Introduction

The Gilbert equation summarizes the standard model for the evolution of the magnetization $\mathbf{m}$ in rigid ferromagnetic bodies. Under common constitutive assumptions, it has the form of a parabolic PDE:

$$\gamma^{-1}\dot{\mathbf{m}} + \mu\mathbf{m} \times \dot{\mathbf{m}} = \mathbf{m} \times (\alpha\Delta\mathbf{m} + \beta(\mathbf{m}\cdot\mathbf{e})\mathbf{e} + \mathbf{h}^{\mathrm{s}} + \mathbf{h}^{\mathrm{e}}) \ .$$

Here $\dot{\mathbf{m}}$ and $\Delta\mathbf{m}$ denote, respectively, the time derivative and the Laplacian of $\mathbf{m}$, and the symbol $\times$ denotes the cross product; γ is the *gyromagnetic ratio*, a negative constant; α, β, μ are positive constants; $\mathbf{e}$ is a unimodular vector (the *easy axis*); $\mathbf{h}^{\mathrm{e}}$ is the *external magnetic field* and $\mathbf{h}^{\mathrm{s}}$ is the *stray field*, the magnetic field generated by the body.[1]

In ferromagnetic bodies, it is possible to observe *magnetic domains*, *i.e.*, regions where the orientation is nearly constant, separated by narrow transitions layers, the *domain walls*. The application of an external magnetic field induces re-orientation and growth of some domains at the expense of others. Our intention is to picture the resulting domain-boundary displacement, accompanied by re-orientation changes in the magnetization, as a process in which domain walls are regarded as surfaces endowed with a mechanical structure, whose motion is ruled by dynamical laws deduced from the Gilbert equation.

The first chapter of this thesis is a brief introduction to *micromagnetics* and *domain theory*, two variational theories intended to describe the equilibrium configurations of the magnetization in a ferromagnetic body.

The second chapter deals with *dynamical micromagnetics*, *i.e.*, the generalization of micromagnetics to a theory describing the evolution of the magnetization vector. First, we illustrate the standard form of the Gilbert equation; next we show that the Gilbert equation can be obtained from a general balance law through specific constitutive assumptions, and explore the generalizations associated with some non-standard constitutive choices.

[1]The notation we here use is only reminiscent of the notation commonly used in the literature on the physics of magnetized matter; in the Appendix we discuss it in the light of the latter, and pay due attention to the relevant dimensional issues.

Domain-wall dynamics is dealt with in Chapter 3, whose first part is drawn, with small changes, from [39]. We consider the *Walker-type solutions*, *i.e.*, one-dimensional exact solutions having the form of a traveling wave. Named after L.R. Walker [49, 43], who first found one, these solutions picture the steady motion of a flat domain wall. We illustrate a method to derive the original Walker's solution for the standard Gilbert equation, and we consider the possibility of extending it to more general constitutive assumptions, showing that this is feasible when dry-friction dissipation is accounted for.

In the second part of the chapter we consider a curved wall, modelled by an oriented regular surface $\mathcal{S}$ evolving smoothly in time in the body Ω under examination. For a subregion Π of Ω crossed by $\mathcal{S}$ (Fig. 1), and for $\varepsilon \equiv \beta^{-1}$ a small parameter, we assume that the solution of the Gilbert equation in Π can be constructed by matching two regular expansions in powers of ε, the one presumed to hold in a tubular neighborhood of $\mathcal{S}$, the other away from $\mathcal{S}$.

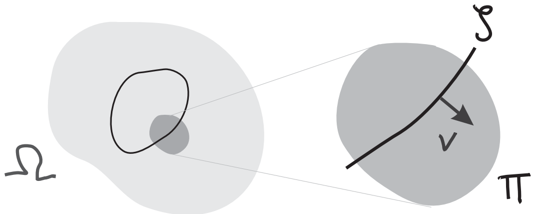

Figure 1: The evolving surface $\mathcal{S}$.

Our main result is that, for $\varepsilon \to 0$, the following condition must hold on $\mathcal{S}$ pointwise:

$$\rho\, v - \sigma\, k = p^e + p^s, \tag{1}$$

where the scalar fields v and k are, respectively, the *normal velocity* and twice the *mean curvature* of the surface $\mathcal{S}$; ρ and σ are scalar quantities which depend on the magnetizations in the two domains; p^e and p^s depend, respectively, on the external magnetic field $\mathbf{h}^{\mathrm{e}}$ and the stray field $\mathbf{h}^{\mathrm{s}}$.