Microfinance with divisible investment projects

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Abstract
In this paper we examine how the traditional results of the microfinance literature change under the project divisibility assumption.
We show that, under standard debt contracts, loan size and borrower profits are unchanged when lending to uncollateralized borrowers with an individual lending or with a group lending/joint liability scheme, as the positive effect of the latter on bank risk is offset by a negative effect on borrowers' optimal loan size.
We also show that participated (debt plus profit sharing) loan contracts which reduce the lending rate (with respect to standard debt contracts) generate higher loan size and output, but lower borrower profits. Such contracts, however, cannot be enforced in presence of ex post hidden information, unless costly state verification by the lender is possible and economically convenient.
We finally show that a problem of borrower heterogeneity may be solved by the lender with a participated loan/group lending scheme since, in this case, it is possible to devise a menu of contracts discriminating among heterogeneous quality groups. In such case we show that, under reasonable parametric conditions, a participated loan/group lending contract ensures higher profits to the high quality borrower than a standard debt individual lending contract.

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1. Introduction

Project indivisibility is a traditional restrictive assumption of many models in the banking literature. According to it, an investment is modeled as having a fixed cost such that a slightly smaller amount invested completely vanishes it, while any amount in excess of the fixed cost is totally unproductive. In many situations of economic life this is obviously not the case since a reduction (an increase) in the monetary value of the investment may reduce (increase) its profitability.

One of the typical examples of divisible investment, akin to the microfinance context, is investment in cultivated land. In such case, given a fixed unit price of land, and the possibility of choosing land size, the investment is clearly divisible.[1] Beyond this example, and on more general terms, any type of investment is always composed by different elements, some of which are divisible. In this perspective, a higher amount of money available for investment may contribute to increase output in many respects, i.e. by hiring more, or more qualified, personnel, or by choosing higher quality investment goods, etc.

Our aim is to bring the issue of investment divisibility into the microfinance literature. Within this framework, our work on microfinance with divisible investment projects is at the crossroad of two different strands. On the one side, models of endogenous loan size (in the standard banking literature) have mainly focused on the problem of ex post hidden information and costly state verification (Townsend, 1979 and 1989; Gale and Hellwig, 1985; Khalil and Parigi, 1998; Simmonds and Garino, 2003).

On the other side, the (indivisible investment) microfinance literature has recently flourished in parallel to the significant expansion of microfinance experiences around the world.[2] This branch has so far focused on various facets of the borrower-lender relationship such as adverse selection (Armendáriz de Aghion and Gollier, 2000; Gangopadhyay, Ghatak and Lensink, 2005; Ghatak, 2000, and Laffont and N'Guessan, 2000), moral hazard (Chowdury, 2005; Conning, 1999 and 2005; Laffont and Rey, 2003, Stiglitz, 1990, Ghatak at al., 1999), ex post hidden information and effects of project correlation under group lending (Armendariz de Aghion, 1999). In this literature there is no work on the relationship between endogenous project size, on the one side, and group lending or individual lending to uncollateralized borrowers, on the other side.

Given the above mentioned considerations, the contribution of our paper is not just in the evaluation of the effects of the introduction of the divisible investment hypothesis

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[1] Another typical example may be advertising, where, with a higher amount of resources invested, both the number of contacts with potential customers and space and/or time length of communication can be increased.

[2] The *Microcredit Summit Campaign* at end 2004 documents the existence of 2,572 microfinance programs around the world reaching approximately 67.6 million borrowers and, among them, 41.5 millions below the absolute poverty threshold.

into the realm of microfinance, but also in the identification of the financial instruments (standard debt contracts or participated loans with our without joint liability) which are most suitable in such framework.
This broadened scope of analysis is important for at least two reasons. First, borrowers are asking more and more microfinance institutions (MFIs) to develop forms of participated loans and the divisible investment assumption may help to explain this phenomenon. Second, even abstracting from such emerging demand, it is important, in principle, to compare the performance of different financial instruments in order to contribute to financial innovation in microfinance.
Beyond this general contribution, our paper provides interesting insights on three fundamental debates in microfinance. The first is on the role of subsidized lending and donor funds whose impact is examined under the assumption of project indivisibility. The second is on the pros and cons of group lending and joint liability in the light of the recent evolution which led the most important microfinance institutions (*Grameen Bank*, *Bancosol*) to overcome the group lending/joint liability approach in direction of alternative solutions (group lending without joint liability, progressive loans, etc.). With this respect, the comparison between participated loans and standard debt contracts provides new insights on this debate by evaluating additional potential advantages (or disadvantages) of individual loans vis-à-vis group loans with joint liability whenever investments are divisible and project output is increasing in loan size.
A third line of interest in our research is the relationship it has with Islamic finance. A significant part of microfinance initiatives are implemented in Muslim countries in which there is a strong demand of consistence with Islamic principles. As it is well known, one of the most important of them is the *Riba,* or the prohibition of the payment or receipt of any predetermined, guaranteed rate of return (Sura II, 279). The prohibition of interest charges hinges on arguments of equality and social justice. Islam considers them an "unfair" cost that is paid ex ante by the borrower, without taking into account the outcome of business operations and with the risk of endangering his success. On the contrary, equity partnership and ex post participation to profits are seen as more ethical as they do not demand money to the borrower before project realisation, while positively contributing to create wealth and to promote successful entrepreneurship. Profit-sharing mechanisms (such as *mudaraba*) are therefore considered among the most favourite alternatives to standard debt contracts.
Without entering into the ethical aspects of this issue, our paper (by comparing standard debt contracts and combinations of debt and full (or partial) profit sharing contracts) provides a framework for evaluating the costs (or benefits) of the constraint imposed by Islamic principles on microfinance, under the specific assumption of divisible investment projects.

The paper is divided into five sections (introduction and conclusions included). We start from the application of the project divisibility assumption to individual lending to uncollateralized borrowers, illustrating how participated loans (which combine

lower lending rates with lender's profit sharing mechanisms) may be superior to standard debt contracts, in terms of borrower output, but may not be enforceable in presence of ex post hidden information which creates an opportunity for profit under representation by the borrower (section 2). We further compare individual lending with group lending with joint liability showing how total profits do not change in this case, since the positive effect of the reduction of the lending rate on the borrower's loan size choice is compensated by the joint liability penalty (section 3).
We finally evaluate the impact of the divisible investment hypothesis in presence of moral hazard and adverse selection (section 4) showing that a problem of borrower heterogeneity may be solved by the lender only with a participated loan/group lending scheme since, in this case, it is possible to devise a menu of contracts which allows to discriminate among groups with different quality. This discrimination capacity allows the bank to set lower loan rates and, under reasonable parametric conditions, ensures higher profits to high quality borrowers than under the standard debt individual lending contracts.

2. Optimal loan size with individual lending

The analysis of the introduction of the investment divisibility hypothesis in a model of individual lending to uncollateralized borrowers leads us to formulate the following proposition.

