The semiclassical limit of the nonlinear Schrödinger equation in a radial potential. (English summary)


In this paper, the authors are concerned with the nonlinear Schrödinger equation

\[ i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \psi + V(x)\psi - \gamma_h |\psi|^{p-2} \psi, \quad \gamma_h > 0, \quad x \in \mathbb{R}^2, \]

where \( \hbar > 0, \ 2 < p < 6, \psi : \mathbb{R}^2 \to \mathbb{C}, \) and the potential \( V \) is radially symmetric. Upon denoting by \((r, \theta)\) the polar coordinates in the plane, the authors’ purpose is to obtain positive solutions of the form \( \psi(r, \theta, t) = e^{i\frac{\hbar}{\theta}(iM_h\theta + iEt)}v(r) \). They assume \( M_h > 0 \), which implies that all such functions have nontrivial angular momentum. This kind of solution exhibits a “spike-layer” pattern as \( \hbar \to 0^+ \); that is, as \( \hbar \to 0^+ \) the solutions concentrate on a circle centered at the origin while approximating uniformly zero away from it. In order to locate the asymptotic peaks, the authors analyze the appearance of such a concentration’s asymptotic behavior by means of a suitable auxiliary functional.

Reviewed by _Alberto Parmeggiani_

---

**References**


11. V. Benci and N. Visciglia, Solitary Waves with non Vanishing Angular Momentum, in preparation.


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

© Copyright American Mathematical Society 2003, 2011