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Optimal Taxation in R&D Driven Endogenous Growth Models

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Abstract

Is it possible to increase growth and welfare by raising lump-sum taxes and disposing of the tax revenues? Is it possible to increase welfare by raising capital income taxes and redistributing the revenue as a subsidy to labor income? This thesis shows these may indeed be the case in standard R&D models with technological change, represented either by an increase in the variety of intermediate goods or by creative destruction. The key mechanism is that with elastic labor supply the tax programs can increase the employment rate in equilibrium. This creates two spillover effects on the R&D pace. In addition the tax programs themselves will have level effect on the instantaneous utility. The relative momentums of the spillovers and the level effect determine the sign of the welfare effect. It is shown that, for parameter values consistent with available estimates, the growth and welfare can both be improved under the wasted lump-sum tax program, and that the welfare effect can be positive even if the long-run growth rate decreases after the increase in the capital income tax rate.

JEL Codes: E62, H21, O41

Keywords: R&D, endogenous growth, lump-sum taxes, capital income taxes, labor income subsidy, growth effect, welfare effect, optimal tax rate.

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Introduction

The prediction that permanent variations in tax rates would give rise to different steady-state growth rates has long been a hallmark of the endogenous growth literature. In contrast to the older neoclassical framework, where long-run growth was exogenously determined by the rate of technical progress, these models predict that increases in tax rates would induce lower growth rates (see, for example the survey in Myles (2000) and Jones and Manuelli (2005)). This negative correlation reflects the distortional effects of taxation. However, empirical cross-country growth studies, notably by Levine and Renelt (1992), Levine and Zervos (1993) and Tanzi and Zee (2000) have not been able to confirm this negative correlation; and more recently Angelopoulos et al (2007) even shows contrary conclusion. Jones (1995) and Stokey and Rebelo (1995) also argued that US time series data was at odds with the implications of linear growth models: on the basis of these models, the dramatic increase in income taxation which took place in the early 1940s would have been expected to contemporaneously decrease the US per capita growth rate, but this did not appear to be the case.

Within the endogenous growth literature, in particular many studies focus on R&D activities, a major driving force for growth, and to fiscal incentives for these activities, which are subsidized in many industrial countries. However, there are some common limitations of these studies. One limitation is that they often treat labor supply as inelastic, thereby abstracting from the decision to allocate time between work and leisure. Since the labor-leisure choice is actual microeconomic phenomenon - individual agents face the tradeoff and make their decision in time allocation, I adopt the labor-leisure choice into my model for the household sector in the economy to show a non obvious result for fiscal policy based on the intratemporal tax distortions. Another limitation is that they often analyze only the effect of the taxes on growth without further looking into their influence on welfare by implicitly assuming that higher growth rate always leads to higher welfare. But in an imperfectly competitive dynamic economy this assumption cannot always be justified because that the growth rate in a decentralized economy may differ from the socially optimal rate and if the former is bigger than the latter, the increase of the former may decrease overall welfare in stead of raising it. Another reason that can nullify this assumption is that the change in instantaneous consumption and that in growth rate may counteract each other in determining the utility level, so even if growth rate increases but if it can be offset by the decrease in the instantaneous consumption, the overall welfare may still be reduced. Therefore, it leads to the necessity of the analysis of both growth effect and welfare effect in my study.

An important debate in the literature of R&D driven endogenous growth model is over with-or-without scale effect. The very baseline endogenous technological change models feature a scale effect in the sense that a larger population, L, translates into a higher interest rate and a higher growth rate. Jones, among others, suggest modified versions, where the scale effect can be removed but higher population growth can still translate into higher per capita output growth. Following the line of research that puts technological innovation at the forefront of explanations of differences in standards of living across countries and time, I use the endogenous growth models as in the Chapter 6 and 7 of Barro and Sala-i-Martin (2004) to emphasize features of the real world like imperfect competition, accumulation of intangibles, economies of scale, creative destruction, and the distinction between quality improvements and the creation of new products. Thus my model incorporates scale effect in the sense that a greater effective labor force, embodied by longer equilibrium working hours assigned to representative agents with an aggregated scale of unit, creates more demand for intermediate goods, making R&D more profitable thus leads to faster growth. But what I am doing does not give answers to the relevant criticisms over either side of the debate, because we cannot assume that the effect of an increase in the number of workers is just the same as an increase in hours worked per worker, on which we focus.

I do not incorporate the variety expansion and creative destruction into a complicatedly comprehensive model for reasons such as that simpler model allows for easy analytical solutions to reaching the optimal welfare effects of the interested policy variables, and that the parallel models give qualitatively the same insights into the real economy as the comprehensive model does, and further clearly show the different momenta of the effects of the taxes whereby I can tell which economic externality the taxes can take effect on by the most.

The other characteristics of my model can be briefly summarized as follows:

• Representative agent with infinite life: this enables me to study the welfare effects of the taxes without considering any distribution effect.

• Balanced fiscal budget: this avoids consideration on the public expenditure financed by deficits and simplifies the analysis by excluding the intertemporal distribution effect of taxes (subsidies).

• No productive use of the taxes: this allows me a closer view of the effect of fiscal policy.

• No capital accumulation: no capital in the usual neoclassical sense of a homogenous, durable, intermediate good accumulated through foregone consumption. Instead, there are differentiated, non-durable, intermediate goods produced through foregone consumption. One can think of these goods as capital, albeit with 100% instantaneous depreciation. This structure allows me to simplify the model with no consideration on the effect of capital accumulation on growth without loss of generality for the purpose of my research.[1]

The fiscal policy variables within the interest of research are the lump-sum taxes and the capital income taxes. I choose to study these two taxes because in the optimal taxation literature, the lump-sum taxation (subsidy) are usually considered non-distortionary so taken as the first-optimal policy instrument but it is not the case in my model, and researches on optimal capital income taxation

[1] As Judd points out, much of the intuition behind the literature on optimal taxation of capital stems from the property that capital income is construed as the income to suppliers of the homogeneous (durable) intermediate good. Thus my model complements his point.

diversify generically in their conclusions thus it leaves much room for reexamination. In the following charpters I will further review the related literature in detail.

My analysis focuses on three aspects. First, I discuss the steady-state equilibrium and analyze the characteristics of the equilibrium, i.e. the determinacy or indeterminacy of the balanced growth path. Second, I derive the effects of the taxes on the long-run labor-leisure allocation and the long-run changes in growth rate. Particular attention is devoted to the welfare of the representative agent as represented by the present value of the benefits. Third, I calibrate the model to a benchmark economy and assess the numerical effects of these taxes relative to this benchmark. And through sensitivity analysis I show the beneficial effect of the taxes on welfare for plausible values of parameters. Finally, I analyse the social planner's economy and compare it with the market economy to show that the fiscal policies affect welfare in a second-best way.

My main conclusions are the following:

• There are no transitional dynamics and the balanced equilibrium is determinate and Pareto suboptimal.

• Lump-sum taxes, whose revenues are thrown to ocean, may increase long-run growth rate as well as welfare.

• Capital income taxes, whose revenues are returned to labor, may improve welfare even if they reduce long-run growth.

• Values of the economic parameters influence not only the direction but also the magnitude of the policies' welfare effect, and thus determine the variant levels of optimal tax rates.

The thesis is organized as follows: in chapter 1 and 2 a variety expansion model and a creative destruction model are set up respectively, and the effects on growth and welfare of the lump-sum taxes are studied in the two models; chapter 3 and 4 are devoted to the analysis of capital income taxes also in the respective models, with the studies on social planner's solution followed each. In the end the conclusion is given.

Chapter I. Lump-sum taxes in a variety expansion model

1 Introduction

In this charpter I show a non obvious result for fiscal policy that is made possible by allowing for flexible labor supply in R&D models: the fact that lump-sum taxes can have positive effects on growth even when the revenue is not used in a productive way.

It should be certainly possible to return the revenue to agents in such a way as to increase their welfare. However assuming as I do that the revenue is not returned allows me a closer view of the effect of fiscal policy. In particular it is often found in theoretical models that growth can be increased by subsidies to R&D financed through lump-sum taxes (see for example Barro and Sala-i-Martin (2004), chapter 6, or Zeng and Zhang (2002)). Here I show that with a flexible labor supply, lump-sum taxes can in themselves increase growth and welfare, i.e. have a direct effect on them.

I conduct my study by using a standard model of endogenous technological progress with an infinitely lived representative agent, originally proposed by Rivera-Batiz and Romer (1991) and known as the "lab-equipment model", and presented in Barro and Sala-i-Martin (2004), chapter 6. Given its flexibility and simplicity this model provides a tractable framework for analyzing a wide array of issues in economic growth.[2] Entrepreneurs spend a fixed cost in order to develop new intermediate goods. Each chooses to produce the same amount of each intermediate good. Output in the final goods production sector is linear in the number of intermediate goods used so unbounded growth is possible. The basic difference in assumptions with respect to this benchmark model is that the decision to supply labor is explicitly analysed.

I analyse the long-run effects of a lump-sum tax whose proceeds are thrown away and find that such a tax will increase growth and will increase welfare for a plausible region of the parameters space. The intuition is simply that in my model, lump-sum taxes have an impact on the allocation of resources, because they influence labor supply and consequently the rate of return on capital and the rate of growth. In the example which I consider, the income effect of a wasted lump-sum tax causes households to consume less leisure and supply more labor. This causes an increase in the interest rate and the long-run rate of growth. Put more explicitly, the mechanism which is at work is the following: a lump-sum tax induces a negative income effect thereby inducing agents to work more; more employment raises the returns to the R&D activity; growth is therefore increased.

The reasoning of the present study is inevitably involved into the debate of with-or-without scale effect associated with the endogenous technological progress models. As stated before, the very baseline models feature that a larger population, L, translates into a higher interest rate and a higher growth rate. However, this is problematic for three reasons as argued in a series of papers by Charles Jones and others:

(1) Larger countries do not necessarily grow faster (though the larger market of the United States or European economies may have been an advantage during the early phases of the industrialization process).

(2) The population of most nations has not been constant. If we have population growth as in the standard neoclassical growth model, e.g., $L(t)=\exp(nt)L(0)$, these models would not feature balanced growth, rather, the growth rate of the economy would be increasing over time.

(3) In the data, the total amount of resources devoted to R&D appears to

[2] See the excellent survey in Gancia and Zilibotti (2005) for a selection of the wide range of applications of this model .

increase steadily, but there is no associated increase in the aggregate growth rate.

These observations have motivated Jones (1995) to suggest a modified version of the baseline endogenous technological progress model. In that model the scale effect can be removed by reducing the impact of knowledge spillovers. While this pattern is referred to as "growth without scale effects", it is useful to note that there are two senses in which there are limited scale effects in these models: First, a faster rate of population growth translates into a higher equilibrium growth rate. Second, a larger population size leads to higher output per capita. It is not clear whether the data support these types of scale effects either. Put differently, some of the evidence suggested against the scale effects in the baseline endogenous technological change models may be inconsistent with this class of models as well. It is also worth noting that these models are sometimes referred to as "semi-endogenous growth" models, because while they exhibit sustained growth, the per capita growth rate of the economy is determined only by population growth and technology, and does not respond to taxes or other policies. Some papers in this literature have developed models of endogenous growth without scale effects, with equilibrium growth responding to policies, but this normally requires a combination of restrictive assumptions (see, among others, Dinopoulos and Thompson (1998), Howitt (1999) and Young (1998)). And further, Aghion and Howitt (1998) and Ha and Howitt (2007) argue that semi-endogenous growth models along these lines also face difficulties when confronted with the time-series evidence.

In fact, each one of the above arguments against scale effects can be debated. Some argued that looking at the 20th century data may not be sufficient to reach a conclusion on whether there is a scale effect or not. Kremer (1993) argues, on the basis of estimates of world population, that there must have been an increase in economic growth over the past one million years. Laincz and Perreto (1996) argue that R&D resources allocated to specific product lines have not increased. Others argued that countries do not provide the right level of analysis because of international trade linkages. These can be seen in the survey of Acemoglu (2009). In addition, a more recent research, Samaniego (2007), reconciles the presumption that R&D is a key driver of economic growth and the empirical evidence by showing that R&D contributes to growth through investment-specific technical change instead of directly forming the total factor productivity (TFP) change. In this way, the empirical "puzzles", including the weak link between measures of knowledge and productivity, and the estimates pointing to the presence of constant or even increasing returns to the production of ideas (which has counterfactual implication that rates of economic growth should increase with the population size), can be easily reconciled.

My model incorporates scale effect in the sense that a greater labor force creates more demand for intermediate goods, making R&D more profitable so that a greater labor force leads to faster growth. I do not deliberately remove the scale effect not only because that the models "without scale effect" themselves meet unsolved problems, but also because that in fact, the "knife-edge" assumptions in those models imply that the responsiveness of long-run growth with respect to policies results at least partly from the inclusion of the scale of labor supply into the factors determining per capita output growth. This can be seen in the survey of Jones (1999), as in Howitt (1999) who shows that while population growth positively affects per capita output growth, population growth itself depends on the amount of labor supply. Jones (2003) also inserts that when population is endogenized, the invariance result in the models "without scale effect", i.e., changes in the allocation of human capital to research have only level effects but no growth rate effects, can be overturned. Therefore, keeping scale effect in my model can be justified because it enables me to analyse how policies affect the endogenous labor-leisure choice. It is worth mentioning that our key argument, i.e., higher labor supply leading to higher growth, varies from those proposed in the baseline endogenous technological progress models in that the effective labor force in my model is represented by the working hours determined by representative agents with an aggregated scale of unit. As for the set-up of my model, considering that if population was growing the economy would not admit a steady state and the growth rate of the economy would increase over time (output reaching infinity in finite time and violating the transversality condition), I abstract my model from population growth and standardize the scale of population to unit, following Zeng and Zhang (2007), therefore the effective labor force in my model is exactly the labor-leisure choice of the representative agent, and is subject to the influence of fiscal policies, which is per se the interest of research of my thesis. Thus we can conclude that what I am doing does not give answers to the relevant criticisms over either side of the debate, because we cannot assume that the effect of an increase in the number of workers is just the same as an increase in hours worked per worker, on which we focus.

The result of my study is an example of second-best theory. The idea that taxes whose revenue is not used productively must reduce welfare is based on the first-best intuition that a waste of resources has a positive social cost. However the withdrawal of resources from productive use may have a social benefit in an economy in which there is imperfect competition, i.e. in a second-best environment.

Another contribution of my analysis is the following: as said above it is very frequent in works studying the effects of fiscal expenditures to assume financing by lump-sum taxes, taken to be non distortionary, or to assume that proceeds of taxes are returned lump-sum (e.g. Devereux and Love (1995), Lin and Russo (1999), Turnovsky (2000), Zeng and Zhang (2002), or Haruyama and Itaya (2006)). However I show that, with elastic labor supply, through general equilibrium effects a lump-sum tax will change relative prices and therefore be indirectly distortionary. In other words the effect on growth of a tax whose revenue is returned lump-sum will be different from the effect on growth of a tax whose proceeds are just thrown away and should therefore be studied separately.

The rest of this charpter is organized as follows: in section 2 the model is presented, section 3 describes the equilibrium conditions which have to hold in the model and analyses the balanced growth path characteristics of the model, section 4 works out the labor supply effect, growth effect and welfare effect of the lump-sum tax in the model, and does numerical calculations to show that such a tax can increase welfare for widely accepted estimates of the relevant parameters, and section 5 gives economic intuitions.

2 The model

2.1 Households

We assume that in the economy there is a continuum of length one of identical households.[3] Each has utility U given by:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \left(\frac{1}{1-\sigma} C^{1-\sigma} h(H) \right) dt \tag{1}$$

where C is consumption and H labor. ρ is rate of time discount. The following conditions ensure non satiation of consumption and leisure: $\sigma > 0$ and

$$h(H) > 0, \tag{2}$$

$$(1-\sigma)h'(H) < 0. \tag{3}$$

Strict concavity of instantaneous felicity imposes:

$$(1-\sigma)h''(H) < 0 \tag{4}$$

and

$$\sigma \frac{h''h}{(\sigma-1)} - h'^2 > 0. \tag{5}$$

The instantaneous budget constraint consumers face is given by:

$$\dot{F} = rF + wH - C - \tau_a \bar{\bar{F}}. \tag{6}$$

Households derive their income by loaning entrepreneurs their financial wealth F (of which all have the same initial endowment) and by supplying labor H to firms, taking the interest rate r and the wage rate w as given. There are lump-sum taxes proportional to average wealth, $\bar{\bar{F}}$, where given our normalization, $F = \bar{\bar{F}}$. Agents, being atomistic, take these averages as variables beyond their control. In this sense these are lump-sum taxes. At an optimum, the marginal rate of substitution between leisure and consumption must equal their relative price:

$$\frac{h'(H)}{h(H)} = \frac{w(\sigma-1)}{C}. \tag{7}$$

[3] As Zeng and Zhang (2007) note, normalizing the population to unity removes from the analysis of taxes the "scale effect" discussed by Jones (1995). For a very balanced view of the debate that followed see chapter 13 of Acemoglu (2009).

Optimal consumption and leisure must also obey this intertemporal condition:

$$-\sigma\frac{\dot{C}}{C}+\frac{h'(H)}{h(H)}\dot{H}=\frac{\dot{\lambda}}{\lambda}=\rho-r \tag{8}$$

where λ is the shadow value of wealth. Given a no Ponzi game condition the transversality condition imposes:

$$\lim_{t\to\infty}\lambda F\exp(-\rho t)=0. \tag{9}$$

2.2 Firms

In this economy there are a final goods sector and an intermediate goods sector. The former is perfectly competitive, whereas the latter is monopolistic. R&D activity leads to an expanding variety of intermediate goods. All patents have an infinitely economic life, that is, we assume no obsolescence of any type of intermediate goods.

Following Spence (1976) and Dixit-Stiglitz (1977) the production function of firm i in the final good sector is given by:

$$Y(i)=AL(i)^{1-\alpha}\int_0^N x(i,j)^{\alpha}di \tag{10}$$

where $Y(i)$ is the amount of final goods produced and $L(i)$ is labor used by firm i and $x(i,j)$ is the quantity this firm uses of the intermediate good indexed by j. For tractability both i and j are treated as continuous variables.[4] We assume $0<\alpha<1$. The final goods sector is competitive and we assume a continuum of length one of identical firms. We can then suppress the index i to avoid notational clutter. Firms maximize profits given by

$$Y-wL-\int_0^N P(j)x(j)dj \tag{11}$$

where w is the wage rate and $P(j)$ is the price of the intermediate good j. By profit maximization we have the demand for good j given by:

$$x(j)=L\left(\frac{A\alpha}{P(j)}\right)^{\frac{1}{1-\alpha}} \tag{12}$$

and labor demand by:

$$w=(1-\alpha)\frac{Y}{L}. \tag{13}$$

Since the firms in the final goods sector are competitive and there are constant returns to scale their profits are zero in equilibrium. In contrast the firms which

[4] For a discussion of the realism of the assumption as regards the intermediate products see Romer (1990).

produce intermediate goods patent which they invent then earn monopoly profits for ever. The cost of production of the intermediate good j, once it has been invented, is given by one unit of final good. The value of the patent for the j^{th} intermediate good $v(j,t)$ at time t is the present discounted value of such profits. The value of the j^{th} patent at time t is then

$$v(j,t) = \int_t^\infty (P(j) - 1)x(j)e^{-\overline{r}(s,t)(s-t)}ds \tag{14}$$

where $\overline{r}(s,t)$ is the average interest rate during the period of time from t to s.

The inventor of the j^{th} intermediate good chooses $P(j)$ to maximize profits $(P(j)-1)x(j)$ where $x(j)$ is given by 12, so for each j, the equilibrium price is:

$$P(j) = P = \frac{1}{\alpha} \tag{15}$$

and

$$x(j) = x = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}. \tag{16}$$

Notice a higher labor supply implies a higher quantity of each intermediate goods in equilibrium. Plugging equation 16 in equation 10 gives us equation

$$Y = NLA^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}} \tag{17}$$

while plugging 17 in 13 we have:

$$w = N(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}. \tag{18}$$

Assuming the interest rate and labor are constant over time, we have substituting 15 and 16 in 14:

$$v(j,t) = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\frac{1}{r}. \tag{19}$$

I will show below that if a balanced growth equilibrium exists, labor supply and the interest rate are indeed constant.

The cost of development of new products is η and there is free entry in the market for inventions, so intermediate goods firms will push the price of an invention to equate its cost and in equilibrium we will have:

$$r = C_1 L \tag{20}$$

where $C_1 \equiv \frac{1-\alpha}{\eta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}}$. Notice that the higher is labor supply the higher is the interest rate, as the scales of each intermediate good are increasing in labor supply, so the entrepreneurs will be willing to pay a higher interest rate.

2.3 Government

We assume no government consumption on goods. We also rule out a market for government bonds and assume that the government runs a balanced budget. The revenue from lump-sum taxes is wasted. The government budget constraint is thus:

$$G = \tau_a \bar{\bar{F}} \tag{21}$$

where on the left-hand side we have outflows and on the right-hand side we have inflows.

