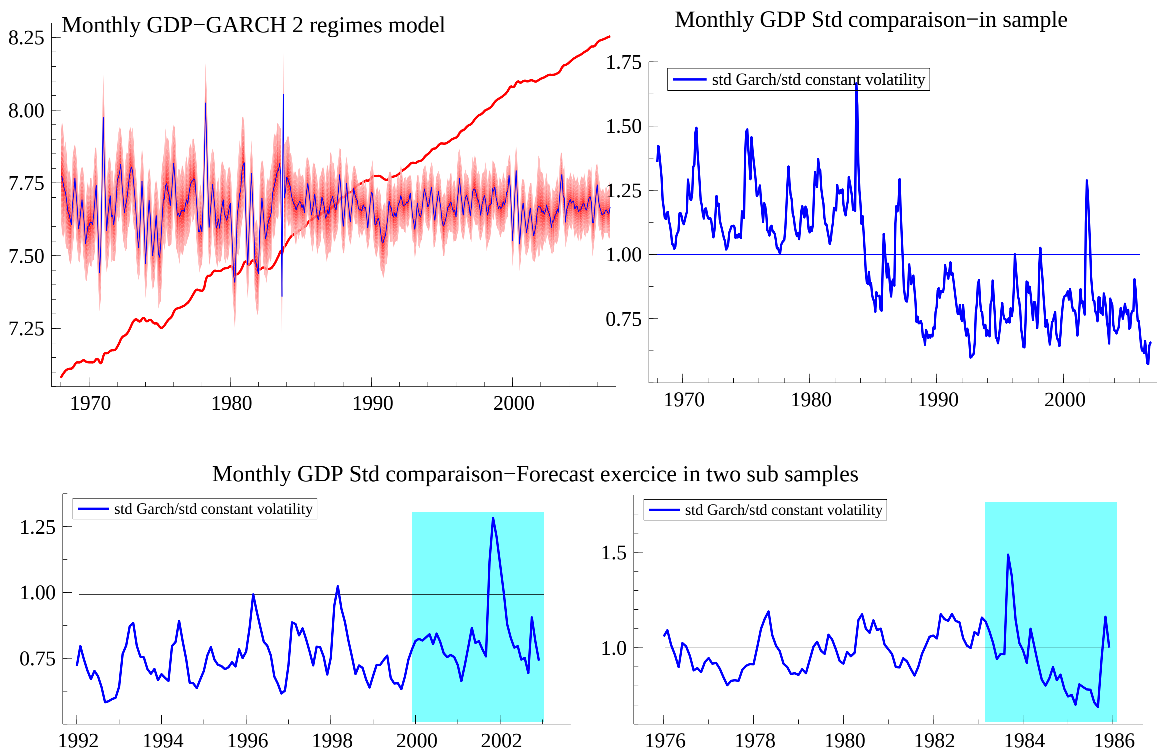
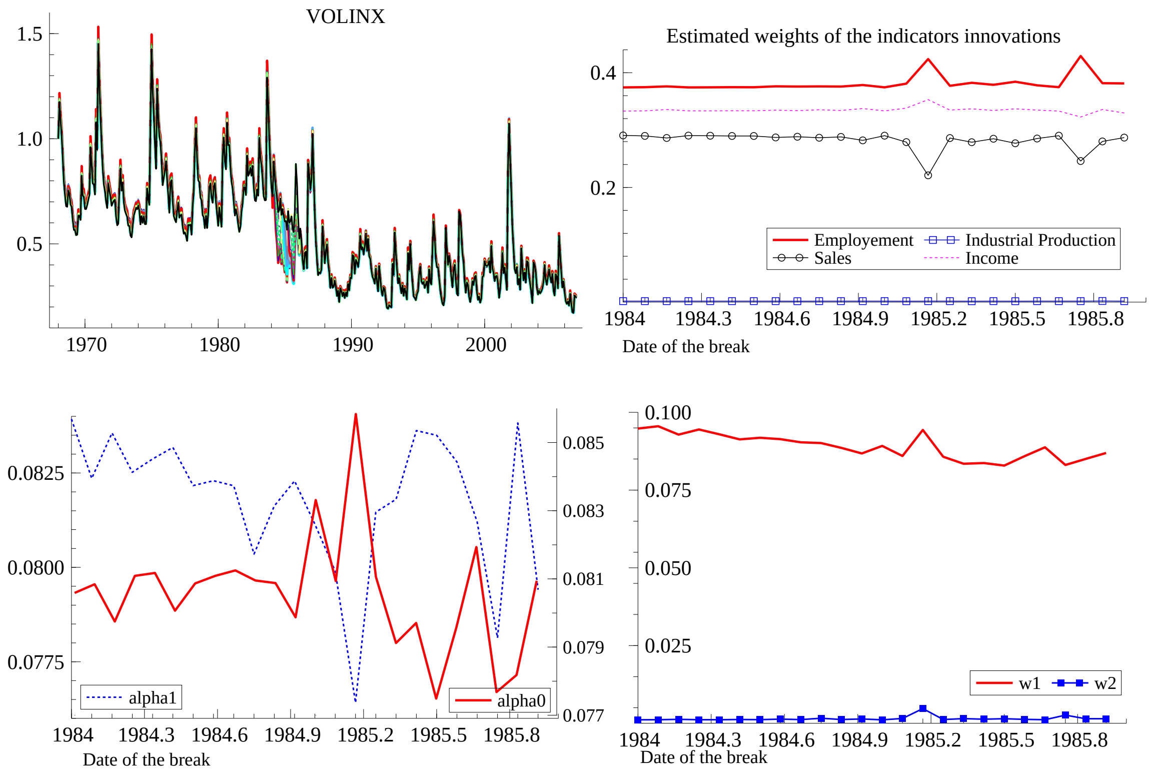


