Neural Networks and Volterra Series for Time-Domain Power Amplifier Behavioral Models

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ABSTRACT: This paper presents a black-box model that can be applied to characterize the nonlinear dynamic behavior of power amplifiers (PAs), including strong nonlinearities and memory effects. Feedforward time-delay Neural Networks (TDNN) are used to extract the model from a large-signal input-output time-domain characterization in a given bandwidth; furthermore, explicit formulas to derive Volterra kernels from the TDNN parameters are also presented. The TDNN and related Volterra models can predict the amplifier response to different frequency excitations in the same bandwidth and power sweep. As a case study, a PA, characterized with a two-tone power swept excitation, is modeled and simulations are found in good agreement with training measurements; moreover, a model validation with two tones of different frequencies and spacing is also performed.

Keywords: dynamic behavioral model; neural network; power amplifier; Volterra series

I. INTRODUCTION

The development of analog RF and microwave circuits for advanced communications requires efficient and accurate CAD modeling approaches for key components, such as amplifiers, mixers, and oscillators, to be exploited within a circuit or system environment to derive overall system performances.

In recent years, the measurement-based modeling of power amplifiers (PAs) has been identified as a particularly critical issue due to the peculiar features of their nonlinear behavior (such as slow and fast memory effects) and the related impact on the system-level distortion and intermodulation. Behavioral models have become the object of extensive research during the last few years as a black-box, technology independent tool to model off-the-shelf PAs starting from conventional or ad-hoc measurements [1, 2]. Behavioral models generally have the structure of a dynamic nonlinear system meant to provide an output variable as a function of one or more input time series. The model is considered as a “black-box,” meaning that no knowledge of the internal structure is required and the modeling information is completely included in the device external responses [3]. The process of converting measured data into model equations relies on fitting techniques [4]; however, many of the available methods are useful when the input and output data are well behaved over a given range of the independent variables, and when the object is known to follow a well-defined, also parameterized, mathematical model structure. On the other hand, problems arise when the complex internal
behavior of the modeled object as well as the model structure are previously unknown. In such cases, techniques that allow a wider degree of adaptability are useful. In particular, a new technique that has received increasing attention for the development of PA behavioral models is the neural network (NN) approach \[5, 6\], since model tailoring to the element under study only needs a training procedure based on input-output time- or frequency-domain data sets.

In this article, feedforward time-delay neural networks (TDNN) with finite memory duration are exploited to enable to learn a nonlinear behavior with medium-to-strong memory effects, along with high-order nonlinearity, by carrying out the training with input-output time-delayed data samples at different power levels, simultaneously \[7, 8\]. This fact turns out of outstanding importance to build behavioral models of PAs, which are able to simulate the nonlinear performance with different input spectra and power levels. This approach aims at extracting a nonlinear relationship from a time-domain characterization set, to build an input-output model able to generalize the nonlinear dynamic behavior of electronic components for input waveforms not used at the training stage.

Volterra series approaches have played an important role in behavioral, black-box modeling of dynamic nonlinear elements. However, behavioral models based on the Volterra series hold their validity only for weak nonlinearities and calculating the kernels is a tiresome exercise that requires heavy characterization efforts, especially when multitone intermodulation is a matter of interest. A further advantage of this approach is that we exploit NNs able to derive equivalent (truncated) Volterra series models. Our proposal is to obtain an analytical expression for the model, either as neural analytical model or as Volterra series expansion, calculated in function of the NN model parameters. A new algorithm to extract the Volterra kernels in the time-domain directly from the NN parameters has been found \[9\], and the resulting model represents a very good approximation of the nonlinear behavior, with only three order kernels, even with medium-to-strong nonlinearities. This fact can be a great asset for large-signal analysis, because Volterra series behavioral models provide open information about the nonlinear amplifier behavior, and their implementation in circuit simulators is generally less time-consuming.

In this article, we show, as a case study, the results obtained from a PA input-output time-domain characterization to build both neuronal- and Volterra-series-based black-box models. The organization of the article is the following: in the next section, the NN model proposed is described; in Section III, the Volterra modeling for PAs is presented; in Section IV, the procedure for Volterra kernels extraction using the parameters of a NN is explained; in Section V, the model implementation into a commercial CAD tool is described; in Section VI, the PA characterization and the modeling results are compared. Finally, some conclusions are reported in Section VII.