3 Market equilibrium

In calculating the equilibrium in the final goods market the total of intermediate goods used xN is subtracted from final production Y to obtain total value added. All investment in the model is investment in research and development of new intermediate goods $\eta\dot{N}$. The economy-wide resource constraint is therefore given by:

$$Y - xN = C + \eta\dot{N} + G. \tag{22}$$

We are now ready for the following:

Definition 1 *In a competitive equilibrium individual and aggregate variables are the same and prices and quantities are consistent with the (private) efficiency conditions for the households 6, 7, 8 and 9, the profit maximization conditions for firms in the final goods sector, 12 and 13 (or 18), and for firms in the intermediate goods sector, 15 (or 16) and 20, with the government budget constraint 21 and with the market clearing conditions for labor $H = L$, for wealth $F = \eta N$, and for the final good, 22.*

In equilibrium the social budget constraint 22 can be written as:

$$\frac{\dot{N}}{N} = \frac{1}{\eta N}(Y - xN - C - \tau_a \eta N). \tag{23}$$

Proposition 2 *If the economy follows a balanced growth path (hence BGP) variables grow at a constant rate, and in particular employment is constant at a value $\tilde{L}$. Along this path, rate of growth of capital and consumption, γ, is then given by:*

$$\gamma = \frac{r(\widetilde{L}) - \rho}{\sigma}. \tag{24}$$

Proof. Totally differentiating 7 we get:

$$\frac{\dot{C}}{C} = \frac{\dot{N}}{N} + (h'/h - h''/h')\dot{L}. \tag{25}$$

From this we deduce that along a BGP, when $\dot{L} = 0$, the rates of growth of C and N will be the same. From 8 and $\dot{L} = 0$, we get 24. ■

In Appendix 1 I show how to deduce from the competitive equilibrium conditions described above the following differential equation for labor, which is the fundamental dynamic equation of the model:

$$\dot{L} = \frac{\rho + \left(\sigma - 1 - \frac{\sigma}{\alpha}\left(\frac{(\sigma-1)h}{h'L} - 1\right)\right) r - \sigma\tau_a}{\sigma\frac{h''}{h'} + (1-\sigma)\frac{h'}{h}} \equiv \frac{B(L)}{A(L)}. \tag{26}$$

Notice that, as r is a linear function of L, 26 is a differential equation in L. The denominator of the fraction on the right-hand side $A(L)$ is always strictly positive for all values of L, by the negative definiteness condition of the hessian of the utility function 4, so the equation is defined for all values of L between 0 and 1. Along a BGP $\dot{L}$ will equal zero, so the numerator $B(L)$ will be zero, i.e. $B(\tilde{L}) = 0$ where $\tilde{L}$ is the BGP labor supply. To study the dynamic nature of a fixed point of 26, i.e. of BGP labor supply, we have to sign $d\dot{L}(\tilde{L})/d\tilde{L}$, the derivative of $\dot{L}$ with respect to L, calculated at the fixed point $\tilde{L}$ implicitly defined by $B(\tilde{L}) = 0$. If the derivative is positive the fixed point $\tilde{L}$ is a repeller and the BGP is locally determinate in the sense that if L were close to but not exactly equal to $\widetilde{L}$, then L would diverge further from $\widetilde{L}$. Thus, the BGP with $\widetilde{L}$ a repeller is a (locally) unique equilibrium path and we can say that there is no (local) indeterminacy in this case. If the equilibrium is unstable there will be no transitional dynamics to it, the economy will always follow the BGP. If $d\dot{L}(\widetilde{L})/d\widetilde{L}$ is negative then $\widetilde{L}$ is an attractor, that is if L is near $\widetilde{L}$ it will eventually approach it. So there is local indeterminacy, i.e. a continuum of equilibrium trajectories all converging to the fixed point. We have: $\frac{d\dot{L}}{dL}(\tilde{L}) = \frac{B'(\tilde{L})}{A(\tilde{L})} - \frac{A'(\tilde{L})B(\tilde{L})}{A^2(\tilde{L})} = \frac{B'(\tilde{L})}{A(\tilde{L})}$ (since $B(\tilde{L}) = 0$). Below we prove that $B(\tilde{L}) = 0$ implies $B'(\tilde{L}) > 0$. Since $d\dot{L}(\widetilde{L})/d\widetilde{L}$ is always positive, we can deduce that if BGP exists it is unique as from the phase diagram of 26 we can easily see that there is no way for $B(L)/A(L)$, which is a continuous function, to cross the horizontal axis from below two times in a row.

From the instability of the equilibrium we can also deduce that there will be no transitional dynamics in the model. Below we prove that $B(\tilde{L}) = 0$ implies $B'(\tilde{L}) > 0$. We are now ready for the following:

Proposition 3 *If a BGP equilibrium defined by $B(\tilde{L}) = 0$ exists, it is unique and locally determinate, so there is no transitional dynamics to it.*

Proof. Since the first derivative of r with respect to L equals C_1, we will then have the derivative of $B(L)$ with respect to L, deriving and rearranging:

$$B'(L) = C_1 \left[\sigma - 1 + \frac{\sigma}{\alpha}\left((1-\sigma)\left(1 - \frac{hh''}{h'^2}\right) + 1\right)\right]. \tag{27}$$

From condition 5, we have

$$(1-\sigma)\left(1 - \frac{hh''}{h'^2}\right) + 1 > (1-\sigma) + \frac{(1-\sigma)^2}{\sigma} + 1 = \frac{1}{\sigma}. \tag{28}$$

With 28, $B'(L)$ is strictly positive: $\sigma - 1 + \frac{\sigma}{\alpha}\left((1-\sigma)\left(1-\frac{hh''}{h'^2}\right)+1\right) > \sigma - 1 + \frac{1}{\alpha} > 0$. ■

4 Effects of taxes

4.1 Effect on labor

It is relatively simple to calculate the effect of taxes on labor supply in this model because the wage rate is not affected by labor supply. As said above equilibrium labor supply can be expressed as the solution to $B(\tilde{L}) = 0$. The effect of our tax program on labor can be deduced by using the total derivative of $B(\tilde{L}) = 0$ with respect to labor and the tax. We then have:

$$\frac{d\tilde{L}}{d\tau_a} = \frac{\sigma}{B'(\tilde{L})} = \frac{\sigma}{C_1\left[\sigma - 1 + \frac{\sigma}{\alpha}\left((1-\sigma)\left(1-\frac{hh''}{h'^2}\right)+1\right)\right]}. \tag{29}$$

As we have shown above that $B'(\tilde{L}) > 0$, this derivative is always positive, i.e. the increase in lump-sum tax will induce bigger BGP labor supply. This can be interpreted as a simple income effect: for fixed labor supply, the tax would make households poorer so since both consumption and leisure are normal goods they consume less and offer more labor.

Proposition 4 *An increase in the lump-sum tax rate whose proceeds are thrown to ocean will increase employment in equilibrium.*

4.2 Effect on growth

It is easy to see that the effect of the lump-sum tax on the rate of growth is positive as well, since the rate of growth of consumption increases one for one with the interest rate and the interest rate is proportional to labor supply. In detail we deduce the growth effect of tax τ_a by equation 20, 24 and 29 as follows:

$$\frac{d\gamma}{d\tau_a} = \frac{\partial\gamma}{\partial r}\frac{\partial r}{\partial\tilde{L}}\frac{d\tilde{L}}{d\tau_a} = \frac{1}{\sigma - 1 + \frac{\sigma}{\alpha}\left((1-\sigma)\left(1-\frac{hh''}{h'^2}\right)+1\right)}. \tag{30}$$

Not surprisingly the condition for the tax to be growth increasing is the same as that for it to be employment increasing so this derivative is also positive. We have the following:

Proposition 5 *An increase in the lump-sum tax rate whose proceeds are thrown to ocean will increase growth in the long-run equilibrium.*

4.3 Effect on welfare

Given γ, the constant rate of growth, and $\widetilde{L}$ the BGP labor supply, it is possible to calculate maximum lifetime utility W along a balanced growth path:

$$W = \int_{t=0}^{\infty} e^{-[\rho-\gamma(1-\sigma)]t}\left(\frac{1}{1-\sigma}C(0)^{1-\sigma}h(\tilde{L})\right)dt \tag{31}$$

where $C(0)$ is consumption at time 0.

In Appendix 3 it is shown how to express W as a differentiable function of τ_a and $\tilde{L}$ (itself a function of τ_a). The effect on welfare of an increase in the tax rate τ_a is then positive if $\frac{dW}{d\tau_a}$ is positive. To simplify calculations, I consider the following monotonically increasing transformation of W: $\frac{\log[(1-\sigma)W]}{1-\sigma}$. $\frac{d(\log[(1-\sigma)W])}{(1-\sigma)d\tau_a}$ signs as $\frac{dW}{d\tau_a}$ but is easier to manipulate algebraically so I will use it. We have:

$$\frac{d(\log[(1-\sigma)W])}{(1-\sigma)d\tau_a} = \frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tau_a} + \frac{d\tilde{L}}{d\tau_a}\frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tilde{L}}. \tag{32}$$

In Appendix 3 I also deduce the expression for $\frac{\log[(1-\sigma)W]}{1-\sigma}$, from which I can derive the following:

$$\frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tilde{L}} = \frac{(1-\sigma)\left(1-\frac{hh''}{h'^2}\right)+1}{\sigma-1}\cdot\frac{\frac{C_1}{\alpha}\left(\frac{h'\tilde{L}}{h}-\sigma\right)-\frac{h'}{h}\tau_a}{\frac{C_1}{\alpha}\left(\frac{(\sigma-1)h}{h'}-\tilde{L}\right)+\tau_a}, \tag{33}$$

$$\frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tau_a} = \frac{1}{\sigma-1}\cdot\frac{1}{\frac{C_1}{\alpha}\left(\frac{(\sigma-1)h}{h'}-\tilde{L}\right)+\tau_a}. \tag{34}$$

Substitution of 29, 33 and 34 for the corresponding terms in 32 leads to:

$$\frac{d(\log[(1-\sigma)W])}{(1-\sigma)d\tau_a} = \frac{\frac{\sigma}{C_1}\frac{h'}{h}\left(\frac{C_1\tilde{L}}{\alpha}\Phi+\tau_a\right)\Psi+1-\sigma}{(1-\sigma)\left(\frac{C_1\tilde{L}}{\alpha}\Phi+\tau_a\right)\left(\sigma-1+\frac{\sigma}{\alpha}\Psi\right)} \tag{35}$$

where Φ denotes $\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)$ and Ψ stands for $\left((1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)+1\right)$. Since $\Phi>0$ (see Appendix 2, the condition 49) and $\left(\sigma-1+\frac{\sigma}{\alpha}\Psi\right)>0$ (see the proof for positive $B'(L)$ in Proposition 3), we arrive at the following:

Proposition 6 *The sufficient and necessary condition for an increase in the lump-sum tax rate to be welfare enhancing is:*

$$(1-\sigma)\left(\frac{\sigma}{C_1}\frac{h'}{h}\left(\frac{C_1\tilde{L}}{\alpha}\Phi+\tau_a\right)\Psi+1-\sigma\right)\geq 0 \tag{36}$$

where $\Phi\equiv\frac{(\sigma-1)h}{h'\tilde{L}}-1$ *and* $\Psi\equiv(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)+1$.

If a value for τ_a exists such that for this value 36 holds as an equality, while it holds strictly for lower tax rates, 36 gives us an implicit expression for the optimal tax rate τ_a, given the tax program. In the next section I will show that for specifications of tastes and technology parameters often used in calibration exercises it is possible for the tax program to induce Pareto improvements as well as promote growth. The example I offer is useful to offer an intuition on the mechanism at work in producing the result.

4.4 Are wasted lump-sum taxes good for reasonable parameter values?

I consider here the following class of functions for the disutility of labor:

$$h(H) = (1 - H)^{1-\chi} \tag{37}$$

where $\chi > 1$ if $\sigma > 1$ or $\chi < 1 < \chi + \sigma$ if $0 < \sigma < 1$.

First we notice that we can now obtain an explicit solution for the equilibrium level of activity. With $B(\tilde{L}) = 0$ and by noting 20 we have:

$$\tilde{L} = \frac{\frac{\sigma}{\alpha}\frac{\sigma-1}{\chi-1}C_1 + \sigma\tau_a - \rho}{C_1\left(\frac{\sigma}{\alpha}\frac{\sigma+\chi-2}{\chi-1} + \sigma - 1\right)}. \tag{38}$$

By 24 and again noting 20 we obtain the BGP growth rate:

$$\gamma = \frac{\frac{1}{\alpha}\frac{\sigma-1}{\chi-1}C_1 - \left(1 + \frac{1}{\alpha}\frac{\sigma+\chi-2}{\chi-1}\right)\rho + \tau_a}{\frac{\sigma}{\alpha}\frac{\sigma+\chi-2}{\chi-1} + \sigma - 1}. \tag{39}$$

By using 38 and 39, the effects of lump-sum taxes on BGP labor supply and growth are therefore the following:

$$\frac{d\tilde{L}}{d\tau_a} = \frac{\sigma}{C_1\left(\frac{\sigma}{\alpha}\frac{\sigma+\chi-2}{\chi-1} + \sigma - 1\right)}$$

and

$$\frac{d\gamma}{d\tau_a} = \frac{1}{\frac{\sigma}{\alpha}\frac{\sigma+\chi-2}{\chi-1} + \sigma - 1}.$$

Both $\frac{d\tilde{L}}{d\tau_a}$ and $\frac{d\gamma}{d\tau_a}$ are in sign the same with the term $\left(\frac{\sigma}{\alpha}\frac{\sigma+\chi-2}{\chi-1} + \sigma - 1\right)$. This term is positive given the conditions of parameter σ and χ in the disutility specification 37: with $\sigma > 1$ the positiveness of this term is easy to be found; with $0 < \sigma < 1$, we can transform this term into $\frac{\sigma}{\alpha} + \left(\frac{1}{\alpha}\frac{\sigma}{1-\chi} - 1\right)(1-\sigma)$, whereby since $\chi < 1 < \sigma + \chi$ we have $\frac{\sigma}{1-\chi} > 1$ thus this term is also positive.

The welfare level can be written as:

$$W = \frac{(\eta N(0))^{1-\sigma}}{1-\sigma}\left(\frac{\sigma-1}{\chi-1}\frac{C_1}{\alpha}\right)^{1-\sigma}\frac{\left(1-\tilde{L}\right)^{2-\sigma-\chi}}{\frac{C_1}{\alpha}\left(\frac{\sigma-1}{\chi-1}\left(1-\tilde{L}\right) - \tilde{L}\right) + \tau_a}.$$

For simplicity in the numerical calculation I normalize the value of $(\eta N(0))^{1-\sigma}$ to 1 by choosing suitable value for $N(0)$ given the value of σ and η.[5]

[5] The value of η is got from the value of C_1 by normalizing $A^{\frac{1}{1-\alpha}}$ to 1 while C_1 equals $\frac{r}{\tilde{L}}$ by definition, where interest rate r and labor supply $\tilde{L}$ can be pinned down with data.

Proposition 6 implies that a positive welfare effect requires:

$$1-\frac{\sigma}{\sigma-1}(\sigma+\chi-2)\left[\frac{1}{\alpha}\left(\frac{\sigma-1}{\chi-1}-\frac{\tilde{L}}{1-\tilde{L}}\right)+\frac{\tau_a}{C_1\left(1-\tilde{L}\right)}\right]\geq 0. \quad (40)$$

Now I try to check whether 40 can hold in reasonable parametrizations of the model. I am completely aware that this model is not rich enough in number of variables, not to mention their dynamics, to fit the data well. Models that are rich enough to fit well become complex and difficult to interpret. The aim of my exercise is not realism but the understanding of mechanisms of action of policy not noticed before in the literature. My choices follow related studies of numerical R&D models (e.g. Jones and Williams (2000), Strulik (2007) and Zeng and Zhang (2007)).

A range of values for labor supply are used in calibration exercises. For example Jones et al. (2005) use $\tilde{L}=0.17$ while a value of 0.3 is often adopted. In 2005 the average US worker used 21% (24%) of her (his) time endowment to work.[6] I choose as benchmark value 0.23 and as range for sensitivity analysis 0.17-0.3.

Coming to $1/\alpha$, which is the monopoly markup in my model, I choose for it the range (1.1, 1.37) and take 1.2 as the benchmark, corresponding to the range of estimated markups of 1.05-1.4 indicated in Jones and William (2000), while Strulik (2007) fixes it at 1.2.[7] Coming to the long-run growth rate, Kenc (2004) chooses 1.5 percent, Strulik (2007) uses 1.75 percent, and Mankiw and Weinzierl (2006) select 2 percent. Therefore I set 1.75 percent as benchmark value and as range for sensitivity analysis 1.5 percent -2 percent. Again following Jones and Williams, the benchmark value for the steady-state interest rate is set to 7.0 percent, which represents the average real return on the stock market over the last century and let it vary between 4.0 percent and 10.0 percent.

σ is the risk aversion parameter and in the constant relative risk aversion (CRRA) preference as in my model equals the reciprocal of intertemporal elasticity of substitution (IES) in consumption. The value of σ can be tracked either in the literature of estimates of the relative risk aversion (RRA) or in the literature of estimates of the IES. There is considerable debate about the magnitude of the IES. Hall (1988) and Campbell (1999) estimate its value to be well below 1. Other studies that estimate the IES to be smaller than unit include Blundell, Meghir and Neves (1993) (0.5), Attanasio and Weber (1995) (0.6-0.7), Ogaki

[6] Source: The US Bureau of Labour Statistics, Current Population Survey, March 2005. For further discussion see chap.2, Borjas(2008).

[7] Jones and Williams note that in Romer (1990) the monopoly markup is equal to the inverse of the capital share $1/\alpha$. Empirically, this implies a gross markup (the ratio of price to marginal cost) of approximately 3, sharply exceeding empirical estimates of 1.05 to 1.4. In our model the capital share is $\alpha/(1+\alpha)$, so the trade off between matching income shares and matching mark-ups is less severe. Taking the data from the IMF's World Economic Outlook (April 2007) and the European Commission's Employment in Europe (2007), in the US capital share of income is 39.7% (2005), in EU-15 it is 42.2% (2006) (among which the highest is in Spain, at 45.5%). With markup 1.2, $\alpha/(1+\alpha)=0.4545$, with mark-up 1.37 it is $\alpha/(1+\alpha)=0.42$.

and Reinhart (1998) (0.27-0.766), Vissing-Jorgensen (2002) (0.3-1), Ziliak and Kniesner (2005) (0.7-1), and Engelhardt and Kumar (2009) (with point estimate at 0.74 and a 95% confidence interval of 0.37-1.21). Hansen and Singleton (1982), Attanasio and Weber (1989), Vissing-Jorgensen and Attanasio (2003) estimate the IES to be well in excess of 1. More recently, Hansen, Heaton, and Li (2008) consider a long-run risk model specification where the IES is pinned at 1, while Bansal and Yaron (2004) and Bansal (2007) estimate the IES over 1. However, it is worth noting that those studies with estimates of IES bigger than unit often employ preferences that allow for a separation between the IES and risk aversion (for example Vissing-Jorgensen and Attanasio (2003), Bansal and Yaron (2004) and Bansal (2007)). The RRA in these papers is not generally equal to the reciprocal of the IES and still has values over unit (Vissing-Jorgensen and Attanasio (2003) find the lowest value for RRA is 5, and Bansal (2007) sets RRA from 7.5-10). Meanwhile, literature of estimating RRA usually reveals over-unit RRA. Mehra and Prescott (1985) argue that a reasonable upper bound for risk aversion is around 10. Barsky et al. (1997) find that the risk aversion of those who hold stocks is around 4.2. Halek and Eisenhauer (2001), using data from life insurance purchases, estimate risk aversion around 3.75. Tödter (2008) get a point estimate close to 3.5. More relevant to our model, Chetty (2006) discusses two natural measures of risk aversion when hours of work are also included into the preferences. In one, hours are held constant; in the other, hours adjust when the random state becomes known. He notes that risk aversion is always greater by the first measure than by the second, finds a mean estimate of the coefficient of RRA almost equal to 1 and then shows that generating a coefficient of RRA bigger than 2 requires that wage increases cause sharper labor supply reductions. Therefore, the value of σ in my model is over unit. Considering both literatures, I take a range of σ from 1.1 to 3 and following Hall (2009) set the benchmark value 2 for it.

In table 1 I report the benchmark parameterization. The 5-tuple $\left\{r, \gamma, \tilde{L}, \sigma, \alpha\right\}$ implies values for ρ (through 24) and for χ (through 38 when $\tau_a = 0$ and $C_1 = r/\tilde{L}$ through 20).

Table 1: Benchmark parameterization

Parameters and Steady State Variables Determined	Parameter	Value
TFP Growth	γ	0.0175
Hours Worked over Time Endowment	$\tilde{L}$	0.23
Mark-up	$1/\alpha$	1.2
Interest rate	r	0.07
Intertemporal Elasticity of Substitution in Consumption	σ	2
Parameters and Steady State Variables Implied		
Labour Supply Parameter	χ	3.06
Rate of Time Discount	ρ	0.035
Optimal Tax Rate	$\widehat{\tau}_a$	-

Under the benchmark parametrization the tax program is welfare decreasing therefore the optimal tax policy in this parametrization should be no tax at all. However, sensitivity test gives out reasonable parametrization under which the wasted lump-sum taxation can be welfare increasing, i.e., $dW/d\tau_a|_{\tau_a=0}$ is positive. These welfare-improving results are stated in Table 2.

The optimal tax rate $\widehat{\tau}_a$ is obtained by plugging in 40 the expression for $\tilde{L}$ given by 38 and equating it to zero:

$$1-\frac{\sigma\left(\sigma+\chi-2\right)\left[\frac{1}{\alpha}\frac{\sigma-1}{\chi-1}C_1+\frac{1}{\alpha}\frac{\sigma+\chi-2}{(\sigma-1)(\chi-1)}\rho+\tau_a\right]}{\left(\frac{\sigma}{\alpha}+\sigma-1\right)C_1-\sigma\tau_a+\rho}=0, \tag{41}$$

to which the solution is:

$$\widehat{\tau}_a=\frac{\left[\frac{\sigma}{\alpha}+\sigma-1-\frac{\sigma}{\alpha}\frac{\sigma-1}{\chi-1}\left(\sigma+\chi-2\right)\right]C_1+\left[1-\frac{\sigma}{\alpha}\frac{(\sigma+\chi-2)^2}{(\sigma-1)(\chi-1)}\right]\rho}{\sigma\left(\sigma+\chi-2\right)}. \tag{42}$$

The root 42 gives us the optimal value of the lump-sum tax, for each five-tuple of the parameters $\{\sigma,\alpha,\rho,\chi,C_1\}$, of which as mentioned above the first two are determined, the other implied. For example, with σ=1.1 and the other parameters the same as their benchmark value (i.e., the third row in Table 2), it is then calculated that the lump-sum tax rate associated with maximum utility is 8.26%, under which the social welfare increases from -245.31 before tax to -242.92 after tax, hence improved by 0.97%.