Figure 4.4: Monthly estimates of GDP in US (1968-2006). Standard errors analysis in and out of sample.



Note: The fan chart refers to confidence bands of 5% level. Unless in the top right panel, it is shown the ratio between the monthly GDP standard error of the Garch model and the constant volatility model. A value of one represents equal no difference in the confidence bands.

Figure 4.5: Rolling estimation of the break date in the GARCH for the sample (1983M6-1985M5). Estimated VOLINX and Coincident Index, overlapped results.



For all panels, the breaking date is rolling from January 1984 to December 1984. In top left panel overlapped VOLINX for all the breaking dates is represented, while in top right plot represents the estimated weights of the forecasting errors of the indicators. The rolling breaking date is represented in the x-axe for the top right and bottom panels. Bottom panels show the estimated parameters α_0, α_1, ω_1 and ω_2. All represented in the left y-axe except α_0.

4.A Appendix: The model in State Space Form

In the sequel we show how to cast the model (4.3.1)-(4.3.3) in the state space form (SSF). We first consider the case of homogeneous frequency of the series and then we explain how to extend it to include the temporal disaggregation constraint. To make exposition clear we present the state space of each component separately, first for the coincident index and then for the idiosyncratics, and finally we put all together to get the complete form. We present here only the specification we followed; for a general SSF see Proietti and Frale (2006).

1. **The coincident index**: Let us start from the single index, $\phi(L)\Delta\mu_t = \eta_t$, that is an autoregressive process of order p, AR(p). As selected by AIC and BIC criteria, we choose an AR(1), for which the SSF of the stationary model in difference, $\Delta\mu_t$, is:

$$\begin{aligned}\Delta\mu_t &= [0,1]\,\mathbf{g}_t,\\ \mathbf{g}_t &= \begin{bmatrix}\phi_1 & 1\\ \phi_2 & 0\end{bmatrix}\mathbf{g}_{t-1} + \begin{bmatrix}1\\0\end{bmatrix}\sigma_t^2\eta_t,\end{aligned}$$

therefore the Markovian representation of the model for μ_t becomes

$$\begin{aligned}\mu_t &= [1,0,0]\,\boldsymbol{\alpha}_{\mu,t}\\ \boldsymbol{\alpha}_{\mu,t} &= \mathbf{T}_\mu\boldsymbol{\alpha}_{\mu,t-1} + [1,1,0]'\,\sigma_t^2\eta_t, \quad \mathbf{T}_\mu = \begin{bmatrix}1 & \phi_1 & 1\\ 0 & \phi_1 & 1\\ 0 & \phi_2 & 0\end{bmatrix}\end{aligned}$$

