Interbank Exchanges, Liquidity Management and Banking Crises

by
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1. Introduction

In a theoretical framework where liquidity crises are not only caused by bank runs, and where there is uncertainty about the proportion of depositors who may want to withdraw deposits, we show that abandoning the hypothesis of a representative bank (as in Diamond and Dybvig (hereafter DD) models), makes interbank exchanges relevant.

In this paper we consider a bank system characterized by two banks ($i=A,B$), and show that the probability of a banking crisis at a single bank decreases when interbank transactions are introduced - relative to an autarchic case where banks stand alone. With given interest rates, the total amount of liquidity is lower than in the autarchic case: the possibility of implementing interbank exchanges implies better bank liquidity management and lower liquidity risk.

Bank investment decisions are influenced by the following economic variables: costs of liquidation, deposit interest rates, loan interest rates and interbank interest rates. In particular, we show that if the cost of access to the interbank market is lower than the expected cost of being forced to liquidate long-run assets, banks gain by participating in the interbank market.

We interpret the banking system as being driven by uncertainty regarding the withdrawal of deposits by "impatient" depositors. Banks discover the true value of the proportion of their impatient depositors only *ex-post*. Once this information becomes available, banks determine whether deposit losses can be financed through interbank exchanges. In particular, if shocks are negatively correlated, interbank exchanges are possible and profitable, at least in our two-period game. The hypothesis that the existence of an interbank market is linked to the different local shocks can be related, as observed by Bhattacharya and Gale [1987, page 74], to the fact that "the intermediaries are banks, distinguished by geographical location. Depositors attach themselves to particular banks by location proximity. Then local economic conditions in the area where a bank operates will have a marked impact on its demand for liquidity". Bhattacharya and Gale (1987)[1] seem to suggest that the existence and the success of the interbank market is strongly related to the degree of spatial diversification of the banking market[2].

In our model we show that bank *A* and bank *B*'s liquidity strategies are strategic substitutes: the reaction functions have a negative slope and the non-cooperative solution is superior to the autarchic one. This can be interpreted as the consequence of strategic choices by banks, which interact by maximizing their expected profit function given the liquidity level of the other bank(s). The resulting liquidity level held by each bank is lower than that it would hold in autarchy. In this sense, a banking system in which banks interact through an interbank market has better liquidity management and a lower probability of bank failure. However, we show that this equilibrium is "inferior" to the cooperative one. A greater liquidity investment would allow banks to obtain, with the same probability of failure, an expected profit greater than they get in the non-cooperative equilibrium.

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[1] See also other authors: for example, Smith [1991], Chari [1989], Donaldson [1992] McAndrews and Roberds [1995]; for some empirical applications see, for instance, Haubrigh [1990] and Calomiris [1993].

[2] On the theory of spatial differentiation of the banking market see, among others, Chiapporri, Perez-Castrillo and Verdier [1995] and the bibliographical references quoted there in.

By assuming that liquidity shocks differ across banks located in different regions, the paper is developed as follows. In section 2 we present the structure of the model. In section 3 we analyze the problem of the bank's optimal liquidity choice, and we derive cooperative and non-cooperative solutions. Section 4 concludes and offers some suggestions for further developments of the model.

2. *The model*

The model is developed over a two-period time horizon, which is defined by three points in time $t=0,1,2$. At $t=0$ there are two equal banks ($i=A,B$); each bank has an initial endowment of resources equal to D. Part of these resources is invested in liquidity, L_i, while the remaining part is invested in long-run assets, $(D-L_i)$. The return on short-run assets is nil; the return on long-term assets is positive and such that a unit investment at $t=0$ returns $R>1$ at $t=2$. We assume that the long-run return is greater than the unit liquidation value of the deposits, *i.e.* $\sqrt{R} > d$, where $d>1$ is the reimbursement value of a unit deposit after one period, at $t=1$. At $t=1$, long-run assets can be liquidated at a cost, by paying a fraction $\tau \in [0,1]$ for each unit of liquidated assets[3]. Finally, differently from an autarchic economy, the existence of "many" banks permits them to interact and to realize profitable interbank exchanges. The interbank return is positive and equal to $n \in [d, \sqrt{R}]$.

At $t=0$ banks face aggregate and individual uncertainty about the proportion of depositors that will want to withdraw funds

[3] The results we obtain in the present paper don't change substantially if we introduce the more realistic hypothesis of increasing marginal cost.

at $t=1$. In particular[4], *ex-ante* banks know neither the aggregate number of impatient depositors b, nor the proportion of impatient depositors that at $t=1$ will withdraw funds from bank i, b_i.

At $t=1$ each bank observes the true value of b_i. Given the investment strategies adopted at $t=0$ banks can now establish if the total value of their liquidity is enough to satisfy the revealed impatient depositors' demand. It is easy to verify that banks can successfully fulfill their liquidity function at $t=1$, if and only if the following condition is satisfied: $L_A + L_B \geq (b_A + b_B)Dd$.

We proceed now to studying the conditions that ensure banks against liquidity risk. For simplicity we focus on one of them, say bank A. If there is an aggregate liquidity crisis at $t=1$ bank A can be either a borrower and a lender in the interbank market, or it can stand alone.

2.1 Interbank loans and bank failure

At $t=1$, if the demand from bank A-depositors is such that $b_A \leq [L_A + L_B - b_B Dd]/Dd$, and that from bank-$B$ depositors is such that $b_B > L_B / Dd$, bank A could be a net lender in the interbank market - so that all depositors are satisfied at $t=1$. However, if it wants to be completely successful in its monetary function, bank A must also be able to satisfy depositors at $t=2$. If bank A wants to be solvent at $t=1$ and at $t=2$ two conditions must be satisfied:

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[4] Bhattacharya and Gale [1987], on the contrary, assume there is only individual uncertainty. The value of b at the aggregate level is known to be $b = \sum_i p_i b_i$, where b_i is the proportion of impatient people among the depositors
of bank i, and p_i is the probability that financial intermediaries will be of type i.

$$\begin{cases} b_A \le [g_A + g_B - b_B d]/d = b(L_A, L_B)\ ,\ b_B > g_B / d \quad \text{and} \\ n(b_B d - g_B) + R(1-g_A) \ge (1-b_A)d^2,\ i.e.\ \ b_A \ge \dfrac{g_A(R-1)-(R-d^2)-(n-1)(b_B d - g_B)}{d(d-1)} = a(L_A, L_B), \end{cases}$$

where g_i is the proportion of deposits initially invested in liquid assets, *i.e.* $g_i = L_i / D$. Since $a(L_A, L_B) < b(L_A, L_B)$, the solution to the above system is $b_A \in [a(L_A, L_B), b(L_A, L_B)]$.

