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Magnetized four-dimensional $\mathbb{Z}_2 \times \mathbb{Z}_2$ Orientifolds

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to Salvo

Chapter 1

Introduction

It has been about fourty years since the era of String Theory [1] began with the seminal work of Veneziano [2] on the four hadron scattering amplitudes. The non-renormalizability of the Albert Einstein General Relativity [3, 4, 5, 6, 7, 8, 9, 10] has actually provided the main hint to reconsider String Theory as a possible candidate embracing the gravitational interaction into the frame of a unified quantum theory containing all known particles and fundamental interactions [11, 12]. In particular, the solution to the problem of the short-distance divergences of the quantum gravity and the unavoidable presence of spin two massless particles in the spectrum, have led to think of String Theory as a promising and finite quantum theory of gravity.

Conceiving a theory of strings as a model for the description of nature, had the immediate consequence of giving up the fundamental ingredient of the quantum field theory, the point particle interpretation.

1.1 The Bosonic String

The underlying idea on which String Theory rests is essentially an extension of the basic principle of Quantum Field Theory, to the case in which the fundamental entities are one-dimensional objects with characteristic length l_s. In this context particles are waves propagating along the string, and the point-like excitations of field theory are replaced by one-dimensional excitations interacting in a geometrical way. Thus, in a D-dimensional space-time, a string sweeps out two-dimensional surfaces, or *world-sheets* Σ, parametrized by the two conventional variables τ and σ ($\tau \in (-\infty, +\infty), \sigma \in [0, \pi]$) and described by the position of the string $X^\mu(\tau, \sigma)$ ($\mu = 0, .., D-1$). Therefore, we are naturally led to study two-dimensional field theories.

The simplest model of relativistic string is the bosonic string. Its motion in a flat D-dimensional space-time is defined by the following action [13, 14]

$$S = -T \int d^2\xi \left(\frac{1}{2}\sqrt{-g} g^{\alpha\beta}(\xi) \eta_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu\right), \tag{1.1}$$

usually known as *Polyakov action*, where $T = \frac{1}{2\pi\alpha'}$ is the string tension, α' is the *Regge slope* ($[T] = L^{-2} = M^2$), $d^2\xi = d\tau d\sigma$, and $g_{\alpha\beta}(\xi)$ $(\alpha, \beta = 0, 1)$ is the metric on the world-sheet.

This action, describing D massless scalar fields coupled to gravity in two-dimensions, is the most convenient action for the quantization procedure, since it is invariant under:

1. *global* D-dimensional Poincarè transformations

$$\delta X^\mu = \Omega^{\mu\nu} X_\nu + a^\mu \,, \tag{1.2}$$

$$\delta g_{\alpha\beta} = 0 \,, \tag{1.3}$$

   where $\Omega^{\mu\nu}$ is an antisymmetric matrix of the D-dimensional representation of the $SO(1, D-1)$ group and a^μ is a constant vector

2. *local*

   - world-sheet diffeomorphisms (*diff*):

$$\delta X^\mu = \zeta^\alpha \partial_\alpha X^\mu \,, \tag{1.4}$$

$$\delta g_{\alpha\beta} = \nabla_\alpha \zeta_\beta + \nabla_\beta \zeta_\alpha \,, \tag{1.5}$$

     i.e. it is independent from the choice of the coordinates ξ^α .

   - Weyl rescalings of the metric on the world-sheets

$$\delta X^\mu = 0 \,, \tag{1.6}$$

$$\delta g_{\alpha\beta} = 2\Lambda(\xi) g_{\alpha\beta} \,. \tag{1.7}$$

Actually there is a residual symmetry of the world-sheet theory, left over by the gauge-fixing of (*diff* $\times$ Weyl)-invariance and that is fundamental for the construction of a consistent quantum theory. It is *conformal invariance*, a key property of the simple action of eq. (1.1), that allows to get rid of negative norm states of the Hilbert space and thus must be preserved by quantum corrections. Conformal anomaly cancellation is thus a consistency condition of string theory. It reflects itself in the exact vanishing of the central charge imposing stringent restrictions on the dimension of the space-time, fixed to $D = 26$ for the bosonic string case.

According to the possible choices of the boundary conditions, strings can come in two very distinctive varieties: **closed** or **open**. In particular, if the coordinate X^μ satisfy periodic boundary conditions, $X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \pi)$, the strings are topologically equivalent to circles. By contrast, for open strings the endpoints are no more coincident, but can be *free* to move, and thus subjected to Neumann boundary conditions $X'^\mu(\tau, 0) = X'^\mu(\tau, \pi)$, or *fixed* in space-time, *i.e.* such to satisfy Dirichlet boundary conditions $X^\mu(\tau, 0) = a^\mu$, $X^\mu(\tau, \pi) = b^\mu$, with a^μ and b^μ constant vectors.

The quantization of the theory can follow several different methods, the simplest of which is the so called *light-cone gauge quantization* [15]. It consists in eliminating two degrees of freedom of the gauge-fixed theory, with the end result that only the transverse space-time directions are allowed to oscillate. As a consequence, are just the corresponding transverse vibrating modes that give rise to the complete spectrum of the theory. Both closed and open string theories present a tachyon at the lowest mass level, a particle with negative mass-squared. As it happens in field theory, this signals a wrong identification of the vacuum. The next mass level consists of massless bosonic space-time fields, that for the closed string are the transverse modes of a two tensor, for a total of $(D-2)^2$ states, while for the open string they are the transverse modes of a vector, *i.e.* $(D-2)$ states. The massless closed spectrum thus describes: a traceless symmetric tensor that can be identified with the *graviton* g_{ij}, a spin two massless particle whose presence made string theory the first candidate able to incorporate gravity in a unifying quantum field theory of all interactions; an antisymmetric tensor B_{ij} and a scalar field, usually called the *dilaton* ϕ and whose vacuum expectation value is the string coupling constant. Beyond this level, one can find a tower of states with higher and higher masses and spins [2, 16, 17, 18]. Their typical mass is conventionally of the order of the Planck scale. This clearly reveals that, even if they are essential for the soft behavior of the string amplitudes at high-energy [19], they are marginal with respect to the low-energy physics description, which is instead dominated by the massless vibrational modes.

It is worth to stress that, in contrast to closed strings, the presence of the two ends allows open strings to carry *gauge* degrees of freedom, known as *Chan-Paton factors* [20]. They are the charges of an internal symmetry, giving rise to non-abelian groups of the type $U(n)$ in the oriented case and $SO(n)$ or $USp(n)$ in the unoriented case, but not of the Exceptional type of the classical Cartan classification [22, 23]. Actually, as will be discussed in the third chapter, open string theories can be obtained as *orientifolds* of the corresponding closed string ones. In particular, in $D = 26$ one can obtain the $SO(8192)$ bosonic string model, by adding, to the perturbative spectrum of the theory, closed and open unoriented string states. The dimension $n = 2^{13} = 8192$ of the Chan-Paton gauge group is fixed by the *dilaton tadpole cancellation condition*, *i.e.* by eliminating a divergent contribution due to the dilaton field [24]. Following the same steps, one can construct the *Type I* superstring theory in $D = 10$, where in this case the (R-R) tadpole condition fixes $SO(32)$ as Chan-Paton group [25, 27].

1.2 The Superstring

The bosonic string suffers of two main problems:

1. absence of space-time fermions, thus not providing a realistic description of nature;

2. presence of tachyons, that indicates at least the need for a a vacuum redefinition and that the bosonic string, as such, is not a viable theory.

This necessarily leads to the formulation of a different string theory, obtained as a generalized version of the previous one. A first attempt in this direction, able to solve the first problem, is the introduction of fermionic partners of the bosonic space-time coordinates X^μ, namely anticommuting fields $\psi^\mu(\tau,\sigma)$. The so called *dual spinor model* [28, 29] is therefore obtained extending the reparametrization invariant action (1.1), consistently with local supersymmetry in two-dimensions (the necessity for local supersymmetry follows from the fact that the commutator of two supersymmetry transformations is a world-sheet translation, thus implying the presence of a metric $g_{\alpha\beta}$ and of a Rarita-Schwinger field χ^α, in addition to the physical coordinates X^μ and ψ^μ). The resulting action is [13]

$$S = -\frac{T}{2}\int d^2\xi\sqrt{-g}[g^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\mu + i\bar{\psi}^\mu\gamma^\alpha\nabla_\alpha\psi_\mu + i\chi^\alpha\gamma^\beta\gamma^\alpha\psi^\mu(\partial_\beta X_\mu + \frac{i}{4}\chi_\beta\psi_\mu)]\,, \qquad (1.8)$$

that now describes two-dimensional supergravity coupled to the fields X^μ and ψ^μ. The fields ψ^μ are two-dimensional Majorana spinors, but space-time vectors. The supersymmetry gauge field, χ^α, is a Majorana gravitino but a world-sheet vector and, as $g^{\alpha\beta}$, is a non-dynamical Lagrange multiplier. Finally the γ^α are two-dimensional Dirac gamma-matrices.

The introduction of fermionic degrees of freedom increases the symmetries of the action that now is invariant under local supersymmetry transformations, too. The cancellation of the Weyl anomaly instead fixes the total space-time dimension to $D = 10$.

The quantization of the theory can be done following the same steps made for the bosonic string. But a crucial point when dealing with superstring is the choice of boundary conditions for the fermions. As for the closed bosonic string, we still have periodic boundary condition for the bosonic degrees of freedom, but for the fermionic fields we can have either periodic (*Ramond*) or antiperiodic (*Neveu-Schwarz*) boundary conditions [28]:

$$\begin{aligned} \psi^\mu(\tau,\sigma) &= +\psi^\mu(\tau,\sigma+\pi) \quad (R)\,, \\ \psi^\mu(\tau,\sigma) &= -\psi^\mu(\tau,\sigma+\pi) \quad (NS)\,. \end{aligned} \qquad (1.9)$$

As a consequence, since for each handness we have two choices of periodicity, *R* and *NS*, for a closed string one needs to distinguish between four different sectors corresponding to the combined boundary conditions for left- and right-moving modes. Two of them, *NS-NS* and *R-R*, describe space-time bosons, while the others, *NS-R* and *R-NS*, describe space-time fermions. On the other hand, open strings have a single set of modes. Thus the *NS* sector describes space-time bosons, while the *R* sector describes space-time fermions. From this point of view, an open (super)string can be thought as "half" of a closed one, since it is always characterized by only one (left or right) independent sector.

The resulting models, both closed and open, still suffer of having a tachyon as the ground state in the NS-NS sector of their spectrum. In 1977 Gliozzi, Scherk and Olive [30] proposed a peculiar truncation for the spectrum, known as *GSO projection*, able to cure the previous pathologies and to impose further conditions on the states. The truncation is related to the observation that even and odd number of anticommuting fermionic modes have opposite statistic. The *GSO* prescription thus consists in projecting out of the spectrum all states created by *even* numbers of fermionic oscillators, in the NS sector, and in projecting onto states of definite world-sheet fermion number and chirality, in the R sector. In this way one obtains three different supersymmetric string theories, known as *Type IIA* and *Type IIB* [31, 26] in the closed case, and *Type I-SO(32)* [25, 27] in the open case.

It is worth to stress that string theory does not intrinsically predict space-time supersymmetry, although it arises quite naturally. This is due to the fact that the *GSO* projection can give rise to supersymmetric spectra from the target space point of view, thus providing, at any mass level, the same number of fermionic and bosonic degrees of freedom. As will be shown in the next chapter, space-time supersymmetry is actually a bonus in the three previous models, related to some mathematical properties satisfied by the (*Theta*) functions describing the theories in consideration. Actually in $D = 10$ there exist two non-supersymmetric string theories too, the *Type 0A* and the *Type 0B* models [32] that provide non-trivial istances of a *generalized* GSO projection, namely compatible with modular invariance but not with supersymmetry. Both the $0A$ and the $0B$ spectra are thus purely bosonic. They are also not chiral and include a tachyonic mode in their spectrum, summarized in Table (1.1).

	0A	**0B**
NS-NS	$T, \phi, g_{\mu\nu}, B_{\mu\nu}$	$T, \phi, g_{\mu\nu}, B_{\mu\nu}$
R-R	$A_\mu, A'_\mu, C_{\mu\nu\rho}, C'_{\mu\nu\rho}$	$\phi', \phi'', B'_{\mu\nu}, B''_{\mu\nu}, D_{\mu\nu\rho\sigma}$

Table 1.1: Spectrum of the ten-dimensional non-supersymmetric strings

As we will see later on, the low-energy field contents of the susy models are the corresponding ten-dimensional Supergravity (SUGRA). In particular, Type IIA superstring has as low-energy limit the non-chiral Type IIA $\mathcal{N} = (1, 1)$ Supergravity, while the Type IIB superstring has as low-energy limit the chiral Type IIB $\mathcal{N} = (2, 0)$ Supergravity. Finally, the Type I superstring low-energy limit is described by a ten-dimensional Supergravity theory with $\mathcal{N} = (1, 0)$ supersymmetry coupled to a Super-Yang-Mills theory with $SO(32)$ gauge group.

Actually there are five known and consistently defined supersymmetric string theories in critical dimension $D = 10$ (summarized, together with their basic properties, in Table (1.2)). Besides the just mentioned Type II and Type I models, there exist the so-called *Heterotic* $E_8 \times E_8$ and $SO(32)$ strings [33]. As the Type IIs, they are closed (super)string theories, whose low-energy limit are described by the chiral $\mathcal{N} = (1,0)$ Supergravity coupled to Super-Yang-Mills theories with gauge groups $E_8 \times E_8$ and $SO(32)$, respectively. The origin of the heterotic strings is an hybrid mixture of the superstring and the bosonic string. The basic observation on which the heterotic construction rests is that, in the closed string theories, left and right modes are mostly each other independent. It is thus possible to imagine a closed string theory whose left and right modes have a different nature. Then to incorporate space-time supersymmetry, that allows the presence of fermions and absence of tachyons, one can take the right modes of the usual ten-dimensional superstring combined with ten out of the twentysix left modes of the bosonic string. The remaining sixteen bosonic coordinates will have to be compactified on some internal manifold that also allows the introduction of the gauge degrees of freedom. Modular invariance again fixes the possible gauge groups.

Let us emphasize that also for the heterotic case it is possible to build non-supersymmetric models in D=10, corresponding to other versions of the same theory with different projections of the spectrum. The *O(16)* $\times$ *O(16)* is indeed an example of non-supersymmetric heterotic string.

Type	Strings	Massless Bosonic Spectra
IIA	*closed oriented*	**NS-NS:** $g_{\mu\nu}, \phi, B_{\mu\nu}$ **R-R:** $C_{\mu\nu\rho}$, A_μ in $U(1)$
IIB	*closed oriented*	**NS-NS:** $g_{\mu\nu}, \phi, B^N_{\mu\nu}$ **R-R:** $\phi', B^R_{\mu\nu}, D^\dagger_{\mu\nu\rho\sigma}$
I-$SO(32)$	*unoriented open and closed*	$g_{\mu\nu}, \phi, A_{\mu\nu}$ and A_μ in $adj SO(32)$
Het. $SO(32)$	*closed oriented*	$g_{\mu\nu}, \phi, C_{\mu\nu}$ and A_μ in $adj SO(32)$
Het. $E_8 \times E_8$	*closed oriented*	$g_{\mu\nu}, \phi, C_{\mu\nu}$ and A_μ in $adj(E_8 \times E_8)$

Table 1.2: The consistent ten dimensional supersymmetric string theories.

1.3 Compactifications and Dualities

So far, we have discussed the so-called *critical* (super)strings, *i.e.* those propagating in a ten dimensional Minkowskian space-time. If our purpose is to explain within this framework the observed physics, it has to be clarified why only four of these ten dimensions are visible in our everyday experience. One possible solution to this problem is to translate in this context the famous idea of Kaluza and Klein [34], according to which a suitable number of space-time dimensions can constitute a compact and very small manifold whose structure is invisible at low-energy [35, 36, 37, 38, 39, 40].

The choice of the compact manifold is not arbitrary, since it is related to the quest for "realistic" D=4 models with reduced (*N=1*) supersymmetry. Moreover, in order to guarantee the vanishing of the cosmological constant and again, to obtain *N=1* theories in D=4, one considers only those manifolds that preserve part of the space-time supersymmetry of the original ten-dimensional theory. These requirements can be satisfied compactifing the theory over *Calabi-Yau manifolds* [41, 42]. Every compactification of this kind is however characterized by a variable number of free parameters, called *moduli*. In the low-energy limit they appear in the effective lagrangian as scalar fields with only derivative couplings, and so with flat potentials and undetermined vacuum expectation values. Moreover the moduli give rise to a great variety of possible models for the description of perturbative vacua, since no fundamental principle able to single out a unique physical vacuum has been found so far. However, non-supersymmetric vacua typically fix some moduli but give rise to curved space-time.

The simplest class of compact manifolds is given by tori, as product of circles. The effect of this kind of compactifications is the periodic identification of some of the bosonic coordinates, that in turn implies the quantization of the momentum carried by the string along the compact direction. This idea, already known in field theory in the form of Kaluza-Klein compactification, when applied to extended objects like strings or p-branes, offers new interesting "stringy" effects, not present in the usual Kaluza-Klein schemas. Indeed there are new states that correspond to strings wrapped around a compact dimension. Their zero-modes, called *winding* numbers, count the number of times the string wraps and play a very important rôle in the study of non-perturbative description of string theory. In the last decade the string-community has done remarkable efforts in attempting to prove that all the five supersymmetric models are not independent but rather different manifestations, in different regimes, of a unique underlying theory. In particular, the discovery and the formulation of equivalences between two or more string theories, known as *dualities* [43], has definitely led towards the fact the five superstrings can be effectively seen as different asymptotic limits of a unified and more fundamental 11-dimensional theory, provisionally indicated as **M-theory** [44]. Unfortunately M-theory is a mysterious and yet poorly understood theory of which only the low-lying massless modes are known to coincide with the degrees of freedom of the $D = 11$ supergravity theory discovered in the seventies by Cremmer, Julia and Scherk [45].

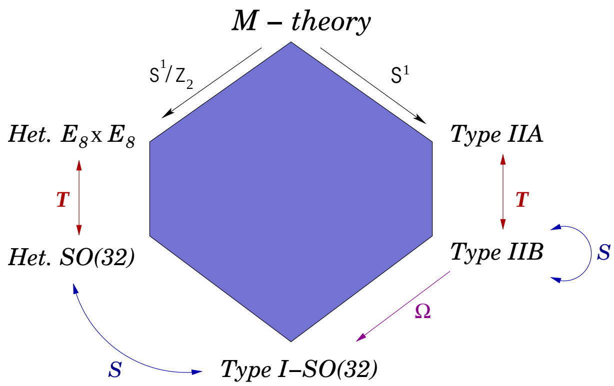

Figure 1.1: The five known consistent Supestring Theories and the Web of Dualities.

By postulating the existence of M-theory, there are two possible dimensional reductions that can give rise to as many superstring theories in $D = 10$. The Type IIA superstring can be recovered by compactifing the M-theory on a circle S^1, while the $E_8 \times E_8$ heterotic superstring can be described by the compactification of the M-theory on the segment $S^1/\mathbb{Z}_2$ [46]. Starting from the Type IIA or from the $E_8 \times E_8$, the remaining three superstring theories may be reached through duality relations. In particular, one can show that the Type IIB theory compactified on a circle with radius R, in the limit $R \to 0$, becomes the Type IIA and viceversa. The two theories are said to be T-dual. *T-duality* indeed, connects theories compactified over spaces with large volume to others compactified over spaces with small volume. Analogously, the heterotic string $E_8 \times E_8$ and the heterotic $SO(32)$ are T-dual [47]. Another kind of duality is the *S-duality* ($g \to \frac{1}{g}$) that identifies the strong-coupling limit of one theory with the weak-coupling limit of the other. The Type I string and the heterotic $SO(32)$ are S-dual [48], while the Type IIB is S-self-dual [49, 50]. The last but not the least of duality relations in ten and eleven dimensions is the world-sheet parity operation $\Omega :\ (x + iy) \to (x - iy)$ that defines the *open descendant* or *orientifold* construction, as will be explained in the third chapter, allows to obtain the Type I superstring theory from the Type IIB one [27]. Let us stress that among the ten dimensional superstrings, the Type I-$SO(32)$ is the only containing in its spectrum closed and open unoriented string in interaction. Thus, while it is possible to build a consistent theory either from closed plus open strings, or from just closed ones (Heterotic and Type II) is not possible to build an interacting theory just from open strings.

The rôle of dualities in string theory is thus twofold. They have highlighted the link between the geometrical and algebraic string notions. T-duality in particular, has showed the unusual way in which strings perceive the space-time geometry, revealing the need for a minimal spacial length. On the other hand, dualities in general connect the perturbative states of two or more models. In the analysis of non-perturbative properties of string theory, it has been crucial the identification of the solitons called *p-branes*, non-perturbative states of the low-energy supergravity spectrum that appear as extended hypersurfaces in p space-time directions [51]. When the p-branes are charged with respect to the fields of the string theory R-R sector, they are called *D-branes* [52, 53]. More precisely, D-branes can be thought as manifolds on which open string endpoints can terminate. T-duality exchanges Neumann with Dirichlet boundary conditions, thus connecting D-branes of various dimensions [54].

Perturbative type I vacua [27, 55, 56] (for reviews see [57, 58]) are a small corner in the moduli space of the eleven dimensional M-theory. Nowadays, however, they are the most promising perturbative models to test possible "stringy" effects at the next generation of accelerators [59]. Indeed, while in the usual heterotic SUSY-GUT scenarios [61] the string scale is directly tied to the Planck scale, making it hard to conceive probes of low-energy effects, the type I string scale is basically an independent parameter, that can be lowered down to a few TeV's [59, 62, 63]. In this setting, and more generally in the context of *brane-world scenarios* [64, 59], the gauge degrees of freedom are confined to some (stacks of) branes while the gravitational interactions invade the whole higher dimensional spacetime. In order to respect the experimental limits on gravitational interactions, the extra dimensions orthogonal to the branes could be up to sub-millimeter size [65], while the extra dimensions longitudinal to the branes should be quite tiny (at least of TeV scale) but still testable in future experiments [66]. In this context, however, several aspects of the conventional Standard Model picture like, for instance, the problem of scale hierarchies and the unification of the running coupling constants at a scale of order $10^{16} GeV$ in the MSSM desert hypothesis, have to be reconsidered [58, 59, 60]. Some other issues, like supersymmetry breaking, find instead new possibilities in type I perturbative vacua.

The mechanisms for breaking supersymmetry offered by Type I models can be essentially grouped into three classes:

1. Breaking by Compactification [67, 59, 62, 68, 69]: beside the conventional Scherk-Schwarz breaking mechanism, where the direction used to separate bosons and fermions is parallel to the D-brane, it is possible to consider orthogonal "breaking" directions. In the low-energy spectrum of the states living on the brane survive, in this case, one or more global supersymmetries (*brane supersymmetry*).

2. *Brane Supersymmetry Breaking* [67, 70]: in these constructions, the consistency conditions require that the supersymmetry is broken at the string scale by brane-antibrane pairs of the same or of different kinds, but it remains exact at tree-level, in the closed sector and on the remaining branes.

3. Breaking by the Introduction of Internal Magnetic Fields [71, 72, 73, 74, 75, 76]: the breaking of supersymmetry is related to the presence of magnetic fields H_i, in the simplest

case, on a torus. The fields couple to the open string endpoints, that under H_i bring charges q_L and q_R, giving rise to different couplings for particles of different spin, that in turn acquire different masses thus breaking the supersymmetry. The mass splitting of the string states can be summarized by the relation $\delta m^2 \sim (2n+1)|H_i| + 2\Sigma_i H_i$, where n is the order of the Landau levels and the Σ_i are the internal helicities of the projected states. The spectrum in general contains tachyons resulting from the scalar fields with $\Sigma_i = -1$ ($\Sigma_i = 1$), for positive (negative) magnetic fields.

The first models with magnetized tori [74] were analysed attempting to find, in the effective field theory of the superstring, chiral four dimensional spectra [71, 77]. More recently, the study of abelian magnetic deformations in open string theories has revealed how the corresponding magnetic couplings can lead to chiral models with supersymmetry breaking [72]. This constructions have been realized assuming a non-vanishing instanton number. Actually it is possible to compensate a non-vanishing instanton density by the introduction of other branes [75, 78]. This is achievable thanks to the peculiar coupling of the branes to the R-R fields [79, 80, 81].

1.4 Outline

The main argument of this thesis is devoted to the study of deformations of $\mathbb{Z}_2 \times \mathbb{Z}_2$ (shift-) orientifolds in four dimensions in the presence of both uniform Abelian internal magnetic fields and quantized NS-NS B_{ab} backgrounds. The organization is as follows. Chapter 2 and the first part of Chapter 3 provide an introduction to the perturbative superstring constructions. The second part of the third Chapter contains a discussion on the relations between shifts and the quantized NS-NS two form B_{ab}, and a review of the basic properties of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ (shift-)orientifolds, that will be useful for the following discussion of the last Chapter. Chapter 4 contains the description of the basic effects induced by the presence of an internal magnetic field in open string theories and a short survey of the $T^4/\mathbb{Z}_2$ six-dimensional models of [75]. Finally, Chapter 5 is devoted to the analysis of four-dimensional examples based on magnetic deformations of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ (shift-)orientifolds previously introduced and to one *brane supersymmetry breaking* example. Notations, conventions and Tables concerning the last two chapters are displayed in the Appendices. In particular, Appendix A collects the relevant lattice sums that enter the one-loop partition functions. The $\mathbb{Z}_2 \times \mathbb{Z}_2$ characters are defined in Appendix B, while Appendix C is a collection of Tables that summarize massless spectra, gauge groups and tadpole cancellation conditions for all the models in this thesis.

Chapter 2

String Perturbation Theory

The Feynman path integral provides the most elegant way to calculate amplitudes in Quantum Field Theory. The computation of amplitudes is related to the sum over all the possible "trajectories" connecting initial and final states. In quantum field theory, in fact, particle interactions are described, perturbatively, by a sum over all the topologically inequivalent Feynman diagrams that can be constructed from elementary vertices and propagators.

In 1981 Polyakov [82, 83] suggested for the (Super)String Theory a generalization of this procedure, where the sum over different paths is replaced by a sum over the world-sheets connecting some (closed and/or open) initial and final curves. Actually, because of conformal invariance, a world-sheet is globally a Riemann surface and thus, in the perturbative Polyakov series, the sum over the world-sheets means, after gauge fixing, the sum over *conformally inequivalent* Riemann surfaces.

Although similar, the string theory perturbative expansion differs from the quantum field theory one in many deep aspects. First of all, string theory is a first-quantized theory (we are dealing with the motion of a string and not with the string fields). Furthermore, in string theory one can build less diagrams than in field theory. In fact, for each diagram of field theory there is a corresponding string diagram obtained "fattening" the world lines of the particles.

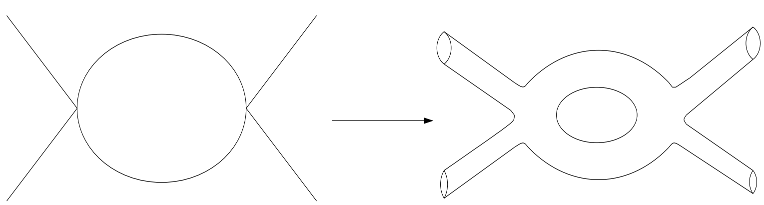

Figure 2.1: Field/String Theory diagram correspondence.

But diagrams that are different for particles in quantum field theory give rise to the same string diagram, when fatten. For example, models of oriented closed strings have the feature of receiving one contribution at each order of perturbation theory as shown in fig. (2.2).

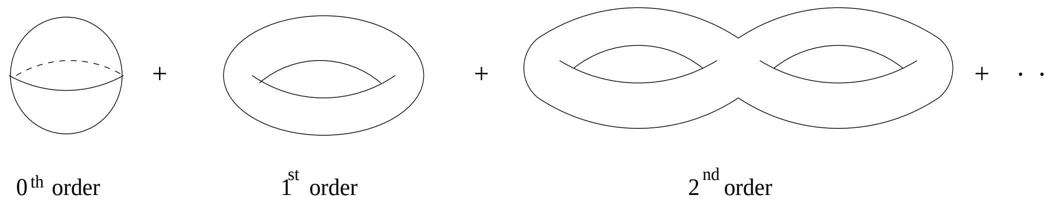

Figure 2.2: Perturbative Series for Closed Oriented String Models.

This is due to the fact that the topology of closed and oriented two-dimensional Riemann surfaces is completely specified by their number of *handles* h [84], that defines the *genus* of the corresponding Riemann surface and counts the number of loops, *i.e.* the order of the perturbative expansion. Actually, the Polyakov series is weighted by the factor $g_s^{-\chi}$, where g_s is the string coupling constant, determined by the vacuum expectation value of the *dilaton* field: $g_s = e^{<\phi>}$. χ is instead the *Euler character* of the surface and for closed orientable Riemann surfaces is defined as $\chi = 2 - 2h$. The classification of the string diagrams becomes richer in models with open and/or unoriented closed strings [85]. The Polyakov expansion involves in fact a sum over unorientable and bordered Riemann surfaces, *i.e.* Riemann surfaces containing a variable number of *boundaries*, b (holes within the surfaces) and *crosscaps*, c (real projective planes). In this case the *Euler character* can be obtained by the relation

$$\chi = 2 - 2h - b - c \ ,$$

while the genus g of the corresponding surface is

$$g = h + \frac{1}{2}\, b + \frac{1}{2}\, c = 1 - \frac{1}{2}\, \chi \ . \tag{2.1}$$

From (2.1) follows that in the closed and open unoriented strings theories the Polyakov perturbative series receives the following contributions:

$\boldsymbol{g}$	χ	Surface	$\boldsymbol{h}$	$\boldsymbol{b}$	$\boldsymbol{c}$	Kind
0	2	*Sphere*	0	0	0	*orientable*
$\frac{1}{2}$	1	*Disk*	0	1	0	*orientable*
$\frac{1}{2}$	1	*Crosscap*	0	0	1	*unorientable*
1	0	*Torus*	1	0	0	orientable
1	0	*Klein bottle*	0	0	2	*unorientable*
1	0	*Annulus*	0	2	0	orientable
1	0	*Möbius strip*	0	1	1	*unorientable*

At genus one there are four possible surfaces with vanishing Euler character ($\chi = 0$): the *Torus*, the *Klein bottle*, the *Annulus* and the *Möbius strip*.

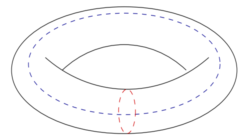

Figure 2.3: The Torus diagram.

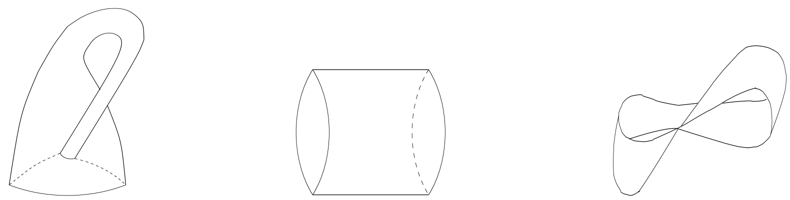

Figure 2.4: The Klein bottle, the Annulus and the Möbius strip Surfaces.

The Torus, *i.e.* the one-loop closed string vacuum amplitude, can be also represented as a flat torus (using a Weyl scaling) defined by a complex plane modulo a two-dimensional lattice generated by two vectors 1 and $\tau = \tau_1 + i\tau_2$ (see figure 2.5). The torus is thus identified by assigning the complex number τ known as *modulus* or *Teichmüller parameter*, that specifies its shape.

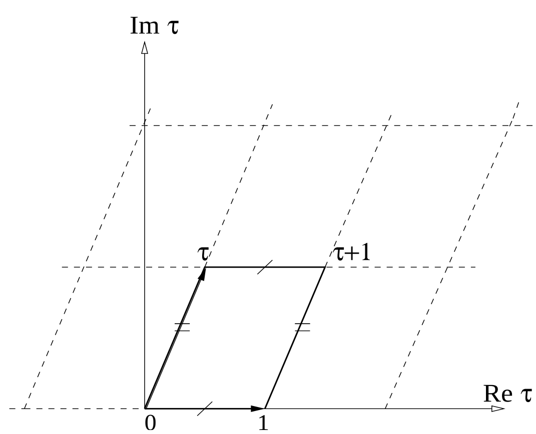

Figure 2.5: The flat torus, defined as a two-dimensional lattice.

In this *fundamental polygon representation* opposite edges are periodically identified by the relation $m + n\tau$, with m and $n \in \mathbb{Z}$. Choosing the imaginary axis as the world-sheet time and the real axis as the spatial direction of the string, then $Im\tau = \tau_2$ is the world-sheet proper time needed by the string to sweep the torus.

However not all τ's in the upper half of the complex plane define different tori. The full family of inequivalent tori can be reached by the lattice automorphism group, known as **Modular Group** $PSL(2,\mathbb{Z}) = SL(2,\mathbb{Z})/\mathbb{Z}_2$ [86] and defined by the transformations

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d} \quad \text{with} \quad ad - bc = 1 \quad (a, b, c, d, \in \mathbb{Z}) \,,$$

or, equivalently, generated by the two modular transformations:

$$T : \tau \longrightarrow \tau + 1$$

$$S : \tau \longrightarrow -\frac{1}{\tau}$$

which satisfy

$$S^2 = (ST)^3 = 1.$$

As a result, the Modular Group reduces the torus moduli space to the *Fundamental Modular Domain* $\mathcal{F}$ defined as

$$\mathcal{F}(\tau) = \{\tau \in \mathbb{C} : |\tau| \geq 1 \; -\frac{1}{2} \leq (Re\,\tau) \leq \frac{1}{2}\} \,,$$

representing only one, out of the infinity, suitable choice of fundamental domain.

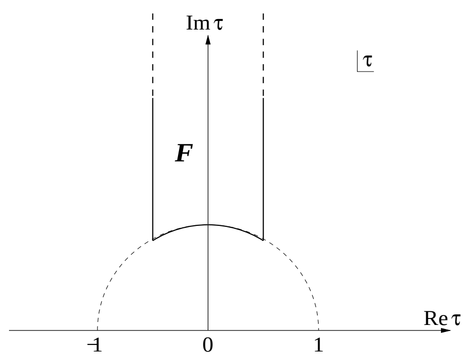

Figure 2.6: Fundamental Modular Domain.

The modular parameter τ, being related to the world-sheet swept by the the string, is also the integration variable entering the vacuum amplitudes in the Polyakov expansion.

As for the Torus, the remaining genus-one surfaces can be described by a lattice of the complex plane, where the vertical parameter τ_2 represents the proper time for the propagation of *closed* strings, in the Klein bottle diagram, and of *open* strings, in the Annulus and Möbius strip diagrams. Let us stress that this vertical time defines the so-called *direct-channel* or *one-loop* diagrams.

Actually there is a further description for the three fundamental polygons, achievable by referring to the doubly-covering torus modded out by suitable involutions. It defines a distinctive choice of the proper time: the horizontal one, l, exhibiting in all three cases the propagation of

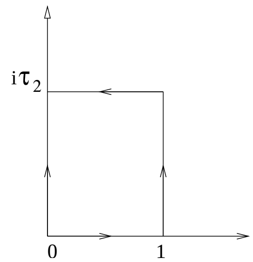

Figure 2.7: Parallelogram representations for the Klein bottle, the Annulus and the Möbius strip surfaces respectively.

closed strings between: a) two crosscaps, for the Klein bottle surface; b) two boundaries, for the annulus diagram; c) one crosscap and one boundary, for the Möbius strip case.

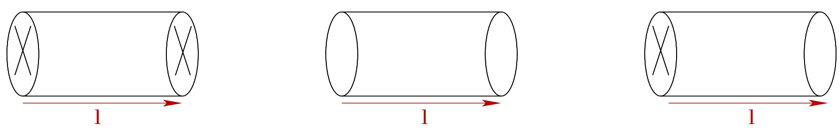

Figure 2.8: Klein bottle, Annulus and Möbius strip diagrams, in the transverse channel representation.