2. **Idiosyncratic components**: A similar representation holds for each individual μ_{it}^*, with ϕ_j replaced by d_{ij}, so that, if we assume that all of them follow an AR(1), we can write:

$$\begin{aligned}\mu_{it}^\star &= [1,0,0]\,\boldsymbol{\alpha}_{\mu i,t}\\ \boldsymbol{\alpha}_{\mu i,t} &= \mathbf{T_i}\boldsymbol{\alpha}_{\mu i,t-1} + [\delta_i,\delta_i,0]' + [1,1,0]'\,\sigma_t^2\eta_{it}^\star, \quad \mathbf{T_i} = \begin{bmatrix}1 & d_{i1} & 1\\ 0 & d_{i1} & 1\\ 0 & d_{i2} & 0\end{bmatrix},\end{aligned}$$

where δ_i is the drift of the $i-th$ idiosyncratic component, and thus of the series, since we have assumed a zero drift for the common factor.

3. **The complete SSF**: Combining all the blocks, we obtain the SSF of the complete model by defining the state vector $\boldsymbol{\alpha}_t$, with dimension $2+2N$, where N is the number of indicators, as follows:

$$\boldsymbol{\alpha}_t = [\boldsymbol{\alpha}'_{\mu,t}, \boldsymbol{\alpha}'_{\mu_1,t}, \ldots, \boldsymbol{\alpha}'_{\mu_N,t}]'. \tag{4.A.1}$$

Consequently, the measurement and the transition equation of the SW model in levels are:

$$\mathbf{y}_t = \mathbf{Z}\boldsymbol{\alpha}_t + \mathbf{X}_t\boldsymbol{\beta}, \quad \boldsymbol{\alpha}_t = \mathbf{T}\boldsymbol{\alpha}_{t-1} + \mathbf{W}\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_t,$$

where $\boldsymbol{\epsilon}_t = [\eta_t, \eta^*_{1,t}, \ldots, \eta^*_{N,t}]'$ and the system matrices are given below:

$$\mathbf{Z} = \left[\boldsymbol{\theta}_0, \vdots\, \boldsymbol{\theta}_1 \;\; \vdots\, \mathbf{0} \vdots \text{diag}(e'_2, \ldots, e'_2)\right], \qquad \mathbf{T} = \text{diag}(\mathbf{T}_\mu, \mathbf{T}_1, \ldots, \mathbf{T}_N),$$
$$\mathbf{H} = \text{diag}(\mathbf{h}_\mu, \mathbf{h}_1, \ldots, \mathbf{h}_N).$$

where $e'_k = [1, 0, \ldots, 0]$ is a $1xk$ vector and $\mathbf{h}_\mu, \ldots, \mathbf{h}_N = [1, e'_2]$

The two matrix $\mathbf{X}_t$ and $\mathbf{W}$ are used to incorporate regression effects and to initialize the system, whereas the $(2N + k)$ vector $\boldsymbol{\beta}$ contains the pairs $\{\mu^\star_{i0}, \delta_i, i = 1, \ldots, N\}$, the starting values at time $t = 0$ of the idiosyncratic components and the constant drifts δ_i.

In particular, the regression matrix $\mathbf{X}_t = [\mathbf{0}, \quad \mathbf{X}^*_t]$, where $\mathbf{X}^*_t$ is a $N \times k$ matrix containing the values of exogenous variables that are used to incorporate k calendar effects (trading day regressors, Easter, length of the month) and intervention variables (level shifts, additive outliers, etc.). The zero block in the matrix X has dimension $N \times 2N$ and corresponds to the elements of $\boldsymbol{\beta}$ that are used for the initialisation and other fixed effects.