Proposition 1. Participated loans, which combine lower lending rates with profit sharing rules, generate higher output (but lower borrower's profits) than standard debt contracts in individual lending to uncollateralized borrowers under the assumption of divisible investment projects.

To demonstrate our proposition we adopt the following steps. We start by calculating borrower profits, the optimal loan size and the equilibrium lending rate under the hypothesis of lender zero profits. We then show that borrower profits and loan size may be higher if the lender charges a rate which is equal to the opportunity cost of funds. We show that, in such case, the lender has losses which he may recover by imposing a tax on borrower's profits (a profit sharing rule). We finally highlight that, after imposing a profit sharing rule which restores the lender's zero profit condition, the borrower produces higher output but lower profits.

Consider a financial intermediary which pools financial resources from risk neutral investors whose opportunity cost is (r). Investment projects are divisible and the intermediary lends a variable amount L to an individual borrower so that his total costs are $R=(1+r)L$. If he charges a lending rate (i), and has a cost of screening projects equal to Ks, his zero profit condition will be $R = pL(1+i) - Ks$.[3]

From this zero profit condition we get the minimum lending rate admissible for the borrower

$$i^* = \frac{(1+r)}{p} + \frac{Ks}{pL} - 1 \tag{1}$$

We assume that the investment project is divisible and the borrower adopts a technology which is continuously differentiable concave in the loan size, yielding the following output $X = f(L)$ with $f'(L) > 0$, $f''(L) < 0$.

We also assume that the borrower may be hit by a negative shock so that his project may fail with probability $(1-p)$. As a consequence, he will choose loan size by maximizing the following function

$$\max_L : p\left[f(L) - (1+i)L\right] \tag{2}$$

which, by replacing for (1), becomes

$$\max_L : pf(L) - (1+r)L - Ks \tag{2'}$$

yielding the following first order condition $pf'(L) = (1+r)$.

We assume that our production function has the following standard specification $f(L) = AL^{\alpha}$, with $0 < \alpha < 1$.

Under such functional form the borrower zero profit condition leads to the following optimal loan size

$$L^* = \left(\frac{1+r}{\alpha A p}\right)^{\frac{1}{\alpha-1}} \tag{3}$$

which yields the following profit for the borrower

$$\pi^* = (1+r)^{\frac{\alpha}{\alpha-1}} \left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}} \left(\frac{1}{p}\right)^{\frac{1}{\alpha-1}} \left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] - Ks \tag{4}$$

Borrower's total profits would be obviously enhanced if the bank could reduce its interest rate. Consider, for instance, the case in which the bank charges the opportunity cost (r) instead of the zero profit interest rate (i^*).

In such case the optimal loan size turns into

$$L^{**} = \left(\frac{1+r}{\alpha A}\right)^{\frac{1}{\alpha-1}} \tag{5}$$

[3] The decision to model the microfinance institution as a zero profit bank (standard in the microfinance literature) starts from the consideration that most MFIs have the goal of extending access to credit to potential borrowers lacking of collateral and subordinate profit maximization to this goal.

and borrower's profits are

$$\pi^{**} = p(1+r)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \quad (6)$$

By comparing (3) and (5) we can easily check that $L^{**}(r) > L^{*}(i^{*})$.

Furthermore, from (4) and (6) we infer that $\pi^{**}(r) > \pi^{*}(i^{*})$ since the borrower would increase his profits by just maintaining the loan size L^{*} in presence of a reduction of the equilibrium interest rate from i to r.

Consider also that, when charging a lending rate of (r), the bank needs the amount $L^{**}(r)\left[i^{*}_{zpc(L^{**})} - r\right]p$ to restore its zero profit condition, where

$$i^{*}_{zpc(L^{**})} = \left[\frac{1+r}{p} + \frac{Ks}{pL^{**}}\right] - 1 \quad (7)$$

By comparing (7) and (1) (with L^{**} replacing L^{*}) we find that $i_{zpc(L^{**})} < i^{*}$ (which is obvious since, *coeteris paribus*, a lower percentage of the loan size is needed to satisfy the zero profit condition when the loan size gets larger).[4]

When comparing differences between output and total loan payment to the MFI under the two different rates we find that $p\left[A\left(L^{**}(r)\right)^{\alpha} - L^{**}(1+r)\right] > p\left[A\left(L^{*}(i^{*})\right)^{\alpha} - L^{*}(1+i^{*})\right]$, but the $\pi^{**}(L^{**}, r)$ case implies losses for the bank. The bank can recover its zero profit condition by applying a tax (t) on borrower's profits. In Appendix 1 we show that $p > \alpha$ is a sufficient[5] condition for $t < 1$ but $\pi^{*}(L^{*}, i^{*}) > (1-t)\pi^{**}(L^{**}, r)$. Hence, the borrower's output (profit) is higher (lower) under the participated loan than under the (L^{*}, i^{*}) standard debt contract case, after paying the tax which satisfies the zero profit condition of the bank.

The results of this proposition may be interpreted in opposite ways in terms of welfare perspectives. If we look at per capita income we observe that participated loans dominate standard debt contracts while if we consider returns to capital the opposite occurs.

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[4] The obvious intuition for it is that administrative (screening) costs on the projects are lump sum and therefore, when the loan size is larger, their weight on bank expected revenues is lower.

[5]In Appendix 1 we show that, in case of zero screening costs (Ks=0), this condition ensures lender's zero profits (when combining the loan and the profit sharing component) and lower borrower profits than under the standard debt contract. When, on the contrary, Ks>0 the condition is further relaxed.

To interpret the $p > \alpha$ condition consider the classical conflict of interest between the bank and the limited liability borrower. The former is worried by the borrower low performance, which may lead to non restitution of the loan, and has no marginal gain from the increase in borrower's revenues above the restitution threshold. The latter has limited liability in case of low performance (and non restitution of the loan), but earns all additional gains in high productivity states in which output is above the restitution threshold. In this framework (p) crucially defines the probability of loan solvency, while α determines the marginal productivity of loan size and therefore is more related to the extra gains of the borrower. A large α and a low (p) create a situation of risk for the bank since a higher α may lead the borrower to increase the loan size (and therefore the value at risk for the bank) without reducing the risk of borrower's failure (even though, in case of success, a higher loan implies more interest payments for the bank).

It is worth nothing that, as it is intuitively reasonable to believe, the condition discussed here becomes tighter as far as we reduce further the interest payment on the standard debt component of the contract. In Appendix 1 we show how it changes under the extreme case of a full profit sharing contract in which $i = 0$. The result provided there outlines the effects of the constraint posed by Islamic finance (no interest payment replaced by profit sharing). Under the general assumptions of our model this constraint generates higher output but lower borrower's profits.