Obviously for $b_A \in [0, a(L_A, L_B)]$ the bank cannot completely satisfy the depositors' demand. In particular, we can observe that in this case the bank’s earning are insufficient, because of a too conservative investment strategy. In fact, the bank has invested too many resources in liquidity, with respect to impatient depositors' demand. Hence, illiquid assets are not enough at *t=2* to satisfy all patient depositors' demand. We define this situation as a *bank failure*, and distinguish it from the liquidity crisis due to an excessive withdrawal of liquid resources by impatient depositors at *t=1*, defined in the literature as *bank runs*.

If we now assume, as in the DD models, that bank *A* is a representative bank, two conditions must be verified for this bank, in order that all depositors' demand be satisfied at *t=1* and *t=2*:

(i) at *t=1* $L_A \ge bDd$, *i.e.* $b_A \le g_A / d$ and

(ii) at *t=2* $R(D - L_A) + L_A - b_A Dd < (1 - b_A) Dd^2$,

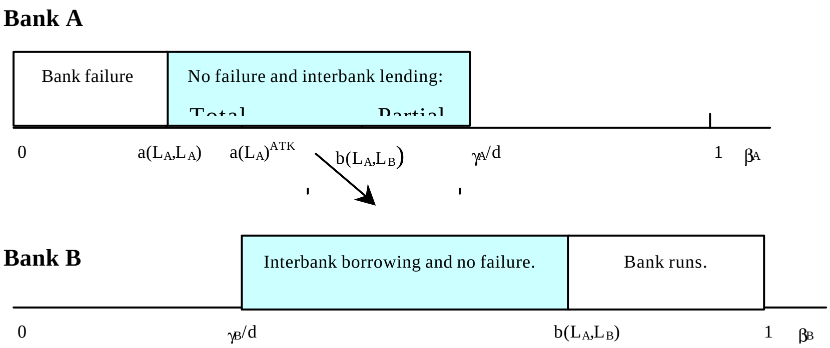
which implies: $b_A < a(L_A)^{ATK}$, where $a(L_A)^{ATK} = \dfrac{g_A(R-1) - (R-d^2)}{d(d-1)}$ and *ATK* means autarky.

The function $a(L_A)^{ATK}$ can take different values, depending upon the bank portfolio composition. In particular $a(L_A)^{ATK}$ can be lower or higher than g_A/d. If higher, deposit withdrawals are larger than those the bank can fund: therefore, condition (i) above is not satisfied. However, this does not apply since $b_A > a(L_A)^{ATK} > g_A/d$: hence the only relevant solutions for b_A (for which the bank is not subject to liquidity crises) are those such that $a(L_A)^{ATK} \le b_A < g_A/d$.

Now, if we proceed to a simple comparison between the value of the lower extreme of this interval and the one calculated in the case of interbank exchanges, we can verify that $a(L_A, L_B) < a(L_A)^{ATK}$. This means that doing away with the assumption of an autarchic system decreases the bank failure risk due to a too prudent bank asset management.

Graphically the interval for which bank A is a net lender on the interbank market and is not subject to any liquidity crisis, provided that $b_B > g_B/d$, can be shown as follows:

Fig. 1 – Interbank exchanges and bank failure



Until now we have considered the case where bank A can completely satisfy bank B's demand for liquidity. However, it is possible that, given the observed values of b_A and b_B, bank A's excess liquidity is enough to satisfy only partially bank B's demand for liquidity. This is true for all values of $b_A \in [b(L_A, L_B), g_A / d]$. In this case the interbank loan is equal to $(L_A - b_A Dd)$, and in order for the lending bank to survive also at $t=2$, the following condition must be satisfied: $(L_A - b_A Dd)n + R(D - L_A) \geq (1 - b_A)Dd^2$, which implies $b_A \geq \dfrac{L_A(R-n) - D(R-d^2)}{Dd(n-d)}$. Thus three conditions must be satisfied, in order that a two bank system with an interbank market is not subject to any crisis:

$$\begin{cases} b_A > b(L_A, L_B); \\ b_A \leq g_A / d; \\ b_A \geq \dfrac{g_A(R-n) - (R-d^2)}{d(n-d)} = w(L_A), \end{cases}$$

where $w(L_A) \leq a(L_A)^{ATK}$ and $a(L_A)^{ATK} < b(L_A, L_B)$, whence it follows that $w(L_A) \leq b(L_A, L_B)$, so that for $b_A \in [b(L_A, L_B), g_A / d]$ partial interbank exchanges and absence of failures hold (see Fig. 1).

2.2 Interbank debt and bank runs

If, contrary to the previous case, bank A liquid assets turned out to be insufficient to satisfy the impatient depositors' demand at $t=1$, but bank B liquidity investment proves to be excessive with respect to its *ex-post* depositors' demand (because $b_B \leq [L_B + L_A - b_A Dd]/Dd$), bank A could choose to borrow from bank B at a cost equal to the interbank interest rate. Because the interbank cost is lower than the expected cost of liquidation of long run assets (given by the sum of the liquidation cost t and the opportunity cost R, *i.e.* $n(b_A Dd - L_A) < a(b_A)(D - L_A)(R + t)$, where $a(b_A) = \frac{b_A d - g_A}{(1-g_A)(1-t)} < 1$ is the proportion of illiquid assets to be liquidated at $t=1$ to satisfy all impatient depositors), bank A borrows from bank B and an interbank market is created. In order for bank A to completely and successfully fulfill its activity, at $t=2$, it must be able to reimburse all funds borrowed at $t=1$ and satisfy all remaining depositors' demand. This result can be guaranteed if and only if the following is satisfied:

$$\begin{cases} b_A > g_A / d; \\ b_A \leq [g_A + g_B - b_B d]/d, \text{ dove } b_B \leq [g_A + g_B - b_A d]/d;. \\ b_A \geq \dfrac{g_A (R-n) - (R - d^2)}{d(n-d)} = w(L_A), \end{cases}$$

from the solution of which we find that bank A is always solvent for $b_A \in [g_A / d, b(L_A, L_B)]$.

If we now assume that bank A is a representative bank and cannot borrow from another bank by interbank exchanges, in order to be successful both at *t=1* and at *t=2* the following conditions have to be satisfied:

(i) at *t=1* $L_A < b_A Dd$, *i.e.* $b > g_A / d$. In order to satisfy the depositors' demand completely, bank *A* has to liquidate partially its illiquid assets, so that the following is satisfied: $L_A + a(D - L_A)(1 - t) = b_A Dd$. Solving this for b_A, we see that the anticipated liquidation of illiquid assets allows the bank to fulfill successfully its liquidity function for all values of $b_A \in [g_A / d, c_1(L_A)]$, where $c_1(L_A) = [1 - t(1 - g_A)] / d$;

(ii) moreover, at *t=2* the condition $(1 - a)(D - L_A)R \geq (1 - b)Dd^2$ must be satisfied, which has a solution for every $b_A \leq \dfrac{(1 - t)(R - d^2) + g_A Rt}{d[R - d(1 - t)]} = c(L_A)^{ATK}$, where $0 < c(L_A)^{ATK} < 1$ and it can assume values both above and below g_A / d. However, if below, the initial hypothesis $b_A > g_A / d$ (for which liquidation of assets is necessary) would not be satisfied. If we eliminate the values for which $c(L_A)^{ATK} \leq g_A / d$, the interval of values of b_A that assure at *t=2* the fulfillment of bank liquidity function are those belonging to the interval: $b_A \in [g_A / d,\ c(L_A)^{ATK}]$.