The horizontal time, l, is related to the vertical one, τ_2, by the modular S transformation and defines the *transverse-channel* or *tree* amplitudes. Actually there is a subtlety related to the double-covering torus of the Möbius strip, whose modulus is not purely imaginary. This implies that for the Möbius case the transition to the transverse channel is implemented by a particular sequence of S and T transformations, called P transformation, defined as

$$P = (TS)T(TS) \ , \tag{2.2}$$

that satisfies

$$P^2 = S^2 = (ST)^3.$$

2.1 One Loop Partition Functions

One loop vacuum amplitudes, (*i.e.* amplitudes of processes with no insertions), also called ***genus-one partition functions***, play a very special rôle in string theory, since they describe in a compact way the complete content of the perturbative spectrum.

One loop vacuum amplitudes can be constructed as generalizations of the one-loop vacuum energy in field theory. To see this, let us consider for sake of simplicity, the theory of a scalar field ϕ, with mass m in D space-time dimensions. The vacuum energy can be obtained from the following generating functional

$$Z = \int [D\phi] e^{-S_E} = \int [D\phi] e^{-\int d^D x \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi + m^2 \phi^2)} \sim [det(-\partial_\mu \partial^\mu + m^2)]^{-1/2} \;, \tag{2.3}$$

from which follows that the effective action is

$$\Gamma = \frac{1}{2} log[det(-\partial_\mu \partial^\mu + m^2)] = \frac{1}{2} tr[log(-\partial_\mu \partial^\mu + m^2)] \,. \tag{2.4}$$

Using the Schwinger parametrization for a generic operator A

$$log A = -\int_\epsilon^\infty \frac{dt}{t} e^{-tA} \,, \tag{2.5}$$

where t is the Schwinger parameter and ϵ is an ultraviolet cut-off (being (2.5) divergent in $t = 0$), one gets the following vacuum energy

$$\Gamma = -\frac{V}{2(4\pi)^{\frac{D}{2}}} \int_\epsilon^\infty \frac{dt}{t^{\frac{D}{2}+1}} e^{-tm^2} \,. \tag{2.6}$$

The result (2.6) can be readily generalized to the case of more particles in the loop with mass m and different spin, as

$$\Gamma = \frac{V}{2(4\pi)^{\frac{D}{2}}} \int_\epsilon^\infty \frac{dt}{t^{\frac{D}{2}+1}} Str(e^{-tm^2}) \,, \tag{2.7}$$

where Str takes into account the signed multiplicities of the fermionic and bosonic states with spin J and mass m_J.

Substituting in (2.7) the mass-spectrum of the string theory under consideration (bosonic or fermionic), and scaling out the volume factors, one can obtain the following general expression, called **Torus amplitude**,

$$\mathcal{T} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{1}{(Im\tau)^{\frac{D-2}{2}}} tr(q^{N-a} \bar{q}^{\bar{N}-\bar{a}}) \,, \tag{2.8}$$

where

$$d^2\tau = d(Re\tau) d(Im\tau) = d\tau_1 d\tau_2 \,,$$

and

$$q = e^{2\pi i \tau} \,, \quad \bar{q} = e^{-2\pi i \bar{\tau}} \,.$$

Furthermore N ($\bar{N}$) is the energy operator for an infinite set of oscillators, that counts the level of the excitations, and $a = \bar{a} = \frac{D-2}{24}$ is the normal ordering constant of the corresponding Virasoro operator, *i.e.* the shift contribution to the zero-point energy.

The restriction of the integration to the fundamental domain in (2.8), introduces an ultra-violet cut-off for the string modes, since the $Im(\tau) = 0$ point is eliminated from the range. It follows that these theories are free of UV divergences.

In order to obtain the explicit form of $\mathcal{T}$, one has to evaluate $tr(q^N)$.

For the bosonic string case the calculus is related to the sum over the Fock space of a set of bosonic oscillators, given by the following expression:

$$tr(q^N) = \prod_{k=1}^{\infty} tr(q^{k a_k^\dagger a_k}) = \prod_{k=1}^{\infty} \frac{1}{1-q^k}.$$

Being the number of transverse dimensions $(D-2) = 24$, for the bosonic string the Torus amplitude reads:

$$\mathcal{T} = \int_{\mathcal{F}} \frac{d\tau^2}{(Im\tau)^2} \frac{1}{(Im\tau)^{12}} \frac{1}{|\eta(\tau)|^{48}} , \tag{2.9}$$

where we have introduced the *Dedekind* η - function

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{k=1}^{\infty} (1-q^k) , \tag{2.10}$$

that under the modular group generated by T and S has the following transformations:

$$\begin{aligned} T: &\quad \eta\,(\tau+1) = e^{\frac{i\pi}{12}}\,\eta\,(\tau) \;, \\ S: &\quad \eta\,(-\frac{1}{\tau}) = \sqrt{-i\tau}\,\eta\,(\tau) \;. \end{aligned} \tag{2.11}$$

Thus, the invariance of the integration measure in (2.9) implies that the Torus amplitude (2.9) is *modular invariant*, *i.e.* it is unaffected by the $SL(2,\mathbb{Z})$ transformations of τ.

Modular invariance is a crucial property for the overall consistency (unitarity) of the quantum string theory, because it certifies the correct counting of inequivalent tori. A well defined theory of closed strings on a torus, has to be *modular invariant* [87, 88, 89, 32].

However, the presence of the tachyonic instability in the bosonic theory makes the modular integral over $\mathcal{F}$ in (2.9) divergent. This pathology is cured by the fermionic (super)string.

For the superstring case, the evaluation of $tr(q^N)$ involves also the sum over fermionic (anti-commuting) oscillators, for which standard results for a Fermi gas can be used. If λ_r is a generic fermionic oscillator, it follows that:

$$tr(q^{N_F}) = tr(q^{\sum_r r\lambda^i_{-r}\lambda_{r,i}}) = \prod_r tr(q^{r\lambda^i_{-r}\lambda_{r,i}}) = \prod_r (1+q^r)^{D-2} \;, \tag{2.12}$$

where $(D-2) = 8$ is the number of transverse coordinates. Let us remind that for the fermionic string we have to distinguish between two sectors of the theory, namely the *Neveu-Schwarz* sector, with half-integer modes $r \in \mathbb{Z} + 1/2$, and the *Ramond* sector, with integer modes $r \in \mathbb{Z}$.

The complete Torus amplitude thus reads

$$\mathcal{T} = \int_{\mathcal{F}} \frac{d^2\tau}{(\tau_2)^2} \frac{1}{(\tau_2)^4} tr(q^{N_{Bose}+N_{Fermi}-a} \bar{q}^{\bar{N}_{Bose}+\bar{N}_{Fermi}-\bar{a}})\,, \tag{2.13}$$

where the shifts to the vacuum energy are in this case

$a = \bar{a} = \frac{D-2}{16}$, in the NS sector;

$a = \bar{a} = 0$, in the R sector.

For this purpose, it is interesting to stress that the Ramond and Neveu-Schwarz sectors can find in this context a geometrical interpretation. This is related to the fact that we are dealing with fermions on a torus, *i.e.* on a Riemann surface with a non-trivial topology, that can give rise to ambiguities in the boundary condition definitions. Since a torus is characterized by two periods, a fermionic theory on a torus is specified by choosing periodic (**P**) or anti-periodic (**A**) boundary conditions around the two independent and non-contractible cycles of the torus, namely by choosing a fixed **spin structure** [32].

There are four possible spin structures, pictorially denoted as

$$time\ \overset{space}{\Box} = P\,\overset{P}{\Box}\ ,\quad P\,\overset{A}{\Box}\ ,\quad A\,\overset{P}{\Box}\ ,\quad P\,\overset{P}{\Box}\ , \tag{2.14}$$

corresponding to the periodicity properties (P and A) in the space (σ_1) or time (σ_2) directions of the torus. They are related to the following transformations of the fermions

$$\begin{aligned} \psi(\sigma_1 + 2\pi, \sigma_2) &= -e^{2\pi i\alpha}\psi(\sigma_1, \sigma_2)\,, \\ \psi(\sigma_1, \sigma_2 + 2\pi) &= -e^{2\pi i\beta}\psi(\sigma_1, \sigma_2)\,, \end{aligned} \tag{2.15}$$

and to the choices for the *twists* (α, β)

$$(A, A) = (0, 0) \quad (A, P) = (0, \frac{1}{2}) \quad (P, A) = (\frac{1}{2}, 0) \quad (P, P) = (\frac{1}{2}, \frac{1}{2}).$$

The relation of these spin structures to the NS and R sectors is given by:

$$A\,\overset{A}{\Box} \quad \equiv \quad tr_{\rm NS}\Big(q^{N_F}\Big) = \prod_{r=1/2}^{\infty} (1+q^r)^8 = \prod_{k=1}^{\infty} \left(1+q^{k-1/2}\right)^8, \tag{2.16}$$

$$A\,\overset{P}{\Box} \quad \equiv \quad tr_{\rm R}\Big(q^{N_F}\Big) = 16 \prod_{k=1}^{\infty} \left(1+q^k\right)^8. \tag{2.17}$$

where the 16 in (2.17) counts for the degeneration of the R vacuum which is a Majorana-Weyl spinor.

To obtain the total superstring amplitude we have to multiply the previous results by the corresponding bosonic contribution $q^{1/3}\,\eta(\tau)^{-8}$, thus arriving to the following expressions:

$$tr_{NS}\Big[q^{(N_B+N_F-a)}\Big] = \frac{\prod_{k=1}^{\infty}\left(1+q^{k-1/2}\right)^8}{q^{1/2}\prod_{k=1}^{\infty}\left(1-q^k\right)^8} = \frac{1}{\eta(\tau)^8}\,\frac{\theta_3^4(0|\tau)}{\eta^4(\tau)}\ , \tag{2.18}$$

$$tr_R\Big[q^{(N_B+N_F-a)}\Big] = 16\,\frac{\prod_{k=1}^{\infty}(1+q^k)^8}{\prod_{k=1}^{\infty}(1-q^k)^8} = \frac{1}{\eta(\tau)^8}\,\frac{\theta_2^4(0|\tau)}{\eta^4(\tau)}\ , \tag{2.19}$$

where we have introduced the **Jacobi θ- functions**, elliptic functions defined by Gaussian sums or by infinite products as

$$\vartheta\left[\begin{array}{c}\alpha\\ \beta\end{array}\right](z|\tau) = \sum_{n\in Z} q^{1/2(n+\alpha)^2} e^{2\pi i(n+\alpha)(z+\beta)} \tag{2.20}$$

$$= e^{2\pi i\alpha(z+\beta)} q^{\frac{\alpha^2}{2}} \prod_{n=1}^{\infty}(1-q^n)(1+q^{n+\alpha-1/2}e^{2\pi i(z+\beta)})(1+q^{n-\alpha-1/2}e^{-2\pi i(z+\beta)}).$$

Of particular interest for our purpose, are the so-called *theta constants i.e.* the expressions for the theta functions evaluated at the origin of the torus ($z = 0$) and with characteristics $\alpha, \beta = 0, 1/2$:

$$\vartheta\left[\begin{array}{c}1/2\\ 1/2\end{array}\right] = \theta_1(0|\tau) = 0, \tag{2.21}$$

$$\vartheta\left[\begin{array}{c}1/2\\ 0\end{array}\right] = \theta_2(0|\tau) = 2q^{1/8}\prod_{n=1}^{\infty}(1-q^n)(1+q^n)^2, \tag{2.22}$$

$$\vartheta\left[\begin{array}{c}0\\ 0\end{array}\right] = \theta_3(0|\tau) = \prod_{n=1}^{\infty}(1-q^n)(1+q^{n-1/2})^2, \tag{2.23}$$

$$\vartheta\left[\begin{array}{c}0\\ 1/2\end{array}\right] = \theta_4(0|\tau) = \prod_{n=1}^{\infty}(1-q^n)(1-q^{n-1/2})^2. \tag{2.24}$$

As mentioned, the right way of describing fermions on a torus is including all the four spin structures, that have to be combined in a modular invariant form. The GSO projection corresponds to some possible modular invariant combination. For supersymmetric strings, the contributions corresponding to spin structures that are *periodic* in the time direction, can be evaluated introducing a suitable projector in the trace. This kind of operator, that has to anticommute with the fermionic modes, can be identified with the fermionic parity operator $(-)^F = e^{2\pi i\beta F}$ with $\beta = 0, 1/2$. In the end we will also have

$$P\ \overset{A}{\square} \equiv tr_{\mathrm{NS}}\Big[(-)^F q^{N_F}\Big] = \prod_{k=1}^{\infty}(1-q^{k-1/2})^8\,,$$

$$P\ \overset{P}{\square} \equiv tr_{\mathrm{R}}\Big[(-)^F q^{N_F}\Big] = 0\,.$$

Again the complete contribution must contain the bosonic part too, namely

$$tr_{NS}\Big[(-)^F\,q^{(N_B+N_F-a)}\Big] = \frac{\prod_{k=1}^{\infty}(1-q^{k-1/2})^8}{q^{1/2}\prod_{k=1}^{\infty}(1-q^k)^8} = \frac{1}{\eta(\tau)^8}\,\frac{\theta_4^4(0|\tau)}{\eta^4(\tau)}\ , \tag{2.25}$$

$$tr_R\Big[(-)^F\, q^{(N_B+N_F)}\Big] = \quad 0 \quad = \quad \frac{1}{\eta(\tau)^8}\,\frac{\theta_1^4(0|\tau)}{\eta^4(\tau)}\ . \tag{2.26}$$

Note that the (P,P) spin structure gives a vanishing contribution to the amplitude, but it has to be included in order to read the correct spectrum. In fact it appears with a relative sign that reflects the corresponding chirality of the Ramond vacuum.

All the previous ingredients allow to finally write the superstring torus amplitude in the following form:

$$\mathcal{T} = \int \frac{d^2\tau}{\tau_2^6}\,\Big|\,\frac{\theta_3^4(0|\tau)-\theta_4^4(0|\tau)-\theta_2^4(0|\tau)\pm\theta_1^4(0|\tau)}{\eta^{12}(\tau)}\,\Big|^2 . \tag{2.27}$$

This expression encodes the two main properties strictly enforced by the GSO projection, namely

1. *Supersymmetry* : the theta-functions satisfy several identities, the most remarkable of which is the *Aequatio Identica Satis Abstrusa*:

   $$\theta_3^4(0|\tau)-\theta_4^4(0|\tau)-\theta_2^4(0|\tau)\pm\theta_1^4(0|\tau) = 0\ , \tag{2.28}$$

   liable for the vanishing of the closed superstring partition function (2.27). Physically this is a very important result because it implies that the theory is supersymmetric and, as such, has the same bosonic and fermionic degrees of freedom at any mass level. Because of the spin-statistics relation, fermions enter the sum with a minus sign and thus the final result vanishes identically.

2. *Modular Invariance* : the behavior of the theta-functions under the S and T modular transformations are

   $$T:\quad \vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(z|\tau+1) = e^{-i\pi\alpha(\alpha-1)}\theta\begin{bmatrix}\alpha\\ \beta+\alpha-1/2\end{bmatrix}(z|\tau), \tag{2.29}$$

   $$S:\quad \vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(z|-\frac{1}{\tau}) = \sqrt{-i\tau}\,\theta\begin{bmatrix}\beta\\ -\alpha\end{bmatrix}(z|\tau). \tag{2.30}$$

   This means that the torus amplitude (2.27) is modular invariant [90], although the individual sectors are not.

It is interesting and useful to introduce at this stage some peculiar theta constant combinations, defining the **level 1 characters** of the $so(8)$ algebra [91] [92]. Namely

$$\begin{aligned} O_8 &= \frac{\theta_3^4+\theta_4^4}{2\eta^4}\,, \qquad & V_8 &= \frac{\theta_3^4-\theta_4^4}{2\eta^4}\,,\\ S_8 &= \frac{\theta_2^4+\theta_1^4}{2\eta^4}\,, \qquad & C_8 &= \frac{\theta_2^4-\theta_1^4}{2\eta^4}\,, \end{aligned} \tag{2.31}$$

where the O_8 and V_8 representations are respectively associated to the scalar and vector conjugacy classes of the algebra belonging to the NS sector of the spectrum, while the S_8 and C_8

are associated to the spinorial representation that belongs to the R sector. The $so(2n)$ algebras are one class of the so called *affine Kac-Moody algebras* [93, 94], that find different applications in field theory and in particular allow the introduction of the gauge symmetry in closed string theories.

In terms of (2.31), the torus partition functions (2.27) and the corresponding spectra acquire a very elegant description. Every superstring sector is in fact associated to an independent character:

$$\begin{aligned} \mathcal{T}_{IIB} &= \int \frac{d^2\tau}{\tau_2^2} \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^8} (\bar{V}_8 - \bar{S}_8)(V_8 - S_8) = \int \frac{d^2\tau}{\tau_2^2} \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^8} |V_8 - S_8|^2 \ , \\ \mathcal{T}_{IIA} &= \int \frac{d^2\tau}{\tau_2^2} \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^8} (\bar{V}_8 - \bar{S}_8)(V_8 - C_8) \ . \end{aligned} \tag{2.32}$$

It is worth to stress that at the lowest mass level, the character O_8 starts with a tachyon, the V_8 with a vector, while the S_8 and C_8 characters start with two spinors of opposite chirality. As a result, Type IIA superstring has, in the R sector, massless fermions of opposite chirality ($\bar{S}_8$ and C_8), while they have the same chirality in Type IIB superstring ($\bar{S}_8$ and S_8).

From this follows that the NS-NS bosonic sector of both theories includes a dilaton ϕ, an antisymmetric tensor $B_{\mu\nu}$ and a graviton $g_{\mu\nu}$. The fermionic fields resulting from the mixed sectors of both theories are two gravitini ψ^{μ}_{α} and two spinors χ_{α}, which have the same and opposite chirality in the Type IIB and Type IIA case, respectively. It follows that the two theories are tachyonic-free and that the massless states of the Type IIA fill the non-chiral multiplet of the $(1,1)$ supergravity in $D = 10$, while the massless states of the Type IIB fill the chiral multiplet of the $(2,0)$ supergravity in $D = 10$.

Let us note, finally, that the Type IIB superstring has a symmetry that exchanges the left and the right moving sectors in the world-sheet. This transformation, known as the *world-sheet parity transformation*, is not a symmetry of Type IIA theory, where the GSO projections give rise to different left and right sectors.

2.2 Toroidal Compactifications

As candidates for the description of nature, the closed superstring theories discussed so far have to face one immediate criticism: the critical space-time dimensions in which superstrings are embedded is still too high in comparison with the observed four-dimensional world.

Consequently one is naturally led to consider the possibility that the true Minkowskian space-time M_D takes the form of a direct product $M_d \times K_{D-d}$, where K_{D-d} is a compact internal manifold, whose characteristic size is extremely tiny to be probed by nowadays accelerators. This idea, with one or more compactified dimensions periodically identified, was already present in field theory in the form of Kaluza-Klein reduction, but string compactifications offer new interesting "stringy" effects not present in the usual Kaluza-Klein schemes.

The simplest and proper context to describe string theory compactifications and relative symmetries is represented by compactifications on d-dimensional tori, T^d. In this case the internal manifold K^d can be thought as the quotient

$$K^d = T^d = \mathbb{R}^d/2\pi\Lambda$$

with internal coordinates y^i periodic along the d homology cycles of the torus. Here $\Lambda = \{e^i_a n^a; n^a \in \mathbb{Z}\}$ is a squared-lattice that can be represented as the product of d cycles, and $e^i_a$is a vielbein that brings the constant metric g_{ab} on the torus in the Euclidean form: $g_{ab} = e^i_a e^j_b \delta_{ij}$.

When the theory is compactified over T^d, the only difference with the flat space are the boundary conditions. The most general solution of the equation of motion for the internal string coordinates X^i

$$\begin{aligned} X^i(\tau,\sigma) &= X^i_L(\tau,\sigma) + X^i_R(\tau,\sigma) \\ &= x^i + 2\alpha' p^i \tau + 2w^i\sigma + oscillators \end{aligned} \tag{2.33}$$

differs from the solution for $\mathbb{R}^d$ in the structures of the zero-modes. Besides the momentum p^i, quantized in terms of the integers m^i, eq.(2.33) reveals in fact the presence of a new quantum number, w^i, that counts the number of times that the string can wind around a non-trivial loop on T^d. It is called *winding number*, and it is quantized in terms of the integers n^i.

The zero-modes change the vacuum energy, adding the towers of states associated to the quantum numbers m^i and n^i. The contributions to the Partition Function due to the oscillators are thus unaffected, while the trace over the states related to the zero-modes gives rise to a sum over the lattice generated by m^i and n^i. The end result is

$$Z_{T^d} = \frac{1}{(\eta(\tau)\bar{\eta}(\bar{\tau}))^d} \sum_{(\mathbf{m},\mathbf{n})\in Z} q^{\frac{p_L^2}{2}} \bar{q}^{\frac{p_R^2}{2}}, \tag{2.34}$$

where

$$\begin{aligned} p^i_L &= m_a \tilde{e}^{ai} + \frac{n^a}{\alpha'} e^i_a - \frac{B_{ab}}{\alpha'} n^b \tilde{e}^{ai}, \\ p^i_R &= m_a \tilde{e}^{ai} - \frac{n^a}{\alpha'} e^i_a - \frac{B_{ab}}{\alpha'} n^b \tilde{e}^{ai}. \end{aligned} \tag{2.35}$$

with $\{e^i_a\}$ a suitable basis in the lattice Λ, $\{\tilde{e}^{ai}\}$ the corresponding one in the dual lattice $\tilde{\Lambda}$ ($\tilde{e}^{ai} = g^{ab} e^i_b$), and where B_{ab} is an antisymmetric tensor. (Let us stress that actually the expansion of the compact coordinates in eq.(2.33) get slightly modified when a tensor B_{ab} is present.

The invariance of the theory (2.34) under the modular transformation T and S calls for an even, Lorentzian and self-dual lattice.

2.3 Orbifold Compactifications

Toroidal compactifications provide the introduction of the gauge symmetry in the heterotic theories (besides the standard gravitational multiplet $(\phi, g_{\mu\nu}, B_{\mu\nu})$, in fact, the compactification of the sixteen bosonic coordinates allows the presence of as many as vectors of the $U(1)$ gauge group), but in general they cannot give rise to realistic theories because of the large number of unbroken supersymmetries in the extended space-time. As a consequence, the resulting theories are not chiral. Actually the torus is a very simple manifold, and to obtain phenomenologically interesting models one has to consider compactifications on more complicated manifolds, known as Calabi-Yau spaces [41, 42]. *Orbifolds* are the simplest type of Calabi-Yau spaces (or better, they can be interpreted as singular limit of the Calabi-Yau manifolds), and, as we will see, the orbifold compactifications provide a good compromise between the simplicity of the analysis and the richness of the results. An orbifold $\mathcal{O}$ is a space that can locally be defined as the quotient of a certain manifold M by the action of some discrete group G, $\mathcal{O} = M/G$ [95, 96, 97]. Such a space is not, in general, a smooth manifold, since it becomes singular at the points left fixed by the group action.

The dynamics of strings on orbifolds implies two basic additional properties of the Hilbert space of the string states [96]. First of all, since points of the space-time are identified under the action of the G-elements, strings that are closed modulo the G-action must be taken into account too. They are defined by the boundary conditions:

$$X(\sigma + 2\pi, \tau) = g \; X(\sigma, \tau)\,,$$

where g is a group element. These give rise to the so called *twisted sectors* of the Hilbert space, that, as a consequence, will be enlarged to be the direct sum of the various sectors $\mathcal{H}_g$, one for each coniugacy class of G.

Moreover, the Hilbert space of the physical states describing closed strings on the orbifold, will have to be G-invariant. Thus, every sector $\mathcal{H}_g$ will be projected onto a G-invariant subspace.

The simplest examples of orbifolds are obtained identifying M with a torus T^k, and G with a $\mathbb{R}^d$ rotation group, $\mathbb{Z}_N$. In these cases the orbifold is *abelian* (since G is abelian), and will be characterized by 2^k fixed points.

A simple illustration of the idea of orbifold compactification is given by the "tetrahedron" $\mathcal{O}_2 = T^2/\mathbb{Z}_2$ that can be obtained starting from the two-dimensional torus T^2 and identifying points under a reflection $X \to -X$ (or equivalently, under a π-rotation around the origin). There are four fixed-points of the rotation and they coincide with the tetrahedron vertices.

Every orbifold fixed-point supplies a possible location for the *twisted* states.

The inclusion of the *twisted* sectors is crucial to obtain a modular invariant partition function. To be explicit, let us introduce the projector that correctly selects the Hilbert space of the G-

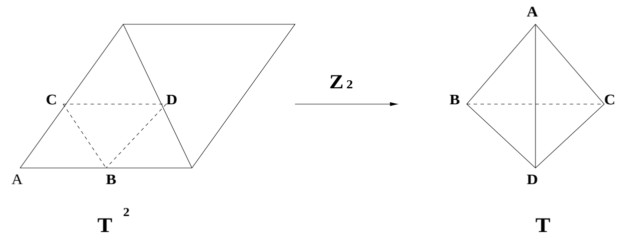

Figure 2.9: The $T^2/\mathbb{Z}_2$ orbifold.

invariant states

$$\mathcal{P} = \frac{1}{(dimG)} \sum_{i=1}^{(dimG)} g_i \ , \qquad G = \{g_i\} \ .$$

The general expression for the partition function, in the case of $T^k/\mathbb{Z}_N$ orbifolds compactifications is given by

$$\Gamma = \frac{1}{(4\pi)^{\frac{D-2}{2}}} \int_0^\infty \frac{dt}{t^{\frac{D-k}{2}+1}} \sum_k tr_k(\mathcal{P} e^{-tm^2}), \tag{2.36}$$

where the index k indicates that the trace is over the $k-th$ *twisted* sector.

In the operator formalism, the *untwisted* sector contribution to the partition function (2.36), is made of $(dimG)$ terms, namely

$$\mathcal{Z}_{untw} = \frac{1}{(dimG)} \sum_{i=1}^{(dimG)} \mathcal{Z}_{(0,g_i)} = \frac{1}{(dimG)} \sum_{i=1}^{(dimG)} tr_{\mathcal{H}_0} \ g_i \ q^{N-a} \bar{q}^{\bar{N}-\bar{a}} \ , \tag{2.37}$$

where $\mathcal{H}_0$ indicates the Hilbert space of the states unaffected by the projection, and where N ($\bar{N}$) is the usual number operator for an infinite set of oscillators, already introduced in sec. 2.1.

Every term in (2.37) corresponds to states satisfying

$$X(\sigma, \tau + 2\pi) = g_i \ X(\sigma, \tau) \, .$$

Although the T modular transformation leaves invariant the (2.37), the S transformation acts exchanging σ with τ. Thus:

$$\{X(\sigma, \tau + 2\pi) = g_i X(\sigma, \tau)\} \ \xrightarrow{S} \ \{X(\sigma + 2\pi, \tau) = g_i X(\sigma, \tau)\} \ ,$$

that asks for the presence of the g_i-*twisted* states

$$\mathcal{Z}_{tw} = \frac{1}{(dimG)} \sum_{i,j=1}^{(dimG)} \mathcal{Z}_{(g_i,g_j)} = \frac{1}{(dimG)} \sum_{i,j=1}^{(dimG)} tr_{\mathcal{H}_{g_i}} \ g_j \ q^{N-a} \bar{q}^{\bar{N}-a} \ .$$

It is useful to stress how the quest of modular invariance for the closed string one-loop amplitude provides a tool for the construction of the models since its violation would imply an inconsistency of the theory. For this purpose, let us consider the partition function of one boson compactified on the segment $S^1/\mathbb{Z}_2$, where S^1 is a circle of radius R. This orbifold has two fixed-points under

$\mathbb{Z}_2$, *i.e.* the endpoints of the segment. The starting point is the amplitude for closed strings compactified on a circle, whose general expression is given by (2.34), but when specialized to one dimension and with vanishing B_{ab} has the following form

$$\mathcal{Z}_{(P,P)} = \sum_{(m,n)} \frac{q^{\frac{p_L^2}{2}} \bar{q}^{\frac{p_R^2}{2}}}{\eta(\tau)\bar{\eta}(\tau)} = \frac{\sum_{(m,n)} q^{\frac{p_L^2}{2}} \bar{q}^{\frac{p_R^2}{2}}}{(q\bar{q})^{1/24} |\prod_{k=1}^{\infty}(1-q^k)|^2} \ . \tag{2.38}$$

To make this expression $\mathbb{Z}_2$-invariant (that means symmetrizing with respect to $X \to -X$), we have to add the following contribution

$$\mathcal{Z}_{(P,A)} = \frac{1}{(q\bar{q})^{1/24} |\prod_{k=1}^{\infty}(1+q^k)|^2} \ , \tag{2.39}$$

obtained from (2.38) by getting rid of the zero-modes and flipping the sign of the oscillators. Thus $\mathcal{Z}_{untw} = \mathcal{Z}_{(P,P)} + \mathcal{Z}_{(P,A)}$. But (2.39) is not modular invariant. As a consequence, we have to add the terms obtained from $\mathcal{Z}_{(P,A)}$ acting with the S and T modular transformations respectively, namely

$$\mathcal{Z}_{(A,P)} = 2^2 \frac{(q\bar{q})^{1/48}}{|\prod_{k=1}^{\infty}(1-q^{k-1/2})|^2} \ , \tag{2.40}$$

and

$$\mathcal{Z}_{(A,A)} = 2^2 \frac{(q\bar{q})^{1/48}}{|\prod_{k=1}^{\infty}(1+q^{k-1/2})|^2} \ , \tag{2.41}$$

where the factor of 2^2 is the fixed-point degeneration.

We can finally write $\mathcal{Z}_{tw} = \mathcal{Z}_{(A,P)} + \mathcal{Z}_{(A,A)}$,

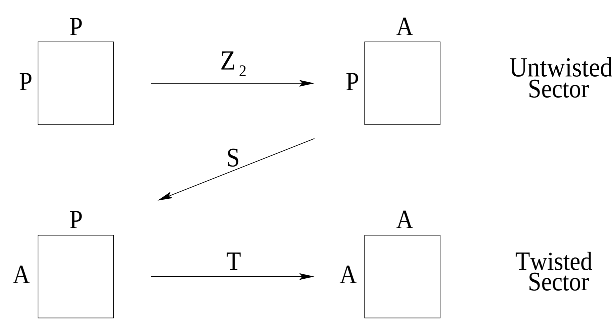

Figure 2.10: Twisted sector + Untwisted sector.

that shows the crucial rôle of the twisted sector for the modular invariance.

It is worth noting that the GSO projection of superstring is a sort of $\mathbb{Z}_2$ orbifold with respect to the space-time fermion number $(-)^F$. Moreover, the $0A$ and $0B$ superstrings can also be constructed modding out the corresponding Type II superstrings by the action of the discrete group $(-)^F$.

This procedure gives rise to the following expressions

$$\mathcal{T}_{0B} = \mathcal{T}_{IIB/(-)^F} = (\ |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2\ ) \ , \tag{2.42}$$

and

$$\mathcal{T}_{0A} = \mathcal{T}_{IIA/(-)^F} = (\ |O_8|^2 + |V_8|^2 + \bar{S}_8 C_8 + \bar{C}_8 S_8\ )\ . \tag{2.43}$$

In the next chapter we will analyze Type I open superstring, showing how they can be constructed as orbifolds in the world-sheet, or *orientifolds*, of theories that are symmetric under the interchange of the corresponding holomorphic and antiholomorphic sectors.

Chapter 3

Type I Superstrings

3.1 Open Descendants or Orientifolds

In this section we would like to discuss more extensively the link between the Type IIB and Type I-$SO(32)$ superstring theories, showing in particular how the latter descends from the former [27, 57, 58].

Let us first remind that between the five consistently defined ten dimensional superstring theories, only the Type I-$SO(32)$ contains open strings in its spectrum. Open strings have the important feature of allowing the introduction of internal symmetry groups. It is in fact quite natural to introduce further non-dynamical degrees of freedom associated to their endpoints [20, 22, 23]. These, in turn, can be equipped by charges of an internal gauge group, known as *Chan-Paton charges*. Moreover, a crucial and well-known characteristic of the open string tree-level amplitudes is their invariance under cyclic permutations of the external legs. This property is still preserved multiplying the amplitudes by the *Chan-Paton factors* $tr(\lambda_1\lambda_2...\lambda_N)$, *i.e.* by traces of product of matrices λ_i valued in the fundamental representation of the classical Lie algebras $so(n), sp(2n)$, and $u(n)$, but the exceptional ones. This result [21, 22], obtained by analyzing the amplitude behaviors under the world-sheet parity operator Ω, implies that the introduction of the Chan-Paton charges gives rise to an oriented open string spectrum in the theory, if the gauge group is $U(n)$, while it is made of unoriented open strings for the $SO(n)$ and $Sp(2n)$ groups. Since the Chan-Paton charges are associated to the open string endpoints, one has to specifies the action of Ω on λ_i. In this way the selected gauge group in $D = 10$ is $SO(32)$. As a consequence, the Type I-$SO(32)$ is the only superstring theory consistently defined at the quantum level in $D = 10$, that contains unoriented closed and open strings in interaction. The presence of the closed sector is required by the unitarity of the theory since as seen before, theories of open strings only cannot occur.

We have already pointed out that the basic tools determining the perturbative spectra of the closed oriented strings, are essentially due to two constraints for the corresponding vacuum amplitudes: modular invariance and spin-statistic relations. For open string models the situation is more complicated, since the presence of boundaries and crosscaps do not allow to appeal to modular invariance. Thus, in the open string case it is necessary to resort to new principles able to guarantee the consistency of the model. As we will see, anomalies or divergences cancellation will provide this basic principle for open string models. An algorithm to build *Open Descendant*, currently known as *Orientifolds*, has been developed during the last 15 years. It provides a well-defined technique for the perturbative constructions of non-orientable open string theories, and is based on the idea, suggested by Sagnotti in 1987 [27], that open string models can be obtained as *orbifolds in the parameter space* (open descendants) of closed models symmetric under the exchange of left and right modes. The orbifold operator is the *world-sheet parity* Ω, that, in the closed string case, acts exchanging the right modes with the left ones ($\Omega: \ \alpha^\mu_m \leftrightarrow \bar{\alpha}^\mu_m$), while in the open string case, where left and right modes are not independent, interchanges the string endpoints ($\Omega : \alpha^\mu_m \leftrightarrow (-)^m \alpha^\mu_m$).

When applied to models in critical dimension, the algorithm provides the bosonic *SO(8192)* string, the *Type-I SO(32)* superstring and the *USp(32) Sugimoto* model [98]. It applies also to Conformal Field Theories in D=2, with or without Boundaries, allowing, for instance, the description of a wide class of two-dimensional Statistical Mechanics models [99].

As we will see, the model building through the orientifold projection follows a strategy that is similar to the one described for the geometrical orbifolds. The starting point is a theory of closed oriented strings, symmetric under Ω. In the one-loop partition function description, we have to project the Torus amplitude onto states that are effectively symmetric with respect to the exchange of left and right modes. This kind of projection is implemented by the inclusion of the Klein bottle amplitude, providing the definition of the analog of the *untwisted sector*. The *twisted sectors* are instead represented by the open unoriented strings, whose spectra are encoded in the Annulus and the Möbius strip amplitude, the latter being the corresponding projection of the Annulus by Ω.

As in the construction of the geometric orbifolds, where the amplitudes in the twisted sectors contain some multiplicities due to the number of fixed points under the gauge group action, in the construction of the open descendants the multiplicities of the Chan-Paton charges are associated to the open string endpoints. As we will see in the next section, these multiplicities can also have a geometrical interpretation in terms of hyperplanes on which open strings can terminate.

It is worth to stress that in the usual orbifold construction, the introduction of the twisted states is sufficient to guarantee the complete consistency of closed string models since it yields a modular invariant theory. The three world-sheet amplitudes $\mathcal{K}, \mathcal{A}$ and $\mathcal{M}$ do not exhibit the property of modular invariance, and in addition present ultraviolet divergences associated with their behavior in $\tau_2 = 0$, that in general can spoil their consistency at the quantum level. These divergencies are called *tadpoles*, in analogy to the field theory picture where a single particle is generated from the vacuum by quantum effects. In string theory a non-vanishing tadpole signals that the equations of motion of some massless fields in the effective theory are not satisfied. Finiteness can be restored by imposing the so called *tadpole cancellation conditions*, that reflect into constraints on the dimensions and types of the Chan-Paton gauge groups. To extract the contribution of each surface to the tadpole, it is convenient to write the amplitudes in the transverse (dual) channel. In this way in fact, one-loop open and closed unoriented string amplitudes are transformed into tree-level amplitudes for closed string states flowing into a tube ending with a boundary and/or a crosscap.