The problem for enforcing all participated loan contracts, such as the one described in proposition 1, is asymmetric information under the form of ex post hidden information. If the bank cannot perfectly monitor the borrower, the latter has the incentive to misrepresent his profits in order to pay lower profit taxes. Hence, the (r,t) contract is not enforceable for the bank. It can be enforceable only when i) the intermediary has local information which eliminates the informational asymmetry, ii) the borrower has costs of misrepresenting profits (i.e. deviation from social norms or personal values) which are higher than the benefit from doing it or iii) the lender may enforce an economically convenient state verification mechanism. More specifically, the problem of state verification may be solved if the lender can pay a cost Cv, to evaluate correctly borrower profits, which satisfies the borrower participation constraints[6] (i.e. is lower than borrower profits) or

$$Cv < (1-t)\, p\left[A\left(L^{**}(r)\right)^{\alpha} - L^{**}(1+r)\right]^{7} \qquad (8).$$

[6] The literature on costly state verification emphasizes the existence of a problem of time inconsistency. Ex post the lender may be tempted not to pay the cost Cv if his threat of doing so has induced the virtuous behaviour of the borrower (Bolton and Sharfstein, 1990; Hart, 1995). But in a repeated game with other borrowers which are informed about what happened in the previous stages of the game, reputation concerns of the lender should prevent time inconsistency (Bulow and Rogoff, 1989b).

[7] Consider also that the problem of ex post hidden information may be partially solved (by abstracting from discounting issues and considering for simplicity an infinitely patient borrower) in a multiperiodal loan scheme when the following condition is respected N[(1-t)[AL**$^{\alpha}$- (1+r)

3. Optimal loan size with group lending and joint liability

In this section we wonder how the assumption of project divisibility affects the hotly debated issue of individual versus group lending in microfinance. The analysis of the effects of such assumption on these two traditional lending forms leads us to formulate the following proposition.

Proposition 2. With divisible projects, and under standard debt contracts, borrower's profits are the same under individual lending and group lending with joint liability. In the latter, the positive effect of the reduction of the lending rate (with respect to individual lending) is in fact compensated by the joint liability penalty.

Consider the loan size problem in a microfinance scheme under a (two borrowers) group lending scheme with joint liability. We assume for the moment that effort is exogenous and fixed at a level which generates an individual probability of success equal to p.
The zero profit condition of the bank becomes

$$R=(1+r)L=p^2(1+i)L+2p(1-p)(1+i)L-Ks \quad (9)$$

and yields the following zero profit lending rate for the MFI

$$i^*_{GL}=\left[\frac{(1+r)L}{(2-p)pL}+\frac{Ks}{(2-p)pL}\right]-1 \quad (9')$$

with $i^*_{GL}<i$ for a given loan size. As it is well known, the zero profit lending rate of the bank is lower than in the single borrower case. This is because the bank reduces its risk with the joint liability clause which allows full repayment when one borrower is successful while the other fails. We outline a scenario in which the joint liability penalty is the highest by assuming that the groupmate project has zero market value in the bad state of affairs and the debt contract provision requires that the successful groupmate fully repays the amount of the other borrower.
In such case borrower maximization becomes

$$\max_L : pf(L)-p^2(1+i^*_{GL})L-2p(1-p)(1+i^*_{GL})L \quad (10)$$

or

$$\max_L : pf(L)+p(1+i^*_{GL})(p-2)L \quad (10')$$

L**]]> AL**$^{\alpha}$ + (N-1)OP, where N is the length of the borrower-lender relationship in number of periods, OP is the borrower outside option and the punishment strategy is such that, in case of default, the borrower does not receive the following loan installments. The problem of such strategy is that it penalizes only the declaration of default and not profit underreporting.

By substituting the optimal value of the interest rate found in (9'), we get

$$\max_L : pf\left(L\right)-\left(1+r\right)L-Ks \qquad (10'')$$

Under the usual project technology $f\left(L\right)=AL^{\alpha}$ - the first order condition is

$$L_{GL}^{*}=\left(\frac{1+r}{\alpha Ap}\right)^{\frac{1}{\alpha-1}} \qquad (11)$$

which is the same as in the individual lending case. This occurs because the positive effect on loan size generated by the lower lending rate is exactly compensated by the higher tax on success determined by the joint liability clause.
As a consequence borrower profits are now

$$\pi_{GL}^{*}=\left(1+r\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}\left(\frac{1}{p}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]-Ks \qquad (12)$$

and are the same as in the individual lending scenario.

The rationale for this result is that the effect of group lending is simply that of shifting part of the risk and of the costs of intermediation from the bank to the borrower. More specifically, the borrower provides an implicit insurance to the bank with his commitment to repay the debt when the groupmate is unsuccessful. With individual lending the cost of this failure is assumed by the bank. This is why the bank can reduce its zero profit lending rate and why, in spite of this, borrower's profits are not higher.

Finally, to obtain optimal lending size and profits under participated loan consider that borrower has to maximize the following function

$$\max_L : pf\left(L\right)-p^{2}\left(1+r\right)L-2p\left(1-p\right)\left(1+r\right)L \qquad (13)$$

loan size is

$$L_{GL}^{**}=\left[\frac{\left(2-p\right)\left(1+r\right)}{\alpha A}\right]^{\frac{1}{\alpha-1}} \qquad (14)$$

and profit

$$\pi_{GL}^{**}=p\left(2-p\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}\left(1+r\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right] \qquad (15)$$

[8]

3.1 The role of donors funds

[8] See Appendix 3.

The more microfinance institutions try to provide credit to the low end beneficiaries, the higher the administrative costs per loan and the more they are likely to incur in operating losses. In many of these cases they are supported either by guarantee funds, financed by domestic or international institutions, or, directly, by the socially responsible (SR) investment of the private sector, provided by associations or individuals.
To quote some relevant examples, *Oikocredit*, one of the leading financial institutions channeling socially responsible investment to microfinance, at 31 December 2004, supported over 398 project partners with an outstanding capital of 114 million euros. Its total share capital was 203.5 million euros invested by 24,000 members and 37 Support Associations. In Italy, *Banca Etica* remunerates deposits below the market level and channels funds to microfinance institutions around the world. *Banca Etica* has reached after five years of operation one of the highest numbers of shareholders (26.000 in 2006) among Italian banks.
It is easy to imagine that pros and cons of the provision of subsidies are a main issue in the microfinance debate. According to Amendariz de Aghion and Morduch (2005) subsidies are beneficial if we assume a non flat distribution of social weights, a demand of credit which is elastic to interest rates, adverse selection effects and positive spillovers on other lenders, while the most traditional argument against subsidies is based on the "infant industry" critique, or on the weakening effect of subsidies on the efficiency of microfinance institutions. In our paper the analysis of the relationship between SR investment and microfinance in presence of divisible investment leads us to formulate the following proposition

Proposition 3. When investment is divisible private donors' funds significantly increase loan size and output, thereby increasing social welfare under both individual lending and group lending with joint liability.

What is the effect of voluntary private donors accepting a lower remuneration on the sums lent to the microfinance institution? Assume that, in presence of donors or socially responsible financiers, the MFI can pay its financial resources at the rate $r' < r$.