Table 2: Alternative parameterizations

σ=1.1	χ	ρ	$dW/d\tau_a\|_{\tau_a=0}$	$d\gamma/d\tau_a\|_{\tau_a=0}$	$\widehat{\tau}_a$	$\Delta W/\|W\|$ (%)
γ=0.015	1.20	0.054	>0	>0	0.074	0.74
γ=0.0175	1.21	0.051	>0	>0	0.083	0.97
γ=0.02	1.21	0.048	>0	>0	0.091	1.26
$\tilde{L}$=0.17	1.30	0.051	>0	>0	0.133	2.27
$\tilde{L}$=0.3	1.14	0.051	>0	>0	0.039	0.22
$1/\alpha$=1.1	1.20	0.051	>0	>0	0.083	1.12
$1/\alpha$=1.37	1.22	0.051	>0	>0	0.082	0.77
r=0.04	1.23	0.021	>0	>0	0.072	3.29
r=0.10	1.20	0.081	>0	>0	0.093	0.54

So we see that for a wide region of the reasonable parameters space, a lump-sum tax whose proceeds are disposed will increase growth as well as welfare. This is sensitive to the value of parameter σ: the welfare-enhancing effect of the wasted lump-sum taxes is made more difficult if σ is higher.

In order to check that the parameter values for χ consistent with welfare improving lump-sum taxation are reasonable, I calculate the corresponding compensated elasticity of labor supply (or, the Frisch elasticity of labor supply, which is obtained by keeping constant the shadow value of wealth) and I compare the results with the available estimates. With the specification of the utility function 37, the Frisch elasticity of labor supply in the BGP is given by (see Appendix 4)

$$\left(1+\frac{\chi-1}{\sigma}\right)^{-1}\frac{1-\tilde{L}}{\tilde{L}}, \tag{43}$$

so it is decreasing in χ and increasing in σ. The values of the Frisch elasticity of labor supply consistent with optimal taxation are located between 0.98 and 1.50.[8] These values are consistent with the estimates of the Frisch elasticity found in the literature, which range from 0.5 to 3 or even higher to 3.8 (see, for example, Imai and Keane (2004), Domeij and Flodén (2006) and Prescott (2006)).

5 Economic intuition

After the numerical calculation it is worth noticing that though the growth rate will be largely increased,[9] the magnitude of possible welfare gains is not that large. It is because the lump-sum taxation, whose revenues are wasted, will reduce leisure and instantaneous consumption according to the income effect. The drop in the instantaneous utility should be compensated by a significant increase in the growth rate so as to achieve welfare improvement. That is, the welfare-enhancing effect of the wasted lump-sum taxes can happen should the dynamic gain overwhelm the static loss.

In the present section I first explain why the BGP growth rate is increased with the lump-sum taxes whose revenue is thrown to ocean. Then I analyse the two externalities that contribute to the welfare effect, and I also study the role of the parameters α, σ and χ in influencing the externalities.

1. Effect on growth

On first impact, the wasted lump-sum taxes reduce the consumers' disposable income without changing the opportunity cost of leisure. Since the substitution effect is zero, the income effect on leisure will cause labor supply to increase. Further, the increased labor supply induces a higher demand for the intermediate goods. This in turn induces a higher demand for investment in R&D so the interest rate will rise. Since the BGP growth rate is a monotonically increasing function of the interest rate, it also increases.

2. Two spillovers

There are two spillovers in the economy. Firstly, increased labor supply causes a positive spillover as it increases the value of patents. The worker considers only the increase in labor income w but output increases by $\frac{w}{\frac{1}{1+\alpha}}$ where $\frac{1}{1+\alpha}$ is the income share of labor. The difference is a spillover. Notice the size of this spillover is positively related with the value of α. This helps

[8] The values of the Frisch elasticity of labor supply associated with the before-tax parameter spaces are mainly located between 2 and 3, with 2.06 the lowest and 3.83 the highest. Only in one case the Frisch elasticity is over 3: with σ=1.1 and $\tilde{L}$ =0.17 (i.e., the fifth row in Table 2).

[9] The value of the increase in the growth rate is not shown in the text.

us to understand why the program which increases equilibrium employment is particularly beneficial when α is high. This spillover occurs because the price of intermediate goods is greater than their marginal cost so increased demand for an intermediate good has a first order benefit for its inventor. Secondly, the introduction of a new intermediate good causes increased welfare because it causes increased wages. The inventor only considers the part of the contribution to production that goes to capital (here income on patents). So the effect of an invention on the present discounted value of income is the cost of inventing divided by the income share of capital, that is $\frac{\eta}{\frac{\alpha}{1+\alpha}}$. When the return to capital is increased after the introduction of lump-sum taxes, the pace of invention of new patents will be accelerated. So this is also positive spillover.

3. Level effect

With the extraction of lump-sum taxes, the instantaneous consumption decreases according to the negative income effect. Considered that the leisure is also reduced, this causes a loss to the static level of welfare. Therefore, though the two positive spillovers can result in higher equilibrium growth rate, their effects are counteracted by the loss of the static level of welfare. The sign of overall welfare effect of the tax program is thus determined by the relatively stronger one given the tradeoff between growth and level.

4. Factors influencing the welfare effect

Two things influence the magnitudes of the spillovers. One is the income share of labor: if it is small the first spillover is large and the second is small. However, since the two spillovers are both positive, the value of α does not alter much the magnitude of their joint effect on welfare. This can be seen from the comparison between row 7 and 8 in table 2 that the optimal rates of the wasted lump-sum taxes do not differ from each other much. The other is the effect of the policy on labor supply, for which the elasticity of labor supply plays an important role. In our specification of the disutility function of labor 37, the elasticity of labor supply is decreasing in the parameter χ and $\tilde{L}$ but increasing in the parameter σ (see 43). A smaller χ or $\tilde{L}$ means higher elasticity of labor supply so a cut in disposable income will cause labor supply to increase much. To see the influence of parameter χ, since χ in our numerical calculation is obtained by the implied value from the function between χ and the other parameters $\{r, \gamma, \tilde{L}, \sigma, \alpha\}$, we should compare the results under different values of χ which are generated with only one determining parameter free and the others fixed. Therefore, we can refer to row 7 and 8 in table 2, where $\{r, \gamma, \tilde{L}, \sigma\}$ are the same with only α varying, that the bigger is χ, the lower is the optimal rate of tax and the smaller is the improvement of welfare. As for the affect of $\tilde{L}$, we can find from row 5 and 6 in table 2 (where the free parameters $\{r, \gamma, \sigma, \alpha\}$ are the same) that the smaller is the before-tax labor supply $\tilde{L}$, the higher is the optimal tax rate as well as the welfare improvement.

A bigger σ also means smaller intertemporal substitution elasticity of consumption, or that consumers weigh more the current consumption (lower) than the future (higher) ones. So, when the instantaneous consumption is decreased with the wasted lump-sum taxes, this reduction is given more weight than the future gain. This can explain why with big σ we can only have welfare reduced by the tax: big σ means small IES in consumption, therefore to keep the same utility level the reduction in instantaneous consumption should be compensated by large increase in future consumption, which renders it difficult to achieve the welfare improvement.

The tradeoff between the opposing growth and level effect of the tax program also depends on the subjective discount rate ρ of the representative household. With smaller ρ the tax program can be more easily to enhance welfare: smaller ρ means that the discounted value of the future consumption, growing at a given growth rate, will be bigger so that it can more easily compensate for the loss in instantaneous consumption. In table 2, bigger γ or lower r implies smaller ρ, and associated with it we can see the bigger potential of the tax to improve welfare.

Appendices

Appendix 1

By 18 and 20 we can get the following relationship between labor income and capital income:

$$wL = \frac{r\eta N}{\alpha}. \tag{44}$$

Using the factor exhaustion condition that the wage bill plus total interest payments is equal to GNP, that is $Y - xN = wL + r\eta N$, substituting for C using equation 7 and noting 44, we can write 23 as:

$$\frac{\dot{N}}{N} = r\left(1 - \frac{1}{\alpha}\left(\frac{(\sigma-1)h(L)}{h'(L)L} - 1\right)\right) - \tau_a. \tag{45}$$

Substituting 25 for $\frac{\dot{C}}{C}$ in 8 we get:

$$-\sigma\left[\frac{\dot{N}}{N} + (h'/h - h''/h')\dot{L}\right] + \frac{h'}{h}\dot{L} = \rho - r. \tag{46}$$

Finally if we substitute in 46 the expression for $\frac{\dot{N}}{N}$ given by 45 we obtain 26 in the text.

Appendix 2

Transversality condition 9 requires $\gamma < r$. In an initially taxless economy, 45 gives in equilibrium

$$\gamma = r\left(1 - \frac{1}{\alpha}\left(\frac{(\sigma-1)h(L)}{h'(L)L} - 1\right)\right)$$

so $\gamma < r$ leads to

$$\frac{(\sigma-1)h(L)}{h'(L)L} > 1. \tag{47}$$

In addition, in a growing economy the investment should be positive, therefore in an initially taxless economy there should be

$$C < Y - xN,$$

for which by using 7, 16, 17 and 18 we get

$$\frac{(\sigma-1)h(L)}{h'(L)L} < 1+\alpha. \tag{48}$$

Combining condition 47 and 48 we have

$$1 < \frac{(\sigma-1)h(L)}{h'(L)L} < 1+\alpha. \tag{49}$$

Appendix 3

By solving the integral in 31 we obtain:

$$W = \frac{1}{1-\sigma}\frac{C(0)^{1-\sigma}h(\tilde{L})}{\rho-\gamma(1-\sigma)}. \tag{50}$$

By using 7, 20 and 44 we can write:

$$C(0) = \eta N(0)\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})}\frac{C_1}{\alpha}$$

where $N(0)$ is the initial stock of patents. Using 24 we have:

$$\rho-\gamma(1-\sigma) = r-\gamma,$$

while by using 45 to get an expression for γ, we obtain:

$$r-\gamma = \frac{r}{\alpha}\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}-1\right)+\tau_a = \frac{C_1\tilde{L}}{\alpha}\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}-1\right)+\tau_a.$$

We can thus rewrite 50 as:

$$W = \frac{(\eta N(0))^{1-\sigma}}{1-\sigma}\left(\frac{\sigma-1}{h'(\tilde{L})}\frac{C_1}{\alpha}\right)^{1-\sigma}\frac{h(\tilde{L})^{2-\sigma}}{\frac{C_1\tilde{L}}{\alpha}\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}-1\right)+\tau_a}.$$

Thus we have the increasing monotonically transformation of W:

$$\begin{aligned}\frac{\log[(1-\sigma)W]}{1-\sigma} &= \log(\eta N(0))+\log\left(\frac{\sigma-1}{h'(\tilde{L})}\right)+\log\left(\frac{C_1}{\alpha}\right)+\frac{2-\sigma}{1-\sigma}\log(h(\tilde{L})) \\ &\quad -\frac{1}{1-\sigma}\log\left(\frac{C_1\tilde{L}}{\alpha}\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}-1\right)+\tau_a\right).\end{aligned}$$

Appendix 4

To calculate the Frisch elasticity of labor supply we have that one optimality condition of the household is:

$$\left(\frac{(\sigma-1)w}{h'(H)}\right)^{-\sigma} h(H)^{1-\sigma} = \lambda. \tag{51}$$

Since the Frisch elasticity of labor supply is the measure under constant marginal utility of wealth (here it is λ), we can take the total derivative with respect to H and w for equation 51, keeping λ fixed and derive the following:

$$\frac{dH}{dw} |_{\bar{\lambda}} = \frac{\sigma h(H) h'(H)}{w\left((1-\sigma)h'(H)^2 + \sigma h(H) h''(H)\right)}. \tag{52}$$

Therefore the Frisch elasticity of labor supply e_F is

$$e_F = \frac{dH}{dw} |_{\bar{\lambda}} \cdot \frac{w}{H} = \frac{\sigma h(H) h'(H)}{H\left((1-\sigma)h'(H)^2 + \sigma h(H) h''(H)\right)}. \tag{53}$$

With the specification of labor-disutility function in our model, the Frisch elasticity of labor supply is thus

$$e_F = \left(1 + \frac{\chi - 1}{\sigma}\right)^{-1} \frac{1-H}{H}. \tag{54}$$

In the BGP the Frisch elasticity of labor supply is exactly that in the text.

Chapter II. Lump-sum taxes in a creative destruction model

6 The model

The last chapter modeled technological progress as an increase in the number of types of products, N. In this chapter, we allow for improvements in the quality or productivity of each type. This approach has come to be known as the Schumpeterian approach to endogenous growth. An important aspect of the Schumpeterian model is that, when a product or technique is improved, the new good or method tends to displace the old one. Thus it is natural to model different quality grades for a good of a given type as close substitutes. We make the extreme assumption that the different qualities of a particular type of intermediate input are perfect substitutes; hence, the discovery of a higher grade turns out to drive out the lower grades completely. For this reason, successful researchers along the quality dimension tend to eliminate or "destroy" the monopoly rentals of their predecessors, a process labeled as "creative destruction" by Schumpeter (1934) and Aghion and Howitt (1992). On the normative side, the process of creative destruction implies a "business-stealing" effect.

With no assumption on either cost advantage or complete property right, a monopolist always has lower incentives to undertake innovation than a competitive firm. This result, which was first pointed out in Arrow's (1962) seminal paper, is referred to as the replacement effect. The terminology reflects the intuition for the result; the monopolist has lower incentives to undertake innovation than the firm in a competitive industry because with its innovation will replace its own already existing profits. In contrast, a competitive firm would be making zero profits and thus had no profits to replace. An immediate and perhaps more useful corollary of this proposition is the following: An entrant will have stronger incentives to undertake an innovation than an incumbent monopolist. The potential entrant is making zero profits without the innovation. The replacement effect and this corollary imply that in many models entrants have stronger incentives to invest in R&D than incumbents. Acemoglu (2009) supplies a good survey of the innovation by entrant, which directly leads to the so-called business-stealing effect.

The economy in the present chapter is the same as in the previous chapter except that the R&D activity contributes to new patents so as a series of increasing quality of intermediate goods. That is, there is obsolescence of any type of intermediate goods when a new patent is invented successfully. The household sector and the government sector are the same as those in the variety expansion model so hereby I avoid repeat. In this section I focus on the model for firms.

6.1 Final good sector

Following again Spence (1976) and Dixit-Stiglitz (1977) the production function of firm i in the final good sector is given by:

$$Y(i) = AL(i)^{1-\alpha} \int_0^N \tilde{x}(i,j)^\alpha di \tag{55}$$

where A is a positive technologic parameter, α is the ratio of intermediate input and $0 < \alpha < 1$, N is the dimension of the varieties of intermediate goods in the economy and is assumed fixed in this model, $Y(i)$ is the amount produced and $L(i)$ is labor used by firm i and $\tilde{x}(i,j)$ is the quality-adjusted quantity this firm uses of the intermediate good indexed by j, and is defined as

$$\tilde{x}(i,j) = q^{k_j} x(i,j) \tag{56}$$

where $x(i,j)$ is the physical input of the intermediate good j by firm i, q is the unit of rung of the quality ladder and $q > 1$ as well as $\alpha q \leq 1$,[10] k_j is the improvements in quality that have occurred in intermediate good sector j. For tractability both i and j are treated as continuous variables. The final good production sector is competitive and I assume a continuum of length one of identical firms. I can then suppress the index i to avoid notational clutter.

[10] Condition $\alpha q \leq 1$ is necessary for keeping a non-explosive economy.

The numeraire in the economy is one unit of final output and the price of any final good is standardised to 1. Firms maximize profits given by

$$Y - wL - \int_0^N P(j)x(j)dj$$

where w is the wage rate and $P(j)$ is the price of the intermediate good j. By profit maximization we have:

$$x(j) = \left(\frac{A\alpha q^{\alpha k_j}}{P(j)}\right)^{\frac{1}{1-\alpha}} L \tag{57}$$

and

$$w = (1-\alpha)\frac{Y}{L}. \tag{58}$$

6.2 Intermediate good sector

6.2.1 Production

Suppose the production of intermediate goods utilizes only final output and by choosing the unit of intermediate goods I can set the marginal cost to 1. The inventor of the j^{th} intermediate good chooses $P(j)$ to maximize profits $(P(j)-1)x(j)$ where $x(j)$ is given by 57, so for each j:

$$P(j) = P = \frac{1}{\alpha} \tag{59}$$

and

$$x(j) = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}q^{\alpha k_j/(1-\alpha)}. \tag{60}$$

I define the aggregate quality index

$$Q \equiv \int_{j=1}^N q^{\alpha k_j/(1-\alpha)}dj. \tag{61}$$

Summing over j the both sides of equation 60 we get the total output of intermediate goods

$$X = \int_{j=1}^N x(j)dj = QLA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}. \tag{62}$$

Using 60 and noticing 61, we get the final goods output

$$Y = QLA^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}. \tag{63}$$

From 62 and 63 we get the relation

$$X = \alpha^2 Y. \tag{64}$$

Substituting 63 for Y into equation 58, the wage rate is now

$$w = Q(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}. \tag{65}$$

I define

$$\bar{\pi} \equiv (1-\alpha)LA^{\frac{1}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}}. \tag{66}$$

By using 59 and 60, the profit of the intermediate good j producer is then

$$\pi(k_j) = \bar{\pi}q^{\alpha k_j/(1-\alpha)}. \tag{67}$$

So the total profit of the intermediate good sector is

$$\pi = \int_{j=1}^{N} \pi(k_j)dj = \bar{\pi}Q. \tag{68}$$

Using 63, 66 and 68 we get the relation

$$\pi = \alpha(1-\alpha)Y. \tag{69}$$

6.2.2 R&D Activity

Let $p(k_j)$ denote the probability per unit of time of a successful innovation in the j^{th} intermediate good sector when the top-of-the-line quality is k_j. In other words $p(k_j)$ is the probability per unit of time that an outside researcher will raise the quality of intermediate good j from k_j to k_j+1. Assume that the probability of the incumbent losing his monopoly position is generated from a Poisson process.

I assume that $p(k_j)$ depends positively on R&D effort $z(k_j)$, which is the aggregate flow of resources expended by potential innovators in sector j when the highest rung available is k_j; and that $p(k_j)$ also depends negatively on $\phi(k_j)$, which captures the increasing difficulty of innovation with the increasing k_j and is represented by

$$\phi(k_j) = \frac{1}{\zeta}q^{-\alpha(k_j+1)/(1-\alpha)} \tag{70}$$

where $\zeta > 0$ is a parameter that represents the cost of doing research. Additionally, I assume that the distribution of the expenditure across researchers does not have any influence on $p(k_j)$. Thus, the probability of research success is endogenized as

$$p(k_j) = z(k_j) \cdot \phi(k_j). \tag{71}$$

As what has described, $p(k_j)$ is increasing in $z(k_j)$ and decreasing in $\phi(k_j)$.

If I let t_{k_j} be the moment when the k_j quality improvement is made and t_{k_j+1} the time of the next improvement by a competitor, flow of profit $\pi(k_j)$ applies only from time t_{k_j} to t_{k_j+1}. Let $T(k_j)$ denote the duration of the monopoly for the inventor of rung k_j, that is, $T(k_j) \equiv t_{k_j+1} - t_{k_j}$.

Denote by $v(k_j)$ the present value of all the profits that the inventor of rung k_j evaluated at time t_{k_j}:

$$v(k_j) = \int_{t_{k_j}}^{t_{k_j+1}} \pi(k_j) e^{-\bar{r}(s,t_{k_j})(s-t_{k_j})} ds \tag{72}$$

where

$$\bar{r}(s, t_{k_j}) \equiv \frac{1}{s - t_{k_j}} \int_{t_{k_j}}^{s} r(\omega) d\omega$$

is the average interest rate between times t_{k_j} and s. In a balanced growth equilibrium, the interest rate should be a constant r. Hence in equilibrium, the present value 72 can be simplified as

$$v(k_j) = \frac{1}{r}\pi(k_j)[1 - \exp(-r \cdot T(k_j)]. \tag{73}$$

Since $T(k_j)$ is a random variable subject to the Poisson process with the arrival rate $p(k_j)$, by noticing the equation 67 for substituting for $\pi(k_j)$ in equation 73, the expected present value is in fact

$$E[v(k_j)] = \frac{\bar{\pi}}{r + p(k_j)} q^{\alpha k_j/(1-\alpha)}. \tag{74}$$

I assume that potential innovators care only about the expected present value $E[v(k_j + 1)]$ and not about the randomness of the return. Free entry is allowed in the R&D activity. Thus the net expected return per unit of time in the R&D investment must be zero. That is, we have the free entry condition

$$p(k_j)E[v(k_j + 1)] - z(k_j) = 0. \tag{75}$$

By substituting for $p(k_j)$ from equation 70 and 71, and using 74 for $E[v(k_j + 1)]$, for any positive expenditure $z(k_j)$, equation 75 becomes

$$r + p(k_j + 1) = \frac{\bar{\pi}}{\zeta}. \tag{76}$$

Therefore, equation 76 means that the probability of research success per unit of time is the same in each sector, independent of the quality-ladder position, and is given by

$$p = \frac{\bar{\pi}}{\zeta} - r. \tag{77}$$

By using equation 71 for $z(k_j)$, and using 70 for $\phi(k_j)$ as well as 77 for p, the amount of total resources devoted to R&D is

$$Z = \int_{j=1}^{N} z(k_j) dj = q^{\alpha/(1-\alpha)} Q(\bar{\pi} - r\zeta). \tag{78}$$

Substituting for p from equation 77 into equation 74, we get the aggregated market value of firms

$$V = \int_{j=1}^{N} E[v(k_j)]dj = \zeta Q. \tag{79}$$

7 Market equilibrium

To obtain total value added, the total of intermediate goods used X is subtracted from final production Y. Again, all investment in the model is the investment in research and development of more advanced intermediate goods Z. The economy-wide resource constraint is therefore given by

$$Y - X = C + Z + G. \tag{80}$$

Notice that in equilibrium the households' financial wealth equals the total value of existing firms in the economy, i.e. $F = V$. Thus, since $G = \tau_a F$ (see 21), the social budget constraint 80 can be rewritten as

$$Z = Y - X - C - \tau_a V. \tag{81}$$

We are now ready for the following:

Definition 7 *In a competitive equilibrium individual and aggregate variables are the same and prices and quantities are consistent with the (private) efficiency conditions for the households 6, 7, 8 and 9, the profit maximization conditions for firms in the final goods sector, 57 and 58 (or 65), and for firms in the intermediate goods sector, 59 (or 60) and 77, with the government budget constraint 21 and with the market clearing conditions for labor $H = L$, for wealth $F = V$, and for the final good, 80.*

I consider only the balanced growth path (hence BGP) of the model: labor supply L, interest rate r, research success probability p are constant while the other variables, including consumption C, aggregate quality index of the patents Q, wealth (or capital) F, and R&D expenditure Z are at the same constant growth rate.