This can be done by the S modular generator, thus transforming the ultraviolet region responsible for the divergences ($\tau_2 = 0$), into the infrared region. In this limit ($l \to \infty$) the vacuum amplitudes factorizes into tadpole-propagator-tadpole.

Since there is no-momentum flowing along the tubes terminating with boundaries and crosscaps, and the propagator is on-shell, the only divergent contributions can come from the massless particles. Factorizing the on-shell propagator, we are left with the square of the one-point function for closed string states (*tadpole*) in front of a boundary and/or a crosscap, whose cancellation corresponds to the required condition. The tadpole cancellation conditions are the counterpart of modular invariance for the consistency of open string models, and are in fact associated to the cancellation of all gauge and gravitational anomalies. The relation between tadpole condition and anomalies cancellation has been highlighted for the first time for the Type I superstring, by Green and Schwarz [26] and then clarified by Polchinski and Cai. They connected anomalies to tadpoles of non-physical massless states [100, 101, 102]. In general there are two types of possible tadpoles: 1) the *NS-NS tadpoles*, that generate potentials for the corresponding fields and are thus a signal for the background redefinition [103] 2) the *R-R tadpoles*, that have to be always cancelled since they signal an anomaly.

In order to clarify how the procedure works, let us illustrate the open descendant of the Type IIB superstring theory, the Type-I superstring in $D = 10$. The starting point is the Torus one-loop partition function of the Type IIB model:

$$\mathcal{T}_{IIB} = (V_8 - S_8)(\tau)\ (\bar{V}_8 - \bar{S}_8)(\bar{\tau})\ , \tag{3.1}$$

where, for sake of simplicity, the contributions of the transverse bosons and the integration measure of eq. (2.32) have been omitted.

The projection of the closed spectrum can be obtained summing to the halved torus partition function (3.1), the the Klein bottle amplitude contribution

$$\mathcal{K}_{IIB} = \frac{1}{2}(V_8 - S_8)(2i\tau_2), \tag{3.2}$$

where τ_2 is the proper time needed by the closed string to sweep the Klein bottle diagram. $\mathcal{K}$ determines the action of the Ω operation onto the various sectors of the closed *unoriented* spectrum. In particular a plus (minus) sign in the characters implies a symmetrization (anti-symmetrization) of the closed sectors under the left-right modes exchange. As a consequence, the closed *untwisted* spectrum

$$\frac{1}{2}\left[\ |V_8 - S_8|^2 + (V_8 - S_8)\ \right]$$

symmetrizes the NS-NS sector and anti-symmetrizes the R-R one. The fermionic modes are instead halved in number, in order to provide the massless excitations $(g_{\mu\nu}, B_{\mu\nu}, \phi) \oplus (\chi_{\dot{\alpha}}, \psi^{\mu}_{\alpha})$, which are exactly the fields constituting the spectrum corresponding to the $N = (1, 0)$ supergravity. It is interesting to stress that, although the bosonic fields coincide with the content of the universal sector of the various string theories, the antisymmetric tensor $B_{\mu\nu}$ has a different origin in the Type I string. It is in fact an R-R field and, as will be clear in the next section, it plays a crucial rôle in the D-brane interpretation of the spectra.

Because of the presence of an R-R tadpole, that would yield an anomalous theory, the next step in the construction involves the *twisted* sector, namely open strings, determined by the projection $\frac{1}{2}(\mathcal{A} + \mathcal{M})$. The amplitudes are respectively given by

$$\mathcal{A}_{IIB} = \frac{1}{2}\ N^2(V_8 - S_8)(\frac{i\tau_2}{2})\ , \tag{3.3}$$

$$\mathcal{M}_{IIB} = \frac{1}{2}\ \epsilon\ N(\hat{V}_8 - \hat{S}_8)(\frac{i\tau_2}{2} + \frac{1}{2})\ . \tag{3.4}$$

Here N is the multiplicity associated to the dimension of the fundamental representation of the Chan-Paton group. In the annulus amplitude it appears squared because of the presence of two boundaries at the ends of the tube. Let us note the presence, in eq. (3.4), of the *hatted* characters. They are related to the dependency of $\mathcal{M}$ on a not-purely imaginary modulus ($\tau_1 \neq 0$). It is thus convenient to introduce a basis of *real* characters that differ from the usual ones by a phase factor. This redefinition affects also the P modular transformation of eq. (2.2) that now becomes $\hat{P} = (T^{1/2}S)T^2(ST^{1/2})$. Moreover, the sign ambiguity $\epsilon = \pm 1$, in $\mathcal{M}$, determines the resulting gauge group and is fixed by the tadpole cancellation conditions. $\epsilon = +1$ implies an orthogonal group $SO(N)$ with $\frac{N(N-1)}{2}$ gauge vectors, while $\epsilon = -1$ implies a symplectic group $USp(N)$ with $\frac{N(N+1)}{2}$ vectors. Moreover N must be 32. To see how this emerges, it is necessary to write the previous amplitudes in the *transverse channel*, that can be achieved by the S modular transformation. Let us stress that in passing from the direct to the transverse channel one has to pay particular attention to the choice of the modulus of the doubly-covering torus. In fact, in order to determine the tadpole cancellations is crucial to refer the three contributions $\mathcal{K}, \mathcal{A}$, and $\mathcal{M}$ to the same covering modular parameter. Thus, taking into account the integration measure in each amplitude, the Klein bottle receives a factor of $2^{D/2}$, the annulus amplitude a factor of

$2^{-D/2}$ and the Möbius strip a factor of 2. Finally, one gets:

$$\tilde{\mathcal{K}}_{IIB} = \frac{2^5}{2}\ (V_8 - S_8)(il)\ , \tag{3.5}$$

$$\tilde{\mathcal{A}}_{IIB} = \frac{2^{-5}}{2}\ N^2\ (V_8 - S_8)(il)\ , \tag{3.6}$$

$$\tilde{\mathcal{M}}_{IIB} = \frac{2}{2}\ N\ \epsilon\ (\hat{V}_8 - \hat{S}_8)(il + \frac{1}{2})\ , \tag{3.7}$$

where the tilda indicates the amplitudes in the transverse channel, and l is the "horizontal" time displaying the three surfaces as tubes terminating at two crosscaps, at two boundaries and at one crosscap and one boundary, respectively. The tadpole cancellation condition is given by the vanishing of the one-point function associated to the R-R sector :

$$\frac{2^5}{2} + \frac{2^{-5}}{2}\ N^2 + \frac{2}{2}\, N\ \epsilon = \frac{2^{-5}}{2}\ \left(N + 2^5\ \epsilon\ \right)^2 = 0\ . \tag{3.8}$$

Because of supersymmetry, this equation provides the cancellation of the NS-NS tadpole too, the one associated to the $V_8\bar{V}_8$ sector. Its solution, $N = 32$ and $\epsilon = -1$, selects uniquely the $SO(32)$ gauge group, and reveals how the anomaly free Type I-$SO(32)$ superstring theory, obtained for the first time by Green and Schwarz in 1984 [26], emerges as open descendant of the Type IIB model. This last result provides the hint for a very short introduction of notions that will be better discussed in the next section. The solution of the tadpole cancellation can be in fact rephrased in terms of space-time **D-branes** and **O(rientifold)-planes**. They can respectively be thought of as the target space counterpart of the world-sheet boundaries traced by the open string endpoints, and of the crosscaps. As a consequence, the transverse Klein bottle amplitude can also be interpreted as the propagation of a closed string starting and ending on two O-planes, while the transverse annulus amplitude describes the propagation between two D-branes. Actually it is possible to obtain from the Type-I SO(32) superstring (where because of supersymmetry, NS-NS and R-R tadpoles cancel at the same time), a non-supersymmetric configuration with a left over dilaton tadpole. The resulting model [98], described by the following transverse-channel amplitudes

$$\tilde{\mathcal{K}} = \frac{2^5}{2}\ (V_8 - S_8)(il)\ , \tag{3.9}$$

$$\tilde{\mathcal{A}} = \frac{2^{-5}}{2}\ N^2\ (V_8 - S_8)(il)\ , \tag{3.10}$$

$$\tilde{\mathcal{M}} = \frac{2}{2}\ N\ (\hat{V}_8 + \hat{S}_8)(il + \frac{1}{2})\ , \tag{3.11}$$

gives rise the following R-R tadpole

$$\frac{2^{-5}}{2}\ \left(\ -N + 2^5\ \right)^2 = 0\,. \tag{3.12}$$

The corresponding gauge group is *USp(32)*, and the model, unlike the previous one, involves *anti*-D-branes (namely branes with negative tension) and "exotic" O-planes configurations.

3.2 D-Branes

In this section we would like to emphasize the rôle of T-duality in the context of open string theories, and how it has provided the intuitive principle for the introduction of the geometric notion of D-branes. Indeed, T-duality turns out to be essential to justify the origin of D-branes and to explain their nature, being a consequence of the extended nature of strings that does not have an analog in field theory.

As already mentioned, a T-duality operation acts exchanging theories compactified over manifolds of large volumes with theories compactified over small volume manifolds. For instance, in the one-dimensional case, it exchanges the radius of the compactified dimension with its inverse: $R \to \tilde{R} = \frac{\alpha'}{R}$. In perturbative bosonic *closed* string theory, a T-duality transformation is an exact quantum symmetry, and the inversion of R must be accompanied by the exchange of the Kaluza-Klein modes $p = \frac{m}{R}$ with the windings $w = \frac{nR}{\alpha'}$. It is worth to stress that T-duality can be also seen as an "asymmetric" world-sheet parity operation, since it acts only on the right-moving modes (in the one-dimensional case, in fact, $TX_L = X_L$ and $TX_R = -X_R$). Furthermore, one has to pay attention to the dilaton field, on which T-duality acts non trivially, since, how can be derived from the effective action, it undergoes the following redefinition:

$$\phi' = \phi + \frac{1}{2} \log \frac{\alpha'}{R^2}. \tag{3.13}$$

Nevertheless, it can be shown that the spectrum, together with the partition function and the correlators of the theory are invariant under such duality.

In open string theories, the string coordinate does not wind around the periodic direction of the space-time, and this implies that in their compactified version there are only the KK modes m, while the windings n are absent. This could suggest that T-duality is not a symmetry of open string theory, and that something very different must happen with respect to the closed string case. The lack of windings means in fact that open string theory behaves more like a quantum field theory, in the $R \to 0$ limit, where the $m \neq 0$ KK modes become infinitely massive, thus decoupling from the spectrum, and no open string oscillations can occur along the zero-radius direction (no continuum of states is generated). In this limit the open string theory effectively loses one direction, and one is left with an apparent paradox since any fully consistent string theory must contain in its spectrum both open and closed strings, and the latter, in the same limit, do not lose any compact direction. The way out of this paradox is arguing that the interior of open strings can still vibrate in all the space-time dimensions, while their *endpoints* are restricted to lie on a nine dimensional space-time hyperplane. This observation can be rephrased noting that in order to avoid a different behavior between the closed and the open sector of a string theory it is essential that the action of a T-duality on an open string is to transform a Neumann boundary condition into a Dirichlet one. This allows to conclude that T-duality asks for **D-branes**, hyperplanes where open string (Dirichlet) endpoints can be attached

[54]. More explicitly: a $\mathbf{D_p}$-brane is a $(p+1)$-dimensional space-time hypersurface specified by $(p+1)$ string coordinates with Neumann boundary conditions in all the tangent directions $(\partial_\sigma X^\mu(\sigma=0,\pi)=0,\ \mu=0,...,p)$ and by $(d-9-p)$ string coordinates with Dirichlet boundary conditions in the directions transverse to the surface $(\delta X^m(\sigma=0,\pi)=0,\ m=p,...,d)$.

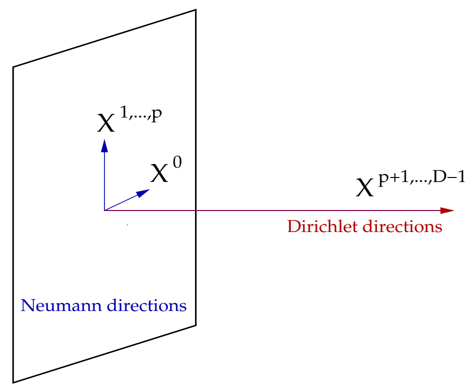

Figure 3.1: D_p-branes.

Let us emphasize how T-duality, exchanging Neumann with Dirichlet boundary conditions, acts on the D-brane by altering its corresponding dimensions: a T-duality along a direction tangent (N) to the D_p-brane reduces its dimension $(D_p \to D_{p-1})$, while a T-duality in a direction orthogonal (D) to the D_p-brane increases its dimension $(D_p \to D_{p+1})$.

So far we have introduced the concept of D_p-branes discussing only their geometric nature. Actually one of the fundamental properties of the D_p-branes and the O_p-planes is the fact that they can carry certain conserved charges: the Ramond-Ramond charges [52]. This means that they are the (electrical) sources for the R-R $(p+1)$-form gauge potentials A_{p+1}. Moreover both D-branes and O-planes have a coupling to the corresponding NS field that is proportional to their tension. This one is always positive for D-branes, while for O-planes can be also negative (O_+). A D-brane with negative R-R charge is called an **anti**-D-brane $(\bar{D})$ as well as an **anti**-O-plane has negative R-R charge $(\bar{O})$. The various possibilities are illustrated in table (3.1).

	O_+	O_-	$\bar{O}_+$	$\bar{O}_-$	D	$\bar{D}$
Charge	-	+	+	-	+	-
Tension	-	+	-	+	+	+

Table 3.1: Charge and Tension of D-branes and of O-planes

As we have briefly mentioned in the previous section, D-branes and O-planes allow a new interpretation of the tadpole cancellation conditions. Let us remind that in superstring theory there are two contributions to the overall tree-channel tadpole: the NS-NS and the R-R tadpoles, coming from the corresponding sectors. Although they appear on the same ground in the partition function, their meanings and rôles are quite different. The R-R tadpole cancellation is equivalent to an overall charge neutrality condition: D_p-branes, as well as O_p-planes, are in fact p-dimensional space-time hyperplanes that couple to R-R $(p+1)$-forms A_{p+1}. As a consequence, consistently with the analog of the Gauss law of electrodynamics, the field lines originating from one charge must either go to infinity or fall down onto an opposite charge. On a compact space they cannot go to infinity and thus must end on an absorber of charge. The theory will be consistent only if this charge exists. This means that in order to cancel the R-R charge due to the presence of an O_p-plane in the theory, one has to introduce suitable D_p-brane configurations (namely suitable open string sectors) that exactly cancel the charge excess. The NS-NS tadpole conditions, instead, are related to a correction to the vacuum energy and are caused by a dependance on the dilaton and graviton fields, whose interaction is responsible for the so-called *dilaton tadpole*. This can be non-vanishing and the resulting theory will be unstable but not inconsistent. Consequently NS-NS tadpoles are not as bad as the R-R counterpart, which are instead related to anomalies in the low-energy effective field theory, and thus have always to be cancelled.

Actually is it possible to give a alternative and *non-perturbative* description of the D-branes, showing how they emerge as solitonic string solutions and thus as dynamical objects, fundamental sectors of the open string vacuum configuration. In this context the massless states of open strings can be interpreted as the fluctuation modes of the D-branes themselves. On the contrary, in the perturbative string theory the O-planes have no string modes associated to their fluctuations, *i.e.* they are non-dynamical objects.

From all of this follows that D-branes are dynamical objects that can move, intersect or even decay into different configurations.

Except the heterotic string, we have seen that all the consistent superstring theories in D=10 contain in their spectrum antisymmetric tensors coming from the R-R sector. This selects their brane content. For the Type IIA are available only supersymmetric (BPS) D_p-branes with even p $(p = 0, 2, 4, 6, 8)$, since the corresponding tensors have an odd number of indices, while for the Type IIB, whose tensors have an even number of indices, are allowed only D_p-branes with odd p $(p = -1, 1, 3, 5, 7, 9)$. Finally, the Type I theory supports, modulo T-duality, only D9, D5 and D1 branes since they are the only consistent with the Ω projection.

So far we have discussed the case of a single D-brane. An interesting and instructive configuration is the one involving a stack of two or more parallel D-branes, since they provide a way to break Chan-Paton gauge symmetry. Moreover, configurations containing multiple D-branes can break supersymmetry further, since they are BPS states and break half of the available supersymmetries. The massless fluctuations of a single D-brane are those related to the open superstrings with their endpoints attached to the brane. In particular, the transverse string oscillations are associated to scalar fields, describing the small displacements of the brane from the equilibrium position. The longitudinal fluctuations, instead, are related to some gauge fields, and when several D-branes coincide a non-abelian gauge symmetry can arise. N parallel branes are thus described by N^2 possible strings stretched between them, generating the spectrum and the interactions of a $U(N)$ (super)Yang-Mills theory. Moving the branes apart causes in general the breaking of the gauge group $U(N)$ to $U(1)^N$ (there is a massless gauge vector for each $U(1)$). Finally, if k D-branes coincide the symmetry will be extended because of the presence of an unbroken $U(k)$ subgroup of the starting theory.

The advent of D-branes has thus allowed the construction of semi-realistic Type-I string theories, even if the constraints of supersymmetry in four-dimensional models are rather restrictive and lead to not fully realistic gauge sectors and matter contents. The quest for Standard Model-like solutions has motivated the analysis of different deformations of this class of constructions, the most studied of which, in the last years, are the so-called *Intersecting Brane-World Models*. Their T-dual version correspond to the introduction of gauge fluxes on some D-brane configurations, and will provide the argument of next chapters.

3.3 Shift-Orbifolds

In the following sections we would like to discuss some example of models derived as descendants of the Type IIB parent string theory compactified on some *shift-orbifolds*. In particular, we will consider the four-dimensional $\mathbb{Z}_2 \times \mathbb{Z}_2$ shift-orientifolds, that constitute one of the basic ingredients of this thesis.

While for conventional orbifold compactifications the string coordinates are identified under some internal inversion operations (the $\mathbb{Z}_2$ case for example corresponds to combined π rotations), *shift-orbifolds* are obtained by their joined action with *discrete shifts* on the basis vectors of the compactification lattice, or more precisely, combining shifts with internal symmetries. This kind of operations play a very interesting röle since they allow to implement, in string theory, the Scherk-Schwarz mechanism for the spontaneous breaking of supersymmetry.

In field theory the Scherk-Schwarz mechanism is essentially a generalization of the standard Kaluza-Klein reduction and it involves shifts of the internal Kaluza-Klein momenta.

This procedure can be extended to string theory by deforming the closed string partition function through momentum or winding shifts along the compact directions, while preserving modular invariance. A momentum shift is longitudinal to the corresponding brane, while a winding shift can be interpreted as an orthogonal momentum shift. The first case is referred to as *Scherk-Schwarz breaking* and the second as *M-theory breaking* [67].

For later convenience, let us analyze the case of a one-dimensional shift-orbifold, more precisely the case of *Scherk-Schwarz model*. It can be obtained constructing the open descendants of the Type IIB theory compactified on a circle with radius R and with shifts in the momentum lattice. In this case the type IIB superstring must be projected by the $\mathbb{Z}_2$ generator $(-)^F\delta$, where $F = F_L + F_R$ is the total space-time fermion number, and δ is the shift $X^9 \to X^9 + \pi R$ along the compact direction, that acts on the states as $(-)^m$. The resulting partition function is

$$\begin{aligned} \mathcal{T}_{S-S} \;&=\; \frac{1}{2}\sum_{m,n} \Big[|V_8 - S_8|^2\Lambda_{m,n} + |V_8 + S_8|^2(-)^m\Lambda_{m,n}\Big] \\ &+\; \frac{1}{2}\sum_{m,n} \Big[|O_8 - C_8|^2\Lambda_{m,n+1/2} + |O_8 + C_8|^2(-)^m\Lambda_{m,n+1/2}\Big]. \end{aligned} \tag{3.14}$$

Introducing the projected (on even or odd momenta) lattice sum

$$\begin{aligned} \mathcal{E}_a \;&=\; \sum_{m,n} \frac{1+(-)^m}{2}\Lambda_{m,n+a} \ , \\ \mathcal{O}_a \;&=\; \sum_{m,n} \frac{1-(-)^m}{2}\Lambda_{m,n+a} \ , \end{aligned} \tag{3.15}$$

where

$$\Lambda_{m,n+a} = \frac{\Big[\sum_{(m,n)\in\mathbb{Z}} \ q^{\frac{\alpha'}{4}(\frac{m}{R}+\frac{(n+a)R}{\alpha'})^2} \ \bar{q}^{\frac{\alpha'}{4}(\frac{m}{R}-\frac{(n+a)R}{\alpha'})^2}\ \Big]}{\eta(\tau)\ \bar{\eta}(\bar{\tau})}, \tag{3.16}$$

the amplitude (3.14) can be written in terms of an orthogonal decomposition of the closed spectrum of the model, namely

$$\begin{aligned} \mathcal{T}_{S-S} &= \mathcal{E}_0(|V_8|^2+|S_8|^2)+\mathcal{E}_{1/2}(|O_8|^2+|C_8|^2) \\ &- \mathcal{O}_0(V_8\bar{S}_8+S_8\bar{V}_8)-\mathcal{O}_{1/2}(O_8\bar{C}_8+C_8\bar{O}_8)\ . \end{aligned} \tag{3.17}$$

Let us note that if $R<\sqrt{\alpha'}$, this model develop a tachyonic mode associated to $|O_8|^2$, while for $R\to\infty$ it reduces to the Type IIB superstring in $D=10$.

It is straightforward to obtain at this point, the one-dimensional *winding*-shift orbifold. In this case the δ-$shift$ acts along the T-dual compact direction appearing in the previous shift model and its action on the states is thus $(-)^n$. The resulting torus amplitude is

$$\begin{aligned} \mathcal{T}_{M-th} &= \tilde{\mathcal{E}}_0(|V_8|^2+|S_8|^2)+\tilde{\mathcal{E}}_{1/2}(|O_8|^2+|C_8|^2) \\ &- \tilde{\mathcal{O}}_0(V_8\bar{S}_8+S_8\bar{V}_8)-\tilde{\mathcal{O}}_{1/2}(O_8\bar{C}_8+C_8\bar{O}_8)\ , \end{aligned} \tag{3.18}$$

where the lattice sums $\tilde{\mathcal{E}}$ and $\tilde{\mathcal{O}}$ are the same of (3.15), but with momenta and *windings* exchanged. Let us note that in this case the torus amplitude develops a tachyonic instability for $R>\sqrt{\alpha'}$, while for $R\to 0$ supersymmetry is restored.

3.4 Shift-Orientifolds

Let us now move to the discussion of the open descendant of the Sherk-Schwarz model. All the Ω-invariant sectors present in (3.17) contribute to the Klein bottle amplitude. They include the states with vanishing windings ($n=0$) only, and thus the corresponding amplitude

$$\mathcal{K}_{S-S}=\frac{1}{2}(V_8-S_8)P_{2m}\ , \tag{3.19}$$

is not affect by the momentum shifts. P_{2m} is the even momenta lattice sum of eq.(3.15) restricted to zero-winding numbers and $a=0$. In the transverse channel, eq. (3.19) becomes

$$\tilde{\mathcal{K}}_{S-S}=\frac{2^{9/2}}{2}\frac{R}{\sqrt{\alpha'}}(V_8-S_8)W_n\ , \tag{3.20}$$

with W_n the winding lattice sum restricted to zero-momenta ($m=0$).

In a similar way, the annulus amplitude in the transverse channel can be deduced by $\mathcal{T}_{S-S}$ restricting the diagonal part of the spectrum to states with zero-momentum ($m=0$). As a consequence, in the tube are allowed to flow only the states associated to V_8 and S_8 with even windings, and those associated to O_8 and C_8 with odd windings. As a result, one can introduce four different kind of Chan-Paton charges, parametrized by four integers, n_1, n_2, n_3 and n_4, thus obtaining

$$\begin{aligned} \tilde{\mathcal{A}}_{S-S} &= \frac{2^{-11/2}}{2}\frac{R}{\sqrt{\alpha'}}\Big[[(n_1+n_2+n_3+n_4)^2V_8-(n_1+n_2-n_3-n_4)^2S_8]W_n \\ &+ [(n_1-n_2+n_3-n_4)^2O_8-(n_1-n_2-n_3+n_4)^2C_8]W_{n+1/2}\Big], \end{aligned} \tag{3.21}$$

with $W_{n+1/2}$ the zero-momenta lattice sum of eq. (3.15) with $a = 1/2$.

Finally the Möbius strip amplitude in the transverse channel is given by the characters common to $\tilde{\mathcal{K}}_{S-S}$ and to $\tilde{\mathcal{A}}_{S-S}$, *i.e.*

$$\tilde{\mathcal{M}}_{S-S} = -\frac{1}{\sqrt{2}}\frac{R}{\sqrt{\alpha'}}\Big[(n_1+n_2+n_3+n_4)^2\hat{V}_8 W_n - (n_1+n_2-n_3-n_4)^2\hat{S}_8(-)^n W_n\Big] \ . \qquad (3.22)$$

The tadpole cancellation conditions can be deduced by eqs. (3.20), (3.21) and (3.22), setting to zero the reflection coefficients for the massless modes that originate from V_8 and S_8:

$$\text{NS-NS} \ : \ \frac{2^{9/2}}{2} + \frac{2^{-11/2}}{2}(n_1+n_2+n_3+n_4)^2 - \frac{1}{\sqrt{2}}(n_1+n_2+n_3+n_4) = 0 \ ,$$

$$\text{R-R} \ : \ \frac{2^{9/2}}{2} + \frac{2^{-11/2}}{2}(n_1+n_2-n_3-n_4)^2 - \frac{1}{\sqrt{2}}(n_1+n_2-n_3-n_4) = 0 \ ,$$

namely:

$$\text{NS-NS} \ : \ (n_1+n_2+n_3+n_4) = 32 \ ,$$

$$\text{R-R} \ : \ (n_1+n_2-n_3-n_4) = 32 \ .$$

From these relations one can infer that n_1 and n_2 fix the number of D9-branes, while n_3 and n_4 determine the number of anti-$\bar{D}9$-branes. The R-R tadpole conditions thus fix the total number of branes in the model. Enforcing also the NS-NS tadpole conditions, the presence of anti-branes is forbidden ($n_3 = n_4 = 0$), and the resulting spectrum, free of tachyons, gives rise to $SO(n_1) \times SO(32 - n_1)$ gauge group.

3.4.1 NS-NS B_{ab} and Shifts

Before moving to the detailed analysis of the four dimensional orientifold models, it is worth to clarify the relation between a non-vanishing discretized NS-NS two form B_{ab} on orientifolds [104, 105] and momentum and winding shifts. In particular, we want to show that the presence of a quantized NS-NS two-form field B_{ab} on a two-torus is exactly equivalent to an asymmetric shift-orbifold in which a momentum shift along the first direction of the torus is accompanied by a winding shift along the other direction For simplicity, let us take for the two-torus a product of circles both of radius R. Parametrizing the discretized two form as [104]

$$B \ = \ \frac{\alpha'}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \qquad (3.23)$$

the generalized momenta of eqs. (A.1, A.2) reduce to the expressions

$$p_{(\mathrm{L,R})\,a} \ = \ m'_a \ \pm \ \frac{1}{\alpha'} \, g_{ab} \, n^b \qquad , \qquad (3.24)$$

where $m'_a \ = \ m_a \ - \ \frac{1}{2}\,\epsilon_{ab}\,n^b$. The presence of B_{ab} makes the m'_a's integers or half-integers depending on the oddness or evenness of the integer n^a's. As a result, omitting the prime on the dummy m-variables for the rest of this section, the torus partition function of eq. (A.3) can be decomposed in the form

$$\Lambda(B) = \Lambda(m_1, m_2, 2n_1, 2n_2) + \Lambda(m_1 + 1/2, m_2, 2n_1, 2n_2 + 1) \\ + \Lambda(m_1, m_2 + 1/2, 2n_1 + 1, 2n_2) + \Lambda(m_1 + 1/2, m_2 + 1/2, 2n_1 + 1, 2n_2 + 1)\,, \tag{3.25}$$

where $\Lambda(m_1, m_2, n_1, n_2)$ denotes the two-dimensional lattice sum over momenta m_a/R and windings $n^a R$.

The same partition function can be obtained as an asymmetric shift-orbifold. Indeed, projecting the conventional ($B = 0$) $\Lambda(m_1, m_2, n_1, n_2)$ under the action of a $p_1 w_2$ shift and completing the modular invariant with the addition of twisted sectors, the resulting partition function is

$$\Lambda(p_1 w_2) = \frac{1}{2} \left[\Lambda(m_1, m_2, n_1, n_2) + (-1)^{m_1+n_2} \Lambda(m_1, m_2, n_1, n_2) \right. \\ \left. + \Lambda(m_1, m_2 + 1/2, n_1 + 1/2, n_2) + (-1)^{m_1+n_2} \Lambda(m_1, m_2 + 1/2, n_1 + 1/2, n_2) \right] . \tag{3.26}$$

This expression is exactly the one in eq. (3.25) after doubling the radius ($R \to 2R$) along the first direction. Thus, it should not come as a surprise that in some cases the effect of the shifts can be compensated by the presence of a quantized B_{ab}. However, all the models appearing in the next chapter display a reduction of the rank of the Chan-Paton group when a non-vanishing quantized B_{ab} is turned on. The reason is that the shifts we consider affect only one *real* coordinate of the tori, rather than two as in the previous asymmetric shift-orbifold construction. Nonetheless, in some cases, the shifts make the multiplicities of the matter multiplets independent of the rank of B_{ab}. By T-duality, other allowed discrete moduli [106, 107, 108] like, for instance, the off-diagonal components of the metric in orientifolds of the type IIA superstring, can also be related to suitable shift-orbifolds.

A nice geometric interpretation of the rank reduction of the Chan-Paton groups can also be given resorting to the asymmetric shift-orbifold description of B_{ab}. As we shall extensively see in the next chapter, a momentum shift orthogonal to D-branes splits them into multiple images. After a T-duality along the second direction of the two-torus, the $p_1 w_2$ becomes a $p_1 p_2$ shift-orbifold, that admits orientifold projections containing $D1$-branes parallel to p_1 and orthogonal to p_2. The corresponding annulus amplitude can be written as

$$\mathcal{A} = \frac{1}{2} N^2 \left(P_1 + P_1^{1/2} \right) \left(W_2 + W_2^{1/2} \right) , \tag{3.27}$$

where P_i and W_i are the usual one-dimensional momentum and winding lattice sums [104], respectively, while $P_i^{1/2}$ and $W_i^{1/2}$ are the corresponding shifted ones, and the consistent Möbius amplitude, describing the unoriented projection, is

$$\mathcal{M} = -\frac{1}{2} N \left(\hat{P}_1 \hat{W}_2 + \hat{P}_1^{1/2} \hat{W}_2^{1/2} \right) . \tag{3.28}$$

Eqs. (3.27) and (3.28) neatly display the expected doublet structure of the $D1$-brane configuration, and the analysis of the tadpole cancellation conditions reveals the related rank reduction of the Chan-Paton group. For instance, an equivalent eight-dimensional p_1p_2 shift-orientifold compactification of the type IIB superstring, would yield type I models with an $SO(16)$ gauge group, thus providing a rank reduction by a factor of two.

3.5 $\mathbb{Z}_2 \times \mathbb{Z}_2$ Orientifolds

In this section we review some four dimensional type I vacua obtained as orientifolds of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds, or of freely acting (*i.e.* without fixed points) $\mathbb{Z}_2 \times \mathbb{Z}_2$ shift-orbifolds. The starting point is an orbifold of the type IIB superstring compactified on an internal six-torus that, without any loss of generality for our purposes, can be chosen to be a product of three two-tori T^{45}, T^{67} and T^{89} along the three complex directions $Z^1 = X^4 + iX^5$, $Z^2 = X^6 + iX^7$ and $Z^3 = X^8 + iX^9$. Each two-torus can be equipped with a NS-NS background two-form B_i of rank r_i (with $r_i = 0$ or 2) that, if the orientifold projection is induced by the world-sheet parity operator Ω, is a discrete modulus and may thus take only quantized values [104, 105]. The orbifold group will be taken to be the combination of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ generated by the elements

$$g : (+, -, -) \qquad \text{and} \qquad h : (-, -, +) \quad , \tag{3.29}$$

where the minus signs indicate the two-dimensional $\mathbb{Z}_2$ inversion of the corresponding coordinates ($Z^i \to -Z^i$), with momenta and/or winding shifts along the real part of (some of) the three complex directions.

The conventional $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds, allowing for the introduction of a *discrete torsion* [109], give rise to supersymmetric orientifolds [110, 111, 112] as well as to orientifolds with *brane supersymmetry breaking* [113]. Moreover, the freely acting $\mathbb{Z}_2 \times \mathbb{Z}_2$ (shift-)orbifolds produce ten classes of orientifolds with different amounts of supersymmetry (models with *brane supersymmetry* [67, 114, 115]), together with a huge number of variants with brane-antibrane pairs and *brane supersymmetry breaking* [113].

3.5.1 $\mathbb{Z}_2 \times \mathbb{Z}_2$ Models and Discrete Torsion

Aside from the identity, the $\mathbb{Z}_2 \times \mathbb{Z}_2$ elements can be grouped together in the matrix

$$\sigma_0 = \begin{pmatrix} + & - & - \\ - & + & - \\ - & - & + \end{pmatrix} \quad , \tag{3.30}$$

whose rows represent the action of g, $f = g \circ h$ and h on the three internal torus coordinates Z^i. The one-loop closed partition function can be obtained supplementing the $\mathbb{Z}_2 \times \mathbb{Z}_2$ projections of the toroidal amplitude with the inclusion of three twisted sectors, located at the three fixed tori, to complete the modular invariant. There are actually two options, related to the freedom of introducing a discrete torsion [109], *i.e.* a relative sign between two disconnected orbits of the modular group. The result is

$$\begin{aligned}
\mathcal{T} \;=\; & \frac{1}{4}\Big\{ |T_{oo}|^2\Lambda_1(B_1)\Lambda_2(B_2)\Lambda_3(B_3) + \big[\, |T_{og}|^2\Lambda_1(B_1) + |T_{of}|^2\Lambda_2(B_2) + |T_{oh}|^2\Lambda_3(B_3) \,\big] \left|\frac{4\eta^2}{\vartheta_2^2}\right|^2 \\
+ & \;\big[\, |T_{go}|^2\Lambda_1(B_1) + |T_{fo}|^2\Lambda_2(B_2) + |T_{ho}|^2\Lambda_3(B_3) \,\big] \left|\frac{4\eta^2}{\vartheta_4^2}\right|^2 \\
+ & \;\big[\, |T_{gg}|^2\Lambda_1(B_1) + |T_{ff}|^2\Lambda_2(B_2) + |T_{hh}|^2\Lambda_3(B_3) \,\big] \left|\frac{4\eta^2}{\vartheta_3^2}\right|^2 \\
+ & \;\omega \left(|T_{gh}|^2 + |T_{gf}|^2 + |T_{fg}|^2 + |T_{fh}|^2 + |T_{hg}|^2 + |T_{hf}|^2\right) \left|\frac{8\eta^3}{\vartheta_2\vartheta_3\vartheta_4}\right|^2 \Big\} \quad ,
\end{aligned} \tag{3.31}$$

where the Λ_i's are the two-dimensional Narain lattice sums for the three internal tori (see Appendix A), that depend on the two-dimensional blocks (B_i) of the NS-NS two-form B_{ab}, and $\omega = \pm 1$ is the sign associated to the discrete torsion. We have expressed the torus amplitude in terms of the 16 quantities ($i = o, g, h, f$)

$$\begin{aligned}
T_{io} &= \tau_{io} + \tau_{ig} + \tau_{ih} + \tau_{if} \quad , \qquad T_{ig} = \tau_{io} + \tau_{ig} - \tau_{ih} - \tau_{if} \quad , \\
T_{ih} &= \tau_{io} - \tau_{ig} + \tau_{ih} - \tau_{if} \quad , \qquad T_{if} = \tau_{io} - \tau_{ig} - \tau_{ih} + \tau_{if} \quad ,
\end{aligned} \tag{3.32}$$

where the 16 $\mathbb{Z}_2 \times \mathbb{Z}_2$ characters τ_{il} [110], combinations of products of level-one SO(2) characters, are displayed in Appendix B. The geometric model, related to the *charge conjugation* modular invariant, corresponds to the choice $\omega = -1$, as can be deduced from the massless spectra reported in Table (C.1). It is a compactification on (a singular limit of) a Calabi-Yau threefold with Hodge numbers ($h_{11} = 51, h_{21} = 3$), while the $\omega = 1$ choice, linked in this context to the T-dual compactification, leads to (a singular limit of) the mirror symmetric Calabi-Yau threefold, with $h_{11} = 3, h_{21} = 51$.