We can now establish the interval of values of b_A for which the bank, by anticipating the liquidation of its long run assets, is subject to failure neither at *t=1* or at *t=2*. The interval is given by the group of values of b_A satisfying the following:

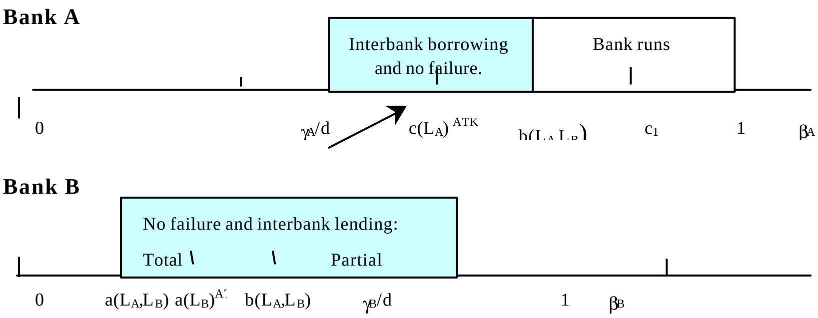
$$\begin{cases} g_A / d < b_A < [1 - t(1 - g_A)]/d = c_1(L_A); \\ b_A \leq \dfrac{(1 - t)(R - d^2) + g_A Rt}{d[R - d(1 - t)]} = c(L_A). \end{cases}$$

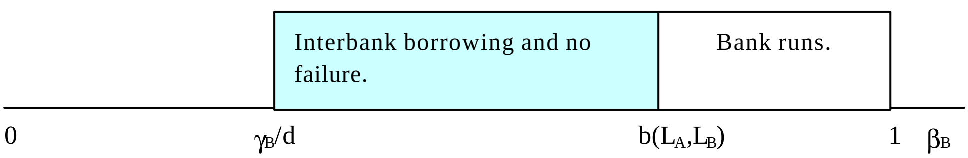
Since $c(L_A) < c_1(L_A)$, the system is always well defined for $b_A \in [g_A / d, c(L_A)^{ATK}]$. Obviously, due to $b_A \in [c(L_A)^{ATK}, 0]$ bank *A*, even by partially or totally liquidating its illiquid assets, cannot completely satisfy its depositors' demand. In particular, for $b_A \in [c(L_A)^{ATK}, c_1(L_A)]$, the bank can accommodate impatient depositors at *t=1*, but the resources it can command at *t=2* are not sufficient to repay the remaining depositors. On the contrary, for $b_A \in [c_1(L_A), 0]$ bank runs cause the total liquidation of bank assets at *t=1*.

If we compare the value of b_A for which the autarchic bank is unable to meet the depositors' demands, with its value when interbank exchanges are possible, it is easy to observe that $b(L_A, L_B) > c(L_A)^{ATK}$. Thus, in this sense the introduction of interbank exchanges reduces bank liquidity risk.

Graphically the interval in which bank *A* borrows from the interbank market and is not subject to any bank failure (provided that $b_B < b(L_A, L_B)$) is:

Fig. 2 – Interbank exchanges and bank runs.





As illustrated in Figure 2, it is possible for the values of b realized *ex-post* to be such that the lending bank can only partially satisfy the other bank's liquidity demand. Provided the cost of access to the interbank market is lower than the anticipated cost of liquidation of long run assets, bank A gains by borrowing from bank B and interbank exchanges are realized. In order for bank A to successfully satisfy all impatient depositors' demand at $t=1$ the following condition has to be satisfied:

$$L_A + (L_B - b_B Dd) + a'(1-t)(D - L_A) = b_A Dd,$$
$$\text{where } a'(b_A, b_B) = \frac{(b_A + b_B)Dd - (L_A + L_B)}{(1-t)(D-L_A)} \leq 1.$$

So interbank borrowing is advantageous for bank A if and only if $n(L_B - b_B Dd) + (R+t)a'(D - L_A) < (R+t)a(D - L_A)$; in other words, only if the costs of the associated interbank borrowing and the necessary liquidation of illiquid assets are lower than the cost the bank faced without interbank exchanges. Given the values of $a(b_A)$ and $a'(b_A, b_B)$, this conclusion is always obtained.

At $t=2$, a borrowing bank must repay its borrowing in the interbank market and a necessary condition for all lenders and depositors to be repaid then is[5]:

$$(1-a')R(D-L_A) \geq (1-b_A)Dd^2 + n(L_b - b_B Dd),$$
$$\text{where } a'(b_A, b_B) = \frac{(b_A + b_B)Dd - (L_A + L_B)}{(1-t)(D-L_A)} \leq 1.$$

[5] Any priority of reimbursement between depositors and other banks exists because irrelevant to our focus.

which is satisfied for all $b_A \leq \dfrac{(1-t)(R-d^2)+Rg_A t+(g_B-b_B d)[R-n(1-t)]}{d[R-d(1-t)]} = z(L_A, L_B)$.

Equivalently, if $b_A \in [g_A/d, z(L_A, L_B)]$ the borrowing bank experiences no liquidity crisis at *t=1* and at *t=2*. Finally, if $z(L_A, L_B) > c(L_A)^{ATK}$ even in the case of partial borrowing from the interbank market the risk of a liquidity crisis is lower than if a bank stands alone, without access to the interbank market (see Fig. 2).

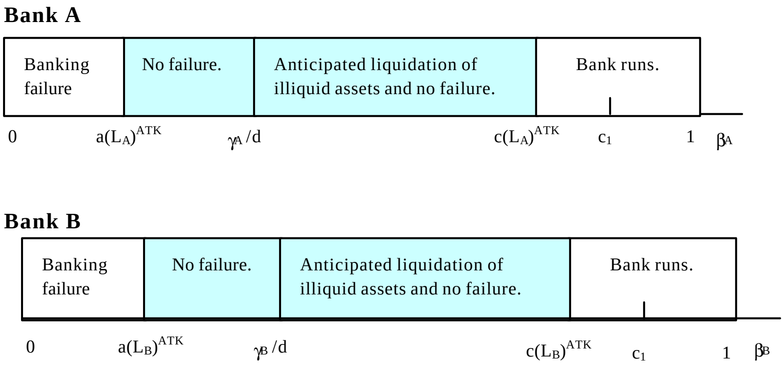
2.3 Absence of interbank exchanges and bank failure.