Since from (1') $\frac{\delta i^*}{\delta r} > 0$, an $r' < r$, even under the standard debt contract, reduces the zero profit lending rate of the microfinance institution and raises the optimal loan size (see equation 3) and borrower's profits (equation 4). If we move to the participated loan contract, as in the case of the passage from i^* to r as a lending rate, the passage from i^* to r' raises even more total output under the $(p > \alpha)$ dominance condition. We must add to this the positive effect determined by the voluntary decision to donate which, for the revealed preference argument, implies that donors are better off by taking this decision than by not taking it.

The same occurs under group lending with joint liability if we analyse the impact of a lower opportunity cost of funds in equations (9'), (11) and (12). As for the case of the passage from i^* to r, the passage from i^* to r' determines the same effect on loan size under individual and group lending. To consider the effect of donors' funds in the perspective of social welfare (SW), we conveniently assume that the latter is given by borrower's revenues (RV) minus the total payment required to satisfy the lender zero profit condition (ZPCosts). Hence we obtain, under the standard debt contract, $SW\left(i^*,t\right)=RV\left(L^*,i^*\right)-ZPCosts\left(i^*,r\right)$ while, under participated loans with donors' funds, we get

$$SW\left(i',r'\right)=RV\left(L',i'\right)-ZPCosts\left(i^*,r\right)+\left[ZPCosts\left(i^*,r\right)-ZPCosts\left(i',r'\right)\right] \qquad (16)$$

where i' is the lending rate which ensures the zero profit condition when the lender fixes the rate of the participated loan at $r'<r$, L' is the corresponding optimal loan size and r' is the (lower) opportunity cost of money for SR investors. In (16) the second term is what the MFI would have paid on the market for the funds lent to the borrower, while the term in square brackets is the implicit subsidy provided by SR investors.
It is easy to check that social welfare in (16) reduces to $SW\left(i',r'\right)=R\left(L',i'\right)-ZPCosts\left(i',r'\right)$ and borrower's welfare is higher in presence of divisible funds because $L'>L^*$ and $i'<i^*$. The difference in loan cost is covered by donors who voluntarily decide to do so and therefore are, by definition, better off than under the alternative of not contributing with their SR savings.

The above commented findings are obviously not conclusive on the general discussion on the role of subsidies and socially responsible finance on microcredit. The contribution of this proposition to the subsidy debate is that, when investment projects are divisible, socially responsible finance may have the positive effect of allowing the borrower to increase the size of his investment with positive effects on social welfare when the latter is defined as the difference between project revenues and the borrower's repayment which satisfies the lender zero profit constraint.

4. Divisible investment projects and group lending in presence of moral hazard

In this section we remove the assumption of exogenous effort by introducing the moral hazard problem in the simplest possible way. The borrower may exert two effort levels yielding, respectively, the probabilities p_H (high effort) and p_L (low effort) of individual project success, with $p_H > p_L$.
In this theoretical framework, and by comparing individual and group lending under the assumption of project divisibility, we may formulate the following proposition.

Proposition 4. In presence of moral hazard and divisible projects, conditions for the satisfaction of the non shirking constraint are more relaxed under standard debt *contract than under participated loans i) in case of group lending and joint liability and ii) in case of individual lending*

When we consider the group lending case under the standard debt contract, the non shirking constraint (NSC), based on the zero profit loan rate[9] in which the lender assumes high effort on behalf of group borrowers, is

$$\left(p_H-p_L\right)\left(1+r\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{p_H}\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]>CE \qquad 17)$$

where CE is the cost differential for the borrower between the high and the low effort level. While the same condition in case of group lending with participated loan scheme, with the tax being calculated assuming high effort, is

$$\left(p_H-p_L\right)\left(1-t_{GL-H}\right)\left(1+r\right)^{\frac{\alpha}{\alpha-1}}\left(2-p_H\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]>CE \qquad (18)$$

It is easy to check that (17) is always higher than (18) since their ratio is always higher than one

$$\frac{p_H^{\frac{1}{1-\alpha}}\left(1-\alpha\right)}{\left(2p_H-p_H^2-\alpha\right)\left(2-p_H\right)^{\frac{1}{\alpha-1}}}>1.$$

In the same way, it is possible to demonstrate that the NSC is relaxed under standard debt contracts than under participated loans in case of individual lending.
In fact, if the bank assumes that the NSC is respected, and the borrower exerts maximum effort, it will derive its loan rate from the following zero profit condition

$$i_H^*=\frac{1+r}{p_H}-1$$

In such case the optimal loan size chosen by the borrower is

$$L_H^*=\left(\frac{1+r}{p_H\alpha A}\right)^{\frac{1}{\alpha-1}} \qquad (19)$$

and its NSC

$$\left(p_H-p_L\right)\left(1+r\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{p_H}\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]>CE \qquad (20)$$

Hence, if CE is small enough and the inequality is respected, the NSC is met as well and the bank will effectively charge i_H^*.

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[9] The rate is equal to $i_{GL-H}^*=\frac{\left(1+r\right)}{p_H\left(2-p_H\right)}-1$

Let us now examine the case of individual lending with a participated loan scheme in which the bank charges the opportunity cost of funds r and fixes a tax on profits (t) in order to restore its zero profit condition.
In this case the NSC is met if

$$\left(p_H - p_L\right)\left(1-t_{I-H}\right)\left(1+r\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] > CE \qquad (21)$$

By comparing (20) and (21) we find that the ratio between the two $\frac{p_H^{\frac{1}{1-\alpha}}\left(1-\alpha\right)}{p_H-\alpha}$ is higher than one and therefore the second part of the proposition is respected.

4.1 Divisible investment projects and group lending in presence of moral hazard and heterogeneity of borrower types

Assume now that borrowers are heterogeneous and, specifically, can be of two different (p and q) types and that, for simplicity, there is an equal chance of meeting a p-type (safe borrower) or a q-type (risky borrower).
Each borrower, whatever his type, may exert high or low effort such that $p_H > q_H > p_L > q_L$. Assume also $p = p_H$ which implies that the high level effort of the good type corresponds to the probability of success in the models in which moral hazard and adverse selection problems do not exist since, in those cases, the lender is able to enforce the high level effort to high quality groupmates or individual borrowers.

Consider that, under the assumption that the NSC holds for all borrowers, and under a guess of a 50 percent probability of meeting one of the two types[10], the zero profit condition of the bank is

$$1+i_{AS}^{GL} = \frac{1+r}{p_{AS}\left(2-p_{AS}\right)} \qquad (22)$$

where $p_{AS} = \frac{1}{2}p_H + \frac{1}{2}q_H$.