The starting point for the orientifold construction are the Klein-bottle amplitudes

$$\begin{aligned}
\mathcal{K} \;=\; & \frac{1}{8}\Big\{ (P_1P_2P_3 + 2^{-4}P_1W_2(B_2)W_3(B_3) + 2^{-4}W_1(B_1)P_2W_3(B_3) + 2^{-4}W_1(B_1)W_2(B_2)P_3)T_{oo} \\
+ & \; 2 \times 16\big[2^{-\frac{r_2}{2}-\frac{r_3}{2}}\omega_1(P_1 + \omega\, 2^{-2}W_1(B_1))T_{go} + 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\omega_2(P_2 + \omega\, 2^{-2}W_2(B_2))T_{fo} \\
+ & \; 2^{-\frac{r_1}{2}-\frac{r_2}{2}}\omega_3(P_3 + \omega\, 2^{-2}W_3(B_3))T_{ho}\big] \left(\frac{\eta}{\vartheta_4}\right)^2 \Big\} \, ,
\end{aligned} \tag{3.33}$$

that project the oriented closed spectra into unoriented ones. The signs ω_i are linked to the discrete torsion through the *crosscap constraint*[99] by the relation [113]

$$\omega_1 \; \omega_2 \; \omega_3 \; = \omega \quad , \tag{3.34}$$

The transverse channel amplitude, obtained performing an S modular transformation, is

$$\begin{aligned}
\tilde{\mathcal{K}} \;=\; & \frac{2^5}{8}\Big\{\big(v_1 v_2 v_3 W_1^e W_2^e W_3^e + 2^{-4}\frac{v_1}{v_2 v_3} W_1^e P_2^e(B_2) P_3^e(B_3) \\
& + 2^{-4}\frac{v_2}{v_1 v_3} P_1^e W_2^e(B_2) P_3^e(B_3) + 2^{-4}\frac{v_3}{v_1 v_2} P_1^e(B_1) P_2^e(B_2) W_3^e\big)\; T_{oo} \\
& + 2\;\big[\; 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\omega_1(\; v_1 W_1^e + \omega\, 2^{-2}\frac{P_1^e(B_1)}{v_1}\;)\; T_{og} \\
& + 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\,\omega_2\,(\; v_2 W_2^e + \omega\, 2^{-2}\frac{P_2^e(B_2)}{v_2}\;)\; T_{of} \\
& + 2^{-\frac{r_1}{2}-\frac{r_2}{2}}\,\omega_3\,(\; v_3 W_3^e + \omega\, 2^{-2}\frac{P_3^e(B_3)}{v_3}\;)\; T_{oh}\,\big]\left(\frac{2\eta}{\theta_2}\right)^2\Big\} \quad , \qquad (3.35)
\end{aligned}$$

where the superscript e denotes the usual restriction of the sums to even subsets and the v_i denote the volumes of the three internal tori. At the origin of the lattices, the reflection coefficients are perfect squares,

$$\begin{aligned}
\tilde{\mathcal{K}}_0 \;=\; & \frac{2^5}{8}\Bigg\{\left(\sqrt{v_1 v_2 v_3} + 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\omega_1\sqrt{\frac{v_1}{v_2 v_3}} + 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\omega_2\sqrt{\frac{v_2}{v_1 v_3}} + 2^{-\frac{r_1}{2}-\frac{r_2}{2}}\omega_3\sqrt{\frac{v_3}{v_1 v_2}}\right)^2 \tau_{oo} \\
& + \left(\sqrt{v_1 v_2 v_3} + 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\omega_1\sqrt{\frac{v_1}{v_2 v_3}} - 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\omega_2\sqrt{\frac{v_2}{v_1 v_3}} - 2^{-\frac{r_1}{2}-\frac{r_2}{2}}\omega_3\sqrt{\frac{v_3}{v_1 v_2}}\right)^2 \tau_{og} \\
& + \left(\sqrt{v_1 v_2 v_3} - 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\omega_1\sqrt{\frac{v_1}{v_2 v_3}} + 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\omega_2\sqrt{\frac{v_2}{v_1 v_3}} - 2^{-\frac{r_1}{2}-\frac{r_2}{2}}\omega_3\sqrt{\frac{v_3}{v_1 v_2}}\right)^2 \tau_{of} \\
& + \left(\sqrt{v_1 v_2 v_3} - 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\omega_1\sqrt{\frac{v_1}{v_2 v_3}} - 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\omega_2\sqrt{\frac{v_2}{v_1 v_3}} + 2^{-\frac{r_1}{2}-\frac{r_2}{2}}\omega_3\sqrt{\frac{v_3}{v_1 v_2}}\right)^2 \tau_{oh}\Bigg\}, \quad (3.36)
\end{aligned}$$

and encode the presence, together with the conventional Orientifold 9-planes ($O9_+$-planes from now on), of three kinds of $O5$-planes, that we shall denote $O5_{1\alpha}$, $O5_{2\alpha}$ and $O5_{3\alpha}$. These (non-dynamical) planes are fixed under the combined action of Ω and the inversion along the directions orthogonal to them, namely g for the $O5_{1\alpha}$, f for the $O5_{2\alpha}$ and h for the $O5_{3\alpha}$, and the index α reflects their R-R charge. We shall use the $+$ sign to indicate O-planes with tension and R-R charge opposite to the corresponding quantities for the D-branes, and the $-$ sign for the "exotic" Orientifold planes with reverted tension and R-R charge. As is evident from eq. (3.36), the ω_i are proportional to the R-R charges of the $O5_i$. While manifestly compatible with the usual positivity requirements, the eight different choices reported, for the case with $B_{ab} = 0$, in table (C.2), affect the tadpole conditions. In particular, the presence of "exotic" $O5_i$ requires the introduction of antibranes in order to globally neutralize the R-R charge of the vacuum configuration. In this respect, according to [70], $\omega = -1$ implies the reversal of at least one of the $O5$-plane charges, producing type I vacua with *brane supersymmetry breaking* [113]. Moreover, the presence of the NS-NS two form blocks B_i affects the reflection coefficients in front of a crosscap by the familiar powers of two, responsible for the rank reduction of the Chan-Paton gauge groups [104, 105].

In this section, we shall limit ourselves to the discussion of the orientifolds of the unique supersymmetric model with $\omega_i = +1$, leaving to section 5 some examples with *brane supersymmetry breaking.* The unoriented closed spectra are reported in Table (C.3), while the annulus amplitude can be written as

$$
\begin{aligned}
\mathcal{A} &= \frac{1}{8}\Big\{ \big(N^2\, 2^{r-6}\, P_1(B_1)P_2(B_2)P_3(B_3) + D_1^2\, 2^{r_1-2}\, P_1(B_1)W_2W_3 \\
&+ D_2^2\, 2^{r_2-2}\, W_1P_2(B_2)W_3 + D_3^2\, 2^{r_3-2}\, W_1W_2P_3(B_3)\big)\, T_{oo} \\
&+ \Big[\, 2^{\frac{r_2}{2}+\frac{r_3}{2}}\, \big(2ND_1\, 2^{r_1-2}\, P_1(B_1) + 2D_2D_3\, W_1\big)\, T_{go} \\
&+ 2^{\frac{r_1}{2}+\frac{r_3}{2}}\, \big(2ND_2\, 2^{r_2-2}\, P_2(B_2) + 2D_1D_3\, W_2\big)\, T_{fo} \\
&+ 2^{\frac{r_1}{2}+\frac{r_2}{2}}\, \big(2ND_3\, 2^{r_3-2}\, P_3(B_3) + 2D_1D_2\, W_3\big)\, T_{ho}\Big] \left(\frac{\eta}{\theta_4}\right)^2 \Big\} \quad , \qquad (3.37)
\end{aligned}
$$

where $r = r_1 + r_2 + r_3$ is the total B-rank. Aside from the standard NN open-strings, there are the three types of open-strings with Dirichlet boundary conditions along two of the three internal directions, as well as mixed ND open strings. The corresponding vacuum-channel amplitude displays four independent squared reflection coefficients, related to the ubiquitous D9-branes on which the NN strings end, and to three types of $D5$-branes. In particular, we call $D5_i$-branes those with world-volume that invades the four-dimensional space-time and the internal Z^i coordinate. Again, the presence of the NS-NS two form reflects itself in the generic appearance of additional matter multiplets whose multiplicities depend on the rank of the B_i-blocks along the directions orthogonal to the fixed tori. N and D_i in eq. (3.37) indicate the traces of the Chan-Paton matrices, or Chan-Paton multiplicities, corresponding to the D9 and D5 branes, respectively.

Standard methods [27, 55, 56, 57] determine the direct-channel Möbius amplitude

$$
\begin{aligned}
\mathcal{M} &= -\frac{1}{8}\Big\{ \big[\, 2^{\frac{r-6}{2}}\, N\, P_1(B_1,\gamma_{\epsilon_1})\, P_2(B_2,\gamma_{\epsilon_2})\, P_3(B_3,\gamma_{\epsilon_3}) \\
&+ 2^{\frac{r_1-6}{2}}\, D_1\, P_1(B_1,\gamma_{\epsilon_1})W_2(B_2,\tilde\gamma_{\epsilon_2})W_3(B_3,\tilde\gamma_{\epsilon_3}) \\
&+ 2^{\frac{r_2-6}{2}}\, D_2\, W_1(B_1,\tilde\gamma_{\epsilon_1})P_2(B_2,\gamma_{\epsilon_2})W_3(B_3,\tilde\gamma_{\epsilon_3}) \\
&+ 2^{\frac{r_3-6}{2}}\, D_3\, W_1(B_1,\tilde\gamma_{\epsilon_1})W_2(B_2,\tilde\gamma_{\epsilon_2})P_3(B_3,\gamma_{\epsilon_3})\,\big]\, \hat T_{oo} \\
&- \big[\, 2^{\frac{r_1-2}{2}}\, (N{+}D_1)P_1(B_1,\gamma_{\epsilon_1}){+}2^{-1}\,(D_2{+}D_3)W_1(B_1,\tilde\gamma_{\epsilon_1})\big]\, \hat T_{og} \left(\frac{2\hat\eta}{\hat\theta_2}\right)^2 \\
&- \big[\, 2^{\frac{r_2-2}{2}}\, (N{+}D_2)P_2(B_2,\gamma_{\epsilon_2}){+}2^{-1}\,(D_1{+}D_3)W_2(B_2,\tilde\gamma_{\epsilon_2})\big]\, \hat T_{of} \left(\frac{2\hat\eta}{\hat\theta_2}\right)^2 \\
&- \big[\, 2^{\frac{r_3-2}{2}}\, (N{+}D_3)P_3(B_3,\gamma_{\epsilon_3}){+}2^{-1}\,(D_1{+}D_2)W_3(B_3,\tilde\gamma_{\epsilon_3})\big]\, \hat T_{oh} \left(\frac{2\hat\eta}{\hat\theta_2}\right)^2 \Big\} \; , \qquad (3.38)
\end{aligned}
$$

where the hatted version of the blocks in eq. (3.32) is linked, as usual, to the choice of a *real* basis of characters. A proper particle interpretation of the annulus and Möbius strip amplitudes requires a rescaling of the charges in such a way that $N = 2n$ and $D_i = 2d_i$. The (untwisted) tadpole conditions reported in Table (C.4) emphasize the usual rank reduction due to the presence of quantized values of B_{ab} and demand that the signs γ_ϵ and $\tilde{\gamma}_\epsilon$ satisfy the conditions

$$\sum_{\epsilon_i=0,1} \gamma_{\epsilon_i} = 2 \quad , \qquad \sum_{\epsilon_i=0,1\,\in \mathrm{Ker}(B)} \tilde{\gamma}_{\epsilon_i} = 2^{(2-r_i)/2} \,. \tag{3.39}$$

There are several solutions for the allowed gauge groups, that depend on the additional signs ξ_i and η_i defined by

$$\sum_{\epsilon_i=0,1} \tilde{\gamma}_{\epsilon_i} = 2\,\xi_i \quad , \qquad \sum_{\epsilon_i=0,1\,\in \mathrm{Ker}(B)} \gamma_{\epsilon_i} = 2^{(2-r_i)/2}\,\eta_i \,. \tag{3.40}$$

As shown in Table (C.4), they are products of four factors, chosen to be USp or SO depending on the values of ξ_i and η_i. The massless unoriented open spectra, encoded in the annulus and Möbius amplitudes at the lattice origin,

$$\begin{aligned}
\mathcal{A}_0 &= \frac{1}{2}\,[\,(\,n^2 + d_1^2 + d_2^2 + d_3^2\,)\,(\tau_{oo} + \tau_{og} + \tau_{oh} + \tau_{of}) \\
&+ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\;(2nd_1 + 2d_2 d_3)(\tau_{go} + \tau_{gg} + \tau_{gh} + \tau_{gf}) \\
&+ 2^{\frac{r_1}{2}+\frac{r_3}{2}}\;(2nd_2 + 2d_1 d_3)(\tau_{fo} + \tau_{fg} + \tau_{fh} + \tau_{ff}) \\
&+ 2^{\frac{r_1}{2}+\frac{r_2}{2}}\;(2nd_3 + 2d_1 d_2)(\tau_{ho} + \tau_{hg} + \tau_{hh} + \tau_{hf})\,]
\end{aligned} \tag{3.41}$$

and

$$\begin{aligned}
\mathcal{M}_0 &= -\frac{1}{2}\Bigg[\;\tau_{oo}\,[\frac{n}{2}\,(\eta_1\eta_2\eta_3 - \eta_1 - \eta_2 - \eta_3) + \frac{d_1}{2}(\,\eta_1\xi_2\xi_3 - \eta_1 - \xi_2 - \xi_3) \\
&+ \frac{d_2}{2}\,(\xi_1\eta_2\xi_3 - \xi_1 - \eta_2 - \xi_3) + \frac{d_3}{2}(\,\xi_1\xi_2\eta_3 - \xi_1 - \xi_2 - \eta_3)] \\
&+ \tau_{og}\,[\frac{n}{2}\,(\eta_1\eta_2\eta_3 - \eta_1 + \eta_2 + \eta_3) + \frac{d_1}{2}(\,\eta_1\xi_2\xi_3 - \eta_1 + \xi_2 + \xi_3) \\
&+ \frac{d_2}{2}\,(\xi_1\eta_2\xi_3 - \xi_1 + \eta_2 + \xi_3) + \frac{d_3}{2}(\,\xi_1\xi_2\eta_3 - \xi_1 + \xi_2 + \eta_3)] \\
&+ \tau_{of}\,[\frac{n}{2}\,(\eta_1\eta_2\eta_3 + \eta_1 - \eta_2 + \eta_3) + \frac{d_1}{2}(\,\eta_1\xi_2\xi_3 + \eta_1 - \xi_2 + \xi_3) \\
&+ \frac{d_2}{2}\,(\xi_1\eta_2\xi_3 + \xi_1 - \eta_2 + \xi_3) + \frac{d_3}{2}(\,\xi_1\xi_2\eta_3 + \xi_1 - \xi_2 + \eta_3)] \\
&+ \tau_{oh}[\frac{n}{2}\,(\eta_1\eta_2\eta_3 + \eta_1 + \eta_2 - \eta_3) + \frac{d_1}{2}(\,\eta_1\xi_2\xi_3 + \eta_1 + \xi_2 - \xi_3) \\
&+ \frac{d_2}{2}\,(\xi_1\eta_2\xi_3 + \xi_1 + \eta_2 - \xi_3) + \frac{d_3}{2}(\,\xi_1\xi_2\eta_3 + \xi_1 + \xi_2 - \eta_3)]\Bigg]\,,
\end{aligned} \tag{3.42}$$

are reported in Table (C.5). Being non chiral, these models are clearly free of anomalies.

3.5.2 $\mathbb{Z}_2 \times \mathbb{Z}_2$ Shift-orientifolds and Brane Supersymmetry

In this section we review how (δ_L, δ_R) shifts can be combined with $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold operations in the open descendants of type IIB compactifications. As in [115, 113], we shall distinguish between symmetric *momentum* shifts $(p) = (\delta, \delta)$ and antisymmetric *winding* shifts $(w) = (\delta, -\delta)$, since the two have very different effects on the resulting spectra. These orbifolds correspond to singular limits of Calabi-Yau manifolds with Hodge numbers $(19, 19)$, $(11, 11)$ and $(3, 3)$ in the cases of one, two and three shifts, respectively, as shown in Table (C.8). Let us begin by introducing a convenient notation to specify the orbifold action $Z_i \to \sigma(Z_i)$ on the complex coordinates of the three internal tori. There are several ways to combine the three operations g, f and h of the matrix (3.30) with shifts consistently with the $\mathbb{Z}_2 \times \mathbb{Z}_2$ group structure. However, up to T-dualities and corresponding redefinitions of the Ω projection, all non-trivial possibilities are captured by [115]

$$\sigma_1(\delta_1, \delta_2, \delta_3) = \begin{pmatrix} \delta_1 & -\delta_2 & -1 \\ -1 & \delta_2 & -\delta_3 \\ -\delta_1 & -1 & \delta_3 \end{pmatrix} \quad , \qquad \sigma_2(\delta_1, \delta_2, \delta_3) = \begin{pmatrix} \delta_1 & -1 & -1 \\ -1 & \delta_2 & -\delta_3 \\ -\delta_1 & -\delta_2 & \delta_3 \end{pmatrix} \quad , \tag{3.43}$$

where the three lines refer to the new operations, that we shall continue to denote by g, f and h, and where $-\delta_i$ indicates the combination of a shift in the real part of the i-th coordinate with the orbifold inversion. Notice that when a line of the table contains p or $-w$, the corresponding $D5$-branes are eliminated. One thus obtains the ten different classes of models reported in Table (3.2), with the $w_2 p_3$ model now linked to the σ_1 action, correcting a misprint in ref. [115]. In listing these models, we have followed the choices of axes made in ref. [115], so that when a single set of $D5$ branes is present, this is always the first, $D5_1$, and when two sets are present, these are always $D5_1$ and $D5_2$. All these freely acting orientifolds have $N = 1$ supersymmetry in the closed sector, but exhibit interesting instances of *brane supersymmetry* in the open part: additional supersymmetries are present for their massless modes, that in some cases extend also to the massive ones [67, 114] confined to some branes. Table (3.2) also collects the number of supersymmetries of the massless modes for the various branes present in each model. The unoriented truncations and the open spectra are generically affected by the shifts, that lift in mass some tree-level closed string terms eliminating the corresponding tadpoles, and determine the brane content of the models, related to the presence of the projectors

$$\Pi_1 \sim 1 + (-1)^{\delta_1+\delta_2} + (-1)^{\delta_2+\delta_3} + (-1)^{\delta_1+\delta_3} \quad , \tag{3.44}$$

$$\Pi_2 \sim 1 + (-1)^{\delta_1} + (-1)^{\delta_2+\delta_3} + (-1)^{\delta_1+\delta_2+\delta_3} \quad , \tag{3.45}$$

for the σ_1 and σ_2 tables respectively, along the tube.

The open-string spectra of the models in Table (3.2) are shown in Table (C.34). They correspond to peculiar and interesting brane configurations, related to the fact that some projections are absent in the NN or $D9 - D9$ string contributions, as well as in the DD or $D5 - D5$ string contributions. These features admit a nice geometrical interpretation: they are linked to the presence of multiplets of branes, associated with multiplets of tori fixed by some $\mathbb{Z}_2 \times \mathbb{Z}_2$ elements

models	shift	$D9$ susy	$D5_1$ susy	$D5_2$ susy
p_3	σ_1	N=1	N=2	N=2
w_2p_3	σ_1	N=2	N=2	N=4
$w_1w_2p_3$	σ_2	N=4	N=4	N=4
p_2p_3	σ_2	N=1	N=2	–
w_1p_2	σ_2	N=2	N=4	–
$w_1p_2p_3$	σ_2	N=2	N=4	–
$w_1p_2w_3$	σ_1	N=4	N=4	–
$p_1p_2p_3$	σ_1	N=1	–	–
$p_1w_2w_3$	σ_2	N=2	–	–
$w_1w_2w_3$	σ_1	N=4	–	–

Table 3.2: Shifts and *brane supersymmetry* for the various models.

and interchanged by the action of the remaining ones. Only the projections introduced by the former elements are thus present, since in these sectors the physical states are combinations of multiplets localized on the image branes. If one attempts to insert all branes at a fixed point, the other operations inevitably move them, giving rise to multiple images. Equivalently, as already discussed in section 2.1, brane multiplets may be traced to the presence of *momentum* shifts orthogonal to the branes [67, 114]. As a consequence, the massless modes (or at times the full spectrum) exhibit enhanced supersymmetry. Figure (3.2) displays the brane configuration

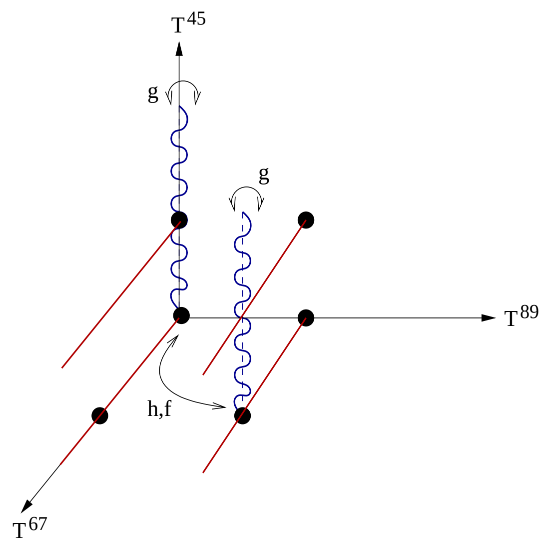

Figure 3.2: $D5_1$ and $D5_2$ brane configurations for the w_2p_3 model.

of the w_2p_3 model. The three axes refer to the three two-tori T^{45}, T^{67} and T^{89}, and the $D5_1$

branes, drawn as wavy blue lines, occupy a pair of fixed tori, while the $D5_2$ branes, the solid red lines, occupy all the four fixed tori along the Z^2 direction. The $D5_1 - D5_1$ configurations correspond to doublets of branes, associated to the pair of tori fixed by g and interchanged by f and h. As expected and as shown in Table (3.2), there is an $N = 2$ supersymmetry associated to the $D5_1$ brane doublets together with an $N = 4$ supersymmetry associated to the quartets of $D5_2$ branes.

Chapter 4

Magnetic Deformations

In this chapter we introduce the basic ingredients on which this second part of the thesis rests: the effect of magnetic deformations in open string models. In order to describe some of the striking features that motivate their presence in some Type-I constructions, we will first illustrate the simple cases of the bosonic string in a constant abelian background field and of open strings on a magnetized torus (showing how they can be related to T-dual versions of some intersecting D-brane models). Then we move to a brief discussion of the most notable results of the magnetized six-dimensional $T^4/\mathbb{Z}_2$ orientifold.

Open strings can be roughly considered as generalizations of the Yang-Mills gauge fields. This makes the analysis of their dynamics in the presence of a background electromagnetic fields particularly interesting [74].

As will be illustrated in the next chapter, the presence of internal magnetic fields in Type-I open string models has several and relevant consequences, such as chirality in some of their low-energy spectra or the breaking of supersymmetry [71, 72]. For this purpose let us remind that there are basically four known ways to break supersymmetry within String Theory. The first is to combine left and right moving modes in a non-supersymmetric fashion, like for instance in the type 0 models [32] and in the corresponding lower dimensional compactifications and orientifolds [55, 133, 134, 135]. Some type I-like instances, the type $0'$B and its compactifications [133], are also free of the tachyons that typically plague this kind of models. The second is the generalization to String Theory of the Scherk-Schwarz mechanism [136], available also in the heterotic case, in which the breaking is due to a generalized Kaluza-Klein compactification that involves different periodicities for bosons and fermions, thereby not respecting supersymmetry [137]. In this framework, the scale of supersymmetry breaking is inversely proportional to the volume of the internal manifold and some residual global supersymmetries may be left at tree level on some branes (*brane supersymmetry*) [67, 114, 115]. The third possibility is related to models with supersymmetry non linearly realized on some branes, as a result of the simultaneous presence of branes and antibranes of the same [138, 113, 139] or of different types

[70, 113, 140], or of the introduction of "exotic" orientifold planes [98]. These are referred to as *brane supersymmetry breaking* models, and the corresponding scale of supersymmetry breaking can be tied generically to the string scale. Finally, supersymmetry can be broken [71, 72, 73] by the introduction of internal magnetic fields [74] in the open sectors [72, 75, 78, 76], since particles of different spins couple differently to them via their magnetic moments, thus giving rise to different masses.

4.1 The bosonic string in a uniform magnetic field

The action of the open bosonic string in an uniform electromagnetic field $F_{\mu\nu}$ is, in the conformal gauge,

$$\begin{aligned} S &= \frac{1}{4\pi\alpha'}\int_{-\infty}^{+\infty} d\tau \int_0^\pi d\sigma \partial_\alpha X^\mu \partial^\alpha X_\mu \\ &- \int_{-\infty}^{+\infty} d\tau [q_L F_{\mu\nu} X^\nu \partial_\tau X^\mu(0) + q_R F_{\mu\nu} X^\nu \partial_\tau X^\mu(\pi)] \ , \end{aligned} \tag{4.1}$$

where $A_\mu = -\frac{1}{2}F_{\mu\nu}X^\nu$ is a possible choice for the vector potential. In general a constant and abelian background field is embedded in the gauge group of the open string. Thus q_L and q_R denote the charges associated to the open string ends that couple to the field. Since the external magnetic field couples solely to the *boundaries* of the string, the string coordinates satisfy the usual free field equations:

$$(\partial_\tau^2 - \partial_\sigma^2)X^\mu = 0\,, \tag{4.2}$$

but with boundary conditions:

$$\begin{aligned} &\partial_\sigma X_\mu - (2\pi\alpha')q_L F_{\mu\nu}\partial_\tau X^\nu = 0 \qquad \text{at } \sigma = 0\ , \\ &\partial_\sigma X_\mu + (2\pi\alpha')q_R F_{\mu\nu}\partial_\tau X^\nu = 0 \qquad \text{at } \sigma = \pi\ , \end{aligned} \tag{4.3}$$

that shows how the field allows the interpolation between Neumann and Dirichlet boundary conditions. Considering the configuration $F_{\mu\nu} = H\epsilon_{\mu\nu}$, with $\mu,\nu = 1,2$, and $\epsilon_{12} = 1 = -\epsilon_{21}$, and introducing the complex coordinate combinations $X_\pm = \frac{1}{\sqrt{2}}(X^1 \pm iX^2)$, with $(X_+)^\dagger = X_-$, eqs. (4.3)) can be written as

$$\begin{aligned} &\partial_\sigma X_+ + i(2\pi\alpha')q_L H\partial_\tau X_+ = 0, \quad \partial_\sigma X_- - i(2\pi\alpha')q_L H\partial_\tau X_- = 0 \quad \text{at}\, \sigma = 0, \\ &\partial_\sigma X_+ - i(2\pi\alpha')q_R H\partial_\tau X_+ = 0, \quad \partial_\sigma X_- + i(2\pi\alpha')q_R H\partial_\tau X_- = 0 \quad \text{at}\, \sigma = \pi. \end{aligned}$$

Defining the total string charge as $Q = q_L + q_R$, it can be shown that it is necessary to treat separately *neutral* and *charged* open sectors. When the total string charge is not vanishing ($Q \neq 0$), because of the linear behavior of the boundary conditions, the $X_\pm$ coordinates can be expanded in terms of the normal modes

$$X_+(\tau,\sigma) = x_+ + i(\sum_{n=1}^{\infty} a_n \phi_n(\tau,\sigma) - \sum_{m=0}^{\infty} b_m^\dagger \phi_{-m}(\tau,\sigma)) \,, \tag{4.4}$$

$$X_-(\tau,\sigma) = x_- + i(\sum_{m=0}^{\infty} b_m \bar{\phi}_{-m}(\tau,\sigma) - \sum_{n=1}^{\infty} a_n^\dagger \bar{\phi}_n(\tau,\sigma)) \,, \tag{4.5}$$

with $(X_+)^\dagger = X_-$ and with normalized mode functions

$$\phi_n = \frac{1}{\sqrt{|n-z|}} e^{-i(n-z)\tau} \cos[(n-z)\sigma + \gamma] \ ,$$

where

$$z = \frac{1}{\pi}[\arctan(2\pi\alpha' q_L H) + \arctan(2\pi\alpha' q_R H)]$$

is the non-linear shift function that summarizes the effect of the magnetic field. By the canonical quantization one can obtain the commutation relations for the oscillators a_n and b_n, but in particular that the zero-mode commutator is not-vanishing

$$[x_+, x_-] = \frac{1}{2(q_L + q_R)H} \qquad \text{with} \quad (x_+)^\dagger = x_- \ , \tag{4.6}$$

which implies that the constant modes x_+ and x_-, that in the classical limit correspond to the coordinates of the center of a Landau orbit, do not commute, and suggests that $2i(q_L + q_R)Hx_-$ behaves as the conjugate momentum operator for x_+. It can be easily shown that the neat effect of a constant background magnetic field is to shift the frequencies of the a and b oscillators by $\pm z$ respectively, modifying at the same time the value of the vacuum energy.

When the total string charge vanishes ($Q = 0$), $z = 0$ and the modes are not altered by $F_{\mu\nu}$ (even though $(\arctan(2\pi\alpha')q_{L,R}H \neq 0)$), but the structure of the zero-modes get modified, since when $Q = 0$ the commutator (4.6) is no more well defined. In this case the expansion for X_+ becomes

$$X_+ = \frac{x_+ + p_-(\tau - iq_L H(\sigma - \frac{\pi}{2}))}{\sqrt{1 + q_L^2 H^2}} + i\sum_{n=1}^{\infty}[a_n \phi_n(\tau,\sigma) - b_n^\dagger \phi_{-n}(\tau,\sigma)] \ . \tag{4.7}$$

where one can note the presence of a linear term in τ allowed by the boundary conditions when $(q_L + q_R)H = 0$. The presence of the total momentum p_- in (4.7) is related to the conserved charges of a particle in a constant magnetic field, that defines the center of their orbits. In fact, imaging that open string ends with opposite charges satisfy the equation

$$m\frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B} = q\frac{d}{dt}(\vec{r} \times \vec{B}) \ , \tag{4.8}$$

one obtains the two constants of motion

$$\begin{aligned} \vec{U}_1 &= \vec{p}_1 - \frac{q}{m}\vec{r}_1 \times \vec{B} \ , \\ \vec{U}_2 &= \vec{p}_2 - \frac{q}{m}\vec{r}_2 \times \vec{B} \ . \end{aligned} \tag{4.9}$$

Defining $\vec{p} = \vec{p}_1 + \vec{p}_2$, and being $q_L = q = -q_R$, it follows that $\vec{p} - q(\vec{r}_1 - \vec{r}_2) \times \vec{B}$ is the conserved quantity. Hence, the relative coordinate of the string ends $\vec{r}_1 - \vec{r}_2$ commutes with $\vec{p}$, as well as the components $\vec{U}_1$ and $\vec{U}_2$.

It is not hard to modify the previous results and to study the behavior of a string in the presence of an electric field [116]. It is sufficient to choose $F_{01} = E$ and to use the coordinates in the light-cone gauge $X^{\pm} = \frac{1}{\sqrt{2}}(X^0 \pm X^1)$. This allows to exactly evaluate the open string analog of the field theory Schwinger effect. In the weak-field limit, the result agrees with the standard one, while when the field tends to a critical value (of the order of the string tension), the probability for the pair creation diverges [116]. This limit behavior can be understood as due to the fact that the electric forces can overcome the tension of the string.

4.2 Open Strings on a Magnetized Torus

As we have just seen, an external electromagnetic field strongly affects the spectrum of the charged ($Q = q_L + q_R \neq 0$) bosonic string, since it modifies the commutation relation of the zero modes, it shifts the oscillation frequencies and it changes the zero-point energy.

Moreover, according to whether the coordinates X^1 and X^2 are or not compact, there is an infinite or finite degeneracy respectively in the spectrum of the theory. In the first case the situation is similar to that of a charged particle moving in the plane under the influence of a orthogonal uniform magnetic field: there are Landau levels with infinite degeneracy, but equally spaced. If the X^1 and X^2 coordinates are instead compact, the zero-modes correspond to those of a charged particle moving in a constant magnetic field on a torus.

A uniform background magnetic field H_i on the i-th torus is actually a monopole field, a $U(1)$ bundle with non-trivial transition functions gluing two local charts, whose consistency with particle dynamics requires a *Dirac quantization condition*

$$2\pi\alpha' q_i H_i = \frac{k_i}{v_i} \ , \tag{4.10}$$

where q_i is the $U(1)$-charge, v_i is the volume of the torus and k_i is an integer defining the (quantized) number of elementary fluxons or, equivalently, the Landau-level degeneracy. For this purpose is intriguing to show how the Dirac relation eq. (4.10) can be obtained in a *geometrical* fashion, using the notion of D-branes and T-duality on a magnetized torus. As discussed in the previous section, for the neutral bosonic string the corresponding zero-modes are, in terms of the X_1 and X_2 coordinates

$$\begin{aligned} X_1^{z.m.} &= \frac{x_1 + (2\alpha')[p_1\tau - q_L H p_2(\sigma - \pi/2)]}{\sqrt{1+q_L^2H^2}} \ , \\ X_2^{z.m.} &= \frac{x_2 + (2\alpha')[-p_2\tau - q_L H p_1(\sigma - \pi/2)]}{\sqrt{1+q_L^2H^2}} \ . \end{aligned} \tag{4.11}$$

By applying a T-duality transformation for example along the X_2 coordinate ($X_2 \to Y_2$), the eqs. (4.11) become

$$
\begin{aligned}
X_1^{z.m.} &= \frac{x_1 + (2\alpha')p_1\tau - 2q_L H w_2(\sigma - \pi/2)}{\sqrt{1+q_L^2 H^2}} \; , \\
Y_2^{z.m.} &= \frac{y_2 - (2\alpha')q_L H p_1\tau - 2w_2(\sigma - \pi/2)}{\sqrt{1+q_L^2 H^2}} \; ,
\end{aligned}
\tag{4.12}
$$

and defining

$$
\cos\theta = \frac{1}{\sqrt{1+q_L^2 H^2}} \; , \quad \sin\theta = \frac{q_L H}{\sqrt{1+q_L^2 H^2}} \; ,
$$
$$
\tilde{x}_1 = x_1 \cos\theta \; , \quad \tilde{y}_2 = y_2 \cos\theta \; ,
$$

the (4.12) can be written as:

$$
\begin{aligned}
X_1^{z.m.} &= \tilde{x}_1 + (2\alpha')p_1 \cos\theta\tau - 2w_2 \sin\theta(\sigma - \pi/2) \; , \\
Y_2^{z.m.} &= \tilde{y}_2 + (2\alpha')p_1 \sin\theta\tau - 2w_2 \cos\theta(\sigma - \pi/2) \; .
\end{aligned}
\tag{4.13}
$$

Noticing that the combination $X_1^{z.m.}\sin\theta + Y_2^{z.m.}\cos\theta$ contains only windings, one can conclude that, with respect to the starting configuration, the D-brane on which the open string ends terminate is roteded by an angle θ. The consistency of the wrapping of the D-brane along the fundamental torus cell implies that

$$
k\tilde{R}\cot\theta = R \; , \tag{4.14}
$$

where k is an integer, and $\tilde{R}$ is the T-dual radius of the squared torus: $\tilde{R} = \frac{\alpha'}{R}$. Reminding that $\tan\theta = (2\pi\alpha')qH$, one obtains again the quantization relation (4.10). From all of this then follows that the Dirac quantization condition in eq. (4.10) can be then interpreted as the requirement that D-branes wrap exactly k_i-times the tori, and that to a uniform Abelian magnetic field can be given a dual interpretation in terms of rotated branes [123].