8 implies that along a BGP with L constant consumption grows as:

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\sigma}. \tag{82}$$

We need track down the technic growth rate $\frac{\dot{Q}}{Q}$. Since in each intermediate good sector, the quality does not change if no innovation occurs but rises up for one rung in the case of research success, and since the probability per unit of time of a success p is the same for all sectors, the expected change rate in Q per unit of time is thus given by

$$\frac{E(\Delta Q)}{Q} = \frac{1}{Q}\int_{j=1}^{N} p[q^{\alpha(k_j+1)/(1-\alpha)} - q^{\alpha k_j/(1-\alpha)}]dj = p\left(q^{\frac{\alpha}{1-\alpha}} - 1\right). \tag{83}$$

Suppose the variety of the intermediate goods, N, is large enough to treat Q as differentiable, by the law of large numbers, the average growth rate of Q measured over any finite interval of time $\frac{\dot{Q}}{Q}$ will be non stochastic and equal to $\frac{E(\Delta Q)}{Q}$. Substituting for p from 77 into 83, we get the growth rate of Q:

$$\frac{\dot{Q}}{Q} = \left(\frac{\bar{\pi}}{\zeta} - r\right)\left(q^{\frac{\alpha}{1-\alpha}} - 1\right). \tag{84}$$

Proposition 8 *If the economy follows a balanced growth path (hence BGP) variables grow at a constant rate, and in particular employment is constant at a value $\tilde{L}$. Along this path, rate of growth of capital and consumption, γ, is then given by*

$$\gamma = \frac{\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)\left(\frac{\bar{\pi}(\tilde{L})}{\zeta} - \rho\right)}{1 + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}. \tag{85}$$

Proof. When labor supply is constant at $\tilde{L}$, the wage w is proportional to the aggregate quality index Q by 65, so w and Q grow at the same rate. And 7 implies that consumption and the wage must grow at the same rate so we have:

$$\frac{\dot{C}}{C} = \frac{\dot{Q}}{Q} \equiv \gamma. \tag{86}$$

By using 82, 84 and 86, we can solve out the BGP growth rate γ as 85, and interest rate r as follows:

$$r = \frac{\rho + \sigma\frac{\bar{\pi}(\tilde{L})}{\zeta}\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}{1 + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}. \tag{87}$$

Substitution for r from equation 87 into 77 helps to get the endogenous innovation success rate p:

$$p = \frac{\frac{\bar{\pi}(\tilde{L})}{\zeta} - \rho}{1 + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}. \tag{88}$$

■

Equations 85, 87 and 88 show that with $q^{\frac{\alpha}{1-\alpha}} > 1$ and $\bar{\pi}$ an increasing function of L (see 66), the BGP γ, r and p are all increasing in L.

In Appendix 5 I show how to deduce from the competitive equilibrium conditions described above the following differential equation of labor, which is the fundamental dynamic equation of the model:

$$\dot{L} = \frac{\rho + (\sigma - 1)r + \frac{\sigma\bar{\pi}}{\alpha\zeta}\left(1 + \frac{(1-\sigma)h}{h'L}\right) - \sigma\tau_a}{\sigma\frac{h''}{h'} + (1-\sigma)\frac{h'}{h}} \equiv \frac{B(L)}{A(L)}. \tag{89}$$

Hereby the denominator of the fraction on the right-hand side $A(L)$ is always strictly positive for all values of L for the same reason as stated in the counterpart in Chapter 1. So the equation is defined for all values of L beween 0 and 1.

Along a BGP $\dot{L}$ will equal zero, i.e. $B(\tilde{L}) = 0$ where $\tilde{L}$ is the BGP labor supply. Again we have to sign $d\dot{L}(\widetilde{L})/d\widetilde{L}$ to study the dynamic nature of a fixed point of 89. We have: $\frac{d\dot{L}}{dL}(\tilde{L}) = \frac{B'(\tilde{L})}{A(\tilde{L})} - \frac{A'(\tilde{L})B(\tilde{L})}{A^2(\tilde{L})} = \frac{B'(\tilde{L})}{A(\tilde{L})}$ (since $B(\tilde{L}) = 0$). Below I prove that $B(\tilde{L}) = 0$ implies $B'(\tilde{L}) > 0$. Since $d\dot{L}(\widetilde{L})/d\widetilde{L}$ is always positive, we can deduce that if BGP exists it is unique as from the phase diagram of 89 we can easily see that there is no way for $B(L)/A(L)$, which is a continuous function, to cross the horizontal axis from below two times in a row. Positive $d\dot{L}(\widetilde{L})/d\widetilde{L}$ also means instability of the equilibrium, therefore we can deduce that there will be no transitional dynamics in the model. Therefore Proposition 3 also serves here and in the following I show the proof for it:

Proof. First, we derive the derivative of $\bar{\pi}$ and that of r with respect to labor as the following:

$$\bar{\pi}'(L) = \frac{\bar{\pi}}{L}$$

and

$$r'(L) = \frac{\bar{\pi}}{\zeta L} \frac{\sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}{1 + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}.$$

Using $\bar{\pi}'(L)$ and $r'(L)$ we will then have, deriving and rearranging:

$$B'(L) = \frac{\sigma\bar{\pi}}{\zeta L}\left[\frac{(\sigma - 1)\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}{1 + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)} + \frac{1}{\alpha}\left((1-\sigma)\left(1 - \frac{hh''}{h'^2}\right) + 1\right)\right]. \tag{90}$$

By condition 28, $B'(L)$ is strictly positive: it can be seen easily with $\sigma > 1$; and with $0 < \sigma < 1$ noting $q^{\frac{\alpha}{1-\alpha}} - 1 > 0$ we have

$$\begin{aligned} & \frac{(\sigma - 1)\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}{1 + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)} + \frac{1}{\alpha}\left((1-\sigma)\left(1 - \frac{hh''}{h'^2}\right) + 1\right) \\ > \quad & \frac{\sigma - 1}{\frac{1}{q^{\frac{\alpha}{1-\alpha}} - 1} + \sigma} + \frac{1}{\alpha\sigma} > \frac{\sigma - 1}{\sigma} + \frac{\frac{1}{\alpha}}{\sigma} > 0. \end{aligned}$$

■

8 Effects of taxes

8.1 Effect on labor

As said above equilibrium labor supply can be expressed as the solution to $B(\tilde{L}) = 0$. The effect of the tax program on labor can be deduced by using the total derivative of $B(\tilde{L}) = 0$ with respect to labor and the tax. The partial derivative of $B(\tilde{L})$ with respect to tax τ_a is

$$B'_{\tau_a} = -\sigma$$

therefore by the formula of total derivative and with equation 90 we have

$$\frac{d\tilde{L}}{d\tau_a} = -\frac{B'_{\tau_a}}{B'(\tilde{L})} = \frac{1}{\frac{\bar{\pi}}{\zeta\tilde{L}}\left[\frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)} + \frac{1}{\alpha}\left((1-\sigma)\left(1-\frac{hh''}{h'^2}\right)+1\right)\right]}. \tag{91}$$

Considered the positiveness of $B'(\tilde{L})$ the effect of the lump-sum tax τ_a on BGP labor supply is thus positive. Hence Proposition 4 also serves here. This can be interpreted as a simple income effect: for fixed labor supply, the tax would make households poorer so since both consumption and leisure are normal goods they consume less and offer more labor.

8.2 Effects on growth

It is easy to see that the effect of the lump-sum tax on the rate of growth is positive as well, since the rate of growth of consumption increases one for one with the interest rate and the interest rate is an increasing function of labor supply. In detail we deduce the growth effect of tax τ_a by equations 85 and 91 as follows:

$$\frac{d\gamma}{d\tau_a} = \frac{\partial\gamma}{\partial\bar{\pi}}\frac{\partial\bar{\pi}}{\partial\tilde{L}}\frac{d\tilde{L}}{d\tau_a} = \frac{q^{\frac{\alpha}{1-\alpha}}-1}{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right) + \frac{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{\alpha}\left((1-\sigma)\left(1-\frac{hh''}{h'^2}\right)+1\right)}. \tag{92}$$

$\frac{d\gamma}{d\tau_a}$ is positive because $q^{\frac{\alpha}{1-\alpha}} > 1$ and the denominator of the RHS of 92 is in sign the same with $B'(\tilde{L})$ thus positive. Therefore Proposition 5 can also be applied here.

8.3 Effect on welfare

Given γ, the constant rate of growth, and $\widetilde{L}$ the BGP labor supply, it is possible to calculate maximum lifetime utility W along a balanced growth path as 31. In Appendix 7 I show how to express W as a differentiable function of τ_a and $\tilde{L}$ (itself a function of τ_a). The effect on welfare of an increase in the tax rate τ_a is then positive if $\frac{dW}{d\tau_a}$ is positive. I consider again the transformation of W: $\frac{\log[(1-\sigma)W]}{1-\sigma}$ which has the same sign as $\frac{dW}{d\tau_a}$ and is more tractable for algebraic calculation.

First, we have the formula for the derivative of $\frac{\log[(1-\sigma)W]}{1-\sigma}$ with respect to τ_a the same as 32. Then we have after deriving and rearranging:

$$\frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tilde{L}} = \frac{(\sigma-1)\left(\frac{hh''}{(h')^2}-1\right)+1}{1-\sigma}\cdot\frac{\frac{\bar{\pi}}{\alpha\zeta}\left(\frac{\sigma}{\tilde{L}}-\frac{h'}{h}\right)+\frac{h'}{h}\tau_a}{\frac{\bar{\pi}}{\alpha\zeta}\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)+\tau_a} \tag{93}$$

and

$$\frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tau_a} = -\frac{1}{1-\sigma}\cdot\frac{1}{\frac{\bar{\pi}}{\alpha\zeta}\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)+\tau_a}. \tag{94}$$

Using 93, 94 and 91, equation 32 can be finally deduced to:

$$\frac{d(\log[(1-\sigma)W])}{(1-\sigma)d\tau_a} = \frac{\frac{h'\tilde{L}}{h}\frac{\zeta}{\bar{\pi}}\left(\frac{\bar{\pi}}{\alpha\zeta}\Phi+\tau_a\right)\Psi - \frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}}{(1-\sigma)\left(\frac{\bar{\pi}}{\alpha\zeta}\Phi+\tau_a\right)\left(\frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}+\frac{1}{\alpha}\Psi\right)} \tag{95}$$

where $\Phi \equiv \left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)$ and $\Psi \equiv \left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)$, the same as in Chapter 1. Remember we have $\frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}+\frac{1}{\alpha}\Psi > 0$ (see 90). We can get the following proposition.

Proposition 9 *: The sufficient and necessary condition for an increase in the rate on lump-sum tax whose proceeds are wasted to increase welfare is:*

$$(1-\sigma)\left[\frac{h'\tilde{L}}{h}\frac{\zeta}{\bar{\pi}}\left(\frac{\bar{\pi}}{\alpha\zeta}\Phi+\tau_a\right)\Psi - \frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right] \geq 0 \tag{96}$$

where $\Phi \equiv \left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)$ *and* $\Psi \equiv \left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)$.

Of course if a value for τ_a exists such that for this value 96 holds as an equality, while it holds strictly for lower tax rates, 96 gives us an implicit expression for the optimal tax rate, given the tax program. In the next section I will show that for specifications of tastes and technology parameters often used in calibration exercises it is possible for the tax program to induce Pareto improvements as well as promote growth. The example I offer is useful to offer an intuition on the mechanism at work in producing the result.

8.4 Are wasted lump-sum taxes good for reasonable parameter values?

The specification of the disutility function of labor 37 is still considered for carrying the numerical calculation. First, an implicit solution for the equilibrium labor supply can be obtained by solving $B(\tilde{L}) = 0$:

$$\tilde{L} = \frac{\frac{\sigma\bar{\pi}}{\alpha\zeta}\frac{\sigma-1}{\chi-1}}{\rho+(\sigma-1)r+\frac{\sigma\bar{\pi}}{\alpha\zeta}\frac{\sigma+\chi-2}{\chi-1}-\sigma\tau_a}. \tag{97}$$

By noting 66 the profit rate $\frac{\bar{\pi}}{\zeta}$ can be written as

$$\frac{\bar{\pi}}{\zeta} = C_2 L \tag{98}$$

where $C_2 \equiv \frac{1-\alpha}{\zeta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}}$. Then substituting 87 and 98 into 97, we get the explicit solution to BGP labor supply as:

$$\tilde{L} = \frac{\tau_a + \frac{C_2}{\alpha}\frac{\sigma-1}{\chi-1} - \frac{\rho q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}}{C_2\left(\frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)} + \frac{\sigma+\chi-2}{\alpha(\chi-1)}\right)}. \tag{99}$$

Substituting 99 into 85, we have the BGP growth rate equal to:

$$\gamma = \frac{\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\left[\tau_a + \frac{C_2}{\alpha}\frac{\sigma-1}{\chi-1} - \rho\left(1+\frac{\sigma+\chi-2}{\alpha(\chi-1)}\right)\right]}{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right) + \frac{\sigma+\chi-2}{\alpha(\chi-1)}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}.$$

The effects of the taxation on BGP labor supply and growth are therefore the following:

$$\frac{d\tilde{L}}{d\tau_a} = \frac{\frac{1}{C_2}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right) + \frac{\sigma+\chi-2}{\alpha(\chi-1)}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}$$

and

$$\frac{d\gamma}{d\tau_a} = \frac{q^{\frac{\alpha}{1-\alpha}}-1}{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right) + \frac{\sigma+\chi-2}{\alpha(\chi-1)}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}.$$

Notice that the signs of both $\frac{d\tilde{L}}{d\tau_a}$ and $\frac{d\gamma}{d\tau_a}$ are the same as the term

$$(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right) + \frac{\sigma+\chi-2}{\alpha(\chi-1)}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right).$$

It can be easily found that with $\sigma > 1$ this term is positive. With $0 < \sigma < 1$, this term equals

$$\frac{\sigma+\chi-2}{\alpha(\chi-1)} + \left[\frac{\sigma}{\alpha} + (1-\sigma)\left(\frac{\sigma}{\alpha\,(1-\chi)} - 1\right)\right]\left(q^{\frac{\alpha}{1-\alpha}}-1\right).$$

Further notice that in disutility function 37, when $0 < \sigma < 1$ there is $\chi < 1 < \sigma + \chi$, so $\frac{\sigma}{1-\chi} > 1$. Therefore we have $(1-\sigma)\left(\frac{\sigma}{\alpha(1-\chi)} - 1\right) > 0$ hence $\frac{\sigma+\chi-2}{\alpha(\chi-1)} + \left[\frac{\sigma}{\alpha} + (1-\sigma)\left(\frac{\sigma}{\alpha(1-\chi)} - 1\right)\right]\left(q^{\frac{\alpha}{1-\alpha}}-1\right) > 0$. Thus $\frac{d\tilde{L}}{d\tau_a}$ and $\frac{d\gamma}{d\tau_a}$ are always positive.

Second, the positive welfare effect condition 96 delivers

$$\frac{q^{\frac{\alpha}{1-\alpha}}-1}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)} - \frac{\sigma+\chi-2}{\sigma-1}\left[\frac{\tau_a}{C_2(1-\tilde{L})} + \frac{1}{\alpha}\left(\frac{\sigma-1}{\chi-1} - \frac{\tilde{L}}{1-\tilde{L}}\right)\right] \geq 0. \tag{100}$$

Now I try to check whether 100 can hold in reasonable parametrizations of the model. My choices for the benchmark values of the parameter set $\left\{r, \gamma, \tilde{L}, \sigma, \alpha\right\}$

and their ranges follow those chosen in Chapter 1. Furthermore, in this model there are three additional parameters, p, q and $\frac{\bar{\pi}}{\zeta}$, need be pinned down.

The value of innovation success rate p is related to the average patent life. Due to the assumption of Poisson distribution, p is approximately equal to the reciprocal of the patent life in years. For example, Caballero and Jaffe (1993) notice that if the life span of the patent is 25 years, then the mean rate of creative destruction (i.e., the p) is 0.04, whereas p is 0.05, 0.1 or 0.2 according to the life span at 20 years, 10 years or 5 years, respectively. I consider lifetimes between 10 and 50 years corresponding to the range of values considered by Stokey (1995) and Jones and Williams (2000), and set the benchmark lifetime to 20 years as Strulik (2007) does. So that p is 0.02-0.1 in my range and its benchmark value is 0.05.

According to equations 85 and 88, we have

$$\gamma = p\left(q^{\frac{\alpha}{1-\alpha}} - 1\right),$$

from which we can work out the implied value for the quality ladder q under the parameter set $\{\gamma, p, \alpha\}$ (notice that the condition $aq \leq 1$, which is a condition for non explosive growth rate, should be satisfied beforehand). And by equation 77 we can calculate the value for $\frac{\bar{\pi}}{\zeta}$ with given r and p, i.e. $\frac{\bar{\pi}}{\zeta} = r + p$.

In table 3 I report the benchmark parameterization. The 6-tuple $\left\{r, \gamma, \tilde{L}, \sigma, \alpha, p\right\}$ implies values for ρ (through 24), for χ (through 99 when $\tau_a = 0$ and $C_2 = (r+p)/\tilde{L}$ through 98) and for q as described above. However, under the benchmark parameterization, the welfare effect of the lump-sum tax τ_a is negative.

Table 3: Benchmark parameterization

Parameters and Steady State Variables Determined	Parameter	Value
TFP Growth	γ	0.0175
Hours Worked over Time Endowment	$\tilde{L}$	0.23
Mark-up	$1/\alpha$	1.2
Interest rate	r	0.07
Intertemporal Elasticity of Substitution in Consumption	σ	2
Innovation Success Rate	p	0.05
Parameters and Steady State Variables Implied		
Labor Supply Parameter	χ	3.45
Rate of Time Discount	ρ	0.035
Optimal Tax Rate	$\widehat{\tau}_a$	-
Quality Ladder	q	1.06

Sensitivity test is done, but under the parameter sets with up-to-two factors variant from the benchmark parameterization, we still cannot see the potential of the tax in welfare-enhancing. Further variation in the parameter sets with combination of lower subjective discount rate (by lower interest rate and higher growth rate), smaller labor supply, higher IES in consumption, and/or smaller innovation success rate allows the lump-sum tax to improve welfare. In table 4 I show these accommodating parameterizations.

Table 4: Alternative parameterizations with the other parameters following the benchmark

r=0.04, γ=0.02, σ=1.1	χ	ρ	$dW/d\tau_a\vert_{\tau_a=0}$	$d\gamma/d\tau_a\vert_{\tau_a=0}$	$\widehat{\tau}_a$	$\Delta W/\vert W\vert$ (%)
$\tilde{L}$=0.17	1.41	0.018	>0	>0	0.071	0.53
$\tilde{L}$=0.23	1.28	0.018	>0	>0	0.031	0.11
$\tilde{L}$=0.3	1.20	0.018	<0	>0	-	-
$1/\alpha$=1.1	1.28	0.018	>0	>0	0.032	0.14
$1/\alpha$=1.37	1.29	0.018	>0	>0	0.030	0.08
p=0.1	1.30	0.018	<0	>0	-	-
p=0.02	1.26	0.018	>0	>0	0.061	1.07

With these parameters the tax program is welfare increasing, i.e. $dW/d\tau_a|_{\tau_a=0}$ is positive. Given $dW/d\tau_a|_{\tau_a=0} > 0$, we pursue the optimal tax rate under which the welfare cannot be further improved by increasing the tax rate. The optimal tax rate $\widehat{\tau}_a$ is obtained by plugging in 100 the expression for $\tilde{L}$ given by 99 and equating it to zero:

$$\frac{q^{\frac{\alpha}{1-\alpha}}-1}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}-\frac{\sigma+\chi-2}{\sigma-1}\frac{\frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\left(\frac{C_2}{\alpha}\frac{\sigma-1}{\chi-1}+\tau_a\right)+\frac{\rho q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\frac{\sigma+\chi-2}{\alpha(\chi-1)}}{C_2\left(\frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}+\frac{1}{\alpha}\right)-\tau_a+\frac{\rho q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}}=0.$$

The root of this equation of τ_a gives us the optimal value of the tax, for each six-tuple of parameters $\{\sigma,\alpha,\rho,\chi,q,C_2\}$, of which as mentioned above the first two are determined, the other implied:

$$\widehat{\tau}_a=\frac{\left(q^{\frac{\alpha}{1-\alpha}}-1\right)C_2\left[\frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}+\frac{1}{\alpha}\left(1-\frac{(\sigma-1)(\sigma+\chi-2)}{\chi-1}\right)\right]+\rho q^{\frac{\alpha}{1-\alpha}}\left[\frac{q^{\frac{\alpha}{1-\alpha}}-1}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}-\frac{(\sigma+\chi-2)^2}{\alpha(\sigma-1)(\chi-1)}\right]}{(\sigma+\chi-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}. \tag{101}$$

Numerical calculation ensures that the stationary point of the welfare function we thus find corresponds to a maximum.

In order to check that the parameter values for χ consistent with welfare improving tax program are reasonable, I calculate the corresponding Frisch elasticity of labor supply and compare the results with the available estimates. The same as in Chapter 1, the Frisch elasticity of labor supply in BGP is given by 43. The values of the Frisch elasticity of labor supply consistent with optimal taxation are mainly located around 2, with 1.39 the lowest and 2.14 the highest.[11] These values are also consistent with the estimates of the Frisch elasticity found in the literature, as stated in Chapter 1.

[11] The values of the Frisch elasticity of labor supply associated with the before-tax parameter spaces are mainly located between 2 and 3, with 2.65 the lowest and 3.55 the highest.