II. NEURAL NETWORK MODEL

The NN used to model the PA is a feedforward TDNN with three layers, the input time-domain voltage samples and their delayed replies in the first one, a nonlinear hidden layer, and a linear output. All the neurons have bias values. The architecture is shown in Figure 1, whereas eq. (1) represents the corresponding input-output analytical expression in the discrete time, when the nonlinear activation function for the hidden neurons is the hyperbolic tangent.

\[
V_{\text{out}}(n) = b_0 + \sum_{k=1}^{M} v_k \tanh \left[ b_k + \sum_{k=0}^{M} w_{k,k+1} V_{\text{in}}(n-k) \right] \quad (1)
\]

This NN is trained with PA time-domain measurements. The input and output waveforms are expressed in terms of their samples in the time domain. \(M\) is the number of the input lags and represents the finite duration of the memory effects of the PA. Since phase delay changes with the input power, a minimum delay element (\(\Delta\)) can be added at the NN output. In this way, input memory has to take into account only the delay variation with the input power. The tap delay is calculated from \((2 \times \text{BW})^{-1}\) to avoid spectral aliasing, where BW is the desired characterization bandwidth. The actual sampling time could be larger (multiple)
than the tap delay. $H$ is the number of hidden neurons to perform the best fitting to the training waveforms without overfitting problems. The NN parameters are optimized with the backpropagation technique, based on the Levenberg–Marquardt algorithm. Once the NN has been trained, both Sigmoidal and Volterra series expansion models (explained in the next Section), calculated in function of the NN parameters, can be extracted. The procedure is presented in Section IV.

### III. VOLterra Modeling of a Power Amplifier

The Volterra approach characterizes a system as a mapping between two function spaces, which represent the input and output spaces of the system [10]. The Volterra series is an extension of the Taylor series representation to cover dynamic systems, and it can be exactly formulated by a converging infinite series

$$y(t) = \sum_{p=1}^{\infty} y_p(t)$$

The terms $h_0, h_1, h_2, \ldots h_p$ are known as the Volterra kernels of the system. In general, $h_p$ is the $p$th-order kernel of the series that completely characterizes the $p$th-order nonlinearity of the system. The series can be described in the time-domain or in the frequency-domain. In the time-domain, the Volterra description can be defined using a continuous domain series in eq. (4) or a discrete-time version in eq. (5), where inputs and outputs are sampled versions of the continuous time input-output signals. In practice, the Volterra series must be truncated to avoid summations over an infinite number of terms. A sufficiently accurate model can be obtained by using a finite number of terms [11]:

$$y(t) = y_0 + \int_{t=0}^{\infty} h_1(\tau)x(t-\tau) \, d\tau$$

$$+ \int_{\tau_1=0}^{\infty} \int_{\tau_2=0}^{\infty} h_2(\tau_1, \tau_2) \times x(t-\tau_1)x(t-\tau_2) \, d\tau_1 \, d\tau_2 + \ldots (4)$$

$$y(n) = h_0 + \sum_{k=0}^{\infty} h_1(k)x(n-k)$$

$$+ \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} h_2(k_1, k_2)x(n-k_1)x(n-k_2) + \ldots (5)$$

A Volterra series model in discrete form that represents the nonlinear dynamic behavior of a PA is presented in eq. (6), where $h_p(k_1, \ldots,k_p)$ is the $p$th-order Volterra kernel. Typically, in the series representation, only terms up to the third-order are included. The summation interval $[0–M]$ is limited to the practically finite duration of the memory effect in the device.

$$y_{out}(n) = h_0 + \sum_{k=0}^{M} h_1(k)y_{in}(n-k)$$

$$+ \sum_{k_1=0}^{M} \sum_{k_2=0}^{M} h_2(k_1, k_2)y_{in}(n-k_1)y_{in}(n-k_2)$$

$$+ \sum_{k_1=0}^{M} \sum_{k_2=0}^{M} \sum_{k_3=0}^{M} h_3(k_1, k_2, k_3) \times y_{in}(n-k_1)y_{in}(n-k_2)y_{in}(n-k_3) (6)$$