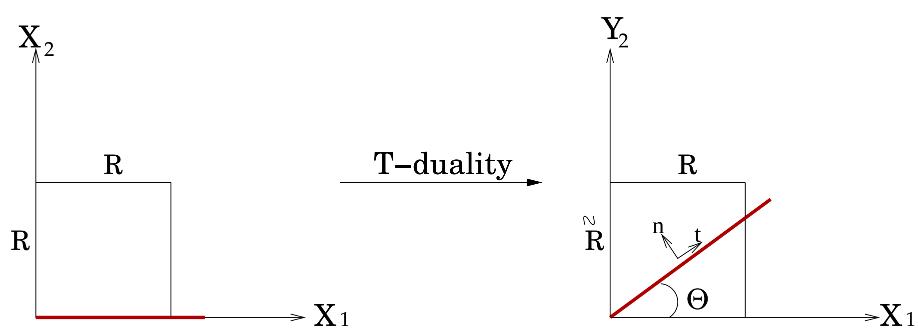

Figure 4.1: Example of a rotated D2-brane.

Notice that we always normalize the electric charge of the open-strings in such a way that it corresponds to the elementary quantum. This is the reason why we can describe all spectra using just one integer k_i for each two-torus or, in other words, setting to one the electric winding number of the corresponding D-branes.

It is instructive to apply the previous results to the Type-I superstring compactified on a two-dimensional squared two-torus, deducing in particular the corresponding *magnetized* open string partition functions. Let us first write the amplitudes that describe the Type I model in eight dimensions. The closed sector is given by the torus amplitude

$$\mathcal{T}^{T^2} = \frac{1}{2}\ |V_8 - S_8|^2(\tau)\ \Lambda_{(2,2)}\ , \tag{4.15}$$

where $\Lambda_{(2,2)}$ is the two-dimensional lattice sum defined as

$$\Lambda_{(2,2)} = \frac{\Big[\sum_{(m,n)\in\mathbb{Z}} q^{\frac{\alpha'}{4}(\frac{m}{R}+\frac{nR}{\alpha'})^2}\ \bar{q}^{\frac{\alpha'}{4}(\frac{m}{R}-\frac{nR}{\alpha'})^2}\ \Big]^2}{|\eta(\tau)|^2}, \tag{4.16}$$

and $q = e^{2\pi i\tau}$. Analogously, the Klein bottle amplitude can be written as

$$\mathcal{K}^{T^2}_{IIB} = \frac{1}{2}\ (V_8 - S_8)(2i\tau_2)\ P_m\ , \tag{4.17}$$

where the lattice sum is restricted to the vanishing winding $n = 0$ states:

$$P_m = \Big[\sum_{m\in Z} (e^{-4\pi\tau_2})^{\frac{\alpha' m^2}{4R^2}}\Big]^2\ . \tag{4.18}$$

The open sector is instead described by the annulus and by the Möbius strip amplitude, that have respectively the following form

$$\mathcal{A}^{T^2}_{IIB} = \frac{1}{2}N^2(V_8 - S_8)(i\tau/2)\ P_m\ , \tag{4.19}$$

where again P_m is the lattice sum restricted to vanishing winding $n = 0$ states

$$P_m = \Big[\sum_{m\in\mathbb{Z}} (e^{-2\pi\tau_2})^{\frac{\alpha' m^2}{2R^2}}\Big]^2\ , \tag{4.20}$$

and

$$\mathcal{M}^{T^2}_{IIB} = -\frac{N}{2}\ (\hat{V}_8 - \hat{S}_8)(\frac{1}{2} + \frac{i\tau_2}{2})\ P_m, \tag{4.21}$$

where

$$P_m = \Big[\sum_{m\in\mathbb{Z}} (e^{-\pi\tau_2})^{\frac{\alpha' m^2}{R^2}}\ \Big]^2\ . \tag{4.22}$$

While the introduction of a constant abelian field does not affect the closed sector, since the field couples only to the open string endpoints, it has crucial effects on the open string states. In particular the Chan-Paton factors N can be thought as splitted between the various charged sectors of the model. Let us remind in fact that one has to distinguish between neutral and charged states. Calling with m and $\bar{m}$ the multiplicities of magnetized branes with a $U(1)$ charge $= \pm 1$, and with m^2 and $\bar{m}^2$ the number of branes with charges $= \pm 2$, then charged strings are described by terms in the amplitudes of the open sectors that are proportional to Nm and $N\bar{m}$, when $Q = \pm 1$, and by terms proportional to m^2 and $\bar{m}^2$ when and $Q = \pm 2$. They are characterized by oscillators with shifted frequencies that reflect into the presence of characters with non-zero argument.

On the other hand, neutral *dipole* strings, with $q_L = -q_R$ ($Q = 0$), have integer-mode frequencies, but the structure of their zero-modes is rather peculiar. Indeed, both momenta and windings are now allowed, but the effect of the magnetic field on this sector is simply to introduce rescalings in the momentum and winding lattices entering the one-loop annulus partition function. An analysis of the resulting projector along the tube, shows that in the open one-loop channel the lattice sums over momenta and windings are indeed subjected to the *complex boosts*

$$m_i \to \frac{m_i}{\sqrt{1 + (2\pi\alpha' q_i H_i)^2}} \ , \quad n_i \to n_i \sqrt{1 + (2\pi\alpha' q_i H_i)^2} \ . \tag{4.23}$$

With this in mind, the expressions (4.19) and (4.21) can be correspondingly deformed and take the following form

$$\begin{aligned} \mathcal{A} \ &= \ \frac{1}{2} \Big[\ [N^2 P_m + 2m\bar{m}\tilde{P}_m] \quad (V_8 - S_8)(0) \\ &- \ i[\ 2Nm(V_8 - S_8)(z\tau;\tau) \ \frac{k\eta}{\theta_1(z\tau)} - 2N\bar{m}(V_8 - S_8)(-z\tau;\tau) \ \frac{k\eta}{\theta_1(-z\tau)} \ ] \\ &- \ i[\ m^2(V_8 - S_8)(2z\tau;\tau) \ \frac{2k\eta}{\theta_1(2z\tau)} - \bar{m}^2(V_8 - S_8)(-2z\tau;\tau) \ \frac{2k\eta}{\theta_1(-2z\tau)} \ ] \Big], \end{aligned} \tag{4.24}$$

where $\tilde{P_m}$ is the deformed lattice sum involving the *boosted* momenta

$$\tilde{P}_m = \Big[\sum_{m\in\mathbb{Z}} (e^{-2\pi\tau_2})^{\frac{\alpha'}{2R^2}(\frac{m}{\sqrt{1+(qH)^2}})^2} \Big]^2 \ . \tag{4.25}$$

Actually these results find a simpler explanation in terms of the T-dual description, that enlightens the constraints on the states flowing into the tube, highlighting at the same time the physical meaning of the rescaled modes.

In general, the zero-modes of the bosonic string on the torus are

$$\begin{aligned} Z_1^{z.m} \ &= \ z_1 + (2\alpha')p_1\tau + 2w_1\sigma \ , \\ Z_2^{z.m} \ &= \ z_2 + (2\alpha')p_2\tau + 2w_2\sigma \ , \end{aligned} \tag{4.26}$$

where

$$p_i = \frac{m_i}{R} = 2\pi\frac{m_i}{L} \ , \quad w_i = n_i R = \frac{n_i L}{2\pi}.$$

By applying a T-duality transformation along the Z_2 coordinate, eqs. (4.26) are transformed into

$$\begin{aligned} Z_1^{z.m} \ &= \ z_1 + (2\alpha')m_1\frac{2\pi}{L}\tau + n_1\frac{L}{\pi}\sigma \ , \\ Z_2^{z.m} \ &= \ \tilde{z}_2 + n_2\frac{L}{\pi}\tau + (2\alpha')m_2\frac{2\pi}{L}\sigma \ . \end{aligned} \tag{4.27}$$

But we have seen that the introduction of a uniform magnetic field on a torus is equivalent to a rotated D-brane by an angle such that $\tan\theta = \frac{\alpha'}{R^2}k$. As a consequence, in the annulus amplitude $\tilde{\mathcal{A}}$ of the theory compactified on a *magnetized* torus can flow only the states with zero momentum, in the direction orthogonal to the brane, and zero winding in the longitudinal direction to the brane, namely

$$\vec{p}\cdot\hat{t} = 0 \ , \quad \vec{w}\cdot\hat{n} = 0 \ ,$$

that lead to

$$\begin{aligned} m_1 &= kn_2 \ , \\ m_2 &= -kn_1 \ . \end{aligned} \tag{4.28}$$

They thus imply modified zero-modes for the globally neutral string, in fact through the relations (4.28), the generic term of the lattice sum $q^{\frac{\alpha'}{4}[\,(\frac{m_1^2}{R^2}+\frac{n_1^2R^2}{\alpha'^2})+(\frac{m_2^2}{R^2}+\frac{n_2^2R^2}{\alpha'^2})\,]}$, can be written as

$$q^{\frac{\alpha'}{4}[(\frac{k^2n_2^2}{R^2}+\frac{n_1^2R^2}{\alpha'^2})+(\frac{k^2n_1^2}{R^2}+\frac{n_2^2R^2}{\alpha'^2})]} = q^{\frac{R^2}{4\alpha'}(n_1^2+n_2^2)[\frac{k^2\alpha'^2}{R^2}+1]} = q^{\frac{R^2}{4\alpha'}(n_1^2+n_2^2)(1+q^2H^2)} \ ,$$

from which follows that the momenta and the windings of the neutral string zero-modes have to be rescaled as in (4.23).

As for the annulus, the Möbius amplitude can be deformed as

$$\begin{aligned} \mathcal{M} = \ & - \ \frac{1}{2}\Big[N(\hat{V}_8-\hat{S}_8)(0)\ P_m \\ & - \ i[\ m(\hat{V}_8-\hat{S}_8)(2z\tau;\tau)\frac{2k\hat{\eta}}{\hat{\theta}_1(2z\tau)} - \bar{m}(\hat{V}_8-\hat{S}_8)(-2z\tau;\tau)\frac{2k\hat{\eta}}{\hat{\theta}_1(-2z\tau)}]\Big] . \end{aligned} \tag{4.29}$$

The transverse channel annulus amplitude can be obtained by an S modular transformation and can be written as

$$\begin{aligned} \tilde{\mathcal{A}} \ &= \ \frac{2^{-5}}{2}\Big[\ [N^2\ v\ W_n + 2m\bar{m}\ (1+q^2H^2)\ v\ \tilde{W}_n]\ (V_8-S_8)(0) \\ &+ \ \ 2\ [\ 2Nm\ (V_8-S_8)(z)\ \frac{k\eta}{\theta_1(z)} - 2N\bar{m}\ (V_8-S_8)(-z)\ \frac{k\eta}{\theta_1(-z)}\ ] \\ &+ \ \ 2\ [\ m^2\ (V_8-S_8)(2z)\ \frac{2k\eta}{\theta_1(2z)} - \bar{m}^2\ (V_8-S_8)(-2z)\ \frac{2k\eta}{\theta_1(-2z)}\ ]\ \Big] \ , \end{aligned} \tag{4.30}$$

where

$$W_n = \Big[\sum_{n\in\mathbb{Z}}(e^{-\pi l})^{\frac{R^2n^2}{2\alpha'}}\Big]^2 \ , \tag{4.31}$$

and

$$\tilde{W}_n = \Big[\sum_{n\in\mathbb{Z}}(e^{-\pi l})^{\frac{R^2}{2\alpha'}[n\sqrt{1+(qH)^2}]}\Big]^2 \ , \tag{4.32}$$

and $v = \frac{\alpha'}{R^2}$ is the volume of the two-dimensional torus.

In the transverse channel, the Möbius strip amplitude becomes

$$\tilde{\mathcal{M}} = - \frac{2}{2} \Big[N(\hat{V}_8 - \hat{S}_8)(0) \; v \; W_{2n} \\ + \; [\; m(\hat{V}_8 - \hat{S}_8)(z;\tau)\frac{2k\hat{\eta}}{\hat{\theta}_1(z)} - \bar{m}(\hat{V}_8 - \hat{S}_8)(-z;\tau)\frac{2k\hat{\eta}}{\hat{\theta}_1(-z)}]\Big], \tag{4.33}$$

with

$$W_{2n} = \Big[\sum_{n\in\mathbb{Z}} (e^{-2\pi l})^{\frac{R^2(2n)^2}{4\alpha'}}\Big]^2 \; . \tag{4.34}$$

From the "transverse" expressions of eqs. (4.31) and (4.34) and from

$$\tilde{\mathcal{K}} = \frac{2^5}{2} \; v \; W_{2n} \; (V_8 - S_8)(0) \; , \tag{4.35}$$

where W_{2n} is as in eq. (4.34), one can derive the tadpole cancellation condition $N+(m+\bar{m}) = 32$ with $m = \bar{m}$, and that also selects a $SO(N) \times U(m)$ gauge group.

4.3 Open Strings on Magnetized Orbifolds

Let us now briefly review how the configurations mentioned in the previous sections manifest themselves in the orientifold of type IIB on a magnetized $(T^2 \times T^2)/\mathbb{Z}_2$ [75] and in the presence of a non vanishing quantized NS-NS B_{ab} background [78]. This is a deformation of the six-dimensional $(T^2 \times T^2)/\mathbb{Z}_2$ [55, 131] model, whose massless oriented closed spectrum is reported in Table (4.1) and comprises $N = (2, 0)$ supergravity coupled to 21 tensor multiplets, as expected for a singular limit of a K_3 compactification. The unoriented spectra, not affected by the constant magnetic backgrounds, are reported in Table (4.2) and exhibit at zero mass $N = (1, 0)$ supergravity modes coupled to hypermultiplets and tensor multiplets whose numbers depend upon the total rank r of B_{ab}. In the open sector, the two magnetic fields H_1 and H_2 turned on inside the two T^2's are aligned along the same $U(1)$ subgroup of the Chan-Paton gauge group. Generic fluxes of H_1 and H_2 produce the breaking of supersymmetry and the presence of Nielsen-Olesen instabilities, signaled by the appearance of tachyonic states. Absence of tachyons and supersymmetry can be attained choosing self-dual field configurations, that at the same time allow to compensate the non-vanishing instanton density with additional lower dimensional branes. As a result, the magnetized D9-branes mimic the behavior of the D5-branes and couple to the R-R six-form, contributing to the tadpole cancellation conditions. This is the *brane transmutation* phenomenon, first described, in this context, in ref. [75] and linked there to the peculiar Wess-Zumino couplings in the D-brane actions [132]. There are several solutions, reported in Tables (4.3), (4.4), (4.5), (4.6) (see Appendix C for notations and conventions used in the Tables), depending on the O-plane configurations, or equivalently on some signs, contained in the Möbius-strip amplitudes. In particular the R-R tadpole cancellation conditions and the resulting Chan-Paton gauge groups are shown in Table (4.3) for the models with complex charges and in Table (4.5) for the models with real charges. Moreover, the untwisted NS-NS tadpoles are related to the derivatives of the Born-Infeld action with respect to the untwisted moduli, while the twisted ones are associated with corresponding couplings in the effective Lagrangian, as in [75, 78]. The resulting open spectra, reported in Table (4.4) for complex charges and in Table (4.6) for real charges, can be easily recognized as deformations of the models without background magnetic fields [55, 131]. Several interesting facts, however, emerge from the analysis of Tables (4.4) and (4.6). First, there is an unusual rank reduction of the Chan-Paton gauge group. Second, some matter multiplets appear in multiple families, because of the degeneracy introduced by the Landau levels. Third, as already stressed the magnetized D9-branes behave exactly like D5-branes. This is particularly evident for the models *without* D5-branes, related for instance to the choice $r = 0$, $k_2 = k_3 = 2$, $d = 0$, $n = 12$ and $m = 4$ in Table (4.4), and corresponds [118] to the fact that the D5-branes can be interpreted as instantons of vanishing size. So, in the presence of self-dual configurations of the internal magnetic fields, a stack of $2^{\frac{r}{2}}|k_2 k_3|$

D5-branes is replaced by a "fat" instanton that invades the whole ten-dimensional space-time, in a transition related by T-duality [112] to the inverse small-instanton transition discussed in [118]. Finally, introducing antiself-dual configurations for the magnetic field, the magnetized $D9$-branes mimic anti-$D5$-branes [78]. Tachyons are again eliminated, but supersymmetry is broken at tree level in the open sector at the string scale or, better, is non-linearly realized in the anti $D5$-brane sector, as discussed in refs. [119, 120].

4.4 Spectra of the $[T^2(H_2) \times T^2(H_3)]/\mathbb{Z}_2$ orientifolds

untwisted SUGRA	untwisted T	twisted T
$N = (2, 0)$	$1 + 4$	16

Table 4.1: Oriented closed spectra of the $[T^2 \times T^2]/\mathbb{Z}_2$ Orbifolds.

B rank r	untwisted SUGRA	untwisted H	untwisted T	twisted H	twisted T
0	$N = (1, 0)$	4	1	16	0
2	$N = (1, 0)$	4	1	12	4
4	$N = (1, 0)$	4	1	10	6

Table 4.2: Unoriented closed spectra of the $[T^2 \times T^2]/\mathbb{Z}_2$ orientifolds.

$U(n) \otimes U(d) \otimes U(m)$
$n+\bar{n}+m+\bar{m}=32\ 2^{-\frac{r}{2}}$
$d+\bar{d}+2^{\frac{r}{2}}\ \lvert k_2k_3\rvert\ (m+\bar{m})=32\ 2^{-\frac{r}{2}}$
$n=\bar{n}$; $d=\bar{d}$; $m=\bar{m}$

Table 4.3: Chan-Paton groups and tadpole conditions for the $[T^2 \times T^2]/\mathbb{Z}_2$ models (complex charges).

Multiplets	Number	Rep.
H	1	$(A+\bar{A},1,1)$
H	1	$(1,A+\bar{A},1)$
H	$(2^r\ \lvert k_2k_3\rvert-4)/2$	$(F,1,F)$
H	$(2^r\ \lvert k_2k_3\rvert+4)/2$	$(\bar{F},1,F)$
H	$(2^r+2^{r/2})\,\lvert k_2k_3\rvert+2$	$(1,1,A)$
H	$(2^r-2^{r/2})\,\lvert k_2k_3\rvert$	$(1,1,S)$
H	$2^{r/2}$	$(F,\bar{F},1)$
H	$2^{r/2}$	$(1,\bar{F},F)$

Table 4.4: Open spectra of the $[T^2(H_2)\times T^2(H_3)]/Z_2$ orientifolds (complex charges).

$USp(n_1)\otimes USp(n_2)\otimes USp(d_1)\otimes USp(d_2)\otimes U(m)$
$n_1+n_2+m+\bar{m}=32\ 2^{-\frac{r}{2}}$
$d_1+d_2+2^{\frac{r}{2}}\ \lvert k_2k_3\rvert\ (m+\bar{m})=32\ 2^{-\frac{r}{2}}$
$n_2=n_1+m+\bar{m}$; $d_1=d_2$; $m=\bar{m}$

Table 4.5: Chan-Paton groups and tadpole conditions for the $[T^2\times T^2]/\mathbb{Z}_2$ models (real charges).

Multiplets	Number	Rep.
H	1	$(F,F,1,1,1)$
H	1	$(1,1,F,F,1)$
H	$(2^r - 2^{r/2})\,\lvert k_2k_3\rvert$	$(1,1,1,1,A)$
H	$(2^r + 2^{r/2})\,\lvert k_2k_3\rvert - 2$	$(1,1,1,1,S)$
H	$2^r\ \ 2\,\lvert k_2k_3\rvert - 2$	$(F,1,1,1,F)$
H	$2^r\ \ 2\,\lvert k_2k_3\rvert + 2$	$(1,F,1,1,F)$
H	$2^{r/2-1}$	$(F,1,F,1,1)$
H	$2^{r/2-1}$	$(1,F,1,F,1)$
H	$2^{r/2}$	$(1,1,F,1,F)$

Table 4.6: Open spectra of the $[T^2(H_2)\times T^2(H_3)]/\mathbb{Z}_2$ orientifolds (real charges).

Chapter 5

Magnetized D=4 Models

In this chapter we extend the construction of [75, 78] to four dimensional models obtained as magnetic deformations of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds, possibly combined with momentum or winding shifts along some of the internal directions [115, 113]. Some preliminary results have already appeared in refs. [121]. As in the six-dimensional models of [75, 78], for generic values of the magnetic fields Nielsen-Olesen instabilities [122] manifest themselves in the presence of tachyonic excitations, and supersymmetry is broken due to the unpairing of states of different spins. However, the compactness of the internal space allows self-dual or antiself-dual Abelian field configurations with non-vanishing instanton number, that can compensate the R-R charge excess, eliminating the tachyons and retrieving supersymmetry if the BPS bound is saturated, or giving rise to *brane supersymmetry breaking* models if the magnetized $D9$-branes transmute into anti $D5$-branes [75, 78]. The resulting models exhibit several interesting features, to wit Chan-Paton gauge groups of reduced rank and several families of matter multiplets, linked in a natural way to the degeneracy of the Landau levels. Moreover, in the presence of D-branes longitudinal to the directions along which the magnetic fields are turned on, the four dimensional models can also be chiral. It should be stressed that these models with internal background (open) magnetic fields are connected by T-duality to orientifolds with D-branes intersecting at angles [123, 124, 125, 112, 126, 127, 128, 129, 130], that have received much attention in the last few years in attempts to recover (extensions of) the Standard Model as low-energy limits of String Theory or M-theory. A byproduct of this analysis is a precise link between a quantized NS-NS B_{ab} and shift-orbifolds.

The four-dimensional models developed in refs. [115, 113] display $D9$ and $D5$ branes in their perturbative spectra, and are thus a natural arena to build consistent magnetized models sharing the qualitative features of the six-dimensional $T^4/\mathbb{Z}_2$ model. As we shall see, their spectra exhibit rank reductions of the gauge groups and multiple matter families. In addition, as in the six-dimensional examples, there is the option of introducing pairs of magnetic fields aligned along the same $U(1)$ subgroup. This is allowed only if the undeformed model contains corresponding $O5$-planes orthogonal to the two magnetized directions, or, equivalently, if there are sources that add to the magnetized $D9$-branes in such a way as to compensate the R-R charge excess. We shall always introduce uniform Abelian background magnetic fields (H_2, H_3) along the (Z^2, Z^3) directions, thus requiring just the presence of $O5_1$-planes to balance the R-R charge excess of magnetized $D9$-branes and $D5_1$-branes.

If $D5$-branes whose world-volume invades coordinates longitudinal to the magnetized directions are also present, one obtains, as a bonus, chiral fermions. Chirality is connected on the one hand to the intersection of two sets of orthogonal $D5$-branes, and on the other to the chiral asymmetry in the "pure magnetic" sector. Moreover, the phenomenon of brane transmutation acquires in this setting its full-fledged form. Indeed, as stressed in section 3.4.2, shift-orientifolds are characterized by the presence of multiplets of defocalized D5-branes. This implies, as mentioned before, that some of the D5-branes cannot be put on the same fixed tori but have to be distributed among the images interchanged by the action of some orbifold group elements. The magnetized D9-branes "remember" the localized distribution of the D5-branes they are mimicking. Indeed, although they invade the whole internal space, the centers of the corresponding classical Landau orbits organize themselves in multiplets that reflect the structure of the D5-branes.

5.1 Magnetized $\mathbb{Z}_2 \times \mathbb{Z}_2$ Orientifolds

Let us begin the discussion of magnetic deformations by considering the "plain" $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\omega_i = 1$ (for related examples in the language of intersecting D-brane models, see [112]). As stated in section 3.4 , the remaining models in Table (C.2) give rise to orientifolds with *brane supersymmetry breaking*, that, for brevity, will not be explicitly discussed here. However, in section 5.3 we shall describe one class of orientifolds with *brane supersymmetry breaking* related to a class of $w_2 p_3$ shift-orbifold models.

The closed string amplitudes, not affected by the introduction of constant background magnetic fields H_2 and H_3 along the Z^2 and Z^3 directions, as well as the O-plane content, are described in section 3.4. In addition, the closed unoriented spectra are collected in Table (C.2). The annulus amplitude can be obtained using the techniques of [75], reviewed in sections 4.2 and 4.3, as a deformation of the annulus amplitudes of eq. (3.37). The result can be cast into the sum of the following three contributions (for notations and conventions see the Appendices)

$$
\begin{aligned}
\mathcal{A}_{(Q=0)} &= \frac{1}{8}\Big\{ \big(N^2\ 2^{r-6}\ P_1(B_1)P_2(B_2)P_3(B_3) + 2m\bar{m}\ 2^{r-6}\ P_1(B_1)\tilde{P}_2(B_2)\tilde{P}_3(B_3) \\
&+ D_1^2\ 2^{r_1-2}\ P_1(B_1)W_2W_3 + D_2^2\ 2^{r_2-2}\ W_1P_2(B_2)W_3 \\
&+ D_3^2\ 2^{r_3-2}\ W_1W_2P_3(B_3)\big)T_{oo}(0;0;0) \\
&+ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ T_{go}(0;0;0)\Big[2ND_1\ 2^{r_1-2}\ P_1(B_1) + 2D_2D_3\ W_1\Big]\Big(\frac{\eta}{\vartheta_4(0)}\Big)^2 \\
&+ 2^{\frac{r_1}{2}+\frac{r_3}{2}}\ T_{fo}(0;0;0)\Big[2ND_2\ 2^{r_2-2}\ P_2(B_2) + 2D_1D_3\ W_2\Big]\Big(\frac{\eta}{\vartheta_4(0)}\Big)^2 \\
&+ 2^{\frac{r_1}{2}+\frac{r_2}{2}}\ T_{ho}(0;0;0)\Big[2ND_3\ 2^{r_3-2}\ P_3(B_3) + 2D_1D_2\ W_3\Big]\Big(\frac{\eta}{\vartheta_4(0)}\Big)^2\Big\}
\end{aligned} \tag{5.1}
$$

for the zero-charge sectors,

$$
\begin{aligned}
\mathcal{A}_{(Q=1)} &= \frac{1}{8}\Big\{-2^{r-2}2mNT_{oo}(0;z_2\tau;z_3\tau)P_1(B_1)\ \frac{k_2\eta}{\vartheta_1(z_2\tau)}\ \frac{k_3\eta}{\vartheta_1(z_3\tau)} \\
&- 2^{r-2}2\bar{m}NT_{oo}(0;-z_2\tau;-z_3\tau)P_1(B_1)\ \frac{k_2\eta}{\vartheta_1(-z_2\tau)}\ \frac{k_3\eta}{\vartheta_1(-z_3\tau)} \\
&+ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ T_{go}(0;z_2\tau;z_3\tau)\Big[2mD_1\ 2^{r_1-2}\ P_1(B_1)\Big]\ \frac{\eta}{\vartheta_4(z_2\tau)}\ \frac{\eta}{\vartheta_4(z_3\tau)} \\
&+ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ T_{go}(0;-z_2\tau;-z_3\tau)\Big[2\bar{m}D_1\ 2^{r_1-2}\ P_1(B_1)\Big]\ \frac{\eta}{\vartheta_4(-z_2\tau)}\ \frac{\eta}{\vartheta_4(-z_3\tau)} \\
&+ 2^{\frac{r+r_2}{2}}\ T_{fo}(0;z_2\tau;z_3\tau)\Big[-2imD_2\Big]\ \frac{\eta}{\vartheta_4(0)}\ \frac{k_2\eta}{\vartheta_1(z_2\tau)}\ \frac{\eta}{\vartheta_4(z_3\tau)} \\
&+ 2^{\frac{r+r_2}{2}}\ T_{fo}(0;-z_2\tau;-z_3\tau)\Big[2i\bar{m}D_2\Big]\ \frac{\eta}{\vartheta_4(0)}\ \frac{k_2\eta}{\vartheta_1(-z_2\tau)}\ \frac{\eta}{\vartheta_4(-z_3\tau)} \\
&+ 2^{\frac{r+r_3}{2}}\ T_{ho}(0;z_2\tau;z_3\tau)\Big[-2imD_3\Big]\ \frac{\eta}{\vartheta_4(0)}\ \frac{\eta}{\vartheta_4(z_2\tau)}\ \frac{k_3\eta}{\vartheta_1(z_3\tau)} \\
&+ 2^{\frac{r+r_3}{2}}\ T_{ho}(0;-z_2\tau;-z_3\tau)\Big[2i\bar{m}D_3\Big]\ \frac{\eta}{\vartheta_4(0)}\ \frac{\eta}{\vartheta_4(-z_2\tau)}\ \frac{k_3\eta}{\vartheta_1(-z_3\tau)}\Big\}
\end{aligned} \tag{5.2}
$$

for the charge-one sectors, and

$$
\begin{aligned}
\mathcal{A}_{(Q=2)} &= \frac{1}{8}\Big\{-2^{r-2}m^2T_{oo}(0;2z_2\tau;2z_3\tau)P_1(B_1)\ \frac{2k_2\eta}{\vartheta_1(2z_2\tau)}\ \frac{2k_3\eta}{\vartheta_1(z_3\tau)} \\
&- 2^{r-2}\bar{m}^2T_{oo}(0;-2z_2\tau;-2z_3\tau)P_1(B_1)\ \frac{2k_2\eta}{\vartheta_1(-2z_2\tau)}\ \frac{2k_3\eta}{\vartheta_1(-2z_3\tau)}\Big\}
\end{aligned} \tag{5.3}
$$

for the total charge-two sectors. The Möbius amplitude can be obtained in a similar way from the undeformed case, distinguishing only the uncharged ($Q = 0$) sector from the charged ($Q = 2$) one, since the $Q = 1$ sector is absent in $\mathcal{M}$ because of the oriented nature of the corresponding open-strings. Thus

$$
\begin{aligned}
\mathcal{M}_{(Q=0)} \;=\; -\frac{1}{8}\Big\{ & \big[\, 2^{\frac{r-6}{2}}\, N\, P_1(B_1,\gamma_{\epsilon_1})\, P_2(B_2,\gamma_{\epsilon_2})\, P_3(B_3,\gamma_{\epsilon_3}) \\
+ \;\; & 2^{\frac{r_1-6}{2}}\, D_1\, P_1(B_1,\gamma_{\epsilon_1}) W_2(B_2,\tilde{\gamma}_{\epsilon_2}) W_3(B_3,\tilde{\gamma}_{\epsilon_3}) \\
+ \;\; & 2^{\frac{r_2-6}{2}}\, D_2\, W_1(B_1,\tilde{\gamma}_{\epsilon_1}) P_2(B_2,\gamma_{\epsilon_2}) W_3(B_3,\tilde{\gamma}_{\epsilon_3}) \\
+ \;\; & 2^{\frac{r_3-6}{2}}\, D_3\, W_1(B_1,\tilde{\gamma}_{\epsilon_1}) W_2(B_2,\tilde{\gamma}_{\epsilon_2}) P_3(B_3,\gamma_{\epsilon_3}) \,\big]\, \hat{T}_{oo}(0;0;0) \\
- \;\; & \big[\, 2^{\frac{r_1-2}{2}}\, (N+D_1) P_1(B_1,\gamma_{\epsilon_1}) + 2^{-1}\, (D_2+D_3) W_1(B_1,\tilde{\gamma}_{\epsilon_1}) \big]\, \hat{T}_{og}(0;0;0) \left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2 \\
- \;\; & \big[\, 2^{\frac{r_2-2}{2}}\, (N+D_2) P_2(B_2,\gamma_{\epsilon_2}) + 2^{-1}\, (D_1+D_3) W_2(B_2,\tilde{\gamma}_{\epsilon_2}) \big]\, \hat{T}_{of}(0;0;0) \left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2 \\
- \;\; & \big[\, 2^{\frac{r_3-2}{2}}\, (N+D_3) P_3(B_3,\gamma_{\epsilon_3}) + 2^{-1}\, (D_1+D_2) W_3(B_3,\tilde{\gamma}_{\epsilon_3}) \big]\, \hat{T}_{oh}(0;0;0) \left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2 \Big\} \quad ,
\end{aligned}
\tag{5.4}
$$

and

$$
\begin{aligned}
\mathcal{M}_{(Q=2)} \;=\; & -\frac{1}{8}\Big\{ -2^{\frac{r-2}{2}} m \hat{T}_{oo}(0;2z_2\tau;2z_3\tau)\, P_1(B_1,\gamma_{\epsilon_1})\, \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(2z_2\tau)} \frac{2k_3\hat{\eta}}{\hat{\vartheta}_1(2z_3\tau)} \\
- \;\; & 2^{\frac{r-2}{2}} \bar{m} \hat{T}_{oo}(0;-2z_2\tau;-2z_3\tau)\, P_1(B_1,\gamma_{\epsilon_1})\, \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(-2z_2\tau)} \frac{2k_3\hat{\eta}}{\hat{\vartheta}_1(-2z_3\tau)} \\
- \;\; & 2^{\frac{r_1-2}{2}} m \hat{T}_{og}(0;2z_2\tau;2z_3\tau)\, P_1(B_1,\gamma_{\epsilon_1})\, \frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_2\tau)} \frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_3\tau)} \\
- \;\; & 2^{\frac{r_1-2}{2}} \bar{m} \hat{T}_{og}(0;-2z_2\tau;-2z_3\tau)\, P_1(B_1,\gamma_{\epsilon_1})\, \frac{2\hat{\eta}}{\hat{\vartheta}_2(-2z_2\tau)} \frac{2\hat{\eta}}{\hat{\vartheta}_2(-2z_3\tau)} \\
+ \;\; & i\, m\, 2^{\frac{r_2}{2}}\, \hat{T}_{of}(0;2z_2\tau;2z_3\tau)\, \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(2z_2\tau)} \frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_3\tau)} \\
- \;\; & i\, \bar{m}\, 2^{\frac{r_2}{2}}\, \hat{T}_{of}(0;-2z_2\tau;-2z_3\tau)\, \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(-2z_2\tau)} \frac{2\hat{\eta}}{\hat{\vartheta}_2(-2z_3\tau)} \\
+ \;\; & i\, m\, 2^{\frac{r_3}{2}}\, \hat{T}_{oh}(0;2z_2\tau;2z_3\tau)\, \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_2\tau)} \frac{2k_3\hat{\eta}}{\hat{\vartheta}_1(2z_3\tau)} \\
- \;\; & i\, \bar{m}\, 2^{\frac{r_3}{2}}\, \hat{T}_{oh}(0;-2z_2\tau;-2z_3\tau)\, \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \frac{2\hat{\eta}}{\hat{\vartheta}_2(-2z_2\tau)} \frac{2k_3\hat{\eta}}{\hat{\vartheta}_1(-2z_3\tau)} \Big\} \quad .
\end{aligned}
\tag{5.5}
$$