9 Economic intuition

Similarly as in Chapter 1, it is worth noticing that though the growth rate will be largely increased, the total welfare can be raised to only a small extent. It is because of the counteraction, as described in Chapter 1, between the loss in the static level of welfare caused by the negative income effect of the wasted lump-sum taxes, and the gain in the growth rate. Again, the results are sensitive to the parameters $\left\{\sigma, \rho, \tilde{L}\right\}$, for which the economic intuitions are the same as explained in Chapter 1 and not repeated here. In this section, the externalities particular with the endogenous growth model of creative destruction will be mentioned and the influence of parameter p, the success rate of innovation, will be explained.

On the one hand, when an entrepreneur considers doing research, she trades off the cost of research with the expected benefit from obtaining the patent. She only considers the expected length the patent can be exploited for, before a new innovation is made. A social planner, on the other hand, would recognize that any new innovation increases the social welfare for all future points in time, as it lays a new foundation on which other discoveries can be made. As the entrepreneur does not take the spillovers of her research into account, too little research is carried out in equilibrium.

On the other hand, we have seen in the very beginning of this chapter that the incumbent monopolist does not engage in research. The expected return from one unit of research is much smaller to her than to a research competitor without a valuable patent. The incentive to do research lies solely in the value of the future patent, and does not internalize that an innovation also results in a loss to the incumbent monopolist. From a societal point of view, stealing business results in too much research.

The net effect of these two forces is unambiguous because they are essentially the same, except that they differ in sign and one comes earlier than the other. The extraction of the monopoly profit is the amount taken from one's predecessor. The treatment of an innovation as temporary is equivalent to ignoring the rents that will be taken by one's followers. The terms are the same in magnitude, except for two considerations: the latter term is higher because of growth of the economy at the rate γ, but it is smaller in present value because of discounting at the rate r. The relation $r > \gamma$ - transversality condition - implies that the first term dominates. Hence, the net effect from incomplete property rights is that the business stealing effect dominates therefore firms have incentives to do more research than is socially optimal.

Parameter p, the innovation success rate, is endogenous in our model. Smaller before-tax p, other parameters given, implies higher rung of quality ladder q, which is instead exogenous. Other things given, higher q means that the innovation, once taken by the entrant successfully, will bring to the innovator more profit (by equation 67) therefore the pace of invention of new patents will be accelerated. As a consequence, the growth effect of the wasted lump-sum taxation in a high-q economy will be larger than in a low-q economy, as we can see from the comparison between row 7 and 8 in table 4.

Appendices

Appendix 5

We establish the factor exhaustion condition

$$Y - X = wL + \pi = wL + \frac{\bar{\pi}}{\zeta}V = wL + (r+p)V \tag{102}$$

where the first equality is got from the equivalence in value between $(Y - X)$ and $(wL + \pi)$ by 58, 64 and 69; the second equality is delivered by 66, 68 and 79; and the third equality exists because of 77. Equation 102 indicates that the value-added in the economy should cover wage bill, interest payments as well as the loss of firm value caused by successful research.

Using factor exhaustion condition 102 and substituting for C using equation 7 we can then write 81 as:

$$w\left(L - \frac{(\sigma-1)h(L)}{h'(L)}\right) + (\frac{\bar{\pi}}{\zeta} - \tau_a)V - Z = 0. \tag{103}$$

From equations 78 and 79, and noticing that $\frac{\dot{V}}{V}$ should be the same as $\frac{\dot{Q}}{Q}$ thus using 84 for $\frac{\dot{V}}{V}$, we can establish the relationship between V and Z as:

$$Z = \dot{V} + pV. \tag{104}$$

The economic intuition of equation 104 is another version of free entry condition: it requires the R&D expenditure by the entrants to be the same as the flow of return from holding the intermediate firms, which is composed by the capital gain and potential wealth transfer from incumbents to entrants. Further substituting for Z with 104 and using 79 for V, and noting the relation $w = \frac{\bar{\pi}Q}{\alpha L}$ (by 65 and 66), equation 103 can be transformed into:

$$\frac{\dot{Q}}{Q} = \frac{\bar{\pi}}{\alpha\zeta}\left(1 - \frac{(\sigma-1)h(L)}{h'(L)L}\right) + r - \tau_a. \tag{105}$$

Totally differentiating 7 we get:

$$\frac{\dot{C}}{C} = \frac{\dot{Q}}{Q} + \left(\frac{h'}{h} - \frac{h''}{h'}\right)\dot{L}.$$

Substituting this expression for $\frac{\dot{C}}{C}$ in 8 we get:

$$-\sigma\frac{\dot{Q}}{Q} + \left(\sigma\frac{h''}{h'} + (1-\sigma)\frac{h'}{h}\right)\dot{L} = \rho - r.$$

Finally if we substitute in this expression for $\frac{\dot{Q}}{Q}$ given by 105 we can obtain the dynamic of labor supply as in 89.

Appendix 6

Transversality condition 9 requires $\gamma < r$. In an initially taxless economy, 105 gives

$$\gamma = \frac{\bar{\pi}}{\alpha\zeta}\left(1 - \frac{(\sigma-1)h(L)}{h'(L)L}\right) + r,$$

so $\gamma < r$ leads to

$$\frac{\bar{\pi}}{\zeta\alpha}\left(1 - \frac{(\sigma-1)h(L)}{h'(L)L}\right) < 0.$$

Hence we get

$$\frac{(\sigma-1)h(L)}{h'(L)L} > 1,$$

which is exactly the counterpart of 47.

In addition, positive growth rate requires positive investment, i.e. $C < Y - X$. Substituting 7 for C and using 102 for $Y - X$ we obtain

$$w\frac{(\sigma-1)h(L)}{h'(L)} < wL + \pi.$$

Further with 58 and 69 this inequality leads to

$$\frac{(\sigma-1)h(L)}{h'(L)L} < 1 + \alpha,$$

which is just the same condition as 48. Hence we arrive at the same condition as 49, i.e.

$$1 < \frac{(\sigma-1)h(L)}{h'(L)L} < 1 + \alpha.$$

Appendix 7

By solving the integral in 31 we obtain 50. By using 7 and the relation $w = \frac{1}{\alpha L}\frac{\bar{\pi}}{\zeta}V$ (by 65, 66 and 79), we can write

$$C(0) = \frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}\frac{\bar{\pi}}{\alpha\zeta}V(0)$$

where $V(0)$ is the aggregate value of firms at time 0. And note that along the BGP $\rho - \gamma(1-\sigma) = r - \gamma$ (by 82 and 86) while $r - \gamma = \frac{\bar{\pi}}{\alpha\zeta}\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}} - 1\right) + \tau_a$ (by using $\gamma = \frac{\dot{Q}}{Q}$ in the BGP and 105 for $\frac{\dot{Q}}{Q}$). Transversality condition 9 is satisfied with $\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}} > 1$ (see also 49 for the suitable range of $\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}$) so that $\gamma < r$ in an initial taxless economy. We can thus rewrite 50 as:

$$W = \frac{1}{1-\sigma} \frac{\left(\frac{\sigma-1}{h'(\tilde{L})\tilde{L}} \frac{\bar{\pi}}{\alpha\zeta} V(0)\right)^{1-\sigma} h(\tilde{L})^{2-\sigma}}{\frac{\bar{\pi}}{\alpha\zeta}\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}} - 1\right) + \tau_a}.$$

Therefore the monotonically increasing transformation of W which we resort to is:

$$\frac{\log(1-\sigma)W}{1-\sigma} = \frac{1}{1-\sigma}\left[\begin{array}{c} \log(V(0)^{1-\sigma}) + (1-\sigma)\log\left(\frac{\sigma-1}{h'(\tilde{L})\tilde{L}}\right) + (1-\sigma)\log\left(\frac{\bar{\pi}}{\alpha\zeta}\right) \\ +(2-\sigma)\log h(\tilde{L}) - \log\left(\frac{\bar{\pi}}{\alpha\zeta}\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}} - 1\right) + \tau_a\right) \end{array}\right].$$

Taking partial derivative of this expression with respect to $\tilde{L}$ and noting that $\frac{d\bar{\pi}}{d\tilde{L}} = \frac{\bar{\pi}}{\tilde{L}}$ we get:

$$\begin{aligned}
\frac{\partial\left(\log(1-\sigma)W\right)}{(1-\sigma)\,\partial\tilde{L}} &= \frac{1}{1-\sigma}\left[(\sigma-1)\left(\frac{h''}{h'} + \frac{1}{\tilde{L}}\right) + \frac{1-\sigma}{\tilde{L}} + \frac{(2-\sigma)h'}{h}\right] \\
&\quad - \frac{1}{1-\sigma} \cdot \frac{\frac{\bar{\pi}}{\alpha\zeta\tilde{L}}\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1\right) + \frac{\bar{\pi}}{\alpha\zeta}\frac{\sigma-1}{\tilde{L}}\left(1 - \frac{hh''}{(h')^2} - \frac{h}{h'\tilde{L}}\right)}{\frac{\bar{\pi}}{\alpha\zeta}\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1\right) + \tau_a} \\
&= \frac{1}{1-\sigma}\left[\frac{(\sigma-1)h''}{h'} + \frac{(2-\sigma)h'}{h} + \frac{\frac{\bar{\pi}}{\alpha\zeta\tilde{L}}\left((\sigma-1)\left(\frac{hh''}{(h')^2} - 1\right) + 1\right)}{\frac{\bar{\pi}}{\alpha\zeta}\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1\right) + \tau_a}\right] \\
&= \frac{1}{1-\sigma}\left[\frac{h'}{h}\left((\sigma-1)\left(\frac{hh''}{(h')^2} - 1\right) + 1\right) + \frac{\frac{\bar{\pi}}{\alpha\zeta\tilde{L}}\left((\sigma-1)\left(\frac{hh''}{(h')^2} - 1\right) + 1\right)}{\frac{\bar{\pi}}{\alpha\zeta}\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1\right) + \tau_a}\right],
\end{aligned}$$

which finally leads to 93. And taking partial derivative of $\frac{\log(1-\sigma)W}{1-\sigma}$ with respect to τ_a we can get very easily 94 in the text.

Chapter III. Capital income taxes in a variety expansion model

10 Introduction

Should the government tax capital income in the long run? The articles by Chamley (1985, 1986) and Judd (1985, 1987), providing the foundations of second-best optimal taxation in modern macroeconomics, answer no. Jones, Manuelli, and Rossi (1997), Atkeson, Chari, and Kehoe (1999) and Chari and Kehoe (1999) show that this negative answer is robust to a relaxation of a number of assumptions made by Chamley (1985, 1986) and Judd (1985, 1987).

Considering imperfect competition, Judd (2002) even argues for a negative long-run capital income tax and a positive labor income tax.

Many works study capital taxation in an endogenous growth framework: one common prediction of the literature is again that taxation has negative effects on welfare and growth, by discouraging work and saving (see the surveys in Myles 2000 and Jones and Manuelli 2005). Lucas (1990) shows that the Chamley result holds in a two-sector endogenous growth model. Jones and Manuelli (1992), Jones et al. (1997), Milesi-Ferretti and Roubini (1998a,b) show that the tax rates on capital, labor (human capital) income and on consumption should all be zero in the long run. The papers focussing on R&D activities also generally find a negative effect of capital taxation on growth (e.g. Lin and Russo 1999 and 2002, Zeng and Zhang 2002).

This paper adds to this literature by considering the effect of taxation on long-run growth and welfare in a model with imperfect competition and R&D leading to an expanding variety of products. Zeng and Zhang (2007) study fiscal issues adopting this same specification of the horizontal innovation model but focus on a different issue, i.e. they compare the effects of subsidizing R&D investment to the effects of subsidizing final output or subsidizing the purchase of intermediate goods in terms of promoting growth. They consider distortionary taxation (i.e. taxes on labor income) but abstract from taxes on interest income.

We show that a long-run capital income tax, when its revenues are returned as a subsidy to labor, can have a positive effect on welfare even if the effect on growth is negative. This result thus complements the group of studies arguing for non-zero capital income taxation: some assumptions have in fact been identified that may invalidate the prescription of a zero capital income tax in the long run. A way in which taxes can be good is when they finance productive government spending, as in Barro (1990), Barro and Sala-i-Martin (1992, 1995), Turnovsky (1996, 2000), Corsetti and Roubini (1996), Baier and Glomm (2001) and Chen (2007), among others. Guo and Lansing (1999) conclude that the steady-state optimal tax on capital income can be zero, negative, or even positive by allowing for depreciation of physical capital, a depreciation tax allowance and endogenous government expenditures. Hubbard and Judd (1986), Aiyagari (1995) and Imrohoroglu (1998) have emphasized that if households face tight borrowing constraints and/or are subject to uninsurable idiosyncratic income risk, then the optimal tax system will in general include a positive capital income tax. Asea and Turnovsky (1998) and Kenc (2004) find that increasing the tax rate on capital income may increase growth in a stochastic environment. Diaz-Gimenez et al. (1992), Erosa and Gervais (2002), Song (2002), Conesa and Garriga (2003) and Yakita (2003) show that in life cycle / OLG models the optimal capital income tax in general is different from zero. Hendricks (2003, 2004) stress the importance of the intergenerational transmission of capital for the effects of taxation. Conesa et al. (2009) quantitatively characterize the optimal capital income tax in an overlapping generations model with idiosyncratic, uninsurable income shocks and find the optimal capital income tax rate is significantly positive at 36 percent.

Some other alternative justifications for a non-zero limiting capital tax in dynamic general equilibrium models include: borrowing constraint and income distribution (Chamley, 2001), capital stock allocation and capital accumulation in open economies (Palomba, 2004), asymmetric information in credit markets (Ho and Wang, 2007).

All these arguments in favor of a positive rate of capital taxation are unrelated to ours as we model a perfect foresight closed economy with infinite lived agents and no public capital.

Two articles closer to our analysis are Pelloni and Waldmann (2000) and de Hek (2006). In the first paper a simple learning by doing model a la Romer (1986) is augmented by endogenous labor supply and it is shown that if the equilibrium is indeterminate capital taxation can increase growth and welfare. However the scope of the result is limited because indeterminacy is only possible with a very high intertemporal elasticity of substitution and because the condition for the tax to be welfare increasing is derived on the assumption the economy is in steady state. But of course with indeterminacy this will not in general be the case, even in the long run. de Hek (2006) studies the effects of taxation on long-run growth in a two-sector endogenous growth model with physical capital as an input in the education sector and leisure as an argument in the utility function. If only capital income is taxed human capital accumulation will be encouraged and the long-run growth rate may be increased. In order to isolate the labor employment factor from these considerations, in our model we do not introduce human capital accumulation, because when labor is in the form of a reproducible human capital, it is less different from physical capital.

Often in the papers on taxation and growth, the welfare effect of the tax experiments considered are not calculated, if in the market equilibrium growth is lower than optimal. In other words, there is an implicit presumption that higher growth means more welfare as, through compounding, growth effects always prevail over level effects. However in the model we study growth is inefficiently low in the absence of taxes, and even when the introduction of the tax lowers growth there might be a positive welfare effect. In our calibrated examples, we show that this counterintuitive effect can arise when choosing values for model parameters consistent with the micro and macro empirical evidence.

The rest of the paper is organized as follows: in section 11 the model is presented, in section 12 the general equilibrium conditions of the model are described, section 13 analyzes the labor supply effect, the growth effect and the welfare effect of a capital income tax whose proceeds are used to subsidize labor, and also carries out numerical calculations to show that even if the growth rate is decreased, such a tax can increase welfare for widely accepted estimates of the relevant parameters, with the optimal tax rates derived for various sets of parameters, section 14 compares the market economy with the social planner's economy to show that the fact that the welfare is improved even if the growth rate is dampened does not result from any possibility of unoptimally higher growth rate in market economy, and finally section 15 explains the economic intuition.

11 The model

In this model, the set-up for the sector of firms follows that described in Chapter 1, i.e. the equations from 10 to 20 also serve in this model. To avoid repeat, this section focuses on the differences of the household sector and government sector from those in Chapter 1.

11.1 Households

Hereby the households share the same utility function as 1 for which the conditions 2, 3 and 4 regulate the consumption and leisure as normal goods and the strict concavity. The instantaneous budget constraint consumers face is given by:

$$\dot{F} = r(1-\tau_k^l)F + w(1+t_w)H - C. \tag{106}$$

Households derive their income by loaning entrepreneurs their financial wealth F (of which all have the same initial endowment) and by supplying labor H to firms, taking the interest rate r and the wage rate w as given. Capital income is taxed at the rate τ_k^l while labor income is subsidized at the rate t_w.

Optimization implies that the marginal rate of substitution between leisure and consumption equals their relative price:

$$\frac{h'}{h} = \frac{w(1+t_w)(\sigma-1)}{C}. \tag{107}$$

Optimal consumption and leisure must also obey this intertemporal condition:

$$-\sigma\frac{\dot{C}}{C} + \frac{h'}{h}\dot{H} = \frac{\dot{\lambda}}{\lambda} = \rho - r(1-\tau_k^l) \tag{108}$$

where λ is the shadow value of wealth. And the transversality condition 9 should be satisfied.

11.2 Government

We assume no government consumption on goods. We also rule out a market for government bonds and assume that the government runs a balanced budget. The revenue from capital income taxes is used for financing the wage subsidy. In equilibrium:

$$t_w wL = r\tau_k^l F \tag{109}$$

where on the left-hand side we have outflows and on the right-hand side we have inflows.

12 Market equilibrium

In calculating the equilibrium in the final goods market the total of intermediate goods used xN is subtracted from final production Y to obtain total value added.

All investment in the model is investment in research and development of new intermediate goods $\eta\dot{N}$. The economy wide resource constraint is therefore given by:

$$Y - xN = C + \eta\dot{N}. \tag{110}$$

We are now ready for the following:

Definition 10 *In a competitive equilibrium individual and aggregate variables are the same and prices and quantities are consistent with the (private) efficiency conditions for the households 106, 107, 108 and 9, the profit maximization conditions for firms in the final goods sector, 12 and 13 (or 18), and for firms in the intermediate goods sector, 15 (or 16) and 20, with the government budget constraint 109 and with the market clearing conditions for labor $H = L$, for wealth ($F = \eta N$), and for the final good, 110.*

We can notice the following relationship between before-tax labor income and before-tax capital income in equilibrium:

$$wL/rF = \frac{1}{\alpha}. \tag{111}$$

From 109 and 111 we can also infer that:

$$t_w = \alpha\tau_k^l. \tag{112}$$

Proposition 11 *If the economy follows a balanced growth path (hence BGP) variables grow at a constant rate, and in particular employment is constant at a value $\widetilde{L}$. Along this path, rate of growth of capital and consumption, γ, is then given by:*

$$\gamma = \frac{r(\widetilde{L})(1-\tau_k^l) - \rho}{\sigma}. \tag{113}$$

Proof. Totally differentiating 107 we get the same expression as 25:

$$\frac{\dot{C}}{C} = \frac{\dot{N}}{N} + (h'/h - h''/h')\dot{L}.$$

From this we deduce that along a BGP, when $\dot{L} = 0$, the rates of growth of C and N will be the same. From 108 and $\dot{L} = 0$, we get 113. ■

In Appendix 8 we show how to deduce from the competitive equilibrium conditions described above the following differential equation for labor, which is the fundamental dynamic equation of the model:

$$\dot{L} = \frac{\rho - C_1 L(1-\tau_k^l - \sigma) + \frac{\sigma}{\alpha} C_1 \left(L + (1+\alpha\tau_k^l)\frac{h(1-\sigma)}{h'}\right)}{\left(\frac{\sigma h"}{h'} + \frac{h'}{h}(1-\sigma)\right)} \equiv \frac{B(L)}{A(L)}. \tag{114}$$

The denominator of the fraction on the right-hand side $A(L)$ is always strictly positive for all values of L, by the negative definiteness condition of the hessian of the utility function 4, so the equation is defined for all values of L beween

0 and 1. Along a balanced growth path (hence BGP) $\dot{L}$ will equal zero, so the numerator $B(L)$ will be zero, i.e. $B(\tilde{L}) = 0$ where $\tilde{L}$ is the BGP labor supply. To study the dynamic nature of a fixed point of 114, i.e. of BGP labor supply, we have to sign $d\dot{L}(\widetilde{L})/d\widetilde{L}$, the derivative of $\dot{L}$ with respect to L, calculated at the fixed point $\widetilde{L}$ implicitely defined by $B(\widetilde{L}) = 0$. If this derivative is positive the fixed point $\widetilde{L}$ is a repeller and the BGP is locally determinate in the sense that if L were close to but not exactly equal to $\widetilde{L}$, then L would diverge further from $\widetilde{L}$. Thus, the BGP with $\widetilde{L}$ a repeller is a (locally) unique equilibrium path and we can say that there is no (local) indeterminacy in this case. If the equilibrium is unstable there will be no transitional dynamics to it, the economy will always follow the BGP. If $d\dot{L}(\widetilde{L})/d\widetilde{L}$ is negative then $\widetilde{L}$ is an attractor, that is if L is near $\widetilde{L}$ it will eventually approach it. So there is local indeterminacy, i.e. a continuum of equilibrium trajectories all converging to the fixed point. We have: $\frac{d\dot{L}}{dL}(\tilde{L}) = \frac{B'(\tilde{L})}{A(\tilde{L})} - \frac{A'(\tilde{L})B(\tilde{L})}{A^2(\tilde{L})} = \frac{B'(\tilde{L})}{A(\tilde{L})}$ (since $B(\tilde{L}) = 0$). Below we prove that $B(\tilde{L}) = 0$ implies $B'(\tilde{L}) > 0$. Since $d\dot{L}(\widetilde{L})/d\widetilde{L}$ is always positive, we can deduce that if BGP exists it is unique as from the phase diagram of 114 we can easily see that there is no way for $B(L)/A(L)$, which is a continuous function, to cross the horizontal axis from below two times in a row.

From the instability of the equilibrium we can also deduce that there will be no transitional dynamics in the model. Below we prove that $B(\tilde{L}) = 0$ implies $B'(\tilde{L}) > 0$, therefore Proposition 3 also applies here.