The Volterra series is the most general, rigorous modeling approach for systems characterized by nonlinear dynamic phenomena. The Volterra series analysis is well suited to the simulation of nonlinear microwave devices and circuits, in particular in the weakly and mildly nonlinear regime, where a low number of kernels (generally up to the third-order) are able to capture the device behavior, e.g. for PA distortion analysis [12]. We consider that the key to system modeling by means of Volterra series is capturing the Volterra kernels that represent the system. Once they are known, the system response to any arbitrary input can be predicted with relative ease. Unfortunately, kernel identification is a difficult process, which has been studied for some time. Several methods have been proposed for kernels identification [13, 14]. However, at microwave frequencies, suitable instrumentation for the measurement of the kernels is still lacking [15]. In spite of this drawback, the Volterra series is actually used for microwave circuit design, by means of complex and time-consuming calculations [16, 17].

We have found a procedure in Ref. 9 that allows generating the Volterra series and its kernels, for the modeling of a nonlinear electronic device with memory, using the weights and bias values of a feedforward TDNN, after it has been trained with time-domain measurements. This is briefly explained in the next Section, where it is also presented a new
IV. VOLterra KERNELS EXTRACTION

The procedure to obtain the Volterra kernels is based on a NN like the one presented in Figure 1 and expressed in analytical form by eq. (1), when the activation function is the hyperbolic tangent. This NN is trained with time-domain device measurements, up to a predetermined number of epochs or a desired accuracy. Once the NN has been trained and its weights and bias values have been fixed, the NN input-output expression is developed as a Taylor series around the bias values of the hidden nodes, that is calculating the hidden neurons activation function derivatives for zero input voltage. If the resulting expression is arranged in common terms, the Volterra kernels can be easily identified and combined to form a Volterra series model representation of the device under study. The general formula that builds the Volterra model, including up to any order kernel, using the NN parameters (weights and bias), is shown in Ref. 7, being \( f^p(b_h) \) the hyperbolic tangent derivative of order \( p \) calculated in the neuron bias. The kernels can be identified as the terms outside brackets.

\[
V_{\text{out}}(n) = \sum_{p=0}^{\infty} \prod_{h=1}^{H} \left( \sum_{k_1=0}^{M} \cdots \sum_{k_h=0}^{M} \right) \times w_{h,k_1+1} \cdots w_{h,k_h+1} f^p(b_h) \times [V_{\text{in}}(n-k_1) \cdots V_{\text{in}}(n-k_h)]^p 
\]

Even if a Sigmoidal neural model can reach a high degree of accuracy in modeling a nonlinear behavior, a Volterra series model extracted from Taylor expansion is close to the neural model only for low-to-medium power signals.

A more straightforward manner to calculate the kernels has been suggested in Ref. 18, where it is proposed a new kind of NN, with a particular topology, having distinct polynomials series with trainable neurons.
coefficients (i.e. $ax + bx^2 + cx^3 + \ldots$) as activation functions in the hidden layer. Volterra kernels, this time, can be directly extracted from neural network parameters at the end of the training process. This approach, however, requires a special training algorithm for updating the polynomials coefficients ($a, b, c$).

In this article, the identification of the Volterra kernels has been achieved using a simpler NN model, like the one presented in Figure 1, where the activation function in each hidden neuron is a $p$-order power (instead of a polynomial series with optimizable coefficients), using also a standard training algorithm such as backpropagation for the NN weights and bias updating. Equation (8) represents the corresponding input-output analytical expression in the discrete time. This simplified form is generally easier to implement in the most used NN software, obtaining very similar results for the purpose of this work.

$$V_{\text{out}}(n) = b_0 + \sum_{h=1}^{H} v_h b_h^n + \sum_{k=0}^{M} w_{h,k+1} V_{\text{in}}(n-k)^p \tag{8}$$

For example, developing the NN output expressed in eq. (8) for third degree polynomials ($p = 3$) yields

$$V_{\text{out}}(n) = b_0 + \sum_{h=1}^{H} v_h b_h + \sum_{k=0}^{M} w_{h,k+1} V_{\text{in}}(n-k) + V_{\text{in}}(n-k) V_{\text{in}}(n-k-1)$$

$$+ \sum_{k=0}^{M} w_{h,k+1} V_{\text{in}}(n-k)^2 V_{\text{in}}(n-k)^2$$

Comparing terms in the Volterra expansion in eq. (6) and the ones obtained in eq. (9), the Volterra kernels are easily identified, and can be calculated using eq. (10).