These amplitudes describe the couplings of conventional $D9$ and $D5_i$ branes with an additional set of (magnetized) $D9$-branes. This natural interpretation is encoded into the four o, g, f and h projections, present in the $(Q = 2)$ sector of the Möbius amplitudes. The tadpole cancellation conditions may be extracted combining the transverse (tree) channel Klein-bottle amplitude of eq. (3.35) with the transverse annulus amplitudes

$$
\begin{aligned}
\tilde{\mathcal{A}} \;&=\; \frac{2^{-5}}{8}\Bigg\{\; [2^{r-6}v_1v_2v_3N^2W_1(B_1)W_2(B_2)W_3(B_3) \;+\; \frac{v_1}{v_2v_3}\,2^{r_1-2}\;D_1^2W_1(B_1)P_2P_3 \\
&+\; \frac{v_2}{v_1v_3}\,2^{r_2-2}\;D_2^2P_1W_2(B_2)P_3 \;+\; \frac{v_3}{v_1v_2}\,2^{r_3-2}\;D_3^2P_1P_2W_3(B_3) \\
&+\; 2^{r-6}v_1v_2v_3\;2m\bar{m}(1+q^2H_2^2)(1+q^2H_3^2)W_1(B_1)\tilde{W}_2(B_2)\tilde{W}_3(B_3)\;]\;T_{oo}(0;0;0) \\
&+\; 2^{r-2}\;8\;N\;[mT_{oo}(0;z_2;z_3)+\bar{m}T_{oo}(0;-z_2;-z_3)]v_1W_1(B_1)\;\frac{k_2\eta}{\vartheta_1(z_2)}\frac{k_3\eta}{\vartheta_1(z_3)} \\
&+\; 2^{r-2}\;4\;[m^2T_{oo}(0;2z_2;2z_3)+\bar{m}^2T_{oo}(0;-2z_2;-2z_3)]v_1W_1(B_1)\;\frac{2k_2\eta}{\vartheta_1(2z_2)}\frac{2k_3\eta}{\vartheta_1(2z_3)} \\
&+\; 2^{\frac{r_2}{2}+\frac{r_3}{2}}\;T_{og}(0;0;0)\Big[2^{r_1-2}\;2ND_1v_1W_1(B_1)+2D_2D_3\frac{P_1}{v_1}\Big]\;\frac{2\eta}{\vartheta_2(0)}\;\frac{2\eta}{\vartheta_2(0)} \\
&+\; 2^{\frac{r_1}{2}+\frac{r_3}{2}}\;T_{of}(0;0;0)\Big[2^{r_2-2}\;2ND_2v_2W_2(B_2)+2D_1D_3\frac{P_2}{v_2}\Big]\;\frac{2\eta}{\vartheta_2(0)}\;\frac{2\eta}{\vartheta_2(0)} \\
&+\; 2^{\frac{r_1}{2}+\frac{r_2}{2}}\;T_{oh}(0;0;0)\Big[2^{r_3-2}\;2ND_3v_3W_3(B_3)+2D_1D_2\frac{P_3}{v_3}\Big]\;\frac{2\eta}{\vartheta_2(0)}\;\frac{2\eta}{\vartheta_2(0)} \\
&+\; 2^{\frac{r_2}{2}+\frac{r_3}{2}}\;T_{og}(0;z_2;z_3)\Big[2^{r_1-2}\;2mD_1v_1W_1(B_1)\Big]\;\frac{2\eta}{\vartheta_2(z_2)}\;\frac{2\eta}{\vartheta_2(z_3)} \\
&+\; 2^{\frac{r_2}{2}+\frac{r_3}{2}}\;T_{og}(0;-z_2;-z_3)\Big[2^{r_1-2}\;2\bar{m}D_1v_1W_1(B_1)\Big]\;\frac{2\eta}{\vartheta_2(-z_2)}\;\frac{2\eta}{\vartheta_2(-z_3)} \\
&+\; 2^{\frac{r_1}{2}+\frac{r_3}{2}}\;T_{of}(0;z_2;z_3)\;2mD_2\;\frac{2\eta}{\vartheta_2(0)}\;\frac{2k_2\eta}{\vartheta_1(z_2)}\;\frac{2\eta}{\vartheta_2(z_3)} \\
&+\; 2^{\frac{r_1}{2}+\frac{r_3}{2}}\;T_{of}(0;-z_2;-z_3)\;2\bar{m}D_2\;\frac{2\eta}{\vartheta_2(0)}\;\frac{2k_2\eta}{\vartheta_1(-z_2)}\;\frac{2\eta}{\vartheta_2(-z_3)} \\
&+\; 2^{\frac{r_1}{2}+\frac{r_2}{2}}\;T_{oh}(0;z_2;z_3)\;2mD_3\;\frac{2\eta}{\vartheta_2(0)}\;\frac{2\eta}{\vartheta_2(z_2)}\;\frac{2k_3\eta}{\vartheta_1(z_3)} \\
&+\; 2^{\frac{r_1}{2}+\frac{r_2}{2}}\;T_{oh}(0;-z_2;-z_3)\;2\bar{m}D_3\;\frac{2\eta}{\vartheta_2(0)}\;\frac{2\eta}{\vartheta_2(-z_2)}\;\frac{2k_3\eta}{\vartheta_1(-z_3)}\;\Bigg\}\quad, \qquad (5.6)
\end{aligned}
$$

and with the transverse Möbius amplitudes

$$
\begin{aligned}
\tilde{\mathcal{M}} \;=\;& -\frac{2}{8}\Bigg\{ [\, 2^{\frac{r-6}{2}}\ N\ v_1v_2v_3\ W_1(B_1,\gamma_{\epsilon_1})\ W_2(B_2,\gamma_{\epsilon_2})\ W_3(B_3,\gamma_{\epsilon_3})]\ \hat{T}_{oo}(0;0;0) \\
+& \ [\, 2^{\frac{r_1-6}{2}}\ \frac{v_1}{v_2v_3}\ D_1\ W_1(B_1,\gamma_{\epsilon_1})\ P_2(B_2,\tilde{\gamma}_{\epsilon_2})\ P_3(B_3,\tilde{\gamma}_{\epsilon_3})\ ]\ \hat{T}_{oo}(0;0;0) \\
+& \ [\, 2^{\frac{r_2-6}{2}}\ \frac{v_2}{v_1v_3}\ D_2\ P_1(B_1,\tilde{\gamma}_{\epsilon_1})\ W_2(B_2,\gamma_{\epsilon_2})\ P_3(B_3,\tilde{\gamma}_{\epsilon_3})\ ]\ \hat{T}_{oo}(0;0;0) \\
+& \ [\, 2^{\frac{r_3-6}{2}}\ \frac{v_3}{v_1v_2}\ D_3\ P_1(B_1,\tilde{\gamma}_{\epsilon_1})\ P_2(B_2,\tilde{\gamma}_{\epsilon_2})\ W_3(B_3,\gamma_{\epsilon_3})\ ]\ \hat{T}_{oo}(0;0;0) \\
+& \ 2^{\frac{r_1}{2}+r_2+r_3-1}\ 4\ [\ m\hat{T}_{oo}(0;z_2;z_3)+\bar{m}\hat{T}_{oo}(0;-z_2;-z_3)\ ]\ v_1W_1(B_1,\gamma_{\epsilon_1})\frac{k_2\hat{\eta}}{\hat{\vartheta}_1(z_2)}\frac{k_3\hat{\eta}}{\hat{\vartheta}_1(z_3)} \\
+& \ \hat{T}_{og}(0;0;0)\ [2^{\frac{r_1}{2}-1}(N+D_1)v_1W_1(B_1,\gamma_{\epsilon_1})\ +\ \frac{2^{-1}}{v_1}(D_2+D_3)P_1(B_1,\tilde{\gamma}_{\epsilon_1})]\ \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \\
+& \ \hat{T}_{of}(0;0;0)\ [2^{\frac{r_2}{2}-1}(N+D_2)v_2W_2(B_2,\gamma_{\epsilon_2})\ +\ \frac{2^{-1}}{v_2}(D_1+D_3)P_2(B_2,\tilde{\gamma}_{\epsilon_2})]\ \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \\
+& \ \hat{T}_{oh}(0;0;0)\ [2^{\frac{r_3}{2}-1}(N+D_3)v_3W_3(B_3,\gamma_{\epsilon_3})\ +\ \frac{2^{-1}}{v_3}(D_1+D_2)P_2(B_2,\tilde{\gamma}_{\epsilon_2})]\ \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \\
+& \ 2^{\frac{r_1}{2}-1}[\ m\hat{T}_{og}(0;z_2;z_3)+\bar{m}\hat{T}_{og}(0;-z_2;-z_3)\ ]\ v_1W_1(B_1,\gamma_{\epsilon_1})\ \frac{2\hat{\eta}}{\hat{\vartheta}_2(z_2)}\ \frac{2\hat{\eta}}{\hat{\vartheta}_2(z_3)} \\
+& \ [\ m\hat{T}_{of}(0;z_2;z_3)-\bar{m}\hat{T}_{of}(0;-z_2;-z_3)\ ]\ \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\ \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(z_2)}\ \frac{2\hat{\eta}}{\hat{\vartheta}_2(z_3)} \\
+& \ [\ m\hat{T}_{oh}(0;z_2;z_3)-\bar{m}\hat{T}_{oh}(0;-z_2;-z_3)\ ]\ \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\ \frac{2\hat{\eta}}{\hat{\vartheta}_2(z_2)}\ \frac{2k_3\hat{\eta}}{\hat{\vartheta}_1(z_3)}\Bigg\} \quad . \qquad (5.7)
\end{aligned}
$$

Apart from the $m = \bar{m}$ condition, automatic for the unitary gauge group selected by the magnetic background, all R-R tadpole cancellation conditions directly tied to four-dimensional non Abelian anomalies arise from the untwisted sector. After the charges are rescaled and parametrized in such a way that $N = 2n$, $D_i = 2d_i$ and $m \to 2m$, the resulting R-R conditions are as in Table (C.6), provided the signs γ_ϵ and $\tilde{\gamma}_\epsilon$ satisfy the same identities (3.39) of the undeformed case. The NS-NS tadpoles, cancelled only at the supersymmetric $H_2 = -H_3$ point, are related to the derivatives of the Born-Infeld action with respect to the moduli, exactly as in the six-dimensional case [75]. Several choices of gauge group are again allowed by the additional signs ξ_i and η_i, in eq.(3.40). Introducing the combinations

$$
\begin{aligned}
2\rho_{\alpha,o} &= a_1a_2a_3 - a_1 - a_2 - a_3 \ , \\
2\rho_{\alpha,g} &= a_1a_2a_3 - a_1 + a_2 + a_3 \ , \\
2\rho_{\alpha,f} &= a_1a_2a_3 + a_1 - a_2 + a_3 \ , \\
2\rho_{\alpha,h} &= a_1a_2a_3 + a_1 + a_2 - a_3 \ , \qquad (5.8)
\end{aligned}
$$

where $a_i = \eta_i$ if $\alpha = n$ while $a_i = \eta_i$ but $a_k = \xi_k, k \neq i$ if $\alpha = d_i$, the massless spectra are encoded in

$$
\begin{aligned}
& \mathcal{A}_0 + \mathcal{M}_0 \;=\; \tau_{oo}(0)\ \Big[\frac{n(n-\rho_{no})}{2} + \frac{d_1(d_1-\rho_{d_1o})}{2} + \frac{d_2(d_2-\rho_{d_2o})}{2} + \frac{d_3(d_3-\rho_{d_3o})}{2} + m\bar{m}\Big] \\
+ \quad & \tau_{og}(0)\Big[\frac{n(n-\rho_{ng})}{2} + \frac{d_1(d_1-\rho_{d_1g})}{2} + \frac{d_2(d_2-\rho_{d_2g})}{2} + \frac{d_3(d_3-\rho_{d_3g})}{2} + m\bar{m}\Big] \\
+ \quad & \tau_{oh}(0)\Big[\frac{n(n-\rho_{nh})}{2} + \frac{d_1(d_1-\rho_{d_1h})}{2} + \frac{d_2(d_2-\rho_{d_2h})}{2} + \frac{d_3(d_3-\rho_{d_3h})}{2} + m\bar{m}\Big] \\
+ \quad & \tau_{of}(0)\Big[\frac{n(n-\rho_{nf})}{2} + \frac{d_1(d_1-\rho_{d_1f})}{2} + \frac{d_2(d_2-\rho_{d_2f})}{2} + \frac{d_3(d_3-\rho_{d_3f})}{2} + m\bar{m}\Big] \\
+ \quad & \Big[\ \tau_{gh}(0)\ +\ \tau_{gf}(0)\ \Big]\ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ (nd_1 + d_2 d_3) \\
+ \quad & \Big[\ \tau_{fg}(0)\ +\ \tau_{fh}(0)\ \Big]\ 2^{\frac{r_1}{2}+\frac{r_3}{2}}\ (nd_2 + d_1 d_3) \\
+ \quad & \Big[\ \tau_{hg}(0)\ +\ \tau_{hf}(0)\ \Big]\ 2^{\frac{r_1}{2}+\frac{r_2}{2}}\ (nd_3 + d_1 d_2) \\
+ \quad & \Big[\ \tau_{oh}(+)\ +\ \tau_{of}(+)\ \Big]\ 2^{r_2+r_3}\ |k_2 k_3|\ nm\ +\ \Big[\ \tau_{oh}(-)\ +\ \tau_{of}(-)\ \Big]\ 2^{r_2+r_3}\ |k_2 k_3|\ n\bar{m} \\
+ \quad & \Big[\ \tau_{gh}(+)\ +\ \tau_{gf}(+)\ \Big]\ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ md_1\ +\ \Big[\ \tau_{gh}(-)\ +\ \tau_{gf}(-)\ \Big]\ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ \bar{m}d_1 \\
+ \quad & \Big[\ \tau_{fh}(+)\ \Big]\ 2^{\frac{r_1}{2}+\frac{r_3}{2}}\ md_2|k_2|\ +\ \Big[\ \tau_{fg}(-)\ \Big]2^{\frac{r_1}{2}+\frac{r_3}{2}}\bar{m}d_2|k_2| \\
+ \quad & \Big[\ \tau_{hg}(+)\ \Big]\ 2^{\frac{r_1}{2}+\frac{r_2}{2}}\ md_3|k_3|\ +\ \Big[\ \tau_{hf}(-)\ \Big]2^{\frac{r_1}{2}+\frac{r_2}{2}}\ \bar{m}d_3|k_3| \\
+ \quad & \Big[\ \tau_{oh}(2+)\ \Big]\ \frac{m(m-1)}{2}\ \Big[2^{r_2+r_3}2|k_2\,k_3| + 2^{\frac{r_2+r_3}{2}}\eta_1|k_2\,k_3| + \eta_1 + 2^{\frac{r_2}{2}}|k_2| - 2^{\frac{r_3}{2}}|k_3|\Big] \\
+ \quad & \Big[\ \tau_{oh}(2+)\ \Big]\ \frac{m(m+1)}{2}\ \Big[2^{r_2+r_3}2|k_2\,k_3| - 2^{\frac{r_2+r_3}{2}}\eta_1|k_2\,k_3| - \eta_1 - 2^{\frac{r_2}{2}}|k_2| + 2^{\frac{r_3}{2}}|k_3|\Big] \\
+ \quad & \Big[\ \tau_{of}(2+)\ \Big]\ \frac{m(m-1)}{2}\ \Big[2^{r_2+r_3}2|k_2\,k_3| + 2^{\frac{r_2+r_3}{2}}\eta_1|k_2\,k_3| + \eta_1 - 2^{\frac{r_2}{2}}|k_2| + 2^{\frac{r_3}{2}}|k_3|\Big] \\
+ \quad & \Big[\ \tau_{of}(2+)\ \Big]\ \frac{m(m+1)}{2}\ \Big[2^{r_2+r_3}2|k_2\,k_3| - 2^{\frac{r_2+r_3}{2}}\eta_1|k_2\,k_3| - \eta_1 + 2^{\frac{r_2}{2}}|k_2| - 2^{\frac{r_3}{2}}|k_3|\Big] \\
+ \quad & \Big[\ \tau_{of}(2-)\ \Big]\ \frac{\bar{m}(\bar{m}-1)}{2}\ \Big[2^{r_2+r_3}2|k_2\,k_3| + 2^{\frac{r_2+r_3}{2}}\eta_1|k_2\,k_3| + \eta_1 + 2^{\frac{r_2}{2}}|k_2| - 2^{\frac{r_3}{2}}|k_3|\Big] \\
+ \quad & \Big[\ \tau_{of}(2-)\ \Big]\ \frac{\bar{m}(\bar{m}+1)}{2}\ \Big[2^{r_2+r_3}2|k_2\,k_3| - 2^{\frac{r_2+r_3}{2}}\eta_1|k_2\,k_3| - \eta_1 - 2^{\frac{r_2}{2}}|k_2| + 2^{\frac{r_3}{2}}|k_3|\Big] \\
+ \quad & \Big[\ \tau_{oh}(2-)\ \Big]\ \frac{\bar{m}(\bar{m}-1)}{2}\ \Big[2^{r_2+r_3}2|k_2\,k_3| + 2^{\frac{r_2+r_3}{2}}\eta_1|k_2\,k_3| + \eta_1 - 2^{\frac{r_2}{2}}|k_2| + 2^{\frac{r_3}{2}}|k_3|\Big] \\
+ \quad & \Big[\ \tau_{oh}(2-)\ \Big]\ \frac{\bar{m}(\bar{m}+1)}{2}\ \Big[2^{r_2+r_3}2|k_2\,k_3| - 2^{\frac{r_2+r_3}{2}}\eta_1|k_2\,k_3| - \eta_1 + 2^{\frac{r_2}{2}}|k_2| - 2^{\frac{r_3}{2}}|k_3|\Big]\ , \qquad (5.9)
\end{aligned}
$$

where (0), $(\pm)$ and $(2\pm)$ are shorthand notations for the arguments $(0,0,0)$, $(0;\pm z_2\tau;\pm z_3\tau)$ and $(0;\pm 2z_2\tau;\pm 2z_3\tau)$, respectively, and the characters with non-vanishing arguments actually denote restrictions to their massless parts.

The resulting gauge groups are reported in Table (C.6), while the open unoriented spectra are displayed in Table (C.7). As expected from the previous discussion, chirality originates from two different sources. The first is the chiral asymmetry in the "pure magnetic" sector, due to the misalignment introduced by the combined action of magnetic backgrounds and orbifold projections on the Möbius amplitudes. The second is the coupling between magnetized $D9$-

branes and $D5$-branes longitudinal to the magnetized complex directions, familiar from the T-dual picture, where chiral fermions live in a natural way at brane intersections [123]. Whenever potentially anomalous $U(1)$'s are present, they call for a generalization of the Dine-Seiberg-Witten mechanism [141], an option that, as in six dimensions [142], requires generalized Green-Schwarz couplings in the Ramond-Ramond sectors [143].

5.2 Magnetized $\mathbb{Z}_2 \times \mathbb{Z}_2$ Shift-orientifolds

In this section we describe the magnetized versions of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ shift-orientifolds introduced in [115] and reviewed in section 3.4.2. As in the "plain" $\mathbb{Z}_2 \times \mathbb{Z}_2$ models of the previous section, chiral matter can be obtained if open strings stretched between magnetized $D9$-branes and $D5$-branes longitudinal to the Z^2 and/or Z^3 directions are present. An inspection of Table (3.2) shows that the p_3, w_2p_3 and $w_1w_2p_3$ models are potentially chiral, while the remaining models are not.

The p_3 model requires a separate discussion, since in its undeformed version [115] it exhibits an f-twisted R-R tadpole condition, corresponding to the action of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ element that fixes the T^{67}-torus, one of the two along which we turn on background magnetic fluxes (see Table (C.9) for the unoriented closed spectra and Table (C.34) for the unoriented open spectra with complex Chan-Paton charges). This tadpole condition can no longer be satisfied if the background magnetic field is present, since some states are lifted in mass by a term depending solely on the field strength H_2 along T^{67}, rather than on the difference $H_2 - H_3$ as in the case for the tadpole conditions coming from the untwisted or from the g-twisted sectors. The resulting models are thus anomalous as string vacua, because the magnetic deformations are, in the aforementioned sense, incompatible with the p_3 shift. The natural geometric interpretation of this phenomenon is as follows: the p_3 shift is introducing a net number of fractional branes [146] that, differently from what happens in the remaining models, are partly longitudinal and partly orthogonal to the magnetic fields carrying a non-vanishing twisted R-R charge, whose excess can be cancelled only turning off the background magnetic field.

In the following we shall analyze the chiral and non-chiral examples with selfdual configurations of the magnetic field, *i.e.* at supersymmetric points, leaving to section 3 the discussion of models with *brane supersymmetry breaking*.

5.2.1 w_2p_3 Models

Let us first analyze in detail the w_2p_3 class of orientifolds, that captures all interesting features of the models discussed in this paper. In the presence of the NS-NS two-form B_{ab}, the unoriented truncation of the closed spectrum is obtained adding to the halved torus amplitude the Klein-bottle amplitude

$$\begin{aligned}
\mathcal{K} \;&=\; \frac{1}{8}\Big\{ \big[\; P_1\, P_2\, P_3 \;+ 2^{-4}\, P_1\, W_2(B_2)\, W_3(B_3) \\
&+\; 2^{-4}\, W_1(B_1)\, P_2\, W_3(B_3) \\
&+\; 2^{-4}\, W_1(B_1)\, (-1)^{n_2} W_2(B_2)\, (-1)^{m_3} P_3 \;\big]\, T_{oo} \\
&+\; 2 \times 16 \,\big[\, 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\, P_1\, T_{go} + 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\, P_2^{\frac{1}{2}}\, T_{fo} \\
&+\; 2^{-\frac{r_1}{2}-\frac{r_2}{2}}\, 2^{-2}\, W_3^{\frac{1}{2}}(B_3)\, T_{ho} \;\big] \left(\frac{\eta}{\vartheta_4}\right)^2 \Big\} \quad .
\end{aligned} \tag{5.10}$$

The resulting massless unoriented closed spectra are reported in Table (C.10). The transverse-channel amplitude

$$\begin{aligned}
\tilde{\mathcal{K}}_0 \;&=\; \frac{2^5}{8}\Bigg\{ \left(\sqrt{v_1 v_2 v_3} + 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\sqrt{\frac{v_1}{v_2 v_3}} + 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\sqrt{\frac{v_2}{v_1 v_3}}\right)^2 \tau_{oo} \\
&+\; \left(\sqrt{v_1 v_2 v_3} + 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\sqrt{\frac{v_1}{v_2 v_3}} - 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\sqrt{\frac{v_2}{v_1 v_3}}\right)^2 \tau_{og} \\
&+\; \left(\sqrt{v_1 v_2 v_3} - 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\sqrt{\frac{v_1}{v_2 v_3}} - 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\sqrt{\frac{v_2}{v_1 v_3}}\right)^2 \tau_{oh} \\
&+\; \left(\sqrt{v_1 v_2 v_3} - 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\sqrt{\frac{v_1}{v_2 v_3}} + 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\sqrt{\frac{v_2}{v_1 v_3}}\right)^2 \tau_{of} \;\Bigg\}
\end{aligned} \tag{5.11}$$

displays very neatly the presence of one conventional $O9$-plane and of the $O5_1$ and $O5_2$ planes, while the $O5_3$-plane of the $\mathbb{Z}_2 \times \mathbb{Z}_2$-models in eq. (3.36) is no longer a fixed manifold of the combined orbifold and shifts, and is thus eliminated.

The annulus amplitude is more subtle. In the presence of B_{ab}, H_2 along T^{67} and H_3 along

T^{89}, it is again the sum of three contributions:

$$
\begin{aligned}
\mathcal{A}_{(Q=0)} &= \frac{1}{8}\Big\{ \big[\frac{N^2}{2}\ 2^{r-6}\ P_1(B_1)\ (\ P_2(B_2)+P_2^{1/2}(B_2)\ )\ P_3(B_3) \\
&+ \frac{2m\bar{m}}{2}\ 2^{r-6}\ P_1(B_1)\ (\ \tilde{P}_2(B_2)+\tilde{P}_2^{1/2}(B_2)\ )\ \tilde{P}_3(B_3) \\
&+ \frac{D_1^2}{2}\ 2^{r_1-2}\ P_1(B_1)W_2(\ W_3+W_3^{1/2}\ ) \\
&+ \frac{D_2^2}{4}\ 2^{r_2-2}\ W_1(\ P_2(B_2)+P_2^{1/2}(B_2)\ )\ (\ W_3+W_3^{1/2}\ )\big]T_{oo}(0;0;0) \\
&+ 2^{r_1-2}\big[\frac{G^2}{2}+\frac{G_1^2}{2}+\frac{2m\bar{m}}{2}\big]T_{og}(0;0;0)\ P_1(B_1)\Big(\frac{2\eta}{\vartheta_2(0)}\Big)^2 \\
&+ 2^{\frac{r_1}{2}+\frac{r_3}{2}}\ T_{go}(0;0;0)\ 2ND_1\ 2^{r_1-2}\ P_1(B_1)\Big(\frac{\eta}{\vartheta_4(0)}\Big)^2 \\
&+ 2^{\frac{r_1}{2}+\frac{r_3}{2}}\ T_{gg}(0;0;0)\ 2GG_1\ 2^{r_1-2}\ P_1(B_1)\Big(\frac{\eta}{\vartheta_3(0)}\Big)^2 \\
&+ 2^{\frac{r_1}{2}+\frac{r_3}{2}}\ T_{fo}(0;0;0)\ ND_2\ 2^{r_2-2}\ (\ P_2^{1/4}(B_2)+P_2^{3/4}(B_2)\ )\ \Big(\frac{\eta}{\vartheta_4(0)}\Big)^2 \\
&+ 2^{\frac{r_1}{2}+\frac{r_2}{2}}\ T_{ho}(0;0;0)\ 2^{r_3-2}D_1D_2\ (\ W_3^{1/4}+W_3^{3/4}\ )\ \Big(\frac{\eta}{\vartheta_4(0)}\Big)^2\Big\}
\end{aligned}
\tag{5.12}
$$

for the $Q=0$ sectors,

$$
\begin{aligned}
\mathcal{A}_{(Q=1)} &= \frac{1}{8}\Big\{ -2^{r-2}\,2\,m\,N\,T_{oo}(0;z_2\tau;z_3\tau)P_1(B_1)\ \frac{k_2\eta}{\vartheta_1(z_2\tau)}\ \frac{k_3\eta}{\vartheta_1(z_3\tau)} \\
&- 2^{r-2}\,2\,\bar{m}\,N\,T_{oo}(0;-z_2\tau;-z_3\tau)P_1(B_1)\ \frac{k_2\eta}{\vartheta_1(-z_2\tau)}\ \frac{k_3\eta}{\vartheta_1(-z_3\tau)} \\
&+ T_{og}(0;z_2\tau;z_3\tau)\Big[\alpha_{mG}\,m\,G\ 2^{r_1-2}\ P_1(B_1)\Big]\ \frac{2\eta}{\vartheta_2(z_2\tau)}\ \frac{2\eta}{\vartheta_2(z_3\tau)} \\
&+ T_{og}(0;-z_2\tau;-z_3\tau)\Big[\bar{\alpha}_{mG}\,\bar{m}\,G\ 2^{r_1-2}\ P_1(B_1)\Big]\ \frac{2\eta}{\vartheta_2(-z_2\tau)}\ \frac{2\eta}{\vartheta_2(-z_3\tau)} \\
&+ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ T_{go}(0;z_2\tau;z_3\tau)\Big[2mD_1\ 2^{r_1-2}\ P_1(B_1)\Big]\ \frac{\eta}{\vartheta_4(z_2\tau)}\ \frac{\eta}{\vartheta_4(z_3\tau)} \\
&+ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ T_{go}(0;-z_2\tau;-z_3\tau)\Big[2\bar{m}D_1\ 2^{r_1-2}\ P_1(B_1)\Big]\ \frac{\eta}{\vartheta_4(-z_2\tau)}\ \frac{\eta}{\vartheta_4(-z_3\tau)} \\
&+ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ T_{gg}(0;z_2\tau;z_3\tau)\Big[\alpha_{mG_1}\ 2mG_1\ 2^{r_1-2}\ P_1(B_1)\Big]\ \frac{\eta}{\vartheta_3(z_2\tau)}\ \frac{\eta}{\vartheta_3(z_3\tau)} \\
&+ 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ T_{gg}(0;-z_2\tau;-z_3\tau)\Big[\bar{\alpha}_{mG_1}\ 2\bar{m}G_1\ 2^{r_1-2}\ P_1(B_1)\Big]\ \frac{\eta}{\vartheta_3(-z_2\tau)}\ \frac{\eta}{\vartheta_3(-z_3\tau)} \\
&+ 2^{\frac{r}{2}+\frac{r_2}{2}}\ T_{fo}(0;z_2\tau;z_3\tau)\Big[-2imD_2\Big]\ \frac{\eta}{\vartheta_4(0)}\ \frac{k_2\eta}{\vartheta_1(z_2\tau)}\ \frac{\eta}{\vartheta_4(z_3\tau)} \\
&+ 2^{\frac{r}{2}+\frac{r_2}{2}}\ T_{fo}(0;-z_2\tau;-z_3\tau)\Big[2i\bar{m}D_2\Big]\ \frac{\eta}{\vartheta_4(0)}\ \frac{k_2\eta}{\vartheta_1(-z_2\tau)}\ \frac{\eta}{\vartheta_4(-z_3\tau)}\Big\},
\end{aligned}
\tag{5.13}
$$

for the $Q=1$ sectors, and

$$\begin{aligned}
\mathcal{A}_{(Q=2)} &= \frac{1}{8}\Big\{-2^{r-2}m^2 T_{oo}(0;2z_2\tau;2z_3\tau)P_1(B_1)\ \frac{2k_2\eta}{\vartheta_1(2z_2\tau)}\ \frac{k_3\eta}{2\vartheta_1(z_3\tau)} \\
&- 2^{r-2}\bar{m}^2 T_{oo}(0;-2z_2\tau;-2z_3\tau)P_1(B_1)\ \frac{k_2\eta}{\vartheta_1(-2z_2\tau)}\ \frac{k_3\eta}{\vartheta_1(-2z_3\tau)} \\
&+ 2^{r_1-2}\alpha_{m^2}\ \frac{m^2}{2}T_{og}(0;2z_2\tau;2z_3\tau)P_1(B_1)\ \frac{2\eta}{\vartheta_2(2z_2\tau)}\ \frac{2\eta}{\vartheta_2(2z_3\tau)} \\
&+ 2^{r_1-2}\bar{\alpha}_{m^2}\ \frac{\bar{m}^2}{2}T_{og}(0;-2z_2\tau;-2z_3\tau)P_1(B_1)\ \frac{2\eta}{\vartheta_2(-2z_2\tau)}\ \frac{2\eta}{\vartheta_2(-2z_3\tau)}\Big\}. \qquad (5.14)
\end{aligned}$$

for the $Q=2$ sectors, where the magnetized Chan-Paton charge multiplicity is denoted by m. The coefficients α_{mG}, $\bar{\alpha}_{mG}$, α_{mG_1}, $\bar{\alpha}_{mG_1}$, α_{m^2} and $\bar{\alpha}_{m^2}$ must be chosen in such a way that the annulus amplitudes become real. The Möbius amplitude can be deduced in a similar way from the undeformed case, adding to the uncharged ($Q=0$) contributions the charged ($Q=2$) ones. The result is

$$\begin{aligned}
\mathcal{M}_{(Q=0)} &= -\frac{1}{8}\Big\{\big[2^{\frac{r-6}{2}}\,N\,P_1(B_1,\gamma_{\epsilon_1})\,P_2(B_2,\gamma_{\epsilon_2})\,P_3(B_3,\gamma_{\epsilon_3}) \\
&+ 2^{\frac{r_1-6}{2}}\,D_1\,P_1(B_1,\gamma_{\epsilon_1})W_2(B_2,\tilde{\gamma}_{\epsilon_2})W_3(B_3,\tilde{\gamma}_{\epsilon_3}) \\
&+ 2^{\frac{r_2-6}{2}}\,D_2\,W_1(B_1,\tilde{\gamma}_{\epsilon_1})P_2(B_2,\gamma_{\epsilon_2})W_3(B_3,\tilde{\gamma}_{\epsilon_3})\big]\,\hat{T}_{oo}(0;0;0) \\
&- \big[2^{\frac{r_1-2}{2}}\,(N+D_1)P_1(B_1,\gamma_{\epsilon_1})\big]\,\hat{T}_{og}(0;0;0)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2 \\
&- \big[2^{\frac{r_2-2}{2}}\,(N+D_2)P_2^{1/2}(B_2,\gamma_{\epsilon_2})\big]\,\hat{T}_{of}(0;0;0)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2 \\
&- \big[2^{-1}\,(D_1+D_2)W_3(B_3,\tilde{\gamma}_{\epsilon_3})\big]\,\hat{T}_{oh}(0;0;0)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2\Big\}\quad , \qquad (5.15)
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{M}_{(Q=2)} &= -\frac{1}{8}\Big\{-2^{\frac{r-2}{2}}m\hat{T}_{oo}(0;2z_2\tau;2z_3\tau)\,P_1(B_1,\gamma_{\epsilon_1})\,\frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(2z_2\tau)}\frac{2k_3\hat{\eta}}{\hat{\vartheta}_1(2z_3\tau)} \\
&- 2^{\frac{r-2}{2}}\bar{m}\hat{T}_{oo}(0;-2z_2\tau;-2z_3\tau)\,P_1(B_1,\gamma_{\epsilon_1})\,\frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(-2z_2\tau)}\frac{2k_3\hat{\eta}}{\hat{\vartheta}_1(-2z_3\tau)} \\
&- 2^{\frac{r_1-2}{2}}m\hat{T}_{og}(0;2z_2\tau;2z_3\tau)\,P_1(B_1,\gamma_{\epsilon_1})\,\frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_2\tau)}\frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_3\tau)} \\
&- 2^{\frac{r_1-2}{2}}\bar{m}\hat{T}_{og}(0;-2z_2\tau;-2z_3\tau)\,P_1(B_1,\gamma_{\epsilon_1})\,\frac{2\hat{\eta}}{\hat{\vartheta}_2(-2z_2\tau)}\frac{2\hat{\eta}}{\hat{\vartheta}_2(-2z_3\tau)} \\
&- 2^{\frac{r_2}{2}}im\hat{T}_{of}(0;2z_2\tau;2z_3\tau)\,\frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\ \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(2z_2\tau)}\frac{2\hat{\eta}}{\hat{\vartheta}_2(2z_3\tau)} \\
&- 2^{\frac{r_2}{2}}(-i)\bar{m}\hat{T}_{of}(0;-2z_2\tau;-2z_3\tau)\,\frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\ \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(-2z_2\tau)}\frac{2\hat{\eta}}{\hat{\vartheta}_2(-2z_3\tau)}\Big\}\quad . \qquad (5.16)
\end{aligned}$$

In order to analyze in some detail the tadpole cancellation conditions, it is worth displaying the transverse (tree) channel amplitudes. The annulus part comprises untwisted and twisted terms

$$\tilde{\mathcal{A}} = \tilde{\mathcal{A}}^U + \tilde{\mathcal{A}}^T\ ,$$

where

$$
\begin{aligned}
\tilde{\mathcal{A}}^U &= \frac{2^{-5}}{8} \Big\{ [2^{r-6} v_1 v_2 v_3 \frac{N^2}{2} W_1(B_1)(W_2(B_2) + (-1)^{n_2} W_2(B_2)) W_3(B_3) \\
&+ \frac{v_1}{v_2 v_3}\, 2^{r_1-2}\, \frac{D_1^2}{2} W_1(B_1) P_2 (P_3 + (-1)^{m_3} P_3) \\
&+ \frac{v_2}{v_1 v_3}\, 2^{r_2-2}\, \frac{D_2^2}{4} P_1 (W_2(B_2) + (-1)^{n_2} W_2(B_2))(P_3 + (-1)^{m_3} P_3) \\
&+ 2^{r-6} v_1 v_2 v_3\, \frac{2m\bar{m}}{2} (1 + q^2 H_2^2)(1 + q^2 H_3^2) W_1(B_1)(\tilde{W}_2(B_2) \\
&+ (-1)^{n_2} \tilde{W}_2(B_2)) \tilde{W}_3(B_3)\ ]\ T_{oo}(0;0;0) \\
&+ 2^{\frac{r_2}{2}+\frac{r_3}{2}} \Big[2^{r_1-2}\ 2 N D_1 v_1 W_1(B_1) \Big]\ T_{og}(0;0;0)\ \frac{2\eta}{\vartheta_2(0)}\ \frac{2\eta}{\vartheta_2(0)} \\
&+ 2^{\frac{r_1}{2}+\frac{r_3}{2}} \Big[2^{r_2-2}\ N D_2 v_2 ((i)^{n_2} W_2(B_2) + (-i)^{n_2} W_2(B_2)) \Big]\ T_{of}(0;0;0)\ \frac{2\eta}{\vartheta_2(0)}\ \frac{2\eta}{\vartheta_2(0)} \\
&+ 2^{\frac{r_1}{2}+\frac{r_2}{2}} \Big[D_1 D_2 \frac{1}{v_3} ((i)^{m_3} P_3 + (-i)^{m_3} P_3) \Big]\ T_{oh}(0;0;0)\ \frac{2\eta}{\vartheta_2(0)}\ \frac{2\eta}{\vartheta_2(0)} \\
&+ 2^{r-2}\ 8\ N\ v_1 W_1(B_1) [m T_{oo}(0; z_2; z_3) + \bar{m} T_{oo}(0; -z_2; -z_3)]\ \frac{k_2 \eta}{\vartheta_1(z_2)} \frac{k_3 \eta}{\vartheta_1(z_3)} \\
&+ 2^{\frac{r}{2}+\frac{r_1}{2}-2}\ 2 D_1 v_1 W_1(B_1) \Big[m T_{og}(0; z_2; z_3) + \bar{m} T_{og}(0; -z_2; -z_3) \Big]\ \frac{2\eta}{\vartheta_2(z_2)}\ \frac{2\eta}{\vartheta_2(z_3)} \\
&+ 2^{\frac{r_2}{2}+\frac{r_3}{2}} \Big[2^{r_1-2}\ 2 \bar{m} D_1 v_1 W_1(B_1) \Big]\ T_{og}(0; -z_2; -z_3)\ \frac{2\eta}{\vartheta_2(-z_2)}\ \frac{2\eta}{\vartheta_2(-z_3)} \\
&+ 2^{\frac{r}{2}+\frac{r_2}{2}}\ \ 2 m D_2\ T_{of}(0; z_2; z_3)\ \frac{2\eta}{\vartheta_2(0)}\ \frac{2 k_2 \eta}{\vartheta_1(z_2)}\ \frac{2\eta}{\vartheta_2(z_3)} \\
&- 2^{\frac{r}{2}+\frac{r_2}{2}}\ \ 2 \bar{m} D_2\ T_{of}(0; -z_2; -z_3)\ \frac{2\eta}{\vartheta_2(0)}\ \frac{2 k_2 \eta}{\vartheta_1(-z_2)}\ \frac{2\eta}{\vartheta_2(-z_3)} \qquad (5.17) \\
&+ 2^{r-2}\ 4\ v_1 W_1(B_1)\ [m^2 T_{oo}(0; 2z_2; 2z_3) + \bar{m}^2 T_{oo}(0; -2z_2; -2z_3)]\ \frac{2 k_2 \eta}{\vartheta_1(2z_2)} \frac{2 k_3 \eta}{\vartheta_1(2z_3)} \Big\} \quad ,
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{\mathcal{A}}^T &= \frac{2^{-5}}{8} \Big\{ 2^{r_1-2}\ 16\ v_1 W_1(B_1) \Big[\frac{G^2}{2} + \frac{{G^2}_1}{2} \frac{2m\bar{m}}{2} \Big]\ T_{go}(0;0;0)\ \frac{\eta}{\vartheta_4(0)}\ \frac{\eta}{\vartheta_4(0)} \\
&- 2^{\frac{r}{2}+\frac{r_1}{2}-2}\ v_1 W_1(B_1)\ 8\ G G_1\ T_{gg}(0;0;0)\ \frac{\eta}{\vartheta_3(0)}\ \frac{\eta}{\vartheta_3(0)} \\
&+ 2^{r_1-2}\ 16\ G\ v_1 W_1(B_1) \Big[\alpha_{mG} m T_{go}(0; z_2; z_3) + \bar{\alpha}_{mG} \bar{m} T_{go}(0; -z_2; -z_3) \Big]\ \frac{\eta}{\vartheta_4(z_2)}\ \frac{\eta}{\vartheta_4(z_3)} \\
&- 2^{\frac{r}{2}+\frac{r_1}{2}-2}\ 8\ G_1\ v_1\ W_1(B_1)\ \Big[\alpha_{mG_1}\ m\ T_{gg}(0; z_2; z_3) \\
&+ \bar{\alpha}_{mG_1}\ \bar{m}\ T_{gg}(0; -z_2; -z_3)\ \Big]\ \frac{\eta}{\vartheta_3(z_2)}\ \frac{\eta}{\vartheta_3(z_3)} \\
&+ 2^{r_1-2}\ 8\ v_1\ W_1(B_1)\ \Big[\alpha_{m^2}\ m^2\ T_{go}(0; 2z_2; 2z_3) \\
&+ \bar{\alpha}_{m^2}\ \bar{m}^2\ T_{go}(0; -2z_2; -2z_3)\ \Big]\ \frac{\eta}{\vartheta_4(2z_2)}\ \frac{\eta}{\vartheta_4(2z_3)} \Big\} \quad . \qquad (5.18)
\end{aligned}
$$