Proof. We have:

$$B'(L) = C_1 \left(B_1(L) + B_2(L)\tau_k^l \right)$$

where:

$$B_1(L) \equiv \sigma - 1 + \frac{\sigma}{\alpha} \left(1 + (1-\sigma) \left(1 - \frac{hh''}{(h')^2} \right) \right)$$

and

$$B_2(L) \equiv 1 + \sigma(1-\sigma) \left(1 - \frac{hh''}{(h')^2} \right).$$

By condition 28, we have:

$$B_1(L) > \sigma - 1 + \frac{1}{\alpha} > \sigma$$

while

$$B_2(L) > 1 + \sigma \left(\frac{1}{\sigma} - 1 \right) = 2 - \sigma.$$

$B'(L)$ is then positive: in fact $B_1(L) + B_2(L)\tau_k^l > \sigma + (2-\sigma)\tau_k^l = \sigma(1-\tau_k^l) + 2\tau_k^l$, which is positive as $\tau_k^l \leq 1$. ■

13 Effects of taxes

13.1 Effect on labor

It is relatively simple to calculate the effect of taxes on labor supply in this model because the wage rate is not affected by labor supply. As said above equilibrium labor supply can be expressed as the solution to $B(\tilde{L}) = 0$. The effect of our tax program on labor can be deduced by using the total derivative of $B(\tilde{L}) = 0$ with respect to labor and the tax. We then have:

$$\frac{d\tilde{L}}{d\tau_k^l} = \frac{r\left(\frac{\sigma(\sigma-1)h}{h'\tilde{L}} - 1\right)}{B'(\tilde{L})} \tag{115}$$

as we have shown above that $B'(\tilde{L}) > 0$, this derivative signs as the term $\left(\frac{(\sigma-1)h}{h'\tilde{L}} - \frac{1}{\sigma}\right)$. To sign this we notice that the condition 9 implies that the BGP rate of growth, γ, is lower than $r(1-\tau_k^l)$. 142 gives us:

$$\gamma = r + \frac{r}{\alpha L}\left(L + \frac{h(1-\sigma)}{h'}(1+\alpha\tau_k^l)\right)$$

so

$$\begin{aligned} 0 &> \gamma - r(1-\tau_k^l) = \frac{r}{\alpha L}\left(L + \frac{h(1-\sigma)}{h'}(1+\alpha\tau_k^l)\right) + r\tau_k^l \\ &= rL\left(\frac{1}{\alpha} + \tau_k^l + \frac{h(1-\sigma)}{h'L}(\frac{1}{\alpha} + \tau_k^l)\right) \end{aligned}$$

leads to

$$\frac{(\sigma-1)h}{h'\tilde{L}} > 1,$$

which is exactly the condition 47. For $\sigma > 1$, we can easily see that we will always have $\frac{d\tilde{L}}{d\tau_k^l} > 0$. We are therefore ready to state the following:

Proposition 12 *An increase in the tax rate on capital income whose proceeds are returned as a subsidy to labor income will increase employment in equilibrium if and only if* $\frac{\sigma(\sigma-1)h}{h'} > L$. *This condition is always satisfied if* $\sigma > 1$.

13.2 Effect on growth

The growth effect of tax τ_k^l is:

$$\begin{aligned} \frac{d\gamma}{d\tau_k^l} &= \frac{\partial\gamma}{\partial r}r'(\tilde{L})\frac{d\tilde{L}}{d\tau_k^l} + \frac{\partial\gamma}{\partial\tau_k^l} \\ &= \frac{1}{\sigma}\frac{r}{\tilde{L}}(1-\tau_k^l)\frac{d\tilde{L}}{d\tau_k^l} - \frac{r}{\sigma} = \frac{r}{\sigma}\left(\frac{(1-\tau_k^l)}{\tau_k^l}\frac{\tau_k^l d\tilde{L}}{\tilde{L}d\tau_k^l} - 1\right). \end{aligned}$$

Not surprisingly the condition for the tax to be growth increasing is stricter than the condition for it to be employment increasing, because for growth to increase we need the net interest rate to increase not just the gross interest rate, which is a linear function of the employment rate. When $\tau_k^l > 0$, the condition for the policy to be growth increasing is that the elasticity of labor supply with respect to the tax $\frac{d\tilde{L}/\tilde{L}}{d\tau_k^l/\tau_k^l}$ is not only positive but bigger than $\tau_k^l/(1-\tau_k^l)$. In general we have, using the derivative of labor with respect to the tax program 20 and 115:

$$\frac{d\gamma}{d\tau_k^l} = \frac{r}{\sigma}\left(\frac{(1-\tau_k^l)r\left(\frac{\sigma(\sigma-1)h}{h'\tilde{L}}-1\right)}{\tilde{L}B'(\tilde{L})}-1\right) =$$

$$\frac{\frac{(\sigma-1)h}{h'\tilde{L}} - 1 - \frac{1}{\alpha}\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right) + \tau_k^l(\sigma-1)\left(1-\frac{hh''}{(h')^2}-\frac{h}{h'\tilde{L}}\right)}{B'(\tilde{L})/\tilde{L}C_1^2}. \tag{116}$$

We have the following:

Proposition 13 *An increase in the tax rate on capital income whose proceeds are returned as a subsidy to labor income will increase growth in equilibrium if and only if* $\frac{(\sigma-1)h}{h'\tilde{L}}-1-\frac{1}{\alpha}\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right)+\tau_k^l(\sigma-1)\left(1-\frac{hh''}{(h')^2}-\frac{h}{h'\tilde{L}}\right) > 0$. *This condition is never satisfied if* $\sigma < 1$.

Proof. The denominator of the right-hand side of 116, $B'(\tilde{L})/\tilde{L}C_1^2$, is always positive, so the sign of $\frac{d\gamma}{d\tau_k^l}$ is the same as that of the numerator. Notice investment I should be positive. We have $I = Y - xN - C$. Since $Y - xN = (1-\alpha^2)Y$ (by 16 and 17), substituting for C its expression given by 107, after expressing the wage in terms of income by 13 we get:

$$I = (1-\alpha^2)Y - \frac{(\sigma-1)h(L)}{h'(L)L}(1+\alpha\tau_k^l)(1-\alpha)Y.$$

So $I > 0$ implies that

$$\frac{(\sigma-1)h(L)}{h'(L)L} < \frac{1+\alpha}{1+\alpha\tau_k^l}, \tag{117}$$

while by 28 we have $-\frac{1}{\alpha}\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right) < -\frac{1}{\alpha\sigma}$. So $\frac{(\sigma-1)h}{h'\tilde{L}} - 1 - \frac{1}{\alpha}\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right) < \frac{\alpha(1-\tau_k^l)}{1+\alpha\tau_k^l} - \frac{1}{\alpha\sigma}$. Moreover, the coefficient of τ_k^l in the numerator of the RHS of 116, $(\sigma-1)\left(1-\frac{hh''}{(h')^2}-\frac{h}{h'\tilde{L}}\right)$, is always negative for $\sigma < 1$, considering the conditions on the utility function, 2, 3 and 4. Since $\frac{\alpha(1-\tau_k^l)}{1+\alpha\tau_k^l} - \frac{1}{\alpha\sigma} < 0$ is always true for $\sigma < 1$ we can conclude that a sufficient condition for the sign of $\frac{d\gamma}{d\tau_k^l}$ to be negative is $\sigma < 1$. ■

13.3 Effect on welfare

Given γ, the constant rate of growth, and $\widetilde{L}$ the BGP labor supply, it is possible to calculate maximum lifetime utility W along a balanced growth path as equation 31. In Appendix 9 it is shown how to express W as a differentiable function of τ_k^l and $\tilde{L}$ (itself a function of τ_k^l). The effect on welfare of an increase in the tax rate τ_k^l is then positive if $\frac{dW}{d\tau_k^l}$ is positive. To simplify calculations, we consider also the following monotonically increasing transformation of W: $\frac{\log[(1-\sigma)W]}{1-\sigma}$ ($\frac{d(\log[(1-\sigma)W])}{(1-\sigma)d\tau_k^l}$ signs as $\frac{dW}{d\tau_k^l}$). We have:

$$\frac{d(\log[(1-\sigma)W])}{(1-\sigma)d\tau_k^l} = \frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tau_k^l} + \frac{d\tilde{L}}{d\tau_k^l} \cdot \frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tilde{L}} \tag{118}$$

In Appendix 9 we also show the following:

$$\frac{\partial(\log[(1-W])}{(1-\sigma)\partial\tilde{L}} = \frac{\frac{h'}{h} - \frac{\sigma}{\tilde{L}}}{\sigma - 1} \cdot \frac{(\sigma-1)\left(\frac{hh''}{(h')^2} - 1\right) + 1}{\frac{(\sigma-1)h}{h'\tilde{L}} - 1} \tag{119}$$

and

$$\frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tau_k^l} = \frac{\sigma\alpha}{(\sigma-1)(1+\alpha\tau_k^l)}. \tag{120}$$

Substituting 115, 119 and 120 in 118, we get:

$$\frac{d(\log[(1-\sigma)W])}{(1-\sigma)d\tau_k^l} = \frac{\alpha\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1\right)(1-\tau_k^l) - \frac{(1+\alpha\tau_k^l)h'\tilde{L}}{\sigma h(\sigma-1)}\left((\sigma-1)\left(\frac{hh''}{(h')^2} - 1\right) + 1\right)}{\frac{(1+\alpha\tau_k^l)}{\sigma C_1}\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1\right)B'(\tilde{L})}. \tag{121}$$

Notice the denominator is always positive given 47 and $B'(\tilde{L}) > 0$. Hence we arrive at the following:

Proposition 14 *The sufficient and necessary condition for an increase in the tax rate on capital income to increase welfare is:*

$$\alpha\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1\right)(1-\tau_k^l) - (1+\alpha\tau_k^l)\frac{h'\tilde{L}}{\sigma h(\sigma-1)}\left((\sigma-1)\left(\frac{hh''}{(h')^2} - 1\right) + 1\right) \geq 0. \tag{122}$$

Of course if a value for τ_k^l exist such that for this value 122 holds as an equality, while it holds strictly for lower tax rates, 122 gives us an implicit expression for the optimal tax rate, given the tax program.

We notice that the condition to improve welfare is less stringent than the condition to improve growth, or in other words that if the latter is satisfied, the first is satisfied as well. In fact, suppose $\frac{d\gamma}{d\tau_k^l} > 0$ (so $\sigma > 1$ by Proposition 13), considering for simplicity the zero tax initial condition. Then: $\frac{(\sigma-1)h}{h'\tilde{L}} - 1 = \frac{1}{\alpha}\left(1 + (1-\sigma)\left(1 - \frac{hh''}{(h')^2}\right)\right) + \frac{\varepsilon}{\alpha}$, for some strictly positive number ε. Substituting in 122 we get:

$$\left(1 + (1-\sigma)\left(1 - \frac{hh''}{(h')^2}\right)\right)\left(1 - \frac{h'\tilde{L}}{\sigma(\sigma-1)h}\right) + \varepsilon.$$

We know from 28 that $\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right) > 0$. Then for the welfare effect to be negative we would need $\left(1-\frac{h'\tilde{L}}{\sigma(\sigma-1)h}\right) < 0$. But this would require $\frac{(\sigma-1)h}{h'\tilde{L}} < \frac{1}{\sigma} < 1$, which by 47 we know is impossible. We have therefore proven:

Proposition 15 *It is possible for an increase in tax rate on capital income to increase welfare and decrease growth.*

In the next section we will show that this is more than a theoretical possibility and that for specifications of tastes and technology parameters often used in calibration exercises it is possible for the tax program to induce Pareto improvements but reduce growth. This can happen even if the market equilibrium before the program the rate of growth is inefficiently low. The example we offer is also useful to offer an intuition on the mechanism at work in producing the result.

13.4 Are capital income taxes good for reasonable parameter values?

The class of functions for the disutility of labor 37 is again considered here. First we notice that we can now obtain an explicit solution for the equilibrium level of activity. With $B(\tilde{L}) = 0$ and by noting 20 we have:

$$\tilde{L} = \frac{\frac{\sigma}{\alpha}(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1} - \frac{\rho}{C_1}}{\frac{\sigma}{\alpha}\left(1+\alpha+(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1}\right) - 1 + \tau_k^l}. \tag{123}$$

By 113 the BGP growth rate is then

$$\gamma = \frac{C_1\frac{\sigma-1}{\chi-1}(1+\alpha\tau_k^l)(1-\tau_k^l) - \rho\left(1+\alpha+(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1}\right)}{\sigma\left(1+\alpha+(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1}\right) - \alpha(1-\tau_k^l)}. \tag{124}$$

By using 123 and 124, the effects of taxes on BGP labor supply and growth are therefore the following:

$$\frac{d\tilde{L}}{d\tau_k^l} = \frac{\frac{1+\alpha}{\alpha}\frac{\sigma(\sigma-1)^2}{\chi-1} + \frac{\rho}{C_1}\left(1+\frac{\sigma(\sigma-1)}{\chi-1}\right)}{\left[\frac{\sigma}{\alpha}\left(1+\alpha+(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1}\right) - 1 + \tau_k^l\right]^2},$$

and

$$\frac{d\gamma}{d\tau_k^l} = \frac{\rho\alpha(1+\alpha)\frac{\sigma+\chi-2}{\chi-1} - C_1\frac{\sigma-1}{\chi-1}\left[\sigma(1+\alpha)(1-\alpha+2\alpha\tau_k^l) + \alpha^2(1-\tau_k^l)^2 + \frac{\sigma(\sigma-1)}{\chi-1}(1+\alpha\tau_k^l)^2\right]}{\left[\sigma\left(1+\alpha+(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1}\right) - \alpha(1-\tau_k^l)\right]^2}.$$

It is easy to find that with $\sigma > 1$, the sign of $\frac{d\tilde{L}}{d\tau_k^l}$ is positive whereas it is ambiguous of the sign of $\frac{d\gamma}{d\tau_k^l}$.

Proposition 14 implies that a positive welfare effect requires:

$$\alpha\left(\frac{\sigma-1}{\chi-1}\frac{1-\tilde{L}}{\tilde{L}}-1\right)(1-\tau_k^l)-(1+\alpha\tau_k^l)\frac{(\sigma+\chi-2)\,\tilde{L}}{\sigma(\sigma-1)(1-\tilde{L})}\geq 0. \tag{125}$$

Now we try to check whether 125 can hold in reasonable parametrizations of the model. We are completely aware that this model is not rich enough in number of variables, not to mention their dynamics, to fit the data well. Models that are rich enough to fit well become complex and difficult to interpret. The aim of our exercise is not realism but the understanding of mechanisms of action of policy not noticed before in the literature. My choices for the benchmark values of the parameter sets and their ranges follow those chosen in Chapter 1, except that hereby I restrict the value of σ to be greater than unity for the reason that with σ smaller than unity the growth rate will definitely be reduced. I consider the range 1.1-3 with the benchmark value 2 for σ.

In table 5 I report the benchmark parameterization. The 5-tuple $\left\{r,\gamma,\tilde{L},\sigma,\alpha\right\}$ implies values for ρ (through 113) and for χ (through `123 when` $\tau_k^l=0$ and $C_1=r/L$ through 20).

Table 5: Benchmark parameterization

Parameters and Steady State Variables Determined	Parameter	Value
TFP Growth	γ	0.0175
Hours Worked over Time Endowment	$\tilde{L}$	0.23
Mark-up	$1/\alpha$	1.2
Interest rate	r	0.07
Intertemporal Elasticity of Substitution in Consumption	σ	2
Parameters and Steady State Variables Implied		
Labor Supply Parameter	χ	3.06
Rate of Time Discount	ρ	0.035
Optimal Tax Rate	$\widehat{\tau}_k^l$	0.032
Steady State Variables under Optimal Taxation	Value	Change
$\tilde{L}$	0.234	1.54%
γ	0.017	-3.36%

With these parameters the tax program is welfare increasing, i.e. $dW/d\tau_k^l|_{\tau_k^l=0}$ is positive, even if growth decreases. It is then calculated that the tax rate associated with maximum utility is 3.17%, under which the welfare increases by 0.07% with the utility level changing from −239.09 before tax to -238.92 after tax. The optimal tax rate is obtained by plugging in 125 the expression for $\tilde{L}$ given by 123 and equating it to zero:

$$(1-\tau_k^l)\alpha\frac{(\sigma-1)(1-\tau_k^l)+\frac{\rho\left(\frac{\sigma}{\alpha}\left(1+\alpha+(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1}\right)-1+\tau_k^l\right)}{C_1\frac{\sigma}{\alpha}(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1}-\rho}}{\frac{\sigma}{\alpha}}$$

$$-\frac{(1+\alpha\tau_k^l)^2}{\sigma}\frac{\frac{\sigma}{\alpha}(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1}-\frac{\rho}{C_1}}{\frac{\sigma}{\alpha}(1+\alpha)-1+\tau_k^l+\frac{\rho}{C_1}}$$

$$-\frac{1}{\alpha}\frac{(1+\alpha\tau_k^l)^3}{\left(\frac{\sigma}{\alpha}(1+\alpha)-1+\tau_k^l+\frac{\rho\left(\frac{\sigma}{\alpha}\left(1+\alpha+(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1}\right)-1+\tau_k^l\right)}{C_1\frac{\sigma}{\alpha}(1+\alpha\tau_k^l)\frac{\sigma-1}{\chi-1}-\rho}\right)}=0.$$

The root of this non linear equation in τ_k^l gives us the optimal value of the tax, for each five-tuple of parameters $\{\sigma, \alpha, \rho, \chi, C_1\}$, of which as mentioned above the first two are determined, the other implied. Notice that for all the parameterizations we consider, the expression is always decreasing in τ_k^l for $0 \leq \tau_k^l \leq 1$, so the stationary point of the welfare function we thus find corresponds to a maximum.

In table 6 we report over our alternative parameterizations and the results of our sensitivity analysis.

Table 6: Alternative parameterizations

	χ	ρ	$dW/d\tau_k^l\vert_{\tau_k^l=0}$	$d\gamma/d\tau_k^l\vert_{\tau_k^l=0}$	$\widehat{\tau}_k^l$	$\Delta W/\vert W\vert$ (%)
γ=0.015	3.02	0.04	>0	<0	0.046	0.14
γ=0.02	3.10	0.03	>0	<0	0.017	0.02
$\tilde{L}$=0.17	4.00	0.035	>0	<0	0.058	0.24
$\tilde{L}$=0.3	2.44	0.035	<0	<0	-	-
$1/\alpha$=1.1	2.99	0.035	>0	<0	0.076	0.44
$1/\alpha$=1.37	3.16	0.035	<0	<0	-	-
r=0.04	3.28	0.005	<0	<0	-	-
r=0.10	2.98	0.065	>0	<0	0.061	0.24
σ=1.1	1.21	0.05	<0	<0	-	-
σ=3	5.12	0.02	>0	<0	0.125	1.73

So we see that for a wide region of the reasonable parameters space, a tax on capital used to subsidize labor will increase welfare, even if it will decrease growth. This is made more difficult if σ is small, the rate of time discount ρ is low, the markup $1/\alpha$ is high or the initial equilibrium labor supply $\tilde{L}$ is big.

In order to check that the parameter values for χ consistent with welfare improving capital taxation are reasonable, I calculate the corresponding Frisch elasticity of labor supply and compare the results with the available estimates. With the specification of the utility function 37, the Frisch elasticity of labor supply in BGP is given by 43:

$$\left(1+\frac{\chi-1}{\sigma}\right)^{-1}\frac{1-\tilde{L}}{\tilde{L}},$$

so it is decreasing in χ, increasing in σ and decreasing in $\tilde{L}$. The values of Frisch elasticity of labor supply consistent with optimal taxation are located between 1 to 2, with 1.30 the lowest and 1.88 the highest.[12] These values are consistent

[12] The values of the Frisch elasticity of labor supply associated with the before-tax parameter spaces are mainly located between 1 and 2, with 1.41 the lowest and 1.95 the highest.

with the estimates of the Frisch elasticity found in the literature, which range from 0.5 to 3 or even higher.

Interestingly, the growth effect of the capital income tax in our model is always found to be negative for parameters consistent with condition 125. This is especially interesting because given that in this model growth is inefficiently low as shown by Zeng and Zhang (2007).

14 Comparison between the market economy and the social planner's economy

In this subsection we study the social planner's problem and compare the social planner's equilibrium with the market equilibrium in order to analyse whether the welfare is improved while the growth rate is reduced is due to the fact that the BGP growth rate in market economy is unoptimally higher than the socially optimal growth rate. This concern comes from the literature that since the incompletely competitive economy may run a higher growth rate than the Pareto optimality, exertion of a tax may pull down the too high growth rate to mimic the Pareto optimality, so as to improve welfare.

Let $X_s \equiv \int_0^N X_s(i)di$, where $X_s(i)$ is the amount of each type of the intermediate goods in the social planner's economy and X_s is the total amount of intermediate goods. Then the final output in equilibrium can be expressed as

$$Y = AL_s^{1-\alpha}\int_0^{N_s} X_s(i)^{\alpha}di. \tag{126}$$

The Hamiltonian for the social planner's problem is:

$$J = \frac{C_s^{1-\sigma}}{1-\sigma}h(L_s)e^{-\rho t} + \frac{\mu}{\eta}\left(AL_s^{1-\alpha}\int_0^{N_s} X_s(i)^{\alpha}di - C_s - \int_0^{N_s} X_s(i)di\right) \tag{127}$$

where μ is the Lagrangian multiplier before the social budget constraint. The social planner decides on the optimal path of the control variable L_s, C_s, and $X_s(i)$, and that of the state variable N_s. The key optimality conditions are:

$$X_s(i) = A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}L_s; \tag{128}$$

$$C_s = \frac{(\sigma-1)h(L_s)}{h'(L_s)}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)N_s; \tag{129}$$

$$-\sigma\frac{\dot{C}_s}{C_s} + \frac{h'(L_s)}{h(L_s)}\dot{L}_s - \rho = \frac{\dot{\mu}}{\mu} = -\frac{1-\alpha}{\eta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}L_s. \tag{130}$$

In the balanced growth path, L_s is constant so $\dot{L}_s = 0$. From 130 we get

$$\frac{\dot{C}_s}{C_s} = \frac{\frac{1-\alpha}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} L_s - \rho}{\sigma}. \tag{131}$$

From equation 131, we can define: in equilibrium, the centralized economy's interest rate r_s is

$$r_s = \frac{1-\alpha}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} L_s. \tag{132}$$

Substituting 128 into 126 we get

$$Y_s = A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} L_s N_s. \tag{133}$$

By using the equations 128, 131 and 133 and the fact that the investment I equals $\eta \dot{N}_s$, the social account can be expressed as

$$\frac{\dot{N}_s}{N_s} = \frac{1}{\eta N_s}(Y_s - C_s - X_s) = \frac{1-\alpha}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} L_s \left(1 - \frac{(\sigma-1)h(L_s)}{h'(L_s)L_s}\right). \tag{134}$$

We use g to denote the BGP growth rate in the centralized economy. In the BGP,

$$\frac{\dot{C}_s}{C_s} = \frac{\dot{N}_s}{N_s} = g.$$

Since the transversality condition requires $0 < g < r_s$, from 132 and 134 we can see that it is equivalent to

$$0 < \frac{(\sigma-1)h(L_s)}{h'(L_s)L_s} < 1. \tag{135}$$

Considering the fact that the investment should be positive for a growing economy, and by 128, 129 and 133, the economic intuition of condition 135 is exactly $C_s < Y_s - X_s$.