The SSF is complete with the equation for the GARCH-type volatility:

$$\sigma^2_t = \varpi_1 I_{[1]} + \varpi_2 I_{[2]} + \alpha_0 \sigma^2_{t-1} + \alpha_1 \sum_{i=1}^{N} \omega_i \tilde{\nu}^2_{i,t-1}$$

where $\varpi_1 \geq 0$, $\varpi_2 \geq 0$, $0 \leq \alpha_0 < 1$, $\alpha_1 \geq 0$, $\omega_i > 0$, and $\sum_{i=1}^{N} \omega_i = 1$ and $I_{[1]}$ $(I_{[2]})$ is an indicator function taking value 1 for t prior (posterior) to the great moderation. The innovations $\nu_{t,i}$ are defined as follow:

$$\nu_{i,t} = y_{i,t} - \text{E}(y_{t,i} | \mathbf{Y}_{t-1}, y_{j,t}, j < i),$$

where $\mathbf{Y}_t$ denotes the information set $\{\mathbf{y}_1, \ldots, \mathbf{y}_t\}$,$i = 1 \ldots N$. In the GARCH formalization we consider the standardized innovation: $\tilde{\nu}_{i,t} = \nu_{i,t}/stdv(\nu_{i,t})$.

3. **Temporal aggregation**: In order to deal with mixed frequency we follow the strategy proposed by Harvey (1989) operating a suitable augmentation of the state vector (4.A.1) using an appropriately defined cumulator variable.

Let us partitioning the set of indicators, $\mathbf{y}_t$, into two groups, $\mathbf{y}_t = [\mathbf{y}'_{1,t}, \mathbf{y}'_{2,t}]'$, of dimension $N = (N1 + N2)$, where $\mathbf{y}_{1,t}$ contains the series observable every period (monthly) and the second block the series partially observable (quarterly). We consider an underlying random sequence $\mathbf{y}^*_{2,t}$ such that

$$\ln(\mathbf{y}_{2,\tau}) = \frac{1}{3} \sum_{i=0}^{2} \ln(\mathbf{y}^*_{2,3\tau-i}), \quad \tau = 1, 2, \ldots, [T/3].$$

or $\mathbf{y}_{2,t}$ is the geometric mean of the unobserved three monthly values $\mathbf{y}^*_{2,t-1}$, $\mathbf{y}^*_{2,t-2}$, $\mathbf{y}^*_{2,t-3}$.

Then define a $N_2 \times 1$ vector $\mathbf{y}^c_{2,t}$, as follows

$$\mathbf{y}^c_{2,t} \quad = \quad \psi_t \mathbf{y}^c_{2,t-1} + \tfrac{1}{3} \ln(\mathbf{y}^*_{2,t})$$

where ψ_t is the cumulator variable, defined by:

$$\psi_t = \begin{cases} 0 & t = 3(\tau - 1) + 1, \quad \tau = 1, \dots, [n/3] \\ 1 & \text{otherwise} \,. \end{cases}$$

In other words the cumulator is equal to the (observed) aggregated series at times $t = 3\tau$, and otherwise it contains the partial cumulative monthly underlying value $\frac{1}{3}\ln(\mathbf{y}^*_{2,t})$ making up the quarter, up to and including the current one. We stress that the monthly GDP in logarithms is obtained from $\mathbf{y}^*_{2,t}$ applying an inverse function which is linear.

The original SSF is augmented by $\mathbf{y}^c_{2,t}$, which is observed at times $t = 3\tau, \tau = 1, 2, \dots, [n/3]$, and is missing at intermediate times:

$$\boldsymbol{\alpha}^*_t = \begin{bmatrix} \boldsymbol{\alpha}_t \\ \mathbf{y}^c_{2,t} \end{bmatrix}, \quad \mathbf{y}^\dagger_t = \begin{bmatrix} \mathbf{y}_{1,t} \\ \mathbf{y}^c_{2,t} \end{bmatrix}$$