If shocks have the same sign and are of equal magnitude and interbank exchanges are not possible, banks have to be able to face two situations. In the first one, every bank holds excess liquidity at *t=1*: as in autarchy the case for $b_A \in [0, a(L_A)^{ATK}]$ banks satisfy impatient depositors' demand, but earn too little to cover their obligations at *t=2*. Alternatively, for $b_A \in [a(L_A)^{ATK}, g_A/d]$, bank asset management is *ex-post* correct, and the bank satisfies the depositors' demands at *t=1* and at *t=2*.

In the second case, the liquid resources invested at *t=0* are not enough at *t=1* to satisfy the impatient depositors' demands. Neither of the two banks has excess liquidity, so neither of them can borrow in the interbank market and they must operate as if they were in a state of autarchy, *i.e.* through liquidation of long run assets. In particular, for $b_A \in [g_A/d, c_A(L)^{ATK}]$, the anticipated liquidation of long run assets is such that banks satisfy both the impatient depositors at *t=1*, and the other depositors at *t=2*; alternatively, for $b_A \in [c_A(L)^{ATK}, 1]$, the liquidation of long run assets is insufficient to satisfy both the impatient depositors at *t=1* and the patient ones at *t=2*. In particular, as previously noticed, for $b_A \in [c_A(L_A)^{ATK}, c_1(L_A)]$ the bank fulfills its liquidity function at *t=1* but not at *t=2*; for $b_A \in [c_1(L_A), 0]$ the resources available to the bank are not enough to satisfy the depositors' demand at *t=1*. The latter case is, in some way, the same as the classical bank run in DD.

Graphically the autarchic case can be shown as follows:

Fig. 3 – Non-interbank exchanges and liquidity risk.



The theoretical framework developed in this section, of which Table 1 offers a summary[6], gives us the opportunity to know *ex-ante* whether or not interbank exchanges can be implemented.

Result 1. *With interbank exchanges the probability of failure and bank runs decrease with respect to the autarchic case. In fact, the interval of* b *values where banks are subject to a liquidity crisis (both because of a conservative management of bank assets,* $a(L_A, L_B) < a(L_A)^{ATK}$*, and risky management of bank assets,* $b(L_A, L_B) > c(L_A)^{ATK}$*), decreases.*

[6] Table 1 shows a summary of the timing structure from whose solution we obtain the interval values of b_i for which the bank is subject to neither bank failure nor to bank runs.

Table 1 - Timing of the model with interbank exchanges and no liquidity crisis.

t=0 $D = L_A + (D - L_A)$ and $D = L_B + (D - L_B)$.
t=1 **i)** $L_A - \boldsymbol{b}_A Dd + L_B - \boldsymbol{b}_B Dd \geq 0 \rightarrow \begin{cases} \textbf{(a)}\ L_A - \boldsymbol{b}_A Dd \geq \boldsymbol{b}_B Dd - L_B \text{ e } \boldsymbol{b}_B Dd > L_B; \\ \textbf{(b)}\ \boldsymbol{b}_A Dd > L_A \text{ e } L_B - \boldsymbol{b}_B Dd \geq \boldsymbol{b}_A Dd - L_A; \\ \textbf{(c)}\ L_A > \boldsymbol{b}_A Dd \text{ e } L_B > \boldsymbol{b}_B Dd. \end{cases}$ **ii)** $L_A - \boldsymbol{b}_A Dd + L_B - \boldsymbol{b}_B Dd < 0 \rightarrow \begin{cases} \textbf{(a)}\ L_A - \boldsymbol{b}_A Dd < \boldsymbol{b}_B Dd - L_B, L_A > \boldsymbol{b}_A Dd \text{ e } \boldsymbol{b}_B Dd > L_B; \\ \textbf{(b)}\ L_A < \boldsymbol{b}_A Dd \text{ e } L_B - \boldsymbol{b}_B Dd < \boldsymbol{b}_A Dd - L_A, L_B > \boldsymbol{b}_B Dd; \\ \textbf{(c)}\ L_A < \boldsymbol{b}_A Dd \text{ e } L_B < \boldsymbol{b}_B Dd \end{cases}$
t=2 **i)** $L_A - \boldsymbol{b}_A Dd + L_B - \boldsymbol{b}_B Dd \geq 0 \rightarrow \begin{cases} \textbf{(a)}\ (\boldsymbol{b}_B Dd - L_B)n + R(D - L_A) \geq (1 - \boldsymbol{b}_A) Dd^2; \\ \textbf{(b)}\ R(D - L_A) \geq n(\boldsymbol{b}_A Dd - L_A) + (1 - \boldsymbol{b}_A) Dd^2; \\ \textbf{(c)}\ (L_A - \boldsymbol{b}_A Dd) + R(D - L_A) \geq (1 - \boldsymbol{b}_A) Dd^2. \end{cases}$ **ii)** $L_A - \boldsymbol{b}_A Dd + L_B - \boldsymbol{b}_B Dd < 0 \rightarrow \begin{cases} \textbf{(a)}\ (L_A - \boldsymbol{b}_A Dd)n + R(D - L_A) \geq (1 - \boldsymbol{b}_A) Dd^2; \\ \textbf{(b)}\ R(1 - \boldsymbol{d})(D - L_A) \geq n(L_B - \boldsymbol{b}_B Dd) + (1 - \boldsymbol{b}_A) Dd^2, \\ \text{where } \boldsymbol{d} = \dfrac{(\boldsymbol{b}_A + \boldsymbol{b}_B) Dd - (L_A + L_B)}{(1 - \boldsymbol{t})(D - L_A)} \leq 1; \\ \textbf{(c)}\ (1 - \boldsymbol{a})(D - L_A) R \geq (1 - \boldsymbol{b}_A) Dd^2 \\ \text{where } \boldsymbol{a} = \dfrac{\boldsymbol{b}_A Dd - L_A (1 - \boldsymbol{t})(D - L_A)}{(1 - \boldsymbol{t})(D - L_A)}. \end{cases}$

3. Interbank exchanges and equilibrium solutions

The different cases emerging from the analysis in the previous section permit us to distinguish the intervals of stochastic variable values b_A and b_B that safeguard the system of two banks from liquidity crises. To some extent, deposit losses can be financed by borrowing from another bank. In particular, if shocks have opposite signs because, for example, banks are spatially diversified, it may be possible to reduce the incidence of failure and bank runs.

On the basis of the above information, every bank can establish its own expected profit *ex-ante*. This formulation depends on the two banks' economic conditions and on the presence of regulatory mechanisms that guarantee the banking system against possible liquidity crises. In this paper, these mechanisms[7] are disregarded, so that the only possible form of protection for banks against the liquidity risk is their own self-regulation: banks are interpreted to behave co-operatively to achieve a better management of bank assets. In the terminology of club theory[8], we define such an agreement as an interbank club.