Loan sizes, when a high quality borrower is in a group with another high quality borrower (SS group) or with a low quality borrower (SR group), are respectively

$$L_{SS}^{*} = \left[\frac{\left(1+r\right)\left(2-p_H\right)}{\alpha A p_{AS}\left(2-p_{AS}\right)}\right]^{\frac{1}{\alpha-1}} \qquad 23)$$

[10] The justification for this assumption will be clear in the page which follows.

$$L_{SR}^{*}=\left[\frac{(1+r)(2-q_H)}{\alpha A p_{AS}(2-p_{AS})}\right]^{\frac{1}{\alpha-1}} \qquad (24)$$

While corresponding profits are

$$\pi_{AS}^{SS}=p_H A\left(L_{SS}^{*}\right)^{\alpha}-p_H(2-p_H)\left(\frac{1+r}{p_{AS}(2-p_{AS})}\right)L_{SS}^{*} \qquad (25)$$

and

$$\pi_{AS}^{SR}=p_H A\left(L_{SR}^{*}\right)^{\alpha}-p_H(2-q_H)\left(\frac{1+r}{p_{AS}(2-p_{AS})}\right)L_{SR}^{*} \qquad (26)$$

The difference between the two profits is:

$$\Delta\left(\pi_{AS}^{SS}-\pi_{AS}^{SR}\right)=p_H\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}(1+r)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{p_{AS}(2-p_{AS})}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\left[(2-p_H)^{\frac{\alpha}{\alpha-1}}-(2-q_H)^{\frac{\alpha}{\alpha-1}}\right]>0$$

Hence, the assortative matching condition is respected since it is more profitable for a safe borrower to make a group with another safe one.
When we compare the profit of a risky borrower matching with a safe one (RS group) and that of the same borrower matching with a risky one (RR group) we obtain

$$L_{RS}^{*}=\left[\frac{(1+r)(2-p_H)}{\alpha A p_{AS}(2-p_{AS})}\right]^{\frac{1}{\alpha-1}} \qquad (27)$$

$$L_{RR}^{*}=\left[\frac{(1+r)(2-q_H)}{\alpha A p_{AS}(2-p_{AS})}\right]^{\frac{1}{\alpha-1}} \qquad (28)$$

and, by comparing profits,

$$\Delta\left(\pi_{RS}-\pi_{RR}\right)=q_H\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}(1+r)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{p_{AS}(2-p_{AS})}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\left[(2-p_H)^{\frac{\alpha}{\alpha-1}}-(2-q_H)^{\frac{\alpha}{\alpha-1}}\right]>0$$

The assortative matching condition is completed by the fact that, also for a risky borrower, it is more profitable to make a group with a safe borrower.
Consider however that the assortative matching literature does not solve a crucial problem. Even though good types have interest to match with other good types and bad types with good types, nothing excludes that bad types, refused by good types, decide to create a group together.
The bank therefore does not know whether it is facing a homogeneous group of high quality or of low quality borrowers and, with a population in which the shares of

good and bad types are equal and even in number, the interest rate he charges is exactly the i_{AS} rate derived in (22).

Under the i_{AS} rate the high and low quality borrowers will ask respectively loan sizes indicated in (23) and (28)
Since (23) and (28) are different the bank may infer from the demanded loan size the quality of the borrower. It can then directly apply, without passing for this first step, a loan rate which ensures the zero profit conditions by applying the correct probability of success to each borrower, thereby charging a higher rate to low quality and a lower rate to high quality borrowers.

More specifically the bank will offer an $\left(i_{GL}^{SS}, L_{AS}^{SS}\right)$ and an $\left(i_{GL}^{RR}, L_{AS}^{RR}\right)$ contract to the two different types of groups of borrowers where L_{AS}^{SS} and L_{AS}^{RR} are given respectively by (23) and (28) and

$$1+i_{GL}^{SS}=\frac{(1+r)}{p_H\left(2-p_H\right)} \tag{29}$$

$$1+i_{GL}^{RR}=\frac{(1+r)}{q_H\left(2-q_H\right)} \tag{30}$$

with $i_{GL}^{SS}<i_{GL}^{RR}$.

In such case profits for the low quality borrowers when they respectively declare their true identity or cheat will be

$$\pi_{RR}^{true}=q_H\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}(1+r)^{\frac{\alpha}{\alpha-1}}\left(2-q_H\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{p_{AS}\left(2-p_{AS}\right)}\right)^{\frac{\alpha}{\alpha-1}}\left[1-\alpha\frac{p_{AS}\left(2-p_{AS}\right)}{q_H\left(2-q_H\right)}\right] \tag{31}$$

and

$$\pi_{RR}^{cheat}=q_H\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}(1+r)^{\frac{\alpha}{\alpha-1}}\left(2-p_H\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{p_{AS}\left(2-p_{AS}\right)}\right)^{\frac{\alpha}{\alpha-1}}\left[1-\alpha\frac{p_{AS}\left(2-p_{AS}\right)\left(2-q_H\right)}{p_H\left(2-p_H\right)^2}\right]$$
(32).

Unfortunately, under this case it is highly likely that low quality borrowers in the low quality group will have an incentive to cheat and ask the same loan size as the high quality borrowers in the high quality group. By choosing between these two options low quality borrowers will trade off an excess amount of loan (with respect to its optimal one) with a lower interest rate.
The ratio between the two possibilities is

$$\frac{\pi_{RR}^{true}}{\pi_{RR}^{cheat}} = \frac{(2-q_H)^{\frac{\alpha}{\alpha-1}}\left[1-\frac{\alpha p_{AS}(2-p_{AS})}{q_H(2-q_H)}\right]}{(2-p_H)^{\frac{\alpha}{\alpha-1}}\left[1-\frac{\alpha p_{AS}(2-p_{AS})(2-q_H)}{p_H(2-p_H)^2}\right]} \quad (33)$$

which is not univocally lower than one but it is so for reasonable parametric values. Let us examine the same problem under the participated loan contract. Under this scenario it is possible to demonstrate the following proposition

Proposition 5. In presence of moral hazard and heterogeneity of borrower types a combination of participated loan plus group lending makes it possible for the MFI, under reasonable parametric conditions, to devise a menu of contracts which allows to discriminate among groups with different quality, thereby ensuring to the high quality borrower higher profits than under the standard debt contract individual lending case.

In such case optimal loan sizes under the different possible groups are

$$L_{SS} = \left[\frac{(2-p_H)(1+r)}{\alpha A}\right]^{\frac{1}{\alpha-1}} \quad (34), \qquad L_{SR} = \left[\frac{(2-q_H)(1+r)}{\alpha A}\right]^{\frac{1}{\alpha-1}} \quad (35),$$

$$L_{RS} = \left[\frac{(2-p_H)(1+r)}{\alpha A}\right]^{\frac{1}{\alpha-1}} \quad (36), \qquad L_{RR} = \left[\frac{(2-q_H)(1+r)}{\alpha A}\right]^{\frac{1}{\alpha-1}} \quad (37).$$

While, under the assumption that the bank correctly identifies different types,[11] the difference in profits for a safe borrower matching with a safe versus matching with a risky borrower is

$$\Delta(\pi_{SS}-\pi_{SR}) = (1-t_{AS})\,p_H\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}(1+r)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\left[(2-p_H)^{\frac{\alpha}{\alpha-1}}-(2-q_H)^{\frac{\alpha}{\alpha-1}}\right]>0 \quad (38)$$

And, correspondingly, the difference in profits for a risky borrower matching with a safe versus matching with a risky borrower is

$$\Delta(\pi_{RS}-\pi_{RR}) = (1-t_{AS})\,q_H\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}(1+r)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{a}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\left[(2-p_H)^{\frac{\alpha}{\alpha-1}}-(2-q_H)^{\frac{\alpha}{\alpha-1}}\right]>0 \quad (39)$$

[11] It is easy to show that the assortative matching condition holds even when the bank does not correctly identifies borrower types since, for any type of interest rate and loan size, both safe and risky borrower always prefer to join another safe borrower.