Therefore the final measurement and transition equations are as follows:

$$\mathbf{y}^\dagger_t = \mathbf{Z}^*\boldsymbol{\alpha}^*_t + \mathbf{X}_t\boldsymbol{\beta}, \quad \boldsymbol{\alpha}^*_t = \mathbf{T}^*\boldsymbol{\alpha}^*_{t-1} + \mathbf{W}^*\boldsymbol{\beta} + \mathbf{H}^*\boldsymbol{\epsilon}_t,$$

with system matrices:

$$\mathbf{Z}^* = \begin{bmatrix} \mathbf{Z}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_2} \end{bmatrix}, \quad \mathbf{T}^* = \begin{bmatrix} \mathbf{T} & \mathbf{0} \\ \mathbf{Z}_2\mathbf{T} & \psi_t\mathbf{I} \end{bmatrix}, \quad \mathbf{W}^* = \begin{bmatrix} \mathbf{W} \\ \mathbf{Z}_2\mathbf{W} + \mathbf{X}_2 \end{bmatrix}, \quad \mathbf{H}^* = \begin{bmatrix} \mathbf{I} \\ \mathbf{Z}_2 \end{bmatrix}\mathbf{H}.$$

where $\mathbf{Z}_2$ is the block of the measurement matrix $\mathbf{Z}$ corresponding to the second set of variables (the cumulator for the quarterly GDP), $\mathbf{Z} = [\mathbf{Z}'_1, \quad \mathbf{Z}'_2]'$ and $\mathbf{y}_{2,t} = \mathbf{Z}_2\boldsymbol{\alpha}_t + \mathbf{X}_2\boldsymbol{\beta}$, where we have partitioned $\mathbf{X}_t = [\mathbf{X}'_1 \quad \mathbf{X}'_2]'$.

Conclusions

The purpose of satisfying the need for timely and disaggregated indicators on the state of the economy steaming from policy makers and many economic actors is the main focus of this analysis. This task has been carried out by providing new methodologies for the GDP disaggregation and estimation and the role of Survey data for nowcasting and forecasting.

In the first chapter two new methods for the quantification of qualitative Survey data, the Spectral Envelope, as well as the Cumulative Logit model with a non linear Kalman filter are presented. Results for the quantified indicator proved to be highly coherent with the cycle of the Industrial production.

The application is limited to Italy, as this methodology largely relies on micro data currently not available at the European level. Future avenue of research may deal with the construction of an indicator for Europe.

In the second chapter the topic of GDP nowcasting is analyzed relying on the Stock and Watson (1991) model cast in state space form as in Mariano and Murasawa (2003). We disaggregate the chained quarterly value added from National Accounts in a monthly base for the Euro area, by using a mixed frequency model based on timely indicators (quarterly and monthly). The computation is made from the output and expenditure side, which both contribute to the final estimates on the basis of their precision. Results, in contrast with part of the recent literature, show that Surveys do not play a role in the estimation of GDP.

The third chapter is dedicated to the investigations of the information content of Survey data and their role in the estimation of GDP. The basic model presented in Chapter 2, is extended to allow for more than one common factor and low frequency cycles. Taking into account real time data and the revisions, the main conclusion is that Surveys indeed are useful to produce more accurate estimates and forecast. The rolling forecast experiment shows that this result is particularly relevant when hard data are not available and the horizon forecast is long (3 steps ahead). This analysis may further extended by including a new indicator for Europe for the level of production, computed according to the framework presented in Chapter 1.

The last Chapter is dedicated to the estimation of a mean-variance coincident index for the US economy. There is broad consensus on the fact that most of the macroeconomic series has become less volatile in the last 20 year in US. Therefore the framework developed for Europe in Chapter 2 is extended to allow for time varying variance by using a Garch-type model with two regimes, that mimic the well known argument of the “great moderation”. The main findings are that the volatility is a reflex of the response of the US economy to negative shock such as wars, oil crises, terroristic attacks. Furthermore, while the level of the economy is mainly driven by the industrial production, the uncertainty is the reflection of other series, mainly income and employment. A limited feature of our model is the fact that the switching time is predefined and not endogenous. A further extension may foresee the use of a multiple regimes framework with endogenous break point to date the “great moderation”. Due to the complexity of this argument, also from the theoretical point of view, we leave it for future research.

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