In the rest of this section we show that the expected profit maximization problem has two solutions. The first one is a non-cooperative solution, similar to that associated with a banking system where banks choose their own investment strategies, taking as given those of other banks[9]. The second one is a cooperative solution, and can be associated to an interbank market

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[7] We assume that the Central Bank does not completely guarantee, as a lender of last resort, the banking system against possible liquidity crises, *i.e.* the "too big to fail" principle does not apply.

[8] Concerning club theories, see Cornes and Sandler [1996]. An application of club theory to the banking system and in particular to the management of liquidity risk in the interbank market is provided by Dowd [1994].

[9] For $N \to \infty$ (where N is the number of banks), the non-cooperative solution coincides with that of a perfectly competitive interbank market. An interpretation of the interbank market similar to this has been given by free banking theorists. On this point see, for example, Timberlake [1984], Selgin and White [1988] and Dowd [1992, 1994].

where single banks realize an investment strategy that guarantees the maximization of expected joint profits - an interbank club[10].

Before proceeding to the analytical solution of the profit maximization problem with two banks, we solve a profit maximization problem of a representative bank, *i.e.* we first consider the autarchic case. This result represents a benchmark, with which one can compare the risk-return performance of a banking system characterized by interbank exchanges.

Given the values of the b's intervals computed in the previous section, we can proceed to the construction of the expected profit function. This function is in general different, according as the banking system is or not regulated in such a way that bank safeness is guaranteed in the case of failure or bank runs. In this paper we assume that banks support completely the losses associated with a possible liquidity crisis, both at the individual and the aggregate levels[11].

If perfect competition is assumed on both the deposit and the loan markets, and the level of deposits D is exogenous, the investment liquidity level, L_A, represents the only bank strategic variable. At $t=0$, the best investment strategy a bank can choose

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[10] This solution is similar to that guaranteed by a cooperative banking club. In particular, for $N\rightarrow$¥ it coincides with a monopolistic solution. All banks belonging to the banking system find it advantageous to delegate the bank liquidity management to a hierarchic structure like a club. However, as largely discussed by free banking theorists this solution differs substantially by that of a traditional Central Bank. For more details on this point see Timberlake [1984], Selgin and White [1988] and Dowd [1992, 1994].

[11] The losses can be interpreted as a private capital reduction of bank shareholders, as a loss of reputation of the bank management (that, being subjected to dismissal, can lose subsequent control revenues), and finally as a criminal sentence on managers and on shareholders.

in the absence of interbank exchanges, is that obtained by solving the following expected profit maximization problem in L_A:

$$\underset{L_A}{Max}E(\Pi)=\int_0^a\{L_A+(D-L_A)R-H(\beta_A)\}f(\beta_A)d\beta_A+\int_a^c[L_A-H(\beta_A)]f(\beta_A)d\beta_A+$$
$$+R\int_a^b(D-L_A)f(\beta_A)d\beta_A+R\int_b^c(D-L_A)[1-\alpha(\beta_A)]f(\beta_A)d\beta_A+(1-\tau)\int_b^c(D-L_A)\alpha(\beta_A)f(\beta_A)d\beta_A+$$
$$+\int_c^{c_1}\{[1-\alpha(\beta_A)](D-L_A)R-(1-\beta_A)D\delta^2\}f(\beta_A)d\beta_A+\int_{c_1}^1\{L_A+(1-\tau)(D-L_A)-H(\beta_A)\}f(\beta_A)d\beta_A,$$

(3)

where, $H(\beta_A)=\beta_A D\delta+(1-\beta_A)D\delta^2$ represents the bank negative payment flows, $a=a(L_A)^{ATK}=\dfrac{\delta^2-R(1-\gamma_A)-\gamma_A}{\delta(\delta-1)}$, $b=\gamma_A/\delta$, $c=c(L_A)^{ATK}=\dfrac{R[1-\tau(1-\gamma_A)]-\delta^2(1-\tau)}{\delta[R-\delta(1-\tau)]}$ and $c_1=c_1(L_A)^{ATK}=[1-\tau(1-\gamma_A)]/\delta$.

We now assume, for analytical tractability, that the stochastic variable β_A is uniformly distributed over the interval [0,1][12]. In this case, it is easy to derive the following first order condition:

$$-a(R-1)-(b-a)(R-1)+[L_A-H(c)]\frac{\partial c}{\partial L_A}-[L_A-H(a)]\frac{\partial a}{\partial L_A}-\frac{R(D-L)(R-\delta)}{D\delta(\delta-1)}+\frac{R\tau(c-b)}{(1-\tau)}-\frac{R(D-L_A)}{D\delta}+$$
$$+R(D-L_A)[1-\frac{c\delta-\gamma_A}{(1-\gamma_A)(1-\tau)}]\frac{\partial c}{\partial L_A}+(1-\tau)(D-L_A)[\frac{c\delta-\gamma_A}{(1-\gamma_A)(1-\tau)}]\frac{\partial c}{\partial L_A}+\frac{R\tau}{(1-\tau)}(c_1-c)+\tau(1-c_1)=0, \quad (3')$$

[12] Abandoning this hypothesis does not modify our results, indeed if we used a *Beta* distribution, $f(\beta)=\dfrac{\beta^{\theta}(1-\beta)^{\theta}}{\int_0^1\beta^{\theta}(1-\beta)^{\theta}}$ our results would not change at all. However, the use of this function would give us the possibility to show that by varying the θ parameter, that can be considered a measure of liquidity risk, the investment decision of our bank would change. In particular, for greater values of θ, in correspondence to which we have smaller standard deviations, the liquidity assets investment decrease. The calculations related to this result are available from the author upon request.

from the solution of which in L_A we obtain:

(4) $$L_A^{ATK} = L_A(D, R, d, \boldsymbol{t}) = \frac{D\boldsymbol{t}[R-(1-\boldsymbol{t})+d]}{(1+\boldsymbol{t})[R-(1-\boldsymbol{t})]},$$

that represents the value of banking liquidity which attains maximum expected profit in autarchy.

3.1 Bank liquidity management and the non-cooperative solution

In this section we keep the assumption of perfect competition on both the deposit and the loan markets; so for a given level of deposits D, the level of liquid investment, L_i, represents once again the only choice variable. Moreover, we assume to know *ex-ante* the interbank interest rate that, for given *ex-post* observed values of b_A and b_B, assures an equilibrium in the interbank market[13], *i.e.* that rate at which $L_A(D,L_B,R,d,n,t)+L_B(D,L_A,R,d,n,t)=Dd(b_A+b_B)$. Notice, however, that this hypothesis does not imply any loss of generality. In fact,

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[13] The endogenization in the model of the variable n would have asked for the solution of the following expected profit maximization problem:

$$\underset{L_{A(B)}}{Max}\, E(\Pi) = E(\Pi(D, L_A, L_B, R, d, n, t)),$$

from which we would have obtained the following two reaction functions $L_A(L_B,n)$ and $L_B(L_A,n)$. Given the values of b_A and b_B observed *ex-post*, the maximization problem would have given us the possibility to determine the value of the equilibrium interbank interest rate, *i.e.* that value of n guaranteeing the equality between supply and demand of reserves on the interbank market, that is:

$$L_A(D,L_B,R,d,n,t)-b_A Dd = b_B Dd - L_B(D,L_A,R,d,n,t) \Rightarrow n(b_A,b_B,L_A,L_B).$$

By substituting this result in the expected profit function, one can obtain the equilibrium values of L_A and L_B.