This is to be contrasted with the transverse Möbius amplitude, that contains only untwisted contributions

$$
\begin{aligned}
\tilde{\mathcal{M}} \;=\; & -\frac{2}{8}\Bigg\{ \Big[\, 2^{\frac{r-6}{2}}\; N\; v_1 v_2 v_3\; W_1^e(B_1,\gamma_{\epsilon_1})\; W_2^e(B_2,\gamma_{\epsilon_2})\; W_3^e(B_3,\gamma_{\epsilon_3}) \\
& +\; 2^{\frac{r_1-6}{2}}\; \frac{v_1}{v_2 v_3}\; D_1\; W_1^e(B_1,\gamma_{\epsilon_1})\; P_2^e(B_2,\tilde{\gamma}_{\epsilon_2})\; P_3^e(B_3,\tilde{\gamma}_{\epsilon_3}) \\
& +\; 2^{\frac{r_2-6}{2}}\; \frac{v_2}{v_1 v_3}\; D_2\; P_1^e(B_1,\tilde{\gamma}_{\epsilon_1})\; W_2^e(B_2,\gamma_{\epsilon_2})\; P_3^e(B_3,\tilde{\gamma}_{\epsilon_3})\,\Big]\; \hat{T}_{oo}(0;0;0) \\
& +\; 2^{\frac{r_1}{2}-1}\; (N+D_1)\; v_1\; W_1^e(B_1,\gamma_{\epsilon_1})\; \hat{T}_{og}(0;0;0)\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \\
& +\; 2^{\frac{r_2}{2}-1}\; (N+D_2)\; v_2\; W_2^e(B_2,\gamma_{\epsilon_2})\; \hat{T}_{of}(0;0;0)\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \\
& +\; \frac{2^{-1}}{v_3}\; (D_1+D_2)\; \phi^{B_3}\; P_3^e(B_3,\tilde{\gamma}_{\epsilon_3})\; \hat{T}_{oh}(0;0;0)\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)} \\
& +\; 2^{\frac{r}{2}-1}\; 4\; v_1 W_1^e(B_1,\gamma_{\epsilon_1})\; [\; m\hat{T}_{oo}(0;z_2;z_3) + \bar{m}\hat{T}_{oo}(0;-z_2;-z_3)\;]\; \frac{k_2\hat{\eta}}{\hat{\vartheta}_1(z_2)}\, \frac{k_3\hat{\eta}}{\hat{\vartheta}_1(z_3)} \\
& +\; 2^{\frac{r_1}{2}-1}\; v_1 W_1^e(B_1,\gamma_{\epsilon_1})\; [\; m\hat{T}_{og}(0;z_2;z_3) + \bar{m}\hat{T}_{og}(0;-z_2;-z_3)\;]\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(z_2)}\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(z_3)} \\
& +\; 2^{\frac{r_2}{2}}\; m\; \hat{T}_{of}(0;z_2;z_3)\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\; \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(z_2)}\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(z_3)} \\
& -\; 2^{\frac{r_2}{2}}\; \bar{m}\; \hat{T}_{of}(0;-z_2;-z_3)\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\; \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(-z_2)}\; \frac{2\hat{\eta}}{\hat{\vartheta}_2(-z_3)}\Bigg\} \quad , \qquad (5.19)
\end{aligned}
$$

where ϕ^{B_3} is a suitable phase that depends on the rank of B_{ab}, not directly relevant for our discussion. The untwisted tadpole cancellation conditions are related to the superposition of $\tilde{\mathcal{K}}$, $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{M}}$, and can be obtained as follows. The residues corresponding to the R-R part of the $\tau_{0\alpha}$ character are

$$
\begin{aligned}
& \sqrt{v_1 v_2 v_3}\,\Big\{ 2^{\frac{r}{2}}\,\Big[N + (m+\bar{m})(1 - 4\pi^2 {\alpha'}^2 q^2 H_2 H_3) + (m-\bar{m})\; 8i\pi^2 {\alpha'}^2 q^2 (H_2+H_3)\Big] - 32\Big\} \\
+\;\; & \lambda_{1\alpha}\sqrt{\frac{v_1}{v_2 v_3}}\;\Big[2^{\frac{r_1}{2}} D_1 \;-\; 32\, 2^{-\frac{r_2+r_3}{2}}\Big] \;+\; \lambda_{2\alpha}\sqrt{\frac{v_2}{v_1 v_3}}\;\Big[2^{\frac{r_2}{2}} D_2 \;-\; 32\, 2^{-\frac{r_1+r_3}{2}}\Big] \;=\; 0, \quad (5.20)
\end{aligned}
$$

together with the complex conjugates, where $\lambda_{1\alpha}$ is $+1$ for $\alpha = o, g$ and is -1 for $\alpha = h, f$ while $\lambda_{2\alpha}$ is $+1$ for $\alpha = o, f$ and is -1 for $\alpha = g, h$. In order to obtain eqs. (5.20), the conditions in (3.39) for the signs γ_ϵ and $\tilde{\gamma}_\epsilon$ must be used, and, in order to ensure the vanishing of the imaginary part the numerical constraint, $m = \bar{m}$ must also be enforced. Using the Dirac quantization condition in eq. (4.10), it is interesting to notice that the magnetized $D9$-branes contribute not only to the tadpole of the R-R ten-form, but also to the tadpole of the R-R six-form. As in the six-dimensional examples, this signals the phenomenon of brane transmutation. In particular, disentangling the diverse contributions, one obtains

$$
\sqrt{v_1 v_2 v_3}\;\Big[2^{\frac{r}{2}}\,(\,N + m + \bar{m})\Big] \;=\; \sqrt{v_1 v_2 v_3}\; 32 \qquad (5.21)
$$

for the $D9$-brane sector,

$$\sqrt{\frac{v_1}{v_2 v_3}}\ \left[2^{\frac{r_1}{2}} D_1\ + 2^{\frac{r}{2}}\, |k_2 k_3|\, (m+\bar{m})\right]\ =\ \sqrt{\frac{v_1}{v_2 v_3}}\ \left[32\, 2^{-\frac{r_2+r_3}{2}}\right] \tag{5.22}$$

for the $D5_1$-brane sector and

$$\sqrt{\frac{v_2}{v_1 v_3}}\ \left[2^{\frac{r_2}{2}} D_2\right]\ =\ \sqrt{\frac{v_2}{v_1 v_3}}\ \left[32\, 2^{-\frac{r_1+r_3}{2}}\right] \tag{5.23}$$

for the $D5_2$-brane sector.

The twisted tadpole conditions determine the nature of the allowed Chan-Paton charges. There are two options, that result in complex or real Chan-Paton charges. In the complex case, one must choose

$$\alpha_{mG} = -\bar{\alpha}_{mG} = \alpha_{mG_1} = -\bar{\alpha}_{mG_1}\ =\ i\quad ;\qquad \alpha_{m^2} = \bar{\alpha}_{m^2}\ =\ -1\quad ,$$

and the tadpole cancellation condition can be written

$$(8-2^{r-r_1+1})\ [G+im-i\bar{m}]^2 + 2\ [2\,G_1 + 2^{\frac{r-r_1}{2}}\ (G+im-i\bar{m}\,)]^2 = 0\quad , \tag{5.24}$$

while the real charges are determined by the choice

$$\alpha_{mG} = -\bar{\alpha}_{mG} = \alpha_{mG_1} = -\bar{\alpha}_{mG_1} = \alpha_{m^2} = \bar{\alpha}_{m^2}\ =\ 1,$$

and the corresponding tadpole cancellation condition can be written in the form

$$(8-2^{r-r_1+1})\ [G+m+\bar{m}]^2 + 2\ [2\,G_1 + 2^{\frac{r-r_1}{2}}\ (G+m+\bar{m}\,)]^2 = 0\quad . \tag{5.25}$$

The analysis of the open spectra is very similar to the one in section 3.4.1, and therefore we shall not repeat it here. With complex charges, after the choice of signs in eq. (3.40), one has to fix

$$\xi_2\ \xi_3 = \eta_2\ \eta_3\ =\ 1\quad , \tag{5.26}$$

while ξ_1 and η_1 are free signs. In order to obtain amplitudes with a proper particle interpretation, the magnetic charges must be rescaled by a factor of two, and a suitable parametrization of the Chan-Paton multiplicities is

$$\begin{aligned} N &= 2\,(\,n\,+\,\bar{n}\,)\quad ,\qquad G\ =\ 2\,i\,(\,n\,-\,\bar{n}\,)\quad ; \\ D_1 &= 2\,(\,d\,+\,\bar{d}\,)\quad ,\qquad G_1\ =\ 2\,i\,(\,d\,-\,\bar{d}\,)\quad ; \\ D_2 &= 4\,d_2\quad . \end{aligned} \tag{5.27}$$

With this choice, the tadpole cancellation conditions are reported in Table (C.11), together with corresponding options for the Chan-Paton gauge groups. The resulting open spectra at the supersymmetric point are reported in Table (C.12), and chirality emerges again both at brane intersections and due to the chiral asymmetry present in the "pure magnetic" sector. It should be noticed that, as in [75], the Möbius-strip amplitudes must be suitably interpreted, since naively they are not compatible with the corresponding annulus amplitudes. As is familiar from rational models, some missing parts must be identified with differences of pairs of identical terms, one symmetrized and the other antisymmetrized by the action of the open "twist"[55, 57, 99]. The real-charge solutions, present only if the B-rank is non-vanishing, correspond to the choice

$$\xi_2\,\xi_3 = \eta_2\,\eta_3 \;=\; -1 \quad , \tag{5.28}$$

with ξ_1 and η_1 again free signs. In this case after rescaling the magnetic charge m by a factor of two, a suitable parametrization for the Chan-Paton multiplicities is

$$\begin{aligned} N &= 2\,(\,n_1 + \,n_2) \quad , \qquad G \;=\; 2\,(\,n_1 - \,n_2\,) \quad ; \\ D_1 &= 2\,(\,d_1 \;+\; d_2\,) \quad , \qquad G_1 \;=\; 2\,(\,d_1 \;-\; d_2\,) \quad ; \\ D_2 &= 4\,d_3 \quad . \end{aligned} \tag{5.29}$$

Table (C.13) displays the tadpole cancellation conditions and the allowed Chan-Paton gauge groups, while the resulting chiral open spectra are exhibited in Table (C.14).

To conclude, let us mention that at the supersymmetric point, *i.e.* for self-dual configurations of the background magnetic fields, the tadpoles originating from the NS-NS sectors are also automatically cancelled. As in ref. [75], they can be traced to corresponding derivatives of the Born-Infeld-type action for the untwisted sectors (the dilaton tadpole, for instance, is one of them). Moreover, the twisted NS-NS tadpoles are subtle: they are not perfect squares because of the behavior of the magnetic field under time reversal [75, 57]. Still, they introduce additional couplings in the twisted NS-NS sectors that are proportional to $H_2 - H_3$ and are thus cancelled at the (self-dual) supersymmetric point.

5.2.2 $w_1w_2p_3$ Models

Another interesting class of chiral orientifolds can be derived from deformations of the $w_1w_2p_3$ models. Since this is very similar to the w_2p_3 case, we shall not perform a detailed description of all the amplitudes as in the previous section, but we shall just quote the results. The Klein bottle amplitude

$$\begin{aligned} \mathcal{K} \;=\; & \frac{1}{8}\Big\{ \big[\, P_1\,P_2\,P_3 \,+ 2^{-4}\,P_1\,W_2(B_2)\,W_3(B_3) \,+\, 2^{-4}\,W_1(B_1)\,P_2\,W_3(B_3) \\ & +\; 2^{-4}\,(-1)^{n_1}\,W_1(B_1)\,(-1)^{n_2} W_2(B_2)\,(-1)^{m_3} P_3\,\big]\,T_{oo} \\ & +\; 2\times 16\,\big[\,2^{-\frac{r_2}{2}-\frac{r_3}{2}}\,P_1^{\frac{1}{2}}\,T_{go} + 2^{-\frac{r_1}{2}-\frac{r_3}{2}}\,P_2^{\frac{1}{2}}\,T_{fo} \\ & +\; 2^{-\frac{r_1}{2}-\frac{r_2}{2}}\,2^{-2}\,W_3^{\frac{1}{2}}(B_3)\,T_{ho}\,\big]\,\left(\frac{\eta}{\vartheta_4}\right)^2 \Big\} \;, \end{aligned} \tag{5.30}$$

produces the unoriented closed spectra, whose massless part is reported in Table (C.15). It should be noticed that the result does not depend on the rank of B_{ab}, in agreement with the considerations made in section 3.3.1 relating quantized values of B_{ab} to shifts. An S transformation of (5.30) yields the transverse channel amplitude, that at the origin of the lattice sums is identical to eq. (5.11). As a result, the models contain $O9_+$, $O5_{1+}$ and $O5_{2+}$ planes. In order to neutralize the R-R charge, (magnetized) $D9$-branes, $D5_1$-branes and $D5_2$-branes are introduced. Due to the triple shifts, the tadpole cancellation conditions derive only from the untwisted sectors, and their analysis is very similar to the one in section 5.2.1. After a proper normalization, the Chan-Paton charge multiplicities are displayed in Table (C.16), where the allowed Chan-Paton gauge groups are also reported. Apart from the m charges, all others are real, as emerges from the open unoriented chiral spectra shown in Table (C.17).

Non-chiral Models

In this section we discuss the remaining models in Table (3.2) that admit magnetic deformations, namely those containing $D5$-branes along T^{45} that can absorb the R-R charge flux of the magnetized $D9$-branes. It is easy to see that the p_2p_3, w_1p_2, $w_1p_2p_3$ and $w_1p_2w_3$ models do admit magnetic deformations, while the $p_1p_2p_3$, $p_1w_2w_3$ and $w_1w_2w_3$ do not. The four models are quite different, but inherit an effective world-sheet parity projection that allows one to express the Klein bottle amplitude in the following form:

$$
\begin{aligned}
\mathcal{K} \;=\; & \frac{1}{8}\Big\{ \big[\, P_1\, P_2\, P_3 \,+ 2^{-4}\, P_1\, W_2(B_2)\, W_3(B_3) \,+\, 2^{-4}\, W_1(B_1)\, (-1)^{\delta_2}\, P_2\, (-1)^{\delta_3}\, W_3(B_3) \\
& +\, 2^{-4}\, (-1)^{\delta_1} W_1(B_1)\, (-1)^{\delta_2}\, W_2(B_2)\, (-1)^{\delta_3}\, P_3 \,\big]\, T_{oo} \\
& +\, 2 \times 16\; 2^{-\frac{r_2}{2}-\frac{r_3}{2}}\, P_1^{\lambda_1}\, T_{go} \left(\frac{\eta}{\vartheta_4}\right)^2 \Big\} \,,
\end{aligned} \tag{5.31}
$$

where λ_1 is 0 if $\delta_1 = p_1$ and $\frac{1}{2}$ if $\delta_1 = w_1$, while obviously the shifts affect the sums only if they are present in the corresponding σ table and are of the same type as the lattice sums. The transverse channel gives the O-plane content in the four cases, that is expected to be the same for the four classes of models. Indeed, at the origin of the lattices

$$
\begin{aligned}
\tilde{\mathcal{K}}_0 \;=\; & \frac{2^5}{8}\Big\{ \left(\sqrt{v_1 v_2 v_3} + 2^{-\frac{r_2}{2}-\frac{r_3}{2}} \sqrt{\frac{v_1}{v_2 v_3}}\right)^2 (\,\tau_{oo} + \tau_{og}\,) \\
& +\; \left(\sqrt{v_1 v_2 v_3} + 2^{-\frac{r_2}{2}-\frac{r_3}{2}} \sqrt{\frac{v_1}{v_2 v_3}}\right)^2 (\,\tau_{oh} + \tau_{of}\,) \Big\} \quad ,
\end{aligned} \tag{5.32}
$$

so that in all four classes of models only $O9_+$ and $O5_{1+}$ are present. On the other hand, both the unoriented closed spectra and the open sectors are quite distinct, as can be deduced from the diverse D-brane multiplet configurations of the undeformed models in Table (C.34). Of course, only (magnetized) $D9$-branes and $D5_1$-branes are needed, with a consequent lack of chirality.

The unoriented closed spectra of the four classes of models can be found in Tables (C.18), (C.19) and (C.20). Only the p_2p_3 models show a dependence on the rank of B_{ab}, while the two models with three shifts have identical massless closed spectra.

There are two different p_2p_3 unoriented partition functions, that differ in the open-string sectors, depending on the sign freedom for the Möbius projections. With complex and properly normalized charges, untwisted and twisted tadpole cancellation conditions are summarized in Table (C.21), where the resulting Chan-Paton gauge groups are also reported. The open spectra can be read from Table (C.22), where R stands for the symmetric representation if $\eta_1 = +1$, or for the antisymmetric representation if $\eta_1 = -1$. The second solution is linked to a real parametrization of the Chan-Paton charges that results into tadpole cancellation conditions and gauge groups as in Table (C.23). It should be noticed that in this case group factors, other than $U(m)$, must be all orthogonal or all symplectic. The massless open spectra can be found in Table (C.24).

The w_1p_2 models and the $w_1p_2p_3$ models are very similar and, independently of the presence of H_i, differ solely in their massive excitations. In other words, the analysis of the massless excitations is not sufficient to distinguish these two classes of models. Their unoriented closed spectra are different, as emerges from Tables (C.19) and (C.20), but they have identical open spectra. The tadpole cancellation conditions and the resulting Chan-Paton groups are reported in Table (C.25) for complex charges and in Table (C.27) for real charges. The non-chiral and coincident open spectra are reported in Table (C.26) for the complex charge cases, and in Table (C.28) for the real charge cases.

Finally, the $w_1p_2w_3$ models exhibit unoriented bulk spectra identical to the one of the $w_1p_2p_3$ models in Table (C.20), but with tadpole conditions and Chan-Paton groups as in Table (C.29). The resulting non-chiral massless open spectra are contained in Table (C.30).

5.3 Brane Supersymmetry Breaking

In this section we discuss one significant class of magnetized orientifolds in which the field configurations are chosen so that magnetized $D9$-branes mimic anti-$D5$-branes rather than $D5$-branes, thus breaking supersymmetry in the open-string sector (*brane supersymmetry breaking*). We analyze in some detail a variant of the w_2p_3 class of models extensively discussed at the supersymmetric point in section 5.2.1. For simplicity, we shall confine ourselves to the $B_{ab} = 0$ case. The oriented closed spectrum is always the one contained in Table (C.8), but the Klein-bottle projection, described by

$$\begin{aligned}\mathcal{K} \;&=\; \frac{1}{8}\Big\{ \big[\, P_1\, P_2\, P_3 \,+\, P_1\, W_2\, W_3 \,+\, W_1\, P_2\, W_3 + W_1\, (-1)^{n_2} W_2\, (-1)^{m_3} P_3 \,\big]\, T_{oo} \\ &-\; 2\times 16 \,\big[\, T_{go}\, P_1 \,-\, T_{fo}\, P_2{}^{\frac{1}{2}} \,+\, W_3^{\frac{1}{2}}\, T_{ho} \,\big] \left(\frac{\eta}{\vartheta_4}\right)^2 \Big\} \,,\end{aligned} \tag{5.33}$$

is now different, due to the inversion of some signs, and produces the massless unoriented closed spectra of Table (C.31). The transverse channel amplitude at the lattice origin,

$$\begin{aligned}
\tilde{\mathcal{K}}_0 &= \frac{2^5}{8}\Bigg\{ \left(\sqrt{v_1 v_2 v_3} - \sqrt{\frac{v_1}{v_2 v_3}} - \sqrt{\frac{v_2}{v_1 v_3}}\right)^2 \tau_{oo} \\
&+ \left(\sqrt{v_1 v_2 v_3} - \sqrt{\frac{v_1}{v_2 v_3}} + \sqrt{\frac{v_2}{v_1 v_3}}\right)^2 \tau_{og} \\
&+ \left(\sqrt{v_1 v_2 v_3} + \sqrt{\frac{v_1}{v_2 v_3}} + \sqrt{\frac{v_2}{v_1 v_3}}\right)^2 \tau_{oh} \\
&+ \left(\sqrt{v_1 v_2 v_3} + \sqrt{\frac{v_1}{v_2 v_3}} - \sqrt{\frac{v_2}{v_1 v_3}}\right)^2 \tau_{of} \Bigg\} \quad , \qquad (5.34)
\end{aligned}$$

displays very neatly the presence of one $O9_+$-plane and of the two "exotic" $O5_{1-}$ and $O5_{2-}$ planes, that require the introduction of anti-$D5_1$-branes and anti-$D5_2$-branes, together with the (magnetized) $D9$-branes. In order to neutralize the global R-R charge, one has to sit at the antiself-dual background field configuration, corresponding to $H_2 = H_3$ in our conventions. Only the R-R tadpole cancellation conditions can be imposed, while the NS-NS tadpoles survive, signaling, as customary, the need for a non-Minkowskian vacuum [145]. The results for the Chan-Paton gauge groups are displayed in Table (C.32), while the open and unoriented massless spectra are displayed in Table (C.33). As usual, in the models with *brane supersymmetry breaking* supersymmetry is exact at tree level on the $D9$-branes but it is only non-linearly realized on the anti-$D5$-branes [119, 120]. This can be foreseen from Table (C.33), where modes originally in a given supermultiplet are assigned to different gauge group representations.

5.4 Conclusions

This thesis contains the detailed analysis of four dimensional orientifolds originating from $\mathbb{Z}_2 \times \mathbb{Z}_2$ toroidal orbifolds and from freely acting $\mathbb{Z}_2 \times \mathbb{Z}_2$ shift-orbifolds of the type IIB superstring, in the presence of uniform background magnetic fluxes along four of the six internal directions and of a quantized NS-NS B_{ab}, that has been shown to be equivalent to an asymmetric shift-orbifold projection. These models are connected by T-duality to models with branes intersecting at angles and contain magnetized $D9$-branes charged also with respect to the R-R six-form, thus exhibiting several interesting novelties. In particular, for suitable self-dual configurations of the internal backgrounds, that in the T-dual picture correspond to suitable angles between the branes, it is possible to obtain non tachyonic four dimensional supersymmetric models with spectra containing in a natural way several families of matter fields whose numbers are related to the multiplicities of the Landau levels. Moreover, the instanton-like behavior of the *magnetized* $D9$-branes that mimic localized $D5$-branes produces an interesting rank reduction of the Chan-Paton gauge groups. As a bonus, if $D5$ branes longitudinal to the directions of the internal magnetic fields are present, the models can acquire chiral spectra, due to the unpairing of fermions at the intersections and to the chiral asymmetry in the "pure magnetic" sectors. Geometrically, chirality is related to configurations in which D-branes are not parallel to the corresponding O-planes, differently from the models of ref. [117], where the $D9$-branes are parallel to the $O9$-planes, but the orbifold projection produces only left-handed fermions. Introducing antiself-dual background fields, it is also possible to obtain non-tachyonic models with *brane supersymmetry breaking*, for which supersymmetry is exact at tree level in the bulk and on the $D9$-branes, but is non-linearly realized and thus effectively broken at the string scale on the anti-$D5$ branes and on the equivalent magnetized $D9$-branes. The chiral four dimensional models can be used to build realistic extensions of the Standard Model in a brane-world like scenario, introducing brane-antibrane pairs or Wilson lines. This is a very interesting and widely pursued effort, but the dynamical stability of all these vacua is still an open question. It would be also interesting to analyze in some detail mechanisms to reduce the number of moduli, for instance introducing background fluxes, as recently discussed in refs. [130].

Appendix A

Lattice Sums in the Presence of a Quantized B_{ab}

In this Appendix we collect the relevant lattice sums that enter the one-loop partition functions. We follow mainly the notation of [57], and display only the sums modified by the presence of an antisymmetric tensor B_{ab}. Since each surface of vanishing Euler number has a different double cover, the sums also differ in their proper time dependence. We will denote with τ the loop channel modulus of each surface and with ℓ the modulus of the doubly covering tori. Let us begin by recalling that, in presence of a B_{ab} background, the generalized d-dimensional momenta $p_{\rm L}$ and $p_{\rm R}$ are [144]:

$$p_{{\rm L},a} = m_a + \frac{1}{\alpha'}(g_{ab} - B_{ab})\, n^b \,, \tag{A.1}$$

$$p_{{\rm R},a} = m_a - \frac{1}{\alpha'}(g_{ab} + B_{ab})\, n^b \,. \tag{A.2}$$

The corresponding lattice sums on the torus take the form

$$\Lambda(B) \ = \ \sum_{m,n} \frac{q^{\frac{\alpha'}{4} p_{\rm L}^T g^{-1} p_{\rm L}}\ \bar{q}^{\frac{\alpha'}{4} p_{\rm R}^T g^{-1} p_{\rm R}}}{|\eta(\tau)|^{2d}} \,, \tag{A.3}$$

as in ref. [144]. For the direct-channel Klein-bottle amplitudes, only the winding sums are modified and become

$$W(B) = \sum_{\epsilon=0,1} \sum_{n} \frac{q^{\frac{1}{2\alpha'} n^{\rm T} g n}\, e^{\frac{2i\pi}{\alpha'} n^{\rm T} B \epsilon}}{\eta^2(2i\tau)} \quad , \tag{A.4}$$

while in the transverse channel the momentum sums are

$$P(B) = \sum_{\epsilon=0,1} \sum_{m} \frac{(e^{-2\pi\ell})^{\alpha'(m+\frac{1}{\alpha'}B\epsilon)^{\rm T} g^{-1} (m+\frac{1}{\alpha'}B\epsilon)}}{\eta^2(i\ell)} \quad . \tag{A.5}$$

In the annulus amplitudes the situation is reverted, and modified momentum sums

$$P(B) = \sum_{\epsilon=0,1} \sum_{m} \frac{q^{\frac{\alpha'}{2}(m+\frac{1}{\alpha'}B\epsilon)^{\rm T} g^{-1} (m+\frac{1}{\alpha'}B\epsilon)}}{\eta^2(i\tau/2)} \quad , \tag{A.6}$$

appear in the direct channel, while modified winding sums

$$W(B) = \sum_{\epsilon=0,1} \sum_{n} \frac{(e^{-2\pi\ell})^{\frac{1}{4\alpha'} n^{\mathrm{T}} g n} e^{\frac{2i\pi}{\alpha'} n^{\mathrm{T}} B \epsilon}}{\eta^2(i\ell)} \tag{A.7}$$

appear in the transverse channel. The direct Möbius amplitudes involve

$$P(B, \gamma_\epsilon) = \sum_{\epsilon=0,1} \sum_{m} \frac{q^{\frac{\alpha'}{2}(m+\frac{1}{\alpha'}B\epsilon)^{\mathrm{T}} g^{-1} (m+\frac{1}{\alpha'}B\epsilon)} \ \gamma_\epsilon}{\hat{\eta}^2(\frac{i\tau}{2}+\frac{1}{2})} \tag{A.8}$$

and

$$W(B, \tilde{\gamma}_\epsilon) = \sum_{\epsilon=0,1} \sum_{n} \frac{q^{\frac{1}{2\alpha'} n^{\mathrm{T}} g n} \ e^{\frac{2i\pi}{\alpha'} n^{\mathrm{T}} B \epsilon} \tilde{\gamma}_\epsilon}{\hat{\eta}^2(\frac{i\tau}{2}+\frac{1}{2})} \quad , \tag{A.9}$$

while the transverse Möbius amplitudes involve

$$W(B, \gamma_\epsilon) = \sum_{\epsilon=0,1} \sum_{n} \frac{(e^{-2\pi\ell})^{\frac{1}{4\alpha'} n^{\mathrm{T}} g n} e^{\frac{2i\pi}{\alpha'} n^{\mathrm{T}} B \epsilon} \ \gamma_\epsilon}{\hat{\eta}^2(i\ell)} \tag{A.10}$$

and

$$P(B, \tilde{\gamma}_\epsilon) = \sum_{\epsilon=0,1} \sum_{m} \frac{(e^{-2\pi\ell})^{\alpha'(m+\frac{1}{\alpha'}B\epsilon)^{\mathrm{T}} g^{-1} (m+\frac{1}{\alpha'}B\epsilon)} \ \tilde{\gamma}_\epsilon}{\hat{\eta}^2(i\ell)} \quad . \tag{A.11}$$

All sums displayed in this Appendix are two-dimensional, since for simplicity the six-dimensional internal torus is chosen to be factorized as a product of two-dimensional tori, while the corresponding antisymmetric two-tensor is also chosen, for simplicity, in a block-diagonal form of two-by-two matrices.

Appendix B

Characters for the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifolds

In this Appendix we list the $\mathbb{Z}_2 \times \mathbb{Z}_2$ characters that enter the one-loop amplitudes. Using the conventions of ref. [57], they may be expressed as ordered products of the four SO(2) level-one characters, O_2, V_2, S_2 and C_2, as follows:

$$
\begin{aligned}
\tau_{oo} &= V_2O_2O_2O_2 + O_2V_2V_2V_2 - S_2S_2S_2S_2 - C_2C_2C_2C_2 \ , \\
\tau_{og} &= O_2V_2O_2O_2 + V_2O_2V_2V_2 - C_2C_2S_2S_2 - S_2S_2C_2C_2 \ , \\
\tau_{oh} &= O_2O_2O_2V_2 + V_2V_2V_2O_2 - C_2S_2S_2C_2 - S_2C_2C_2S_2 \ , \\
\tau_{of} &= O_2O_2V_2O_2 + V_2V_2O_2V_2 - C_2S_2C_2S_2 - S_2C_2S_2C_2 \ , \\
\tau_{go} &= V_2O_2S_2C_2 + O_2V_2C_2S_2 - S_2S_2V_2O_2 - C_2C_2O_2V_2 \ , \\
\tau_{gg} &= O_2V_2S_2C_2 + V_2O_2C_2S_2 - S_2S_2O_2V_2 - C_2C_2V_2O_2 \ , \\
\tau_{gh} &= O_2O_2S_2S_2 + V_2V_2C_2C_2 - C_2S_2V_2V_2 - S_2C_2O_2O_2 \ , \\
\tau_{gf} &= O_2O_2C_2C_2 + V_2V_2S_2S_2 - S_2C_2V_2V_2 - C_2S_2O_2O_2 \ , \\
\tau_{ho} &= V_2S_2C_2O_2 + O_2C_2S_2V_2 - C_2O_2V_2C_2 - S_2V_2O_2S_2 \ , \\
\tau_{hg} &= O_2C_2C_2O_2 + V_2S_2S_2V_2 - C_2O_2O_2S_2 - S_2V_2V_2C_2 \ , \\
\tau_{hh} &= O_2S_2C_2V_2 + V_2C_2S_2O_2 - S_2O_2V_2S_2 - C_2V_2O_2C_2 \ , \\
\tau_{hf} &= O_2S_2S_2O_2 + V_2C_2C_2V_2 - C_2V_2V_2S_2 - S_2O_2O_2C_2 \ , \\
\tau_{fo} &= V_2S_2O_2C_2 + O_2C_2V_2S_2 - S_2V_2S_2O_2 - C_2O_2C_2V_2 \ , \\
\tau_{fg} &= O_2C_2O_2C_2 + V_2S_2V_2S_2 - C_2O_2S_2O_2 - S_2V_2C_2V_2 \ , \\
\tau_{fh} &= O_2S_2O_2S_2 + V_2C_2V_2C_2 - C_2V_2S_2V_2 - S_2O_2C_2O_2 \ , \\
\tau_{ff} &= O_2S_2V_2C_2 + V_2C_2O_2S_2 - C_2V_2C_2O_2 - S_2O_2S_2V_2 \ . \qquad \text{(B.1)}
\end{aligned}
$$

Appendix C

Massless Spectra

This Appendix collects the massless spectra of all the models in the paper. N indicates the number of supersymmetries and H, V, C and $C_{L,R}$ denote hypermultiplets, vector multiplets and chiral multiplets with a Majorana or a Weyl (left or right) spinor, respectively. $CY(h_{11}, h_{12})$ is referred to the fact that the related orbifold is a singular limit of a Calabi-Yau compactification with Hodge numbers (h_{11}, h_{12}), ω is the discrete torsion while ω_i are signs that satisfy $\omega_1\,\omega_2\,\omega_3 = \omega$ (Cfr. section 2). k_i are the integer multiplicities of the Landau Level degeneracies, r_j is the rank of the two-by-two j-th block of the B_{ab} NS-NS antisymmetric tensor and $r = r_1 + r_2 + r_3$ is the total B-rank. The η_i and ξ_i, introduced in eq. (3.40) can be ± 1, and both choices are allowed if their values are not specified. For what concerns the Chan-Paton gauge groups, F, S, A and Adj denote respectively the Fundamental, Symmetric, Antisymmetric and Adjoint representations. When two Chan-Paton groups or two multiplets are within brackets, one of the two factors can be chosen independently. Finally, the notation related to the Chan-Paton charges or to the number of branes reserves the n's to the uncharged $D9$-branes, the m's to the magnetized $D9$-branes and the d's to the $D5_i$-branes.

C.1 Closed Spectra of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

ω	untwisted SUGRA	untwisted H	untwisted V	twisted H	twisted V	
$+1$	$N=2$	$1+3$	3	$16+16+16$	0	CY $(3,51)$
-1	$N=2$	$1+3$	3	0	$16+16+16$	CY $(51,3)$

Table C.1: Oriented closed spectra of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds.