Again, under the assumption that the NSC is respected, all borrower types will have the incentive to make a group with safe borrowers. This does not imply that only SS groups will arrive to the MFI since low quality borrowers, after being refused by high quality ones will nonetheless participate to the venture by forming RR groups. Hence the two relevant loan sizes will be (34) and (37).
Will RR groups correctly reveal their type? In this case profits for the low quality group borrower, which are correctly revealing their quality with the loan size choice, are

$$\pi_{RR-PS}^{true} = \left(1-t_{qH}\right) q_H \left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}} \left(1+r\right)^{\frac{\alpha}{\alpha-1}} \left(2-q_H\right)^{\frac{\alpha}{\alpha-1}} \left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \tag{40}$$

where t_{qH} is the tax on profits which satisfies the zero profit condition of the bank under the assumption that the group is made by two low quality borrowers.
The same profits when the low quality group cheats by demanding the optimal loan size of high quality borrowers are

$$\pi_{RR-PS}^{cheat} = \left(1-t_{pH}\right) q_H \left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}} \left(1+r\right)^{\frac{\alpha}{\alpha-1}} \left(2-p_H\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \left[1-\alpha\frac{\left(2-q_H\right)}{\left(2-p_H\right)}\right] \tag{41}$$

where t_{pH} is the tax on profits which satisfies the zero profit condition of the bank under the assumption that the group is made by two high quality borrowers.
Remember that in the participated loan contract the interest rate is the same. Hence, cheating does not imply an advantage in terms of interest rates, but in term of a lower tax on profits (the bank believes that the borrowers are high quality and underestimates the risk of default hence $t_{pH} < t_{qH}$).
The ratio between the two profits is

$$\frac{\left(1-t_{qH}\right)}{\left(1-t_{pH}\right)} \frac{\left(2-q_H\right)^{\frac{\alpha}{\alpha-1}}}{\left(2-p_H\right)^{\frac{\alpha}{\alpha-1}}} \frac{\left(1-\alpha\right)}{\left[1-\alpha\frac{\left(2-q_H\right)}{\left(2-p_H\right)}\right]} \tag{42}$$

Numerator is always positive while denominator is negative when $\alpha > \frac{\left(2-p_H\right)}{\left(2-q_H\right)}$, so in this range cheating is not the best strategy. Since (42) holds under reasonable parametric conditions, a participated loan with group lending allows to solve the problem of heterogeneity of borrower types.
Consider also that this discriminating capacity of the bank ensures higher profits to the borrower than in the standard debt contract individual lending under reasonable parametric condition.
This is because with the latter the bank can not discriminate between a good and a bad quality borrower and charges the loan rate

$$1+i_{AS}=\frac{1+r}{p} \tag{43}$$

with $p=\frac{1}{2}q_H+\frac{1}{2}p_H$

The profit of the high quality borrower is

$$\pi_{I-H}=p_H\left(\frac{1}{p}\right)^{\frac{\alpha}{\alpha-1}}(1+r)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \tag{44}$$

On the other hand, when the bank correctly identifies the borrower type, the profit of a high quality borrower in a homogeneous group under group lending and participated loan is

$$\pi_{SS-GL}^{true}=\left(1-t_{pH}\right)p_H\left(2-p_H\right)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}(1+r)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \tag{45}$$

The ratio between the two profits is

$$\frac{\pi_{I-H}}{\pi_{SS-GL}^{true}}=\frac{\left(0.5q_H+0.5p_H\right)^{\frac{\alpha}{1-\alpha}}p_H\left(1-\alpha\right)}{\left(2-p_H\right)^{\frac{1}{\alpha-1}}\left(2p_H-p_H^2-\alpha\right)} \tag{46}$$

which is lower than one under reasonable parametric conditions (when the distance between p_H and q_H is large enough).[12]

The property of participated loans with group lending of solving the problem of borrowers heterogeneity demonstrated in this proposition is not shared by individual lending. In such case the bank which does not know the quality of the borrower charges a loan rate i_{AS} which solves the following zero profit condition

$$1+i_{AS}=\frac{1+r}{p_{AS}} \tag{47}$$

Optimal loan size and profits for the high quality borrower will be

$$\pi_{AS}^{S}=p_H AL^{\alpha}-p_H\left(\frac{1+r}{p_{AS}}\right)L \tag{48}$$

and

$$L_{AS}^{S}=\left(\frac{1+r}{p_{AS}\alpha A}\right)^{\frac{1}{\alpha-1}} \tag{49}$$

[12] Consider that, if output is verifiable ex post, the lender may threaten to impose a penalty in case he observes that a group of declared high quality borrowers realises the output of a group of low quality borrowers in order to induce truthful declarations. The participated loan/group lending scheme allows to obtain the same result without paying ex post verification costs.

While those for the low quality borrower

$$\pi_{AS}^{R} = p_L A L^{\alpha} - p_L \left(\frac{1+r}{p_{AS}} \right) L \tag{50}$$

and

$$L_{AS}^{R} = \left(\frac{1+r}{p_{AS} \alpha A} \right)^{\frac{1}{\alpha - 1}} \tag{51}$$

Somewhat surprisingly the optimal loan size demanded by borrowers of different quality is the same. Hence, it is not possible for the MFI to discriminate borrower quality.

The rationale is that the optimal loan size does not depend on the quality of the borrower (probability of success) in the individual lending case as borrower quality determines the probability of project success and therefore equally affects both revenues and costs of the limited liability borrower. On the contrary, under group lending borrower quality has the additional role of affecting the joint liability component of the groupmate and therefore different borrower qualities determine different optimal loan sizes.

To sum up, the problem of assortative matching in presence of heterogeneous quality asymmetric information can be solved only by the combination of project divisibility and group lending.

5. Conclusions

Lending is a complex activity whose success is crucially affected by conflicts of interest and informational asymmetries between borrowers and lenders. Theoretical models usually look at only one or a few of these aspects at a time, at the cost of sacrificing most of the reality and providing dubious policy suggestions. This may typically occur when the interaction of a policy advice, which is optimal when considering only one of these aspects, has a perverse effect on other relevant aspects, neglected in the specific theoretical framework.

This is exactly what we show in our paper where, with progressive approximations, we highlight that conclusions in the simpler scheme may be reversed when we consider the integrated framework in which we combine heterogeneity of borrower types, moral hazard, ex post hidden information and divisible investment projects.

Results of our paper are resumed in Table 1. We first find that, when projects are divisible, under reasonable parametric conditions, standard debt contracts are dominated (in terms of output but not in terms of profits) by participated loans in which lower lending rates are charged in exchange of a share of the uncollateralised borrower's profits (LOAN SIZE AND OUTPUT (BUT NOT PROFIT) DOMINANCE OF

PARTICIPATED LOANS OVER STANDARD DEBT CONTRACTS IN CASE OF INDIVIDUAL LENDING).