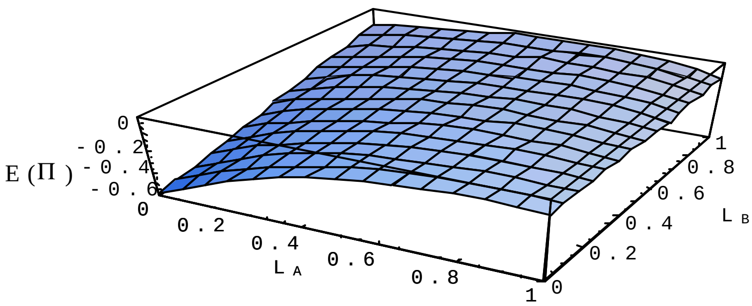
our results hold for every value of $n \in [d, \sqrt{R}]$. In other terms, for every equilibrium value of the interbank interest rate a cooperative and a non-cooperative equilibrium solution exist and they are coherent with results we obtain.

By proceeding now to the solution of bank A's expected profit maximization problem, we can derive the best investment strategy at $t=0$, given the liquid investment of the other bank, L_B.

(5) $$\underset{L_{A(B)}}{Max} E(\Pi)_{A(B)} = E(\Pi(D, L_A, L_B, R, d, n, t)) .$$

Given the analytical complexity of the expected profit function[14], we solve the maximization problem by numerical simulation. Assume for example, $D=1$, $R=1.3$, $d=1.05$, $n=1.1$ and $\tau=0.5$; then it is possible to show that the expected profit function of bank A(B), given the liquidity level of the other bank $L_{B(A)}$, is strictly concave in $L_{A(B)}$ (see Fig. 4). The concavity guarantees the existence of an internal solution for $L_{A(B)} \in [0, D]$: the bank portfolio is diversified between liquid and illiquid assets. Furthermore, we show that this solution depends negatively on the liquid investment of the other bank, *i.e.* $\frac{\partial^2 E(\Pi)_A}{\partial L_A \partial L_B} \leq 0$ (or $\frac{\partial^2 E(\Pi)_B}{\partial L_B \partial L_A} \leq 0$ for bank B).

Fig. 4 – Expected profit function in the presence of interbank exchanges (D=1, R=1.3, d=1.05, n=1.1, τ=0.5).



If we apply the intuition on which our model is based to a simple game, where bank A and B represent two players interacting in a interbank market, this is the same as saying that the reaction functions are negatively sloped, *i.e.* L_A and L_B are strategic substitutes (see Fig. 5). If, moreover, we indicate with L_A and L_B the moves that each player can take in response to the other player's move and with $E(p)$ the payoff associated to each combination of playable strategies, then the non-cooperative equilibrium (L_A^{NC}, L_B^{NC}) is given by the intersection between the two reaction functions defined as follows[15]:

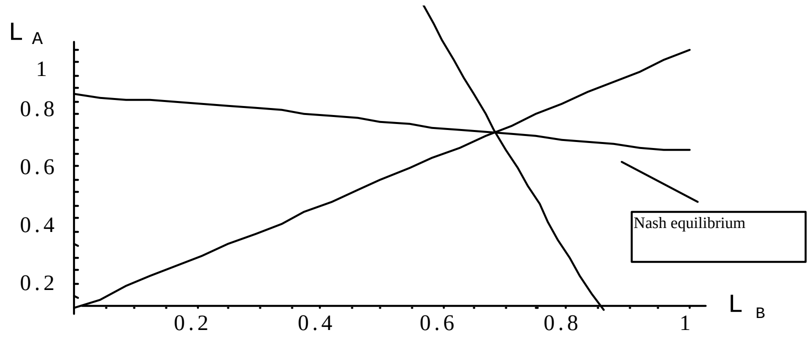
$$\text{Nash equilibrium} \quad \begin{cases} \dfrac{\partial E(\Pi)_A}{\partial L_A} = 0 \quad \Rightarrow \quad L_A = L_A(L_B, R, d, n, t); \\ \dfrac{\partial E(\Pi)_B}{\partial L_B} = 0 \quad \Rightarrow \quad L_B = L_B(L_A, R, d, n, t). \end{cases}$$

---

[15] For the values used for our numerical simulations (see Fig. 5) the reaction functions intersect for $L_A=L_B=0.6797$. The banking system liquidity degree with interbank exchanges, *i.e.* $(L_A+L_B)/2D=0.6797$ is lower than the autarchic one (see equation (4)) equal to $L_A/D=0.7708$. This is true for every $n \in [d, \sqrt{R}]$; if in the limit we assume $n=1.14$ (*i.e.* the highest possible value), the non-cooperative equilibrium liquidity level would be equal *to 0.6858* and the banking system liquidity degree would be *0.6858*.

Graphically the resulting Nash equilibrium can be shown as follows:

Fig. 5 – Interbank liquidity and Nash equilibrium (D=1, R=1.3, d=1.05, n=1.1, t=0.5).



From the study of the liquidity functions resulting from the solution of the maximization problem, we can now establish that the investment strategies are actually influenced by both the market interest rates, and the bank liquidation costs. In particular, we can show that the function of the banking liquidity $L_{A(B)} = L_{A(B)}(L_{B(A)}, d, n, t, R)$, depends positively on the deposit interest rate, the interbank interest rate and the bank liquidation costs, and negatively on the loan interest rate. The non-cooperative solution can be interpreted as an equilibrium solution of an interbank market, within which each bank interacts with every other.

Result 2*. For every $n \in [d, \sqrt{R}]$, at the same deposit and loan interest rates and at the same cost of liquidation, the possibility to interact on the interbank market guarantees a* *better liquidity management ($L_i^{ATK} > L_i^{NC}$), leading to a lower failure risk and a greater expected profit ($E(\pi)_i^{ATK} < E(\pi)_i^{NC}$)*[16].

