



*Centre for
International
Studies
on Economic
Growth*

CEIS Tor Vergata
RESEARCH PAPER SERIES

Working Paper No. 2

January 2003

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CEIS Tor Vergata - Research Paper Series, Vol. 2003, No. 2, January 2003

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Population dynamics and life-cycle consumption*

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January 13, 2002

Abstract

This paper presents a model where aggregate consumption depends on both the level of wealth and the age structure of population. The explicit consideration of an endogenous rate of time preference permits to analyze the important role of population ageing as a determinant of aggregate saving.

Keywords: population dynamics, consumption, endogenous time preference.

JEL codes: D91, E21.

1 Introduction

The present paper proposes a formulation of the life-cycle model in the continuous-time Blanchard (1985) framework of agents endowed with finite horizons.

The life-cycle theory attempts at describing the dynamics of the propensity to consume as a function of accumulating wealth, since the individuals tend to save more while young in order to finance a smooth consumption path when old.

*I would like to thank an anonymous referee for his comments. Financial support from CNR and MURST is gratefully acknowledged.

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Under this respect, the Blanchard (1985) formulation does not appear capable of capturing the effect of declining saving with age. Existing cohorts are homogeneous both in terms of income allocation and life expectancy, and only differ in the birth date.

The "perpetual youth" nature of the model prevents analyses of the relationships between demography, ageing and consumption/saving choices.

The present paper reconciles life-cycle theory with the Blanchard (1985) framework by assuming that agents have recursive preferences in the sense of Epstein and Hynes (1983). More exactly, a class of utility functionals is used where future consumption is weakly separable from past consumption but, as opposed to the additive utility case, past consumption is not weakly separable from future consumption. Hence, preferences are generalized in such a way as to allow for a dependency of the marginal rate of substitution between consumption at times t_1 and t_2 on the consumption level at any time $t \geq \min(t_1, t_2)$. As a consequence, the actual rate of pure time preference depends on the flow of future consumption. This permits to obtain an endogenously varying time preference rate that is distinct for every cohort and adds to the (constant) effect of finite horizons on the propensity to consume. As a result, when agents accumulate and grow older, the share of income saved is reduced.

The formulation proposed also enables to obtain an exact aggregation of consumption choices from a continuum of cohorts that differ in age, as in the spirit of Weil (1989)'s model, where a difference between birth and death rates is allowed for. It is possible, then, to analyze the joint influence of the life-cycle and permanent income hypotheses on aggregate saving, both in and out of the steady-state.

The effect of varying non-human wealth (across the life-cycle) on aggregate saving is described as a generalization of the Blanchard (1985) formulation, where preferences are time-invariant and both the propensity to save and private impatience are independent of non-human wealth. The accumulation of non-human wealth decreases the propensity to save through increases in aggregate impatience.

In particular, the model permits to obtain an exact aggregation of consumption choices without neglecting the systematic relationship between the level of wealth, its composition and the propensity to save, that originate from differences both in age and in time horizon across agents.

The explicit expression for the endogenous time preference rate permits

to analyze the important role of population dynamics and its age distribution as determinants of aggregate saving. An important implication of the model is that the aggregate propensity to consume is an increasing function of the share of elderly people in the population. This makes the model suited for the study of fiscal policy issues where the age-dependency of the saving behavior is an important feature of the analysis.

The scheme of the paper is the following. Section 2 formulates the maximization problem faced by every cohort and derives endogenous impatience. Section 3 aggregates variables over cohorts and analyzes the effects of both population dynamics and age distribution on saving. Section 4 ends the paper with a summary of the main results.

2 The individual's problem

In the standard finite horizon model (see Blanchard, 1985), at each instant t , the representative cohort born at time $s \leq t$ chooses a generalized sequence $\{c(s, z)\}_{z \in [t, \infty)}$ so as to maximize

$$\int_t^\infty u(c(s, z)) e^{-[\theta + p(s, z)](z-t)} dz \quad (1)$$

where $c(s, z)$ is consumption at time z , θ is the exogenous and constant subjective time preference rate, and $p(s, z)$ is the instantaneous death probability. Intertemporal welfare is additively separable and the utility function is $u(c(s, z))$.

The dynamic budget constraint is

$$\dot{v}(s, z) = [r(z) + p(s, z)]v(s, z) + y(s, z) - c(s, z) \quad (2)$$

where v is non-human wealth, r is the interest rate, and y is income.

Unintended bequests are ruled out due to the additional assumption that agents subscribe life insurance contracts from perfectly competitive insurance companies. Hence, they receive a premium on non-human wealth in addition to the market interest rate when alive, and their wealth is transferred to the insurance company at the time of death.

The cohort is prevented from doing Ponzi games by the transversality condition

$$\lim_{z \rightarrow \infty} v(s, z) e^{-\int_t^z [r(\sigma) + p(s, \sigma)] d\sigma} = 0 \quad (3)$$

We depart from the above model by assuming that the cohort maximizes

$$U \left[\{c(s, z)\}_{z \in [t, \infty)} \right] = - \int_t^\infty e^{-\int_t^z u(c(s, \sigma)) d\sigma} dz \quad (4)$$

under the same constraints.

As shown in the Appendix, this implies that the time preference rate at time τ associated with the cohort born at s is

$$\delta [U [\tau c(s, z)]] = - [U [\tau c(s, z)]]^{-1} \quad (5)$$

where $U [\tau c(s, z)]$ is utility from the consumption sequence $\{c(s, z)\}_{z \in (\tau, \infty)}$. Since the optimal sequence of consumption is a function of the stock of non-human wealth, $\delta [U [\tau c(s, z)]]$ is an indirect function of $v(s, z)$ and, consequently, increasing with age (decreasing with the birth date s).