A second group of results shows that, under standard debt contracts, loan size and borrower's profits are exactly equal under individual and under group lending with joint liability, with the difference that the lending rate which ensures the zero profit condition of the borrower in presence of group lending is lower (LOAN SIZE AND PROFIT EQUIVALENCE BETWEEN INDIVIDUAL AND GROUP LENDING UNDER STANDARD DEBT CONTRACTS).

When we introduce the moral hazard problem in our framework we find that (under given parametric conditions) participated loans, by reducing borrower's profits, may strengthen the non shirking constraint, thereby making less easy the realization of the conditions for optimal borrower's effort (DOMINANCE OF STANDARD DEBT CONTRACTS OVER PARTICIPATED LOANS IN PRESENCE OF MORAL HAZARD). The additional problem is that, if informational asymmetries are severe, and not confined to interim hidden action, but extended to ex post hidden information, profit underreporting may vanify the benefits of the participated loan scheme.

When we finally introduce heterogeneity of types we find that group lending with individual loan may not provide the incentive for a group of low quality borrowers of signaling correctly their type, while participated loans plus group lending have the advantage of making it easier separating equilibria which allow virtuous selection of borrower groups under group lending with moral hazard and heterogeneity of borrower types. Such discrimination capacity allows the bank to charge lower loan rates and ensures, under reasonable parametric conditions, higher profits to the high quality borrower than under the standard debt individual lending case (DOMINANCE OF PARTICIPATED LOAN/ GROUP LENDING SCHEMES OVER INDIVIDUAL LENDING/ STANDARD DEBT CONTRACTS IN TERMS OF DISCRIMINATION OF BORROWER QUALITY IN PRESENCE OF MORAL HAZARD AND HETEROGENEITY OF TYPES).

As a side issue, our paper provides interesting insights in the evaluation of the constraints posed by Islamic principles to the adoption of standard financial instruments, given their preference for full profit sharing contracts with respect to standard debt contracts in which interests are paid on loans. With this respect, it shows that, in presence of divisible investment, this constraint has the effect of increasing output at the cost of lower borrower profits in the perfect information framework or when only moral hazard is considered, but, in presence of group lending, it may also lead to higher profits when heterogeneity of types is added to the picture for the superior capacity of participated loans of ensuring a virtuous selection of borrowers.

As a final remark consider that our conclusions crucially depend from the (reasonable) hypothesis of a risk of non restitution which is invariant in project size. Additional theoretical work removing this hypothesis may bring further the reflection on the relationship among investment divisibility, informational asymmetries and microfinance.

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Table 1. Main results in the divisible loan microfinance framework

	Standard debt contract		Participated Loan	
	Individual lending	Group lending	Individual lending	Group lending
Loan size	L^*	L^*_{GL}	L^{**}	L^{**}_{GL}
Borrower's profits	Π^*	Π^*_{GL}	Π^{**}	Π^{**}_{GL}
Lending rate	i^*	i^*_{GL}	r	r

1) LOAN SIZE AND OUTPUT (BUT NOT PROFIT) DOMINANCE OF PARTICIPATED LOANS OVER STANDARD DEBT CONTRACTS IN CASE OF INDIVIDUAL LENDING ($L^{**}>L^*$, $f(L^{**}) > f(L^*)$and $\Pi^{**}< \Pi^*$)

2) LOAN SIZE EQUIVALENCE BETWEEN INDIVIDUAL AND GROUP LENDING UNDER STANDARD DEBT CONTRACTS ($L^*=L^*_{GL}$)

3) PROFIT EQUIVALENCE BETWEEN INDIVIDUAL AND GROUP LENDING UNDER STANDARD DEBT CONTRACTS ($\Pi^*= \Pi\ ^*_{GL}$)

4) INTEREST RATE DOMINANCE OF INDIVIDUAL LENDING W.R.T. GROUP LENDING UNDER STANDARD DEBT CONTRACTS ($i^*_{GL}<i^*$)

5) DOMINANCE OF STANDARD DEBT CONTRACTS OVER PARTICIPATED LOANS IN GROUP LENDING WITH MORAL HAZARD

6) DOMINANCE OF GROUP LENDING OVER INDIVIDUAL LENDING IN PRESENCE OF MORAL HAZARD AND HETEROGENEITY OF TYPES UNDER STANDARD DEBT CONTRACTS

7) DOMINANCE OF PARTICIPATED LOAN/ GROUP LENDING SCHEMES OVER INDIVIDUAL LENDING/ STANDARD DEBT CONTRACTS IN TERMS OF DISCRIMINATION OF BORROWER QUALITY IN PRESENCE OF MORAL HAZARD AND HETEROGENEITY OF TYPES

Appendix 1. Borrower profit (output) is lower (higher) under participated loan than under standard debt contract

To evaluate the difference between $\pi^*\left(L^*,i^*\right)$ and $\pi^{**}\left(L^{**},r\right)$ we compute $\pi^*\left(L^*,i^*\right)$ by considering that

$$R=\left(1+r\right)L=p\left(1+i\right)L-Ks \qquad \text{(A1.1)}$$

The lending rate which satisfies the bank zero profit condition is

$$1+i^* = \frac{\left(1+r\right)L}{pL}+\frac{Ks}{pL}=\frac{\left(1+r\right)}{p}+\frac{Ks}{pL} \qquad \text{(A1.2)}$$

The borrower chooses the optimal loan size by maximising his profit function $\max_L : \left\{p\left[f\left(L\right)-\left(1+i^*\right)L\right]\right\}$ which, by replacing the production function $f\left(L\right)=AL^\alpha$, turns into

$$\max_L : \left\{pAL^\alpha - p\left(\frac{1+r}{p}+\frac{Ks}{pL}\right)L\right\}=\left\{pAL^\alpha-\left(1+r\right)L-Ks\right\} \qquad \text{(A1.3)}$$

First order condition gives

$$\alpha ApL^{\frac{\alpha}{\alpha-1}}-\left(1+r\right)=0 \qquad \text{(A1.4)}$$

The optimal loan size is

$$L^* = \left(\frac{1+r}{\alpha Ap}\right)^{\frac{1}{\alpha-1}} \qquad \text{(A1.5)}$$

and borrower profits are

$$\pi^* = \left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}\left(\frac{1}{p}\right)^{\frac{1}{\alpha-1}}\left(1+r\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]-Ks \qquad \text{(A1.6)}$$

To calculate $\pi^{**}\left(L^{**},r\right)$ consider that, if the bank applies an interest rate equal to the opportunity cost, the borrower maximizes the following profit function

$$\max_L : \left\{p\left[f\left(L\right)-\left(1+r\right)L\right]\right\}=pAL^\alpha-p\left(1+r\right)L \qquad \text{(A1.7)}$$

First order condition implies that
$\alpha ApL^{\alpha-1}-p\left(1+r\right)=0$.