3.2 Banking liquid management and the cooperative solution

Even if the non-cooperative solution is superior to the autarchic one, it is possible to show that (as should be expected) it is inferior to a cooperative solution that can be guaranteed, for example, through the creation of a cooperative club[17]. The solution to the problem of joint expected profit maximization obviously guarantees the existence of a cooperative equilibrium superior to the non-cooperative one. In analytical terms, we solve the following system:

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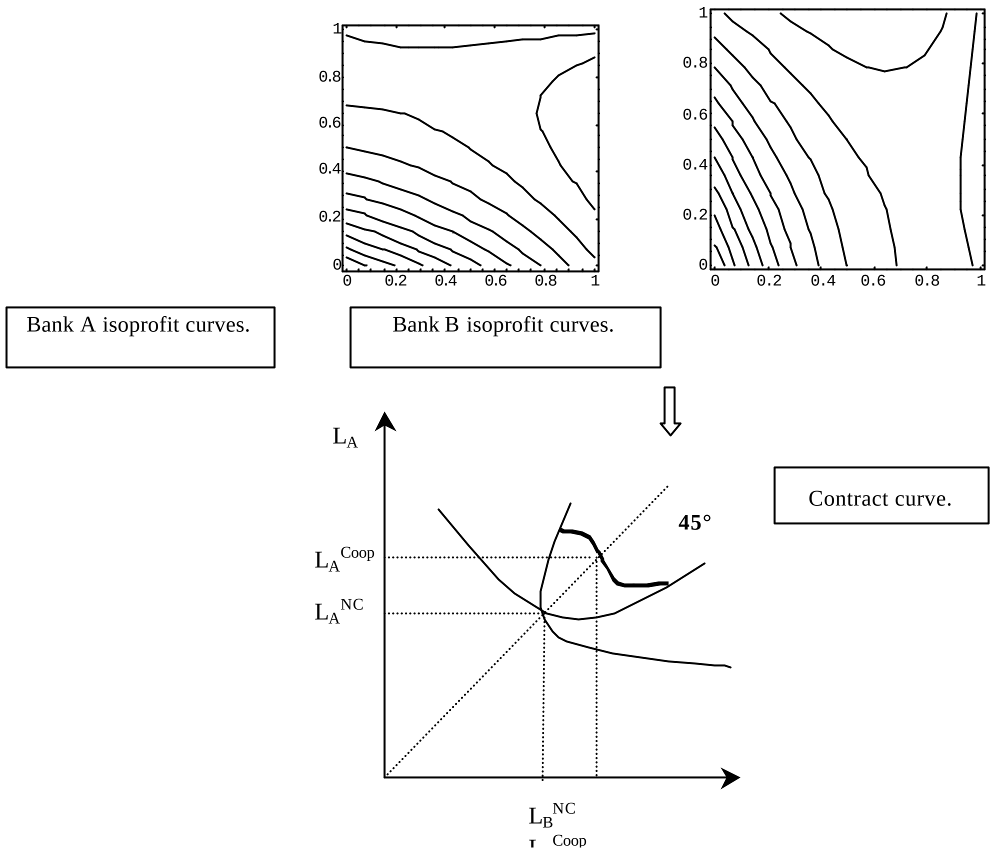
[16] For the grid of values used to formulate our numerical simulations and for $n=1.05$ (*i.e.*, n's lowest possible value), it is easy to show that the autarchic expected profit ($E(\pi)^{ATK} = -0.462401$) is lower than that which can be reached with interbank exchanges and in the absence of cooperation ($E(\pi)^{NC} = 0.109$).

[17] Provided the hypothesis of duopoly in our model, we cannot establish the optimal number of interbank members of the club. As a consequence the cooperative club solution we obtain coincides with a monopoly solution. A possible solution if all banks decided to "freely" adhere to a sort of clearing house managed at a national level. Following free banking theory this solution differs substantially from that one of a Central Bank for at least three reasons: a) it is a volunteer agreement; b) the rules are established by the same member banks; c) no institutional rigidity and total guarantees of system stability exist. For more details on this point see, for instance, Dowd [1994].

$$\text{Symmetric cooperative equilibrium}\quad \begin{cases} \dfrac{\partial^2 E(\Pi)}{\partial^2 L_A} + \dfrac{\partial^2 E(\Pi)}{\partial L_A \partial L_B} = 0, & \Rightarrow\ L_A = L_A(D,R,d,n,\tau) \\ \dfrac{\partial^2 E(\Pi)}{\partial^2 L_B} + \dfrac{\partial^2 E(\Pi)}{\partial L_B \partial L_A} = 0, & \Rightarrow\ L_B = L_B(D,R,d,n,\tau). \end{cases}$$

Given the isoprofit curves' shape (see Fig. 6) and the perfect symmetry between the two players (bank *A* and bank *B* are equal), the cooperative equilibrium is surely superior to the non-cooperative one. The cooperative solution internalizes the positive externalities associated with the higher liquid investment. The consequence is an increase in the expected profits of both banks.

Fig. 6 – Interbank liquidity and cooperative equilibrium (D=1, R=1.3, d=1.05, n=1.1, τ=0.5).



Result 3*. The internalization of positive externalities implies a greater investment in liquid assets in the cooperative equilibrium, with respect to the non-cooperative equilibrium.*

This result depends on the fact that we are maximizing an expected profit function built on the hypothesis that bank activity can be both successful and unsuccessful. A greater investment in liquid assets on the one hand decreases the losses due to the risk of bank runs while on the other hand increases the losses due to the risk of bank failure. Since this last effect is smaller than the first one the expected profit increases. At a credible cooperative solution the liquid investment is greater and so the expected profit is higher.

However, in order to reach a sustainable cooperative solution each bank payoff has to be such that it has no incentive to deviate from the cooperative agreement. In a system structured like a club, this payoff can be represented by the fact that belonging to it guarantees the bank against the liquidity risk that is not guaranteed to banks not belonging to it[18].

4. Conclusion

Relaxing the hypothesis of a representative bank is obviously crucial to show that interbank exchanges are possible

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[18] That a non-club member bank can be the first one to fail in crises periods has been shown in the case of New York trust company during the National Banking Era. The incapability of the trust company to liquidate depositors revealed a negative sign on the banking system's capability to perform its monetary function, so that bank runs spread very quickly to all the system causing a bank panic. For more details on bank panics during the National Banking Era see, among others, Chari [1989].

and profitable. A better management of the bank assets can reduce the liquidity risk due both to a too prudent portfolio diversification, and/or to bank runs. Moreover, the possibility to invest the excess liquidity in the interbank market at a positive interest rate increases expected bank profits.

In particular, we have shown that a non-cooperative solution is superior to the autarchic one. The non-cooperative set up is given by an interbank market within which every bank behaves autonomously, and considers the other banks' investments as given. Increasing the number of banks, we can interpret this solution as a market solution, that, however, is affected by a potential free riding problem. The fact that every bank can free ride on other banks' liquid resources implies an under-investment in liquidity.

The cooperative equilibrium emerging from maximizing the joint expected profit maximization problem can be interpreted as the realization of a cooperative club; given, however, the duopoly hypothesis of our model, this solution coincides with that of a cooperative club of which every bank is a member. A possible extension of the model to *N* banks, and the introduction of positive monitoring costs could represent a good point of departure for further research in this direction.