Even if third-order polynomial NNs perform less degree of global accuracy respect to sigmoidal ones in modeling a nonlinear behavior, we will demonstrate that they perform a tradeoff between small- and large-signal accuracy, when trained with small- to large-signal waveforms, simultaneously, thus ena-
bling Volterra models, directly extracted from polynomial NN parameters as in eq. (10), to efficiently represent low-to-high distortion behaviors as well.

\[
\begin{align*}
h_0 &= b_0 + \sum_{h=1}^{H} v_h b_h^3 \\
h_1(k) &= \sum_{h=1}^{H} 3v_h b_h^2 w_{h,k} \\
h_2(k_1, k_2) &= \begin{cases} \\
\sum_{h=1}^{H} 6v_h b_h w_{h,k_1} w_{h,k_2} & (k_1 = k_2 = k) \\
\sum_{h=1}^{H} 3v_h b_h (w_{h,k})^2 & (k_1 = k_2 = k) \\
\sum_{h=1}^{H} 3v_h w_{h,k_1} w_{h,k_2} w_{h,k_3} & (k_1 = k_2 = k_3 = k) \\
\end{cases} \\
h_3(k_1, k_2, k_3) &= \begin{cases} \\
\sum_{h=1}^{H} v_h (w_{h,k})^3 & (k_1 = k_2 = k_3 = k) \\
\end{cases}
\end{align*}
\]

(10)

V. IMPLEMENTATION OF THE MODEL IN ADS

The extracted neural model for the PA voltage transfer function have been implemented on a commercial CAD simulator such as Agilent advanced design system (ADS), where the PA input-output behavior can be simulated and compared with measurements.

The behavioral model has been implemented on the circuit schematic of the ADS simulator using the symbolically defined device (SDD) from the equation-based nonlinear device library. A proprietary code is used to extract, from the NN file, the corresponding analytical model in its general form shown in eqs. (1) or (8), for Sigmoidal or Volterra models, respectively. The model equation for the voltage at the output port, expressed as a function of the voltage at the first input port and its delayed replies at the other input ports, is then printed in text format on a netlist file linked to the circuit schematic. A constant delay, corresponding to the minimum envelope delay (∆) in Figure 1, is added at the output port. The complete model schematic is shown in Figure 2. Efforts in model implementation are greatly reduced in this way. Validation of the models against measurements is then carried out in ADS. Both models have demonstrated very fast simulation convergence in the characterization power range.

VI. EXPERIMENTAL RESULTS

A. Measurement Setup

As a case study, we consider a Cernex 2267 PA, with a 2–6 GHz bandwidth, a 42 dB gain, and 1 dB compression at 32 dBm, stimulated with two tones at center frequency 4.1 GHz and frequency spacing 100 MHz, generated by an HP83640A synthesized sweeper and an Anritsu MG3692 CW generator, respectively. Each power is swept from −30 to −8 dBm, that is 4 dBm over the 1 dB compression point for the combined input power.

The amplifier output has been connected to a Tek11801B digital sampling oscilloscope (DSO),

![Figure 4. MSE comparison between Polynomial and Sigmoidal NN.](image)

![Figure 5. Time-domain comparison between measurements (○), Sigmoidal (solid) and Volterra (dashed) model simulation of global (upper) and detailed (lower) voltage response, with the training input tones at 4.05 and 4.15 GHz, for P_{in} = −25, −13, −5 dBm, respectively.](image)
that has been triggered with the common 10 MHz RF reference from the generators; despite of this low frequency trigger, repetitive sampling allows the instrument to exploit the full 50 GHz analog bandwidth of the testing probe. The waveform oversampling is used to decrease the noise level through average settings.