3 Endogenous impatience and aggregate consumption

Aggregate consumption can be derived as

$$C(t) = \int_{-\infty}^t c(s, t) e^{-p(t-s)} ds$$

where (see Appendix)

$$c(s, \tau) = \{\delta [U [\tau c(s, z)]] + p\} [h(\tau) + v(\tau)] \quad (6)$$

and

$$C(t) = p(H(t) + V(t)) + \int_{-\infty}^t \delta [U [\tau c(s, z)]] [h(t) + v(t)] e^{-p(t-s)} ds \quad (7)$$

Both population dynamics and its age distribution are important determinants of consumption as can be seen in (7). It is clear from (5) that

$$\delta [U [\tau c(s, z)]] < \delta [U [\tau c(s', z)]] \Leftrightarrow s > s' \quad (8)$$

Aggregate consumption is greater than in the case of constant time preference, i.e.

$$\theta = \delta [U [t c(s, z)]] \implies \int_{-\infty}^t \delta [U [\tau c(s, z)]] [h(t) + v(t)] e^{-p(t-s)} ds > \theta (H(t) + V(t))$$

implies

$$C(t) > (p + \theta)(H(t) + V(t))$$

The aggregation coincides with the standard solution of the finite lives model only if one restricts the analysis to constant and age independent consumption paths.¹

A relevant implication of our model is that there exists a relationship between the age structure of population and the propensity to save. In particular, economies with a larger share of elderly people are characterized by a greater aggregate propensity to consume. It can be seen from (8), that private impatience is increasing with age. Hence, the integral in (7) is greater the larger the share of elderly people. The model thus suggests a negative correlation between saving and the ageing of population.

4 Conclusions

This paper presented a continuous time overlapping generations model with finite lives and endogenous time preference. The explicit consideration of time varying impatience makes it possible to replicate the familiar life cycle consumption pattern, otherwise impossible in the standard "perpetual youth" exogenous time preference model.

Our model also stresses the role of age structure and population dynamics over the life cycle. A notable prediction is that the propensity to save should fall, *ceteris paribus*, in economies with ageing population.

¹In this case the definition of time preference would be (see Epstein and Hynes, 1983)

$$\delta = u(\bar{c})$$

where \bar{c} is the stationary level of individual's consumption. This implies the same solution as in Blanchard (1985)

$$C(t) = (p + \delta)(H(t) + V(t))$$

A Appendix

Following Epstein and Hynes (1983), introduce the new state variable

$$\eta(s, z) = \int_t^z u(c(s, \sigma)) d\sigma$$

which is total utility enjoyed from t up to time z by the cohort born at s . The problem can be reformulated as

$$\begin{aligned} \max & - \int_t^\infty e^{-\eta(s, z)} dz \\ \text{sub } \frac{\partial v(s, z)}{\partial z} &= [r(s, z) + p(s, z)] v(s, z) + y(s, z) - c(s, z), \\ \frac{\partial \eta(s, z)}{\partial z} &= u(c(s, z)) \end{aligned}$$

The Hamiltonian is

$$L = -e^{-\eta(s, z)} + \lambda(s, z) ([r(s, z) + p(s, z)] v(s, z) + y(s, z) - c(s, z)) + \mu(s, z) u(c(s, z))$$

If the generalized sequence $\{\eta_z\}_{z \in [t, \infty)}$ is optimal, then

$$\mu(s, t) = \int_t^\infty e^{-\eta(s, z)} dz$$

Hence

$$\mu(s, \tau) = \int_\tau^\infty e^{-\eta(s, z)} dz$$

and

$$\frac{\partial \mu(s, \tau)}{\partial \tau} = -\frac{1}{e^{-\eta(s, \tau)}}$$

Then

$$\frac{\partial \mu(s, \tau)}{\partial \tau} = -\frac{\mu(s, \tau)}{\int_\tau^\infty e^{-(\eta(s, z) - \tau)} dz}$$

Since

$$\int_\tau^\infty e^{-(\eta(s, z) - \tau)} dz$$

is total welfare arising from the sequence $\{\eta_z\}_{z \in [\tau, \infty)}$, then, from the definition of endogenous impatience by Epstein and Hynes (1983), the time preference rate at time τ associated with the cohort born at s is

$$\delta [U [{}_{\tau}c(s, z)]] = - [U [{}_{\tau}c(s, z)]]^{-1}$$

which is eq. (5) in the text.

Individual consumption evolves according to

$$\frac{\dot{c}(s, \tau)}{c(s, \tau)} = \gamma \{r(s, \tau) - \delta [U [{}_{\tau}c(s, z)]]\}$$

where γ denotes the elasticity of substitution implied by the utility function u . Hence, denoting individual human wealth with h , consumption at τ can be written as

$$c(s, \tau) = \{\delta [U [{}_{\tau}c(s, z)]] + p\} [h(\tau) + v(\tau)]$$

which is eq. (6) in the text.

References

- [1] Blanchard, O.J. (1985), Debts, deficits, and finite horizons, *Journal of Political Economy* 93, 223-247
- [2] Epstein, L.G. and J.A. Hynes (1983), The rate of time preference and dynamic economic analysis, *Journal of Political Economy* 91, 611-635
- [3] Weil, P. (1989), Overlapping families of infinitely-lived agents, *Journal of Public Economics* 38, 183-198