Equalizing 131 and 134 in the BGP and noting 132, we get

$$r_s = \frac{\rho}{1 - \sigma\left(1 - \frac{(\sigma-1)h(L_s)}{h'(L_s)L_s}\right)} \tag{136}$$

and

$$L_s \left(1 - \sigma + \sigma \frac{(\sigma-1)h(L_s)}{h'(L_s)L_s}\right) = \frac{\rho}{\frac{1-\alpha}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}}}$$

which under our specification of the disutility function of labor leads to

$$L_s = \left(\sigma + \frac{\rho\,(\chi-1)}{(1-\sigma)\left(\frac{1-\alpha}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}}\right)}\right) / (\chi + \sigma - 1). \tag{137}$$

To ensure a positive r_s, there should be

$$\frac{(\sigma-1)h(L_s)}{h'(L_s)L_s} > 1 - \frac{1}{\sigma}. \tag{138}$$

With $\sigma > 1$, combining 135 and 138 we have

$$1 - \frac{1}{\sigma} < \frac{(\sigma-1)h(L_s)}{h'(L_s)L_s} < 1. \tag{139}$$

From condition 139 we can find that it is different from the counterpart condition 47 in the market equilibrium. Now we can compare the steady state labor supply in the social planner's economy and that in the decentralized economy. Notice that $\frac{(\sigma-1)h(L)}{h'(L)L}$ is a continuous and differentiable function of L so we can derive the first derivative of $\frac{(\sigma-1)h(L)}{h'(L)L}$ with respect to L. With respect to our specification of the disutility function of labor ($h(L) = (1-L)^{1-\chi}$), $\frac{(\sigma-1)h(L)}{h'(L)L}$ equals $\frac{\sigma-1}{\chi-1}\frac{1-L}{L}$, which is a strictly decreasing function of L. since $\frac{(\sigma-1)h(L_s)}{h'(L_s)L_s} < 1 < \frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}$ (by 47 and 139) we deduce that the steady state labor supply in the social planner's economy is larger than in the market economy. In fact we can show this in general. Suppose $L_s > \tilde{L}$. We know that $\frac{(\sigma-1)h(L_s)}{h'(L_s)L_s} < 1 < \frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}$. We also have:

$$\begin{aligned}
\frac{d\left(\frac{(\sigma-1)h(L)}{h'(L)L}\right)}{dL} &= \frac{\sigma-1}{L}\left(1 - \frac{hh''}{h'^2} - \frac{h}{h'L}\right) \\
&= \frac{-\left((1-\sigma)\left(1-\frac{hh''}{h'^2}\right)+1\right)+1-\frac{(\sigma-1)h}{h'L}}{L} \\
&< \frac{-\frac{1}{\sigma}+1-\frac{(\sigma-1)h}{h'L}}{L},
\end{aligned}$$

then we have $\frac{d\left(\frac{(\sigma-1)h(L)}{h'(L)L}\right)}{dL}|_{L=L_s} < 0$ (by using $\frac{(\sigma-1)h(L)}{h'(L)L}|_{L=L_s} > 1-\frac{1}{\sigma}$) and $\frac{d\left(\frac{(\sigma-1)h(L)}{h'(L)L}\right)}{dL}|_{L=\tilde{L}} < 0$ (by using $\frac{(\sigma-1)h(L)}{h'(L)L}|_{L=\tilde{L}} > 1$). This in itself does not prove that $L_s > \tilde{L}$, because $\frac{d\left(\frac{(\sigma-1)h(L)}{h'(L)L}\right)}{dL}$ could be positive inside the interval $(L_s, \tilde{L})$ (provided it goes to zero twice inside it). However this is not possible. In fact: notice that L is a decreasing function of ρ. Now starting from an equilibrium with $\rho = \rho_1$ suppose ρ goes up. Then $\tilde{L}$ would go down to $\tilde{L}_1$. But still we would have: $\frac{d\left(\frac{(\sigma-1)h(L)}{h'(L)L}\right)}{dL}|_{L=\tilde{L}_1} < 0$. So basically we can rest assured that $\frac{(\sigma-1)h(L)}{h'(L)L}$ must be a decreasing function of L, in the relevant interval, and therefore from any given configurations of parameters deduce from $\frac{(\sigma-1)h(L_s)}{h'(L_s)L_s} < 1 < \frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}$ that:

$$L_s > \tilde{L}. \tag{140}$$

In an initially taxless economy, using 20 in 113 (which equals exactly $\frac{\dot{C}}{C}$ in the BGP of market economy) and then comparing equation 113 and 131, we can find that the curve $\frac{\dot{C}}{C}$ of the social planner's economy lies higher than that of the market economy in the space with labor supply at the horizontal axis and growth rate at the vertical axis. Furthermore, with 140, we can see that the social planner's economy enjoys higher BGP growth rate than the market economy does. In fact, the social BGP growth rate can be derived as by substituting 137 into 131:

$$g = \left(\frac{1-\alpha}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} + \rho(\chi + \sigma - 2) \right) / (\chi + \sigma - 1). \tag{141}$$

Therefore, the conclusion is that, with the capital income tax cum wage subsidy, the fact that welfare may be improved even if the growth rate is decreased does not result from any possibly unoptimally higher growth rate in the market economy.

15 Economic intuition

In the present section I first explain why the BGP growth rate is decreased with the capital income tax cum wage subsidy. Then I analyse the two externalities that determine the sign of the welfare effect, and I also study the role of the parameters α, σ, χ and ρ in influencing the externalities.

1. Effect on growth

On first impact, the capital income tax cum wage subsidy does not influence the consumers' disposable income but increases the opportunity cost of leisure. Since the income effect is zero, the increasing wage only has a substitution effect on leisure, which causes labor supply to increase. Further, the increased labor supply induces a higher demand for the intermediate goods. This in turn induces a higher demand for investment in R&D so the interest rate will rise. But the after-tax interest rate is still smaller than the interest rate in a no-tax economy. Since the BGP growth rate is a monotonically increasing function of the after-tax interest rate, it also decreases.

2. Two spillovers

Similarly as in Chapter 1 and 2 there are two spillovers in the economy but these spillovers are contrary in sign. Firstly, increased labor supply generates a positive spillover through increasing the value of patents. The same as stated in Chapter 1, the size of the spillover is positively related with the value of parameter α because of the markup pricing of intermediate goods over their marginal costs. So when α is high the tax program that enlarges equilibrium employment will be especially beneficial. Secondly, though the effect of an invention on the present discounted value of income is the cost of inventing divided by the income share of capital, that is $\frac{\eta}{\frac{\alpha}{1+\alpha}}$, the inventor only considers the part of the contribution to production that goes to capital (here income on patents). When the return to capital is decreased after the introduction of capital income tax cum wage subsidy, the pace of invention of new patents will be slowed down. So this is a negative spillover. The sign of welfare effect is thus determined by the relatively stronger spillover.

3. Level effect

The response of consumption to changes in the wage, or the consumption-hours cross effect, is positive, implying substitutability between consumption and hours of nonwork or complementarity between consumption and hours of work. People consume more when wages are high because they work more and consume less leisure. The strength of the comlementarity between consumption and hours of work determines the size of the increase in instantaneous consumption with capital income tax cum wage subsidy exerted. The higher the elasticity of consumption with respect to wage rate, the more increase in the instantaneous consumption and in the level of static welfare.

4. Factors influencing the welfare effect

Which of the two spillovers described above is stronger depends on two factors, the same as in Chapter 1 and 2. One is the income share of labor: lower income share of labor makes the first spillover larger whereas the second smaller. So with α bigger, it is easier to achieve a positive welfare effect. The contrast between row 6 and 7 in table 6 illustrates this point. The other is the policy's effectiveness on labor supply, largely determined by the value of labor supply elasticity. In our specification the elasticity of labor supply is decreasing in parameters χ and $\tilde{L}$ but increasing in σ. Remember that the value of χ in our model is implied by the function of other parameters that can be pinned down from data so it is not free. To see the influence of the elasticity in labor supply, we can refer to row 4 and 5 to see that with lower value of before-tax $\tilde{L}$, it is more promising for the tax to be welfare-improving. It is because smaller $\tilde{L}$ means higher elasticity of labor supply so a small subsidy to labor income will cause labor supply to increase much.

Essentially, the tradeoff between the opposing growth and level effect says that households are happier in the present but suffer slower consumption growth. A bigger σ means lower intertemporal substitution elasticity of consumption, or that consumers weigh more the current consumption (lower) than the future (higher) ones. So, when the instantaneous consumption is increased with the capital income tax cum wage subsidy, this increment is given more weight than the future loss. Row 10 and 11 in table 6 give a good illustration for the influence of the magnitude of σ.

With higher subjective discount rate ρ, although consumption will grow at a lower rate under the capital income tax scheme, this dynamic loss is discounted more heavily therefore it will be more likely for the increase in the static level of welfare to overwhelm the effect resulted from the reduced growth rate, and thus the overall welfare effect may be positive. Other parameters given, a smaller before-tax γ or a larger before-tax r implies bigger discount rate ρ, so we can see from row 2 and 3 or row 8 and 9 that it is more easily for the tax to induce welfare improvement under bigger ρ.

Appendices

Appendix 8

Using the factor exhaustion condition that the wage bill plus total interest payments is equal to GNP, that is $Y - xN = wL + r\eta N$, and substituting for C using equation 107, given 111 and 112 we can write 110 as:

$$\frac{\dot{N}}{N} = r + \frac{r}{\alpha L}\left(L + (1 + \alpha\tau_k^l)\frac{h((1-\sigma)}{h'}\right). \tag{142}$$

Substituting 25 for $\frac{\dot{C}}{C}$ in 108 we get:

$$-\sigma\left[\frac{\dot{N}}{N} + (h'/h - h''/h')\dot{L}\right] + \frac{h'}{h}\dot{L} = \rho - r(1 - \tau_k^l). \tag{143}$$

Finally if we substitute in 143 the expression for $\frac{\dot{N}}{N}$ given by 142 we obtain 114 in the text, where we also use 20.

Appendix 9

By solving the integral in 31 we obtain:

$$W = \frac{1}{1-\sigma}\frac{C(0)^{1-\sigma}h(\tilde{L})}{\rho - \gamma(1-\sigma)}.$$

By using 20, 107 and 112 we can write:

$$C(0) = \eta N(0)\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})}\frac{C_1(1+\alpha\tau_k^l)}{\alpha}.$$

Using 113 we have:

$$\rho - \gamma(1-\sigma) = r(1-\tau_k^l) - \gamma,$$

while by using 142 to get an expression for γ, we obtain, again using 20:

$$r(1-\tau_k^l) - \gamma = \frac{r}{\alpha}\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}(1+\alpha\tau_k^l) - 1\right) - r\tau_k^l = C_1\tilde{L}\left(\frac{1}{\alpha} + \tau_k^l\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1\right).$$

We can thus rewrite 31 as:

$$W = \frac{(\eta N(0))^{1-\sigma}}{(1-\sigma)}\left(\frac{\sigma-1}{h'(\tilde{L})}\frac{C_1(1+\alpha\tau_k^l)}{\alpha}\right)^{1-\sigma}\frac{h^{2-\sigma}}{C_1\tilde{L}\left(\frac{1}{\alpha} + \tau_k^l\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1\right)}. \tag{144}$$

We have:

$$\frac{\log[(1-\sigma)W]}{1-\sigma} = \log(\eta N(0)) + \log\left(\frac{\sigma-1}{h'}\right) + \log\left(\frac{C_1(1+\alpha\tau_k^l)}{\alpha}\right) + \frac{2-\sigma}{1-\sigma}\log(h) - \frac{1}{1-\sigma}\log\left(\frac{1}{\alpha}+\tau_k^l\right)\left(\frac{(\sigma-1)h}{h'} - L\right).$$

From here we calculate:

$$\frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tilde{L}} = -\frac{h''}{h'} + \frac{(2-\sigma)h'}{(1-\sigma)h} + \frac{(\sigma-1)\left(1-\frac{hh''}{(h')^2}\right)-1}{(\sigma-1)\left(\frac{(\sigma-1)h}{h'}-L\right)}$$

which, after reordering becomes 119 in the text. We also have:

$$\frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tau_k^l} = \frac{\alpha}{1+\alpha\tau_k^l} + \frac{1}{\sigma-1}\frac{1}{\frac{1}{\alpha}+\tau_k^l},$$

which, after reordering becomes 120 in the text.

Chapter IV. Capital income taxes in a creative destruction model

16 Introduction

Using Schumpetarian model to analyze the long-run growth and welfare effect of fiscal policies has long become a line of research. Lai (1998) builds up a Schumpetarian growth model with gradual obsolescence and semiendogenous rate of innovation to eliminate the scale effect, compares the research duplication effect and intertemporal knowledge spillover effect in R&D and concludes that a small subsidy (tax) to innovation is welfare improving. Zeng and Zhang (2002), by considering saving and the trade-off between labor and leisure in a non-scale Schumpetarian model, study only the growth effect of the taxes and conclude on the negativeness of the growth effect of the capital income tax. Peretto (2003) shows that the only fiscal instruments that affect steady-state growth are taxes on asset and corporate income and that the effective growth-enhancing policies operate through the interest rate. Peretto (2007) examine corporate taxes in a non-scale Schumpeterian economy and shows that interventions such as eliminating the corporate income tax and/or the capital gains tax and reducing taxes on labor raise welfare. His reason is that in all these cases the endogenous increase in the tax on dividends necessary to balance the budget has a positive effect on growth. Peretto (2009) proposes a Schumpeterian analysis of the effects of a deficit-financed cut of the tax rate on distributed dividends and finds negative growth effect and welfare loss despite the fact that the economy's saving and employment ratios rise. Different from the set-up of my model, Peretto's researches focus on the role of market structure and stress the trade-off between the investment in the growth of existing product lines and in the development of new product lines, whereas in my model market structure plays no role in influencing growth. Also with consideration on labor-leisure trade-off, my model simplifies with no population growth, therefore the effective labor force equals the labor choice for representative agent and is within [0,1] horizon and in fact represents the employment ratio stated in Peretto (2007). In my model the effective labor force influences not only the levels of the equilibrium variables such as interest rate, but also affects the steady state growth rate. And the capital income taxes, whose revenues are subsidised to labor income, have long-run effects on growth and welfare, and more specifically, the increase in the capital income tax cum wage subsidy may improve welfare though it reduces growth. And it is worth pointing out that this welfare-enhancing effect is the case in general in the benchmark economy and shows strong robustness to parametric variation in the model.

In this model, the households have the utility function 1 for which conditions 2 and 3 indicate that consumption and leisure are normal goods and 4 ensures the concavity, behave under the optimalization conditions 107 and 108 with budget constraint 106 and obey the transversality condition 9. The firms follow the model set-up as in Chapter 2, i.e. equations from 55 to 79 also apply here. The government, in turn, runs balanced budget constraint as 109, generating fiscal revenue from the capital income tax and expending the revenue as subsidies to labor income.

17 Market equilibrium

The economy-wide resource constraint is

$$Y - X = C + Z \tag{145}$$

where total intermediate goods used X is subtracted from final production Y to obtain total value added, and all investment in the model is the investment in research and development of more advanced intermediate goods Z. In equilibrium the total wealth equals the total market value of the firms, i.e. $F = V$. The relation between t_w and τ_k^l is

$$t_w = \tau_k^l \frac{\alpha \zeta r}{\bar{\pi}} \tag{146}$$

for which we have used 58, 69 and 79 and 109.

Definition 16 *In a competitive equilibrium individual and aggregate variables are the same and prices and quantities are consistent with the (private) efficiency conditions for the households 106, 107, 108 and 9, the profit maximization conditions for firms in the final good sector, 57 and 58 (or 65), and for firms in the intermediate goods sector, 59 (or 60) and 66, with the government budget* *constraint 109 and with the market clearing conditions for labor ($H = L$), for wealth ($F = V$), and for the final good 145.*

I consider only the balanced growth path of the model, where labor supply L, interest rate r, research success probability p are constant while the rates of growth of other variables are constant as well.

Proposition 17 *If the economy follows a balanced growth path (hence BGP) variables grow at a constant rate, and in particular employment is constant at a value $\tilde{L}$. Along this path, rate of growth of capital and consumption, γ, is then given by*

$$\gamma = \frac{\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)\left(\frac{\bar{\pi}(\tilde{L})}{\zeta}(1 - \tau_k^l) - \rho\right)}{1 - \tau_k^l + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}. \tag{147}$$

Proof. When labor supply is constant, the wage w is proportional to the aggregate quality index Q by 65, so w and Q grow at the same rate. And 107 implies that consumption and the wage must grow at the same rate so we have the same equation as 86. 108 implies that along a BGP:

$$\frac{\dot{C}}{C} = \frac{r(1 - \tau_k^l) - \rho}{\sigma}. \tag{148}$$

On the other hand, the technic growth rate $\frac{\dot{Q}}{Q}$ follows the pattern indicated in 84. By using 84, 86 and 148, we can solve out the BGP growth rate γ as 147 and the BGP interest rate r as follows:

$$r = \frac{\rho + \sigma\frac{\bar{\pi}(\tilde{L})}{\zeta}\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}{1 - \tau_k^l + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}. \tag{149}$$

Substitution for r from equation 149 into 77 helps to get the endogenous innovation success rate p:

$$p = \frac{\frac{\bar{\pi}(\tilde{L})}{\zeta}(1 - \tau_k^l) - \rho}{1 - \tau_k^l + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}. \tag{150}$$

■

Equations 147, 149 and 150 show that γ, r and p are all increasing in L because $\bar{\pi}$ is an increasing function of L (see 66) and $q^{\frac{\alpha}{1-\alpha}} > 1$. By using 146 and 149 the relation between t_w and τ_k^l is now:

$$t_w = \frac{\tau_k^l\alpha\left(\frac{\zeta}{\bar{\pi}}\rho + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)\right)}{1 - \tau_k^l + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}. \tag{151}$$

In Appendix 10 I show how to deduce from the competitive equilibrium conditions described above the following differential equation of labor, which is the fundamental dynamic equation of the model:

$$\dot{L} = \frac{\rho - r(1 - \tau_k^l - \sigma) + \frac{\sigma\bar{\pi}}{\alpha\zeta}\left(1 + (1 + t_w)\frac{(1-\sigma)h}{h'L}\right)}{\left(\frac{\sigma h"}{h'} + (1 - \sigma)\frac{h'}{h}\right)} \equiv \frac{B(L)}{A(L)}. \tag{152}$$

Hereby the denominator of the fraction on the right-hand side $A(L)$ is always strictly positive for all values of L for the same reason as stated in the counterpart in Chapter 1. So the equation is defined for all values of L beween 0 and 1. Along a BGP $\dot{L}$ will equal zero, i.e. $B(\tilde{L}) = 0$ where $\tilde{L}$ is the BGP labor supply. Again we have to sign $d\dot{L}(\widetilde{L})/d\widetilde{L}$ to study the dynamic nature of a fixed point of 152. We have: $\frac{d\dot{L}}{dL}(\tilde{L}) = \frac{B'(\tilde{L})}{A(\tilde{L})} - \frac{A'(\tilde{L})B(\tilde{L})}{A^2(\tilde{L})} = \frac{B'(\tilde{L})}{A(\tilde{L})}$ (since $B(\tilde{L}) = 0$). Below I prove that $B(\tilde{L}) = 0$ implies $B'(\tilde{L}) > 0$. Since $d\dot{L}(\widetilde{L})/d\widetilde{L}$ is always positive, we can deduce that if BGP exists it is unique as from the phase diagram of 152 we can easily see that there is no way for $B(L)/A(L)$, which is a continuous function, to cross the horizontal axis from below two times in a row. Positive $d\dot{L}(\widetilde{L})/d\widetilde{L}$ also means instability of the equilibrium, therefore we can deduce that there will be no transitional dynamics in the model. Therefore Proposition 3 also serves here and in the following I show the proof for it:

Proof. By 66 and 149, we derive the first derivative of $\bar{\pi}$ and that of r with respect to labor as the following:

$$\bar{\pi}'(L) = \frac{\bar{\pi}}{L} \tag{153}$$

and

$$r'(L) = \frac{\bar{\pi}}{\zeta L} \frac{\sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}{1 - \tau_k^l + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}. \tag{154}$$

Moreover, using 151 and 153, we have the first derivative of t_w with respect to labor as

$$t_w'(L) = -\frac{\tau_k^l \alpha \frac{\zeta\rho}{\bar{\pi}L}}{1 - \tau_k^l + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}. \tag{155}$$