Hence, the optimal loan size is $L^{**}=\left(\dfrac{1+r}{\alpha A}\right)^{\frac{1}{\alpha-1}}$ (A.1.8)

and profits are

$$\pi^{**} = p\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}(1+r)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \tag{A.1.9}$$

Consider that, if the bank fixes a lending rate equal to the opportunity cost $\left(i^{*}=r\right)$, it incurs in a loss is equal to

$$(1+r)L^{**}-p(1+r)L^{**}=(1+r)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}[1-p] \tag{A1.10}$$

To restore its zero profit condition it must enforce a profit sharing clause which imposes a tax t on borrower profits such that:

$$t\left(\pi^{**}\right)=(1+r)L^{**}(1-p)$$

Hence, the profit share must be such that

$$t=\frac{(1+r)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}[1-p]}{p(1+r)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]}= \tag{A1.11}$$

which, by rearranging and dividing by $\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$, yields

$$t=\frac{\alpha(1-p)}{p(1-\alpha)} \tag{A1.12}$$

The share must be lower than one in order to ensure borrower's nonnegative profits. This condition is respected only if $p>\alpha$.

We now wonder whether in the participated loan scheme borrower's profits, net of the lender's profit share, are higher than under the standard debt contract. This is the case if:

$$(1-t)\pi^{**}\left(L^{**},r\right)>\pi^{*}\left(L^{*},i^{*}\right)$$

or

$$t<\frac{\pi^{**}-\pi^{*}}{\pi^{**}} \tag{A1.13}$$

which may be rewritten as

$\frac{\alpha(1-p)}{p(1-\alpha)} < 1 - p^{\frac{\alpha}{1-\alpha}}$. By simplifying we obtain the following sufficient[13] condition

$(p-\alpha) > (1-\alpha)\, p^{\frac{1}{1-\alpha}}$, which is never met.

Hence, borrower's profit after satisfying the bank zero profit condition are lower under participated loans than under standard debt contract.
On the contrary, borrower output is higher under participated loans. In fact

$$pA\left(L^{**}(r)\right)^{\alpha} > pA\left(L^{*}\left(i^{*}\right)\right)^{\alpha}$$

Where $L^{**} = \left(\frac{1+r}{\alpha A}\right)^{\frac{1}{\alpha-1}}$ and $L^{*} = \left(\frac{1+r}{\alpha A p}\right)^{\frac{1}{\alpha-1}}$.

[13] It is possible to show that this condition ensures borrower zero profits when combining the loan and the profit sharing component in case of zero screening costs Ks=0. When, on the contrary, Ks>0 the condition is further relaxed.

Appendix 2. Lender zero profit condition under a full profit sharing (zero interest participated loan) contract

Assume that the MFI offers to the borrower a full profit sharing /zero interest participated loan contract (0, $\bar{t}$) and that the borrower productive function has a satiation point in $L=\bar{L}$, where $\bar{L}>L^{**}$ in (5). In this case the borrower can chose the optimal loan size such that $\alpha pA\bar{L}^{\alpha-1}=0$ or $\bar{L}=\left(\frac{1}{Ap\alpha}\right)^{\frac{1}{\alpha-1}}$. To satisfy its zero profit condition the MFI assigns the following share of profits to the borrower (1-$\bar{t}$)Π(0)= p(Aα$\bar{L}^{\alpha-1}$-(1+$i_{zpc(\bar{L})}$) $\bar{L}$) or (1-$\bar{t}$) = [p(Aα$\bar{L}^{\alpha-1}$-(1+$i_{zpc(\bar{L})}$)$\bar{L}$)]/pα$\bar{L}^{\alpha-1}$.[14]
Consider however that, in this case, the loss of the bank is obviously higher than under the case presented in section 2 and equal to $Ls=(1+r)\left(\frac{1}{aA}\right)^{\frac{1}{\alpha-1}}-p\left(\frac{1}{aA}\right)^{\frac{1}{\alpha-1}}$ The share of borrower's profits needed to restore the bank zero profit condition is in this case equal to (1-$\bar{t}$)$=\frac{a\left[(1+r)-p\right]}{p(1-a)}$, which implies $p>a(1+r)$ a condition which is stronger than the one described in Appendix 1.

Appendix 3. Borrower profits in participated loan contracts after satisfying the lender zero profit condition in presence of group lending with joint liability (a comparison with the individual lending case)

As in the case of individual lending we calculate the share of profits that the lender must earn in order to satisfy his zero profit condition in the participated loan scheme:
$t\pi_{GL}^{**}=LSS$
with
$$LSS=(1+r)L^{**}-p(1+r)L^{**}-p(1-p)(1+r)L^{**}=(1+r)L^{**}\left[1-p(2-p)\right] \quad \text{(A3.1)}$$
where LSS is the bank loss when charging an interest rate equal to the opportunity cost.
By replacing for L^{**} we get
$$LSS=(1+r)^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}}(2-p)^{\frac{1}{\alpha-1}}\left[1-p(2-p)\right]$$

[14] Following the reasoning of lemma 1 the difference between [p(Aα$\bar{L}^{\alpha}$-(1+izpc($\bar{L}$))$\bar{L}$)]-p[AL*$^{\alpha}$ – L* (1+i*)] is also positive as it is that between [p(Aα$\bar{L}^{\alpha}$-(1+$i_{zpc(\bar{L})}$)$\bar{L}$)]-p[AL**$^{\alpha}$ –L** (1+ $i_{zpc(L^{**})}$)].

Hence, the lender's profit share is equal to (A3.2)

$$t_{GL} = \frac{(1+r)^{\frac{\alpha}{\alpha-1}} \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}} (2-p)^{\frac{1}{\alpha-1}} \left[1-p(2-p)\right]}{p\left[(1+r)(2-p)\right]^{\frac{\alpha}{\alpha-1}} \left(\frac{1}{A}\right)^{\frac{1}{\alpha-1}} \left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]} \quad \text{(A3.3)}$$

or, simplifying

$$t_{GL} = \frac{\alpha\left[1-p(2-p)\right]}{p(2-p)(1-\alpha)} \quad \text{(A3.3')}$$

The share is lower than one if:

$$1-\sqrt{1-\alpha} < p < 1+\sqrt{1-\alpha} \quad \text{(A3.4)}$$

when we compare this condition with the one obtained in case of individual lending $(p > \alpha)$ we find that, for a given α, the inequality in (A3.4) is verified for relatively lower levels of p. The rationale is that group lending reduces the expected loss for the bank which therefore requires a lower profit tax to satisfy its zero profit condition.
Consider also that the borrower profit in a profit lending sharing is never larger than under the standard debt contract, since the ratio between $\pi^{*}_{GL}\left(L^{*}_{GL}, i^{*}_{GL}\right)$ and $\pi^{**}_{GL}\left(L^{**}_{GL}, r\right)$ is $\frac{p^{\frac{1}{1-a}}(1-a)}{(2-p)^{\frac{1}{\alpha-1}}\left(p^2-2p+a\right)} > 1$.