$(\omega_1, \omega_2, \omega_3)$	ω	untwisted SUGRA	untwisted C	twisted C	twisted V
$(+,+,+)$	$+$	$N=1$	$1+3+3$	$16+16+16$	0
$(+,-,-)$ $(-,+,-)$ $(-,-,+)$	$+$ $+$ $+$	$N=1$	$1+3+3$	16	$16+16$
$(-,-,-)$ $(+,+,-)$ $(+,-,+)$ $(-,+,+)$	- - - -	$N=1$	$1+3+3$	$16+16+16$	0

Table C.2: Unoriented closed spectra of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds.

B rank			untwisted	untwisted	twisted	twisted
r_1	r_2	r_3	SUGRA	C	C	V
0	0	0	$N=1$	$1+3+3$	$16+16+16$	0
2	0	0	$N=1$	$1+3+3$	$16+12+12$	$0+4+4$
0	2	0	$N=1$	$1+3+3$	$12+16+12$	$4+0+4$
0	0	2	$N=1$	$1+3+3$	$12+12+16$	$4+4+0$
2	2	0	$N=1$	$1+3+3$	$12+12+10$	$4+4+6$
0	2	2	$N=1$	$1+3+3$	$10+12+12$	$6+4+4$
2	0	2	$N=1$	$1+3+3$	$12+10+12$	$4+6+4$
2	2	2	$N=1$	$1+3+3$	$10+10+10$	$6+6+6$

Table C.3: Unoriented closed spectra of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds with $\omega_i = +1$.

C.2 Open spectra of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds with $\omega_i = +1$

$\begin{pmatrix} USp(n) \\ SO(n) \end{pmatrix} \otimes \begin{pmatrix} USp(d_1) \\ SO(d_1) \end{pmatrix} \otimes \begin{pmatrix} USp(d_2) \\ SO(d_2) \end{pmatrix} \otimes \begin{pmatrix} USp(d_3) \\ SO(d_3) \end{pmatrix}$
$n = d_1 = d_2 = d_3 = 16\ 2^{-\frac{r}{2}}$

Table C.4: Chan-Paton groups and tadpole conditions for the $[T^2 \times T^2 \times T^2]\mathbb{Z}_2 \times \mathbb{Z}_2$ models.

Multiplets	Number	Rep.
C	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ if USp ; $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ if SO	$\begin{pmatrix} \text{(A,1,1,1)} \\ \text{(S,1,1,1)} \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ if USp ; $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ if SO	$\begin{pmatrix} \text{(1,A,1,1)} \\ \text{(1,S,1,1)} \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ if USp ; $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ if SO	$\begin{pmatrix} \text{(1,1,A,1)} \\ \text{(1,1,S,1)} \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ if USp ; $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ if SO	$\begin{pmatrix} \text{(1,1,1,A)} \\ \text{(1,1,1,S)} \end{pmatrix}$
C	$2^{\frac{r_2+r_3}{2}}$	$(F,F,1,1),(1,1,F,F)$
C	$2^{\frac{r_1+r_3}{2}}$	$(F,1,F,1),(1,F,1,F)$
C	$2^{\frac{r_1+r_1}{2}}$	$(F,1,1,F),(1,F,F,1)$

Table C.5: Open spectra of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with $\omega = 1$.

C.3 Open Spectra of the Magnetized $\mathbb{Z}_2 \times \mathbb{Z}_2$ Orientifolds with $\omega_i = +1$

$\begin{pmatrix} USp(n) \\ SO(n) \end{pmatrix} \otimes \begin{pmatrix} USp(d_1) \\ SO(d_1) \end{pmatrix} \otimes \begin{pmatrix} USp(d_2) \\ SO(d_2) \end{pmatrix} \otimes \begin{pmatrix} USp(d_3) \\ SO(d_3) \end{pmatrix} \otimes U(m)$
$n + m + \bar{m} = 16\ 2^{-\frac{r}{2}}$
$d_1 + 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ \lvert k_2 k_3 \rvert\ (m + \bar{m}) = 16\ 2^{-\frac{r}{2}}$
$d_2 = 16\ 2^{-\frac{r}{2}} \quad ; \quad d_3 = 16\ 2^{-\frac{r}{2}} \quad ; \quad m = \bar{m}$

Table C.6: Chan-Paton groups and tadpole conditions for the magnetized $[T^2 \times T^2 \times T^2]/\mathbb{Z}_2 \times \mathbb{Z}_2$ models.

Multiplets	Number	Rep.
C	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ if USp ; $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ if SO	$\begin{pmatrix} \text{(A,1,1,1,1)} \\ \text{(S,1,1,1,1)} \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ if USp ; $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ if SO	$\begin{pmatrix} \text{(1,A,1,1,1)} \\ \text{(1,S,1,1,1)} \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ if USp ; $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ if SO	$\begin{pmatrix} \text{(1,1,A,1,1)} \\ \text{(1,1,S,1,1)} \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ if USp ; $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ if SO	$\begin{pmatrix} \text{(1,1,1,A,1)} \\ \text{(1,1,1,S,1)} \end{pmatrix}$
C	3	$(1,1,1,1,Adj)$
C	$2^{\frac{r_2+r_3}{2}}$	$(F,F,1,1,1),(1,1,F,F,1)$
C	$2^{\frac{r_1+r_3}{2}}$	$(F,1,F,1,1),(1,F,1,F,1)$
C	$2^{\frac{r_1+r_2}{2}}$	$(F,1,1,F,1),(1,F,F,1,1)$
C	$2^{\frac{r_2+r_3}{2}}$	$(1,F,1,1,F+\bar{F})$
C	$2^{r_2+r_3}\ \lvert k_2\,k_3\rvert$	$(F,1,1,1,F+\bar{F})$
C_L	$2^{r_2+r_3+1}\lvert k_2\,k_3\rvert + 2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert + \eta_1 + 2^{\frac{r_2}{2}}\ \lvert k_2\rvert - 2^{\frac{r_3}{2}}\ \lvert k_3\rvert$	$(1,1,1,1,A)$
C_L	$2^{r_2+r_3+1}\lvert k_2\,k_3\rvert - 2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert - \eta_1 - 2^{\frac{r_2}{2}}\ \lvert k_2\rvert + 2^{\frac{r_3}{2}}\ \lvert k_3\rvert$	$(1,1,1,1,S)$
C_L	$2^{r_2+r_3+1}\lvert k_2\,k_3\rvert + 2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert + \eta_1 - 2^{\frac{r_2}{2}}\ \lvert k_2\rvert + 2^{\frac{r_3}{2}}\ \lvert k_3\rvert$	$(1,1,1,1,\bar{A})$
C_L	$2^{r_2+r_3+1}\lvert k_2\,k_3\rvert - 2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert - \eta_1 + 2^{\frac{r_2}{2}}\ \lvert k_2\rvert - 2^{\frac{r_3}{2}}\ \lvert k_3\rvert$	$(1,1,1,1,\bar{S})$
C_L	$2^{\frac{r+r_2}{2}}\ \lvert k_2\rvert$	$(1,1,F,1,F)$
C_R	$2^{\frac{r+r_3}{2}}\ \lvert k_3\rvert$	$(1,1,1,F,F)$

Table C.7: Open spectra of the magnetized $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds with $\omega = 1$.

C.4 Oriented Closed Spectra of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ Shift-orientifolds

model	untwisted SUGRA	untwisted H	untwisted V	twisted H	twisted V	
p_3	$N=2$	$1+3$	3	16	16	CY $(19,19)$
p_2p_3	$N=2$	$1+3$	3	8	8	CY $(11,11)$
w_2p_3	$N=2$	$1+3$	3	8	8	CY $(11,11)$
w_1p_2	$N=2$	$1+3$	3	8	8	CY $(11,11)$
$p_1p_2p_3$	$N=2$	$1+3$	3	0	0	CY $(3,3)$
$p_1w_2w_3$	$N=2$	$1+3$	3	0	0	CY $(3,3)$
$w_1p_2p_3$	$N=2$	$1+3$	3	0	0	CY $(3,3)$
$w_1p_2w_3$	$N=2$	$1+3$	3	0	0	CY $(3,3)$
$w_1w_2p_3$	$N=2$	$1+3$	3	0	0	CY $(3,3)$
$w_1w_2w_3$	$N=2$	$1+3$	3	0	0	CY $(3,3)$

Table C.8: Oriented closed spectra of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ shift-orbifolds.

C.5 Unoriented Closed Spectra of the p_3 Models

B rank			untwisted	untwisted	twisted	twisted
r_1	r_2	r_3	SUGRA	C	C	V
0	0	0	$N=1$	$1+3+3$	$16+16$	0
2	0	0	$N=1$	$1+3+3$	$14+14$	$2+2$
0	2	0	$N=1$	$1+3+3$	$14+14$	$2+2$
0	0	2	$N=1$	$1+3+3$	$12+12$	$4+4$
2	2	0	$N=1$	$1+3+3$	$12+12$	$4+4$
0	2	2	$N=1$	$1+3+3$	$11+11$	$5+5$
2	0	2	$N=1$	$1+3+3$	$11+11$	$5+5$
2	2	2	$N=1$	$1+3+3$	$10+10$	$6+6$

Table C.9: Unoriented closed spectra of the p_3 models.

C.6 Orientifolds of the w_2p_3 Models

B rank $r_2 + r_3$	untwisted SUGRA	untwisted C	twisted C	twisted V
0	$N = 1$	$1 + 3 + 3$	$8 + 8$	0
2	$N = 1$	$1 + 3 + 3$	$6 + 6$	$2 + 2$
4	$N = 1$	$1 + 3 + 3$	$5 + 5$	$3 + 3$

Table C.10: Unoriented closed spectra of the w_2p_3 models.

$$U(n) \otimes U(d_1) \otimes \begin{pmatrix} USp(d_2) \\ SO(d_2) \end{pmatrix} \otimes U(m)$$

$$n + \bar{n} + m + \bar{m} = 16\ 2^{-\frac{r}{2}}$$

$$d_1 + \bar{d}_1 + 2^{\frac{r_2}{2}+\frac{r_3}{2}}\ |k_2k_3|\ (m + \bar{m}) = 16\ 2^{-\frac{r}{2}}$$

$$d_2 = 8\ 2^{-\frac{r}{2}} \quad ; \quad n = \bar{n} \quad ; \quad d_1 = \bar{d}_1 \quad ; \quad m = \bar{m}$$

Table C.11: Chan-Paton groups and tadpole conditions for the w_2p_3 models (complex charges).

Mult.	Number	Rep.
C	1	$(Adj,1,1,1),(1,Adj,1,1)$ $(1,1,1,Adj)$
C	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ if USp ; $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ if SO	$\begin{pmatrix} (1,S+\bar{S},1,1),(S+\bar{S},1,1,1) \\ (A+\bar{A},1,1,1),(1,A+\bar{A},1,1) \end{pmatrix}$
C	3	$(1,1,A,1)$ or $(1,1,S,1)$
C	$2^{r_2+r_3}\ \lvert k_2\,k_3\rvert+2$	$(F,1,1,F)$, $(\bar{F},1,1,\bar{F})$
C	$2^{r_2+r_3}\ \lvert k_2\,k_3\rvert-2$	$(\bar{F},1,1,F)$, $(F,1,1,\bar{F})$
C	$2\ 2^{\frac{r_2+r_3}{2}}$	$(F,F,1,1)$,$(\bar{F},\bar{F},1,1)$
C	$2\ 2^{\frac{r_2+r_3}{2}}$	$(1,F,1,F)$, $(1,\bar{F},1\bar{F})$
C_L	$2^{r_2+r_3}\ 2\ \lvert k_2\,k_3\rvert+1+2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert+\eta_1+2^{\frac{r_2}{2}}\lvert k_2\rvert$	$(1,1,1,A)$
C_L	$2^{r_2+r_3}\ 2\ \lvert k_2\,k_3\rvert+1-2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert-\eta_1-2^{\frac{r_2}{2}}\lvert k_2\rvert$	$(1,1,1,S)$
C_L	$2^{r_2+r_3}\ 2\ \lvert k_2\,k_3\rvert+1+2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert+\eta_1-2^{\frac{r_2}{2}}\lvert k_2\rvert$	$(1,1,1,\bar{A})$
C_L	$2^{r_2+r_3}\ 2\ \lvert k_2\,k_3\rvert+1-2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert-\eta_1+2^{\frac{r_2}{2}}\lvert k_2\rvert$	$(1,1,1,\bar{S})$
C_L	$2^{\frac{r_1+r_3}{2}+r_2}\ 2\ \lvert k_2\rvert$	$(1,1,F,F)$

Table C.12: Open spectra of the w_2p_3 models (complex charges).

$$\begin{pmatrix} USp(n_1)\ \otimes\ USp(n_2)\ \otimes\ USp(d_1)\ \otimes USp(d_2) \\ SO(n_1)\ \otimes SO(n_2)\ \otimes SO(d_1)\ \otimes SO(d_2) \end{pmatrix} \otimes \begin{pmatrix} USp(d_3) \\ SO(d_3) \end{pmatrix} \otimes\ U(m)$$

$$n_1+n_2+m+\bar{m}=16\ 2^{-\frac{r}{2}}$$

$$d_1+d_2+2^{\frac{r_2}{2}+\frac{r_3}{2}}\ \lvert k_2k_3\rvert\ (m+\bar{m})=16\ 2^{-\frac{r}{2}}$$

$$d_3=8\ 2^{-\frac{r}{2}} \quad ; \quad n_2=n_1+m+\bar{m} \quad ; \quad d_1=d_1 \quad ; \quad m=\bar{m}$$

Table C.13: Chan-Paton groups and tadpole conditions for the w_2p_3 models (real charges).

Mult.	Number	Rep.
C	1	$(1,1,1,1,1,Adj)$
C	2	$(F,F,1,1,1,1),(1,1,F,F,1,1)$
C	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ if USp ; $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ if SO	$\begin{pmatrix} (S,1,1,1,1,1),(1,S,1,1,1,1) \\ (A,1,1,1,1,1),(1,A,1,1,1,1) \end{pmatrix}$
C	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ if USp ; $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ if SO	$\begin{pmatrix} (1,1,S,1,1,1),(1,1,1,S,1,1) \\ (1,1,A,1,1,1),(1,1,1,A,1,1) \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ if USp ; $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ if SO	$\begin{pmatrix} (1,1,1,1,1,S) \\ (1,1,1,1,1,A) \end{pmatrix}$
C	$2^{r_2+r_3}\ \lvert k_2\,k_3\rvert + 2$	$(1,F,1,1,1,F+\bar{F})$
C	$2^{r_2+r_3}\ \lvert k_2\,k_3\rvert - 2$	$(F,1,1,1,1,F+\bar{F})$
C	$2\ 2^{\frac{r_2+r_3}{2}}$	$(F,1,1,F,1,1),(1,F,F,1,1,1)$
C	$2\ 2^{\frac{r_2+r_3}{2}}$	$(1,1,1,F,1,F+\bar{F})$
C_L	$2^{r_2+r_3}\ 2\ \lvert k_2\,k_3\rvert - 1 + 2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert + \eta_1 + 2^{\frac{r_2}{2}}\lvert k_2\rvert$	$(1,1,1,1,1,A)$
C_L	$2^{r_2+r_3}\ 2\ \lvert k_2\,k_3\rvert - 1 - 2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert - \eta_1 - 2^{\frac{r_2}{2}}\lvert k_2\rvert$	$(1,1,1,1,1,S)$
C_L	$2^{r_2+r_3}\ 2\ \lvert k_2\,k_3\rvert - 1 + 2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert + \eta_1 - 2^{\frac{r_2}{2}}\lvert k_2\rvert$	$(1,1,1,1,1,\bar{A})$
C_L	$2^{r_2+r_3}\ 2\ \lvert k_2\,k_3\rvert - 1 - 2^{\frac{r_2+r_3}{2}}\eta_1\lvert k_2\,k_3\rvert - \eta_1 + 2^{\frac{r_2}{2}}\lvert k_2\rvert$	$(1,1,1,1,1,\bar{S})$
C_L	$2^{\frac{r_1+r_3}{2}+r_2}\ 2\ \lvert k_2\rvert$	$(1,1,1,1,F,F)$

Table C.14: Open spectra of the w_2p_3 models (real charges).

C.7 Orientifolds of the $w_1 w_2 p_3$ Models

untwisted SUGRA	untwisted C	twisted C	twisted V
$N = 1$	$1 + 3 + 3$	0	0

Table C.15: Unoriented closed spectra of the $w_1 w_2 p_3$ models.

$$\begin{pmatrix} USp(n) \\ SO(n) \end{pmatrix} \otimes \begin{pmatrix} USp(d_1) \\ SO(d_1) \end{pmatrix} \otimes \begin{pmatrix} USp(d_2) \\ SO(d_2) \end{pmatrix} \otimes U(m)$$

$$n + m + \bar{m} = 8\ 2^{-\frac{r}{2}}$$

$$d_1 + 2^{-\frac{r_2}{2} - \frac{r_3}{2}}\ |k_2 k_3|\ (m + \bar{m}) = 8\ 2^{-\frac{r}{2}}$$

$$d_2 = 8\ 2^{-\frac{r}{2}} \qquad ; \qquad m = \bar{m}$$

Table C.16: Chan-Paton groups and tadpole conditions for the $w_1 w_2 p_3$ models.

Mult.	Number	Rep.
C	3	$(Adj,1,1,1),(1,Adj,1,1)$ $(1,1,Adj,1),\ (1,1,1,Adj)$
C	$2^{r_2+r_3}\ 2\ \lvert k_2\,k_3\rvert$	$(F,1,1,F+\bar{F})$
C_L	$2^{r_2+r_3}\ 4\ \lvert k_2\,k_3\rvert+2^{\frac{r_2+r_3}{2}}\eta_1 2\lvert k_2\,k_3\rvert+2^{\frac{r_2}{2}}2\lvert k_2\rvert$	$(1,1,1,A)$
C_L	$2^{r_2+r_3}\ 4\ \lvert k_2\,k_3\rvert-2^{\frac{r_2+r_3}{2}}\eta_1 2\lvert k_2\,k_3\rvert-2^{\frac{r_2}{2}}2\lvert k_2\rvert$	$(1,1,1,S)$
C_L	$2^{r_2+r_3}\ 4\ \lvert k_2\,k_3\rvert+2^{\frac{r_2+r_3}{2}}\eta_1 2\lvert k_2\,k_3\rvert-2^{\frac{r_2}{2}}2\lvert k_2\rvert$	$(1,1,1,\bar{A})$
C_L	$2^{r_2+r_3}\ 4\ \lvert k_2\,k_3\rvert-2^{\frac{r_2+r_3}{2}}\eta_1 2\lvert k_2\,k_3\rvert+2^{\frac{r_2}{2}}2\lvert k_2\rvert$	$(1,1,1,\bar{S})$
C_L	$2^{\frac{r_1+r_3}{2}+r_2}\ 4\ \lvert k_2\rvert$	$(1,1,F,F)$

Table C.17: Open spectra of the $w_1w_2p_3$ models.

C.8 Non-chiral Orientifolds

B rank $r_2 + r_3$	untwisted SUGRA	untwisted C	twisted C	twisted V
0	$N = 1$	$1 + 3 + 3$	$8 + 8$	0
2	$N = 1$	$1 + 3 + 3$	$6 + 6$	$2 + 2$
4	$N = 1$	$1 + 3 + 3$	$5 + 5$	$3 + 3$

Table C.18: Unoriented closed spectra of the p_2p_3 models.

untwisted SUGRA	untwisted C	twisted C	twisted V
$N = 1$	$1 + 3 + 3$	8	8

Table C.19: Unoriented closed spectra of the w_1p_2 models.

untwisted SUGRA	untwisted C	twisted C	twisted V
$N=1$	$1+3+3$	0	0

Table C.20: Unoriented closed spectra of the $w_1p_2p_3$ and $w_1p_2w_3$ models.

$U(n_1)\ \otimes\ U(n_2)\ \otimes\ U(d)\ \otimes\ U(m)$
$n_1+\bar{n}_1+n_2+\bar{n}_2+m+\bar{m}=32\ 2^{-\frac{r}{2}}$
$d+\bar{d}+2^{\frac{r_2}{2}+\frac{r_3}{2}}\ \lvert k_2k_3\rvert\ (m+\bar{m})=32\ 2^{-\frac{r}{2}}$
$n_1=\bar{n}_1\quad;\quad n_2=\bar{n}_2\quad;\quad d=\bar{d}\quad;\quad m=\bar{m}$

Table C.21: Chan-Paton groups and tadpole conditions for the p_2p_3 models (complex charges).

Multiplets	Number	Rep.
C	1	$(F,\bar{F},1,1)$,$(\bar{F},F,1,1)$ $(1,1,Adj,1)$
C	1	$(F,F,1,1)$,$(\bar{F},\bar{F},1,1)$ $(1,1,R+\bar{R},1)$
C	1	$(R+\bar{R},1,1,1),(1,R+\bar{R},1,1)$ $(1,1,R+\bar{R},1)$
C	$\frac{\lvert k_2\,k_3\rvert}{4}\ 2^{r_2+r_3}+1$	$(F,1,1,F),(1,F,1,F)$ $(\bar{F},1,1,\bar{F}),(1,\bar{F},1,\bar{F})$
C	$\frac{\lvert k_2\,k_3\rvert}{4}\ 2^{r_2+r_3}-1$	$(\bar{F},1,1,F),(1,\bar{F},1,F)$ $(F,1,1,\bar{F}),(1,F,1,\bar{F})$
C	$2^{\frac{r_2+r_3}{2}}$	$(1,1,F,F),(1,1,\bar{F},\bar{F})$
C	$\frac{\lvert k_2k_3\rvert}{2}\ (2^{r_2+r_3}+\eta_1\ 2^{\frac{r_2+r_3}{2}})+1$	$(1,1,1,A+\bar{A})$
C	$\frac{\lvert k_2k_3\rvert}{2}\ (2^{r_2+r_3}-\eta_1\ 2^{\frac{r_2+r_3}{2}})$	$(1,1,1,S+\bar{S})$

Table C.22: Open spectra of the magnetized p_2p_3 models (complex charges).

$\begin{pmatrix} USp(n_1)\ \otimes\ USp(n_2)\ \otimes\ USp(n_3)\ \otimes\ USp(n_4)\ \otimes\ USp(d_1)\ \otimes\ USp(d_2) \\ SO(n_1)\ \otimes\ SO(n_2)\ \otimes\ SO(n_3)\ \otimes\ SO(n_4)\ \otimes\ SO(d_1)\ \otimes\ SO(d_2) \end{pmatrix}\ \otimes\ U(m)$
$n_1+n_2+n_3+n_4+m+\bar{m}=32\ 2^{-\frac{r}{2}}$
$d_1+d_2+2^{\frac{r_2}{2}+\frac{r_3}{2}}\ \lvert k_2k_3\rvert\ (m+\bar{m})=32\ 2^{-\frac{r}{2}}$
$n_3+n_4=n_1+n_2+m+\bar{m}\quad;\quad d_1=d_2\quad;\quad m=\bar{m}$

Table C.23: Chan-Paton groups and tadpole conditions for the p_2p_3 models (real charges).

Multiplets	Number	Rep.
C	1	$(F,F,1,1,1,1,1)$,$(1,1,F,F,1,1,1)$ $(1,1,1,1,1,Adj,1)$,$(1,1,1,1,Adj,1,1)$
C	1	$(F,1,F,1,1,1,1)$,$(1,F,1,F,1,1,1)$ $(1,1,1,1,F,F,1)$
C	1	$(F,1,1,F,1,1,1)$,$(1,F,F,1,1,1,1)$ $(1,1,1,1,F,F,1)$
C	$2^{\frac{r_2+r_3}{2}}$	$(F,1,1,1,1,F,1)$, $(1,F,1,1,1,F,1)$ $(1,1,F,1,F,1,1)$, $(1,1,1,F,F,1,1)$
C	$2^{\frac{r_2+r_3}{2}}$	$(1,1,1,1,1,F,F+\bar{F})$
C	$\frac{\lvert k_2\,k_3\rvert}{4}\ 2^{r_2+r_3}+1$	$(F,1,1,1,1,1,F+\bar{F})$, $(1,F,1,1,1,1,F+\bar{F})$
C	$\frac{\lvert k_2\,k_3\rvert}{4}\ 2^{r_2+r_3}-1$	$(1,1,F,1,1,1,F+\bar{F})$, $(1,1,1,F,1,1,F+\bar{F})$
C	$\frac{\lvert k_2k_3\rvert}{2}\ (2^{r_2+r_3}+\eta_1\ 2^{\frac{r_2+r_3}{2}})+1$	$(1,1,1,1,1,1,A+\bar{A})$
C	$\frac{\lvert k_2k_3\rvert}{2}\ (2^{r_2+r_3}-\eta_1\ 2^{\frac{r_2+r_3}{2}})$	$(1,1,1,1,1,1,S+\bar{S})$

Table C.24: Open spectra of the magnetized p_2p_3 models (real charges).

$U(n)\otimes\begin{pmatrix} USp(d) \\ SO(d)\end{pmatrix}\otimes U(m)$
$n+\bar{n}+m+\bar{m}=16\ 2^{-\frac{r}{2}}$
$2d_1+2^{\frac{r_2}{2}+\frac{r_3}{2}}\ \lvert k_2k_3\rvert\ (m+\bar{m})=16\ 2^{-\frac{r}{2}}$
$n=\bar{n}\qquad;\qquad m=\bar{m}$

Table C.25: Chan-Paton groups and tadpole conditions for the w_1p_2 and $w_1p_2p_3$ models (complex charges).

Multiplets	Number	Rep.
C	1	$(Adj,1,1)$, $(1,Adj,1)$ $(1,1,Adj)$
C	2	$(1,Adj,1)$ $(A+\bar{A},1,1)$ or $(S+\bar{S},1,1)$
C	$\frac{\lvert k_2\,k_3\rvert}{2}\ 2^{r_2+r_3}$	$(F+\bar{F},1,F+\bar{F})$
C	$2\ \lvert k_2k_3\rvert\ (2^{r_2+r_3}+\eta_1\ 2^{\frac{r_2+r_3}{2}})$	$(1,1,A+\bar{A})$
C	$2\ \lvert k_2k_3\rvert\ (2^{r_2+r_3}-\eta_1\ 2^{\frac{r_2+r_3}{2}})$	$(1,1,S+\bar{S})$

Table C.26: Open spectra of the magnetized w_1p_2 and $w_1p_2p_3$ models (complex charges).

$$\left(\begin{array}{c} USp(n_1) \ \otimes \ USp(n_2) \\ SO(n_1) \ \otimes \ SO(n_2) \end{array}\right) \otimes \left(\begin{array}{c} USp(d_1) \\ SO(d_1) \end{array}\right) \otimes \ U(m)$$

$$n_1 + n_2 + m + \bar{m} = 16 \ 2^{-\frac{r}{2}}$$
$$2d_1 + 2^{\frac{r_2}{2}+\frac{r_3}{2}} \ |k_2 k_3| \ (m + \bar{m}) = 16 \ 2^{-\frac{r}{2}}$$
$$m = \bar{m}$$

Table C.27: Chan-Paton groups and tadpole conditions for the $w_1 p_2$ and the $w_1 p_2 p_3$ models (real charges).

Multiplets	Number	Rep.
C	1	$(Adj, 1, 1, 1),\ (1, Adj, 1, 1)$ $(1, 1, Adj, 1),\ (1, 1, 1, Adj)$
C	2	$(1, 1, Adj, 1),\ (F, F, 1, 1)$
C	$\frac{\lvert k_2\, k_3\rvert}{2}\ 2^{r_2+r_3}$	$(F, 1, 1, F + \bar{F}),\ (1, F, 1, F + \bar{F})$
C	$2\ \lvert k_2 k_3\rvert\ (2^{r_2+r_3} + \eta_1\ 2^{\frac{r_2+r_3}{2}})$	$(1, 1, 1, A + \bar{A})$
C	$2\ \lvert k_2 k_3\rvert\ (2^{r_2+r_3} - \eta_1\ 2^{\frac{r_2+r_3}{2}})$	$(1, 1, 1, S + \bar{S})$

Table C.28: Open spectra of the magnetized $w_1 p_2$ and $w_1 p_2 p_3$ models (real charges).

$$\left(\begin{array}{c} USp(n) \\ SO(n) \end{array}\right) \otimes \left(\begin{array}{c} USp(d) \\ SO(d) \end{array}\right) \otimes \ U(m)$$

$$n + m + \bar{m} = 8 \ 2^{-\frac{r}{2}}$$
$$d + 2^{\frac{r_2}{2}+\frac{r_3}{2}} \ |k_2 k_3| \ (m + \bar{m}) = 8 \ 2^{-\frac{r}{2}}$$
$$m = \bar{m}$$

Table C.29: Chan-Paton groups of the $w_1 p_2 w_3$ models.

Multiplets	Number	Rep.
C	3	$(Adj, 1, 1),\ (1, Adj, 1)$ $(1, 1, Adj)$
C	$2\ \lvert k_2\, k_3\rvert\ 2^{r_2+r_3}$	$(F, 1, F + \bar{F})$
C	$2\ \lvert k_2 k_3\rvert\ (2\ 2^{r_2+r_3} + \eta_1\ 2^{\frac{r_2+r_3}{2}})$	$(1, 1, A + \bar{A})$
C	$2\ \lvert k_2 k_3\rvert\ (2\ 2^{r_2+r_3} - \eta_1\ 2^{\frac{r_2+r_3}{2}})$	$(1, 1, S + \bar{S})$

Table C.30: Open spectra of the magnetized $w_1 p_2 w_3$ models.

C.9 w_2p_3 Models with *Brane Supersymmetry Breaking*

model	untwisted SUGRA	untwisted C	twisted C	twisted V
w_2p_3	$N=1$	$1+3+3$	8	8

Table C.31: Unoriented closed spectra of the w_2p_3 models with *brane supersymmetry breaking*.

$SO(n_1) \otimes SO(n_2) \otimes Usp(d_1) \otimes Usp(d_2) \otimes Usp(d_3) \otimes U(m)$
$n_1 + n_2 + m + \bar{m} = 16$
$d_1 + d_2 + \lvert k_2 k_3 \rvert \ (m + \bar{m}) = 16$
$d_3 = 8 \quad ; \quad n_2 = n_1 + m + \bar{m} \quad ; \quad m = \bar{m}$

Table C.32: Chan-Paton groups of the w_2p_3 models with *brane supersymmetry breaking*.

States	Number	Rep.
Scalars	2	$(Adj,1,1,1,1,1),(1,Adj,1,1,1,1),(1,1,Adj,1,1,1)$ $(1,1,1,Adj,1,1),(1,1,1,1,Adj,1),(1,1,1,1,1,Adj)$
C	2	$(Adj,1,1,1,1,1),(1,Adj,1,1,1,1),(1,1,A,1,1,1)$ $(1,1,1,A,1,1),(1,1,1,1,A,1),(1,1,1,1,1,Adj)$
Scalars	4	$(F,F,1,1,1,1),(1,1,F,F,1,1),(1,1,1,1,Adj,1)$
C	2	$(F,F,1,1,1,1),(1,1,F,F,1,1),(1,1,1,1,Adj,1)$
Scalars	4	$(F,1,F,1,1,1),(1,F,1,F,1,1)$
C	2	$(1,F,F,1,1,1),(F,1,1,F,1,1)$
C	$\|k_2\,k_3\|/2+2$	$(F,1,1,1,1,F+\bar{F})$
C	$\|k_2\,k_3\|/2-2$	$(1,F,1,1,1,F+\bar{F})$
Scalars	$\|k_2\,k_3\|-4$	$(F,1,1,1,1,F+\bar{F})$
Scalars	$\|k_2\,k_3\|+4$	$(1,F,1,1,1,F+\bar{F})$
C	2	$(1,1,1,F,1,F+\bar{F})$
Scalars	4	$(1,1,F,1,1,F+\bar{F})$
Scalars	$3\ \|k_2\,k_3\|-2-\|k_2\|$	$(1,1,1,1,1,A+\bar{A})$
Scalars	$\|k_2\,k_3\|+\|k_2\|$	$(1,1,1,1,1,S+\bar{S})$
Scalars	$3\ \|k_2\,k_3\|-2+\|k_2\|$	$(1,1,1,1,1,A+\bar{A})$
Scalars	$\|k_2\,k_3\|-\|k_2\|$	$(1,1,1,1,1,S+\bar{S})$
C_L	$3\ \|k_2\,k_3\|+2+\|k_2\|$	$(1,1,1,1,1,A)$
C_L	$\|k_2\,k_3\|-\|k_2\|$	$(1,1,1,1,1,S)$
C_R	$3\ \|k_2\,k_3\|+2-\|k_2\|$	$(1,1,1,1,1,A)$
C_R	$\|k_2\,k_3\|+\|k_2\|$	$(1,1,1,1,1,S)$
C_L	$\|k_2\|$	$(1,1,1,1,F,F)$
Scalars	$\|k_2\|$	$(1,1,1,1,F,F+\bar{F})$

Table C.33: Open spectra of the w_2p_3 models with *brane supersymmetry breaking*.

model	CP group	constraints	susy	chiral multiplets
p_3	$[U(n_1) \times U(n_2)]_9 \times$	$n_1 + n_2 = 16$	N=1	$(A + \bar{A}, 1, 1, 1)_{99}$ $(1, A + \bar{A}, 1, 1)_{99}$
	$U(d_1)_{5_1} \times U(d_2)_{5_2}$	$d_1 = d_2 = 8$		$(F + \bar{F}, F + \bar{F}, 1, 1)_{99}$
				$(1, 1, A, 1)_{55}$ $(1, 1, 1, A)_{55}$
				$(F, 1, F, 1)_{59}$ $(F, 1, 1, F)_{59}$
				$(1, \bar{F}, 1, F)_{59}$ $(1, F, F, 1)_{59}$
p_{23}	$[U(n_1) \times U(n_2)]_9 \times$	$n_1 + n_2 = 16$	N=1	$(A + \bar{A}, 1, 1)_{99}$ $(1, A + \bar{A}, 1)_{99}$
	$U(d)_{5_1}$	$d = 8$		$(F + \bar{F}, F + \bar{F}, 1)_{99}$ $(1, 1, A + \bar{A})_{55}$
				$(F, 1, F)_{59}$ $(\bar{F}, 1, \bar{F})_{59}$
				$(1, F, F)_{59}$ $(1, \bar{F}, \bar{F})_{59}$
p_{123}	$SO(n_o) \times SO(n_g) \times$	$\sum_i n_i = 32$	N=1	$(F, F, 1, 1)_{99}$ $(F, 1, F, 1)_{99}$
	$SO(n_h) \times SO(n_f)$			$(F, 1, 1, F)_{99}$ $(1, F, F, 1)_{99}$
				$(1, F, 1, F)_{99}$ $(1, 1, F, F)_{99}$

model	CP group	constraints	susy	hypermultiplets
$w_2 p_3$	$U(n)_9 \times$	$n = 8$	N=2	$2\ (A, 1, 1)_{99}$ $2\ (1, A, 1,)_{5_1 5_1}$
	$U(d_1)_{5_1} \times SO(d_2)_{5_2}$	$d_1 = d_2 = 8$		$2\ (F, F, 1)_{95_1}$
$w_1 p_2$	$U(n)_9 \times SO(d_1)_{5_1}$	$n = d_1 = 8$	N=2	$2\ (A, 1)_{99}$
$w_1 p_2 p_3$	$U(n)_9 \times SO(d_1)_{5_1}$	$n = d_1 = 8$	N=2	$2\ (A, 1)_{99}$
$w_1 w_2 p_3$	$SO(n)_9 \times$	$n = 8$	N=4	–
	$SO(d_1)_{5_1} \times SO(d_2)_{5_2}$	$d_1 = d_2 = 8$		–
$w_1 p_2 w_3$	$SO(n)_9 \times SO(d_1)_{5_1}$	$n = d_1 = 8$	N=4	–
$p_1 w_2 w_3$	$U(n)_9$	$n = 8$	N=4	–
$w_1 w_2 w_3$	$SO(n)_9$	$n = 8$	N=4	–

Table C.34: Open spectra of the undeformed $\mathbb{Z}_2 \times \mathbb{Z}_2$ shift-orientifolds.

C.10 Open Spectra of the Undeformed $\mathbb{Z}_2 \times \mathbb{Z}_2$ Shift-orientifolds

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