A part from any normative judgment that can emerge from an extension of the model to an enlarged *N*-bank system, the results we obtain allow us to conclude that a compulsory agreement structured as a volunteer club could provide incentives for banks towards a greater investment of liquid assets, so that the expected profit is greater and the bank crisis risk is lower than in an autarchic system. On the other hand, belonging to a club may be attractive for banks, as it creates a better reputation, a greater number of customers and a better management of bank liquidity, in such a way that the same level of expected profit is associated with lower liquidity risk[19].

Appendix

Equation (5) page 17 sums up the relation between the dependent variable $E(p)$ and the independent variables D, L_A, L_B, R, d, n and t. In this appendix we are going to explain its analytical components.

In section 2 we outlined the intervals of variation of stochastic variables b_A and b_B.. We associated to each of those intervals a different bank economic state: interbank loan and/or debt, failure and/or bank runs. We stressed that to assess the economic results of bank activity we have to consider both its depositor behavior and other banks depositors' behavior. Depending on the assumed values of stochastic variables b_A and b_B, we can outline different components of the expected profit function.

(i) The first component of the expected profit function is associated with the state of excess liquidity for both banks. This state of nature can be observed for $b_A \in [0, g_A / d]$ and $b_B \in [0, g_B / d]$. In this case, as in autarchy, we do not have interbank exchanges and bank A expected profit function is the following:

[19] Concerning the hypothesis of the club as a regulatory set-up superior to those imposed by outside institutions and on the incentives that can push economic agents to belong to it see, among others, Gherig and Jost [1995] with reference to the problem of minimum quality standards in the industrial sector; with reference to European financial regulation see Gual and Neven, [1992] and Scarpa [1997].

(1A) $$\int_0^{b_B}\int_0^{b_A}\left[L_A + R(D - L_A) - H(\boldsymbol{b}_A)\right] f(\boldsymbol{b}_A) f(\boldsymbol{b}_B) d\boldsymbol{b}_A d\boldsymbol{b}_B ,$$

where, as in autarchy, $b_A = g_A / d$ and $b_B = g_B / d$.

(ii) When one of the two banks, say bank A, doesn't have enough liquidity to satisfy the depositors' demand (but the other bank, say bank B, is in the opposite situation and can completely finance bank A) bank A's expected profit function is:

(2A)
$$\int_0^{b(L_A,L_B)}\int_{b_A}^{1}\left[R(D - L_A) - n(\boldsymbol{b}_A Dd - L_A) - Dd^2(1 - \boldsymbol{b}_A)\right] f(\boldsymbol{b}_A) f(\boldsymbol{b}_B) d\boldsymbol{b}_A d\boldsymbol{b}_B \cdot$$

(iii) In case of partial lending, bank A expected profit function can be rewritten as follows:

(3A)
$$\int_{b(L_A,L_B)}^{b_B}\int_{b_A}^{1}\left[R(D - L_A)(1 - \boldsymbol{a}') - n(L_B - \boldsymbol{b}_B Dd) - Dd^2(1 - \boldsymbol{b}_A)\right] f(\boldsymbol{b}_A) f(\boldsymbol{b}_B) d\boldsymbol{b}_A d\boldsymbol{b}_B$$
.

(iv) If, alternatively, bank A has excess liquidity and can completely finance bank B, its expected profit function is:

(4A) $$\int_{b_B}^{1}\int_0^{b(L_A,L_B)}\left[L_A - (n-1)(L_B - b_B Dd) + R(D - L_A) - H(b_A)\right] f(b_A) f(b_B) db_A db_B .$$

(v) In the case of partial financing, bank A expected profit function can be rewritten, as follows:

(5A) $$\int_{b_B}^{1}\int_{b(L_A,L_B)}^{b_A}\left[R(D - L_A) - n(b_A Dd - L_A) - Dd^2(1 - b_A)\right] f(b_A) f(b_B) db_A db_B .$$

(vi) Finally, if neither of two banks has enough liquidity to finance its own depositors, as in autarchy a bank run becomes inevitable and bank A expected profit function, or more general the loss expected function, can be written as follows:

(6A)

$$\int_{b_B}^{1}\left\{\int_{b_A}^{c_{1A}}\left[R(1-a)(D-L_A)-Dd^2(1-b_A)\right]f(b_A)db_A+\int_{c_{1A}}^{1}\left[L_A+(1-t)(D-L_A)-H(b_A)\right]f(b_A)db_A\right\}f(b_B)db_B\,,$$

where, as in autarchy, $c_{1A}=[1-t\,(1-g_A)]/d$.

Summing up the above functions, it is easy to obtain bank A expected profit function for every value of b_A and b_B belonging to the interval [0,1]:

$$E(\Pi(D,L_A,L_B,R,d,n,t))=(1A)+(2A)+(3A)+(4A)+(5A)+(6A)\,,$$

that, given the hypothesis of uniform distribution of b_A and b_B over the interval [0,1], can be rewritten as follows:

$$E(\Pi)=\frac{L_BL_A\left[D(R-d^2)-L_A(R-1)\right]}{D^2d^2}+\frac{L_A^{\,2}L_B(d-1)}{2D^2d^2}+b(L_A,L_B)\left\{\left(1-\frac{L_A}{Dd}\right)\left[D(R-d^2)-L_A(R-n)\right]-\frac{Dd(n-d)}{2}\left(1-\frac{L_A^{\,2}}{D^2d^2}\right)\right\}+$$

$$+\left[\frac{L_B}{Dd}-b(L_A,L_B)\right]\left\{\left[\frac{L_B[R-n(1-t)]}{(1-t)}+D(R-d^2)+\frac{R\,tL_A}{(1-t)}\right]\left(1-\frac{L_A}{Dd}\right)-\frac{Dd[R-d(1-t)]}{2(1-t)}\left(1-\frac{L_A^{\,2}}{D^2d^2}\right)\right\}+$$

$$-\frac{Dd[R-n(1-t)]}{2(1-t)}\left(1-\frac{L_A}{Dd}\right)\left[\frac{L_B^{\,2}}{D^2d^2}-b(L_A,L_B)^2\right]+\frac{(n-1)(L_A+L_B)}{2}\left(1-\frac{L_B^{\,2}}{D^2d^2}\right)-\frac{Dd(n-1)}{6}\left(1-\frac{L_B^{\,3}}{D^3d^3}\right)+$$

$$+\left(1-\frac{L_B}{Dd}\right)\left[-\frac{L_B^{\,2}(n-1)}{2Dd}-\frac{2L_AL_B(n-1)}{2Dd}+\frac{[R-(1-t)][-L_A^{\,2}(1+t)+D^2(1-t)+2tDL_A]}{2Dd}+tL_A-D[d^2-(1-t)]+\frac{Dd(d-1)}{2}\right].$$

where $b(L_A,L_B)$ can be substituted by the following expression:

$$b(L_A, L_B) = \int_{b_{B(A)}}^{1} \left[(b_A + b_B) - b_{B(A)}\right] f(b_{B(A)}) db_{B(A)} = \left[(b_A + b_B)(1 - b_B) - 1/2(1 - b_B^2)\right].$$

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