To compare the frequency response of the amplifier measurements and simulations of the two behavioral models, an HP71000 spectrum analyzer (SA) has been connected to the amplifier output, and spectra have been measured for different input power levels. For full automatic control of the measurement setup and data acquisition, controlling programs throughout the GPIB link have been developed using the Matlab\textsuperscript{1} instrument drivers. The characterization setup is shown in Figure 3.

### B. Model Training

Experimental sampled data, for each input power, have been linked together in a single input and output vector, to simultaneously train the NN with the full power range. To build the NN input matrix, the input vector has been copied and delayed as many times, as necessary to take into account the duration of memory effects. A characterization bandwidth of 22 GHz has been used to calculate the tap delay, to take into account up to fifth-order harmonic distortion.

Both the Sigmoidal and the Volterra TDNN, represented by eqs. (1) and (8), respectively, have been trained with the full power sweep, with $M = 4$ memory deep and $N = 5$ hidden neurons, for sake of comparison. For larger memory duration the NNs tend to become unstable. Results are shown in Figure 4. As expected, the MSE as function of the iteration number has better reduction at the end of the training process in the TDNN using the Sigmoidal model than the Volterra model.

At the end of the training process, measurements and ADS transient simulation waveforms of both models have been compared in Figure 5 for three input levels.

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**Figure 6.** Power comparison between measurements (○), Sigmoidal model (solid) and Volterra model (dashed) simulations of intermodulation response, with the training input tones at 4.05 and 4.15 GHz, for $P_{in}$ (at $f_1 = 4.15$ GHz), IMD3 (at $2f_2 - f_1 = 4.25$ GHz) and, only for the Sigmoidal model, IMD5 (at $3f_2 - 2f_1 = 4.35$ GHz).

**Figure 7.** Time-domain comparison between measurements (○), Sigmoidal (solid) and Volterra (dashed) model simulation of global (upper) and detailed (lower) voltage response, with the validation input tones at 4.29 and 4.31 GHz, for $P_{in} = -25, -13, -5$ dBm, respectively.

**Figure 8.** Power comparison between measurements (○), Sigmoidal model (solid) and Volterra model (dashed) simulations of intermodulation response, with the validation input tones at 4.29 and 4.31 GHz, for $P_{out}$ (at $f_1 = 4.31$ GHz), IMD3 (at $2f_2 - f_1 = 4.33$ GHz) and, only for the Sigmoidal model, IMD5 (at $3f_2 - 2f_1 = 4.35$ GHz).
To see the modeling performance in the frequency domain, ADS harmonic balance simulations and spectra measurements from an HP7000 spectrum analyzer have been compared. The results are presented in Figure 6, where power amplitudes at fundamental, third-order and, only for the Sigmoidal model, fifth-order intermodulation have been plotted against the input power. Concluding, both models show well matched behaviors both in the time- and frequency-domain in the full power range. Better performance of the Sigmoidal model is hardly discriminated.

C. Model Validation

To demonstrate the validity of the modeling approach for signals not used in the training stage, measurement data were obtained by stimulating the amplifier with two-tone input signals at different frequencies, in particular 4.3 GHz center frequency, 20 MHz frequency spacing, and the same power sweep. Time-domain envelopes from ADS model simulations and experimental measurements are shown in Figure 7 for three input levels, whereas frequency-domain comparisons are presented in Figure 8. As it can be seen, the third-order Volterra model is less accurate than the Sigmoidal model around the 1 dB compression point, still demonstrating a good modeling performance for low-to-high distortion, especially in the time-domain.

VII. CONCLUSIONS

New large-signal behavioral models for the nonlinear dynamic modeling of PAs, based on feed-forward TDNN, have been presented. Measurements have demonstrated the validity of the modeling approach for high distortion and memory effect characterization in PA. Although the method employs time-domain characterization data, also frequency domain simulations are consistent with intermodulation measurements.

Moreover, an efficient procedure to extract time-domain Volterra kernels from the parameters of a polynomial neural network has been proposed, thus providing a simple way to construct very compact and accurate Volterra models to be used over 1 dB compression. Even if Sigmoidal models are often more accurate to characterize hard nonlinearities, Volterra models provide open information about the nonlinear amplifier behavior, and their implementation in circuit simulators is generally less time-consuming, especially when a Volterra model dedicated library is provided.

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