Appendix

Notation and Units. In SI units, the Gilbert equation reads

$$\gamma_e^{-1}\dot{\mathbf{m}} = \mu_0 \mathbf{m} \times \mathbf{H}\,, \tag{3.168}$$

where $\gamma_e = g\, q_e/m_e$ is the *gyromagnetic ratio*, $q_e < 0$ is the *signed electron charge*, m_e is the *electron mass* and $g > 0$ is the *Landé factor*; μ_0 denotes the magnetic permeability of the vacuum, and

$$\mathbf{H} = -(\mu_0 M_s)^{-1}\delta_{\mathbf{m}}\Phi\,, \tag{3.169}$$

is the *effective field*, with M_s the saturation magnetization and Φ the *Gibbs free energy*. The standard free energy is written as

$$\Phi = \int_\Omega \phi\,, \tag{3.170}$$

with

$$\phi = A|\nabla\mathbf{m}|^2 - K(\mathbf{e}\cdot\mathbf{m})^2 - \mu_0 M_s(\frac{1}{2}\,\mathbf{H}^{\mathrm{s}} + \mathbf{H}^{\mathrm{e}})\cdot\mathbf{m}\,,$$

where $\mathbf{H}^{\mathrm{s}}$ and $\mathbf{H}^{\mathrm{e}}$ are, respectively, the stray field and the external magnetic field, A is the exchange constant and K is the anisotropy constant.

Our notation is recovered upon defining the dimensionless quantities

$$\begin{aligned} \psi &= (\mu_0 M_s^2)^{-1}\phi\,, \\ \beta &= 2(\mu_0 M_s^2)^{-1}K\,, \\ \mathbf{h} &= M_s^{-1}\mathbf{H}\,, \\ \mathbf{h}^{\mathrm{s}} &= M_s^{-1}\mathbf{H}^{\mathrm{s}}\,, \\ \mathbf{h}^{\mathrm{e}} &= M_s^{-1}\mathbf{H}^{\mathrm{e}}\,, \end{aligned} \tag{3.171}$$

and

$$\begin{aligned} \alpha &= 2(\mu_0 M_s^2)^{-1}A\,, \\ \gamma &= M_s\mu_0\gamma_e \end{aligned} \tag{3.172}$$

which have, respectively, the dimensions of (length)^2 and (time)$^{-1}$.