We will then have, deriving and rearranging:

$$\begin{aligned} B'(L) &= \frac{\bar{\pi}}{\zeta L}\left(B_1 + \frac{\sigma}{\alpha}B_2(L)\right) \\ &= \frac{\bar{\pi}}{\zeta L}\left[\begin{array}{c} (\sigma-1)(1-\tau_k^l)\frac{\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)} + \\ \frac{\sigma}{\alpha}\left[(1+t_w)\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right) + \left(\frac{(\sigma-1)h}{h'L} - 1\right)\frac{\tau_k^l\frac{\alpha\zeta\rho}{\bar{\pi}}}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right] \end{array}\right] \end{aligned} \tag{156}$$

where

$$B_1 \equiv (\sigma-1)(1-\tau_k^l)\frac{\sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}{1 - \tau_k^l + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}$$

and

$$B_2(L) \equiv (1+t_w)\left(1 + (1-\sigma)\left(1 - \frac{hh''}{(h')^2}\right)\right) + \left(\frac{(\sigma-1)h}{h'L} - 1\right)\frac{\tau_k^l \frac{\alpha\zeta\rho}{\bar{\pi}}}{1 - \tau_k^l + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}.$$

Transversality condition 9 requires that the BGP growth rate γ should be smaller than the after-tax interest rate $r(1-\tau_k^l)$. Using 105 and noticing that in the BGP $\frac{\dot{Q}}{Q} = \gamma$ (by 86) we get:

$$\gamma = \frac{\bar{\pi}}{\alpha\zeta}\left(1 + (1+t_w)\frac{(1-\sigma)h(L)}{h'(L)L}\right) + r < r(1-\tau_k^l).$$

Therefore

$$\frac{\bar{\pi}}{\alpha\zeta}\left((1+t_w)\frac{(\sigma-1)h(L)}{h'(L)L} - 1\right) - r\tau_k^l > 0,$$

in which we substitute 146 for t_w so it leads to

$$\left(\frac{\bar{\pi}}{\alpha\zeta} + r\tau_k^l\right)\left(\frac{(\sigma-1)h(L)}{h'(L)L} - 1\right) > 0.$$

Since $\frac{\bar{\pi}}{\alpha\zeta} + r\tau_k^l > 0$, we get

$$\frac{(\sigma-1)h(L)}{h'(L)L} > 1,$$

which is exactly the same condition as 47. By conditions 28 and 47 we get

$$B_2(L) > \frac{1+t_w}{\sigma},$$

therefore we have

$$\begin{aligned} B'(L) &> \frac{\bar{\pi}}{\zeta L}\left(B_1 + \frac{1+t_w}{\alpha}\right) \\ &= \frac{\bar{\pi}}{\zeta L}\left(\frac{1-\tau_k^l + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)(1-\alpha(1-\sigma)(1-\tau_k^l))}{\alpha\left(1-\tau_k^l + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)\right)} + \frac{t_w}{\alpha}\right). \end{aligned}$$

It is obvious that as long as $0 \leq \tau_k^l \leq 1$, no matter whether σ is bigger or smaller than 1, $B'(L)$ is positive (notice that when $0 < \sigma < 1$, there is $1 - \alpha(1-\sigma)(1-\tau_k^l) > 1 - (1-\sigma) = \sigma > 0$). ■

18 Effects of taxes

18.1 Effect on labor

As said above equilibrium labor supply can be expressed as the solution to $B(\tilde{L}) = 0$. The effects of taxes on BGP labor supply can be achieved by applying the total derivative formula with respect to $\tilde{L}$ and the tax rate τ_k^l. In Appendix 11 we show that the BGP labor supply effect of the tax τ_k^l is:

$$\frac{d\tilde{L}}{d\tau_k^l} = \frac{r\zeta\tilde{L}}{\Gamma\bar{\pi}}\left(1 + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}} - \frac{q^{\frac{\alpha}{1-\alpha}}}{1 + \sigma\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}\right) \tag{157}$$

where we define

$$\Gamma \equiv \left[\begin{array}{c} (\sigma-1)(1-\tau_k^l)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)+\tau_k^l \frac{\zeta\rho}{\bar{\pi}}\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)+ \\ \frac{1}{\alpha}\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)(1+t_w)\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right) \end{array} \right]. \quad (158)$$

And further we arrive at the following:

Proposition 18 *An increase in the tax rate on capital income whose proceeds are returned as a subsidy to labor income will increase employment in equilibrium if and only if* $\frac{(\sigma-1)h}{h'\tilde{L}} - \frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)} > 0$. *This condition is always satisfied if* $\sigma > 1$.

Proof. Note that Γ equals $\frac{B'(\tilde{L})\zeta\tilde{L}}{\sigma\bar{\pi}}\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)$ (see 156) so is positive. Therefore the sign of $\frac{d\tilde{L}}{d\tau_k^l}$ is the same as that of the term $\left(\frac{(\sigma-1)h}{h'\tilde{L}} - \frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)$. With $\sigma > 1$, we have $1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right) > 1+q^{\frac{\alpha}{1-\alpha}}-1 = q^{\frac{\alpha}{1-\alpha}}$. Therefore we have $\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)} < 1$. By condition 47, we have thus $\frac{(\sigma-1)h}{h'\tilde{L}} > \frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}$.
■

We can understand this as that the capital income taxes, whose revenue is returned to labor, cause only substitution effect between labor and leisure. Since the wage rate does not change with the tax, the after-tax labor income per hour rises therefore labor supply increases.

18.2 Effect on growth

With BGP labor supply increased by the tax, interest rate r also increases because larger demand for intermediate goods should be satisfied with more investment in R&D. However, BGP growth rate moves along with the after-tax return rate of capital, which may decrease. We show also in Appendix 11 the BGP growth effect of the tax τ_k^l as

$$\frac{d\gamma}{d\tau_k^l} = \frac{r\left[\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\left(\frac{\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)}{\Gamma}-1\right)+\frac{\tau_k^l}{\sigma}\right]}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)} \quad (159)$$

where Γ is defined in 158. And we obtain the following:

Proposition 19 *An increase in the tax rate on capital income whose proceeds are returned as a subsidy to labor income will increase growth in equilibrium if* *and only if* $\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\left(\frac{\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)}{\Gamma}-1\right)+\frac{\tau_k^l}{\sigma}\geq 0$

where Γ *is defined in 158. For an initial-zero-tax economy, this condition is never satisfied if* $\sigma<1$.

Proof. Note that in a growing economy, the investment I should be positive. Since $Y-X=(1-\alpha^2)Y$ (by 64), substituting for C its expression given by 107, after expressing the wage in terms of income by 58 we get:

$$I=(1-\alpha^2)Y-\frac{(\sigma-1)h(L)}{h'(L)L}(1+t_w)(1-\alpha)Y.$$

So $I>0$ implies that

$$\frac{(\sigma-1)h(L)}{h'(L)L}<\frac{1+\alpha}{1+t_w}. \tag{160}$$

We are interested in analysing in an economy with initial zero tax whether the increase in the tax can raise or reduce the growth rate. With $\tau_k^l=0$, the term $\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\left(\frac{\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)}{\Gamma}-1\right)+\frac{\tau_k^l}{\sigma}$ becomes

$$\frac{\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left[\frac{(\sigma-1)h}{h'\tilde{L}}-1-\frac{1}{\alpha}\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)\right]}{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)+\frac{1}{\alpha}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)}.$$

By condition 28, we have

$$\begin{aligned}
&(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)+\frac{1}{\alpha}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)\\
>\ &(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)+\frac{1}{\alpha\sigma}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\\
=\ &\frac{1}{\alpha\sigma}+\left(\frac{1}{\alpha}+\sigma-1\right)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)>0,
\end{aligned} \tag{161}$$

so the sign of the growth effect of the increase in the tax rate from zero is the same as that of the term

$$\frac{(\sigma-1)h}{h'\tilde{L}}-1-\frac{1}{\alpha}\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right).$$

Notice that with $\sigma<1$, this term is always negative because by condition 28 and 160 this term is smaller than

$$\alpha-\frac{1}{\alpha\sigma},$$

which is negative with $\sigma < 1$. Therefore we reach the conclusion that with $\sigma < 1$, the growth effect of the tax in an initial-zero-tax economy can never be positive. ■

18.3 Effect on welfare

Given γ, the constant rate of growth, and $\widetilde{L}$ the BGP labor supply, we can calculate maximum lifetime utility W along a balanced growth path as equation 31. In Appendix 12 it is shown how to express W as a differentiable function of τ_k^l and $\tilde{L}$ (itself a function of τ_k^l). The effect on welfare of an increase in the tax rate τ_k^l is then positive if $\frac{dW}{d\tau_k^l}$ is positive. To simplify calculations, we consider again the monotonically increasing transformation of W: $\frac{\log[(1-\sigma)W]}{1-\sigma}$ (see also Appendix 12). We have the formula for $\frac{d(\log[(1-\sigma)W])}{(1-\sigma)d\tau_k^l}$ the same as 118. We obtain the partial derivative of $\frac{\log[(1-\sigma)W]}{1-\sigma}$ with respect to $\tilde{L}$ and τ_k^l after deriving and rearranging as:

$$\frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tilde{L}} = \frac{\frac{\sigma}{\tilde{L}} - \frac{h'}{h}}{1-\sigma} \cdot \frac{(\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1}{\frac{(\sigma-1)h}{h'\tilde{L}}-1} + \frac{\frac{\sigma}{1-\sigma}\frac{\rho\tau_k^l}{\tilde{L}}}{\frac{\bar{\pi}}{\alpha\zeta}\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)+\left(\rho+\sigma\frac{\bar{\pi}}{\zeta}\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\tau_k^l} \tag{162}$$

and

$$\frac{\partial(\log[(1-\sigma)W])}{(1-\sigma)\partial\tau_k^l} = \frac{\sigma}{\sigma-1}\frac{r\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}{\frac{\bar{\pi}}{\alpha\zeta}\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)+\left(\rho+\sigma\frac{\bar{\pi}}{\zeta}\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\tau_k^l}, \tag{163}$$

respectively (see also Appendix 12). Thus, substituting 157, 162 and 163 in equation 118, we can finally get:

$$\frac{d(\log[(1-\sigma)W])}{(1-\sigma)d\tau_k^l} = \frac{\sigma r}{\Gamma}\left[\frac{q^{\frac{\alpha}{1-\alpha}}-1}{\Theta}\left(\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(1-\tau_k^l\right)-\frac{\tau_k^l\rho\zeta}{\bar{\pi}}\right)+\frac{1-\frac{h'\tilde{L}}{(\sigma-1)h}q^{\frac{\alpha}{1-\alpha}}}{\sigma\frac{\bar{\pi}}{\zeta}}\frac{\Psi}{\Phi}\right] \tag{164}$$

(see again Appendix 12) where we define the following:

$$\Theta \equiv \frac{\bar{\pi}}{\alpha\zeta}\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)+\left(\rho+\sigma\frac{\bar{\pi}}{\zeta}\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\tau_k^l,$$

$$\Phi \equiv \frac{(\sigma-1)h}{h'\tilde{L}}-1$$

and

$$\Psi \equiv (\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1.$$

Since Θ, Γ, Φ and Ψ are all positive, we arrive at the following:

Proposition 20 *The sufficient and necessary condition for an increase in the tax rate on capital income to increase welfare is:*

$$\frac{q^{\frac{\alpha}{1-\alpha}}-1}{\Theta}\left(\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(1-\tau_k^l\right)-\frac{\tau_k^l\rho\zeta}{\bar{\pi}}\right)+\frac{1-\frac{h'\tilde{L}}{(\sigma-1)h}q^{\frac{\alpha}{1-\alpha}}}{\sigma\frac{\bar{\pi}}{\zeta}}\frac{\Psi}{\Phi}\geq 0.$$

Especially to our interest, in an economy with initial zero tax the welfare effect of the increase in the rate of tax τ_k^l is therefore:

$$\frac{d(\log[(1-\sigma)W])}{(1-\sigma)d\tau_k^l}|_{\tau_k^l=0}=\frac{\zeta r}{\bar{\pi}\Phi}\cdot\frac{\alpha\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\Phi+\left(1-q^{\frac{\alpha}{1-\alpha}}\frac{h'\tilde{L}}{(\sigma-1)h}\right)\Psi}{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)+\frac{1}{\alpha}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\Psi}.$$

Since $(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)+\frac{1}{\alpha}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\Psi>0$ (by 161) and $\Phi>0$ (by 47), we arrive at the following:

Proposition 21 *The sufficient and necessary condition for an increase in the tax rate on capital income to increase welfare in an economy with initially zero taxes is:*

$$\alpha\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)+\left(1-q^{\frac{\alpha}{1-\alpha}}\frac{h'\tilde{L}}{(\sigma-1)h}\right)\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)>0. \tag{165}$$

We notice that the condition to improve welfare is less stringent than the condition to improve growth, or in other words that if the latter is satisfied, the first is satisfied as well. In fact, in an initially no-tax economy, we assume positive growth effect, so $\sigma>1$ is assumed. Then: $\frac{(\sigma-1)h}{h'\tilde{L}}-1=\frac{1}{\alpha}\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right)+\frac{\varepsilon}{\alpha}$ for some strictly positive number ε. Substituting in the LHS of the condition 165 we get:

$$\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)\left(\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)-\left(q^{\frac{\alpha}{1-\alpha}}\frac{h'\tilde{L}}{(\sigma-1)h}-1\right)\right)+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\varepsilon.$$

We know from 28 that $\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)>0$, and by 47 $0<\frac{h'\tilde{L}}{(\sigma-1)h}<1$ therefore $-\left(q^{\frac{\alpha}{1-\alpha}}\frac{h'\tilde{L}}{(\sigma-1)h}-1\right)>-\left(q^{\frac{\alpha}{1-\alpha}}-1\right)$. Then we can find that $\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)-\left(q^{\frac{\alpha}{1-\alpha}}\frac{h'\tilde{L}}{(\sigma-1)h}-1\right)>(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)>0$ for $\sigma>1$. The welfare effect is thus positive even without adding the positive term $\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\varepsilon$. We have therefore proven that it is possible for an increase in tax rate on capital income to increase welfare and decrease growth. Hence Proposition 15 also applies here.

In the next section we will show that this is more than a theoretical possibility and that for specifications of tastes and technology parameters often used in calibration exercises it is possible for the tax program to induce Pareto improvements but reduce growth. This can happen even if the market equilibrium before the program the rate of growth is inefficiently low. The example we offer is also useful to offer an intuition on the mechanism at work in producing the result.

18.4 Are capital income taxes good for reasonable parameter values?

The class of functions for the disutility of labor 37 is again considered here.

First, we notice that we can now obtain an explicit solution for the equilibrium level of activity. By solving $B(\tilde{L}) = 0$ we obtain

$$\tilde{L} = \left[1 + \frac{\chi - 1}{(\sigma - 1)(1 + t_w)} \left(1 + \frac{\alpha\zeta}{\sigma\bar{\pi}} \left(r(\sigma - 1 + \tau_k^l) + \rho\right)\right)\right]^{-1}. \tag{166}$$

Since $\frac{\bar{\pi}}{\zeta}$, r and t_w are all functions of L, the equation 166 shows an implicit solution to the BGP labor supply $\tilde{L}$. Using 98 (with $C_2 \equiv \frac{1-\alpha}{\zeta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}}$), 149 and 151, the equation 166 can be transformed into an explicit function of $\tilde{L}$:

$$\tilde{L}^2 C_3 + \tilde{L} C_4 + C_5 = 0 \tag{167}$$

where we define the constants

$$C_3 \equiv \frac{\chi + \sigma - 2}{\sigma - 1} \left(1 - \tau_k^l + \sigma \left(q^{\frac{\alpha}{1-\alpha}} - 1\right)\right) + \alpha \left(q^{\frac{\alpha}{1-\alpha}} - 1\right) \left(\left(\sigma + \frac{\chi - 1}{\sigma - 1}\right) \tau_k^l + \chi - 1\right),$$

$$C_4 \equiv \frac{\alpha\rho}{C_2} \left(\tau_k^l + \frac{\chi - 1}{\sigma - 1} q^{\frac{\alpha}{1-\alpha}}\right) - 1 + \tau_k^l - \sigma \left(q^{\frac{\alpha}{1-\alpha}} - 1\right) \left(1 + \alpha\tau_k^l\right),$$

and

$$C_5 \equiv -\frac{\alpha\rho}{C_2} \tau_k^l.$$

We can prove that $C_3 > 0$.

Proof. With $\sigma > 1$, it is easy to find that $C_3 > 0$. With $0 < \sigma < 1$, after reordering we have

$$C_3 = \frac{\chi + \sigma - 2}{\sigma - 1} \left(1 - \tau_k^l\right) + \left(q^{\frac{\alpha}{1-\alpha}} - 1\right) \left[\sigma \frac{\chi - 1}{\sigma - 1} + \alpha \left(\sigma + \frac{\chi - 1}{\sigma - 1}\right) \tau_k^l + \sigma + \alpha (\chi - 1)\right].$$

Note that with $\chi < 1 < \sigma + \chi$ (see the condition for the disutility function 37), we have $\frac{\chi+\sigma-2}{\sigma-1} > 0$, $\frac{\chi-1}{\sigma-1} > 0$ and $\sigma + \alpha(\chi - 1) > \sigma + \chi - 1 > 0$, therefore $C_3 > 0$. ■

With C_3 positive, though the sign of C_4 is ambiguous, considered that $C_5 < 0$, the solution to the quadratic equation 167 should only take the root

$$\tilde{L} = \frac{-C_4 + \sqrt{C_4^2 - 4C_3C_5}}{2C_3} \tag{168}$$

to ensure the positiveness of $\tilde{L}$.

Second, we start by considering the economy with initial zero tax. As for the welfare effect of the capital income tax cum wage subsidy τ_k^l, Proposition 21 requires

$$\alpha\sigma \left(q^{\frac{\alpha}{1-\alpha}} - 1\right) \left(\frac{\sigma - 1}{\chi - 1} \frac{1 - \tilde{L}}{\tilde{L}} - 1\right) + \left(\frac{1}{\chi - 1} - \frac{q^{\frac{\alpha}{1-\alpha}} \tilde{L}}{(\sigma - 1)\left(1 - \tilde{L}\right)}\right) (\sigma + \chi - 2) > 0. \tag{169}$$

Additionally, by Proposition 19, in the economy with initial zero tax, the growth effect of the tax τ_k^l is in sign the same with

$$\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\frac{\frac{\sigma-1}{\chi-1}\frac{1-\tilde{L}}{\tilde{L}}-1-\frac{1}{\alpha}\frac{\sigma+\chi-2}{\chi-1}}{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)+\frac{1}{\alpha}\frac{\sigma+\chi-2}{\chi-1}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}. \tag{170}$$

In the following numerical calculation, substituting 168 into the LHS of 169 and into the expression 170, we can judge whether an increase in tax τ_k^l can lead to an improvement in welfare and whether the growth effect is positive or negative.

Then we take account of the economy going on with positive tax rate. By Proposition 20, the following condition should be satisfied:

$$\frac{\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\left(\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(1-\tau_k^l\right)-\frac{\tau_k^l\rho}{C_2\tilde{L}}\right)}{\frac{1}{\alpha}\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)+\left(\frac{\rho}{C_2\tilde{L}}+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\tau_k^l}+\frac{\left(1-\frac{(\chi-1)\tilde{L}}{(\sigma-1)\left(1-\tilde{L}\right)}q^{\frac{\alpha}{1-\alpha}}\right)(\sigma+\chi-2)}{\sigma\left((\sigma-1)\frac{1-\tilde{L}}{\tilde{L}}-\chi+1\right)}\geq 0. \tag{171}$$

Plugging 168 for $\tilde{L}$ in the LHS of 171, we can get a pure function of τ_k^l. By adjusting the value of τ_k^l, we can get different value of the LHS of 171. We define the optimal capital income tax rate $\hat{\tau}_k^l$ as the value of τ_k^l that makes the LHS of 171 zero and with the tax rate smaller than this value, the LHS of 171 is always positive.

Now I try to check whether 169 and 171 can hold in reasonable parametrizations of the model. We are completely aware that this model is not rich enough in number of variables, not to mention their dynamics, to fit the data well. Models that are rich enough to fit well become complex and difficult to interpret. The aim of our exercise is not realism but the understanding of mechanisms of action of policy not noticed before in the literature. My choices for the benchmark values of the parameters $\{r,\gamma,\tilde{L},\alpha,p\}$ and their ranges follow those chosen in Chapter 2 and the benchmark value and ranges of parameter σ are the same as those taken in Chapter 3.

In table 7 I report the benchmark parameterization. The 6-tuple $\left\{r,\gamma,\tilde{L},\sigma,\alpha,p\right\}$ implies values for ρ (through 148 when $\tau_k^l=0$), for q (through $\gamma=p\left(q^{\frac{\alpha}{1-\alpha}}-1\right)$ by 147 and 150) and for χ (through 166 when $\tau_k^l=0$ and $C_2=(r+p)/\tilde{L}$ through 77 and 98).

Table 7: Benchmark Parameterization

Parameters and Steady State Variables Determined	Parameter	Value
TFP Growth	γ	0.0175
Hours Worked over Time Endowment	$\tilde{L}$	0.23
Mark-up	$1/\alpha$	1.2
Interest rate	r	0.07
Intertemporal Elasticity of Substitution in Consumption	σ	2
Innovation Success Rate	p	0.05
Parameters and Steady State Variables Implied		
Labour Supply Parameter	χ	3.45
Rate of Time Discount	ρ	0.035
Quality Ladder	q	1.06
Optimal Tax Rate	$\widehat{\tau}_k^l$	0.329
Steady State Variables under the Optimal Taxation	Value	Change (%)
$\tilde{L}$	0.244	6.14
γ	0.0129	-26.42
r	0.091	29.39

With these parameters the tax program is welfare increasing, i.e. $dW/d\tau_k^l|_{\tau_k^l=0}$ is positive, even if growth decreases. It is then calculated that the tax rate associated with maximum utility is 32.92%. The utility level is -184.06 before tax, and increases to -180.03 after the exertion of the tax at the optimal rate. The rate of increment in the welfare is thus 2.19%.

In table 8 I report over the alternative parameterizations and the results of the sensitivity analysis.

Table 8: Alternative Parameterizations

	χ	ρ	$dW/d\tau_k^l\|_{\tau_k^l=0}$	$d\gamma/d\tau_k^l\|_{\tau_k^l=0}$	$\widehat{\tau}_k^l$	$\Delta W/\|W\|(\%)$
γ=0.015	3.42	0.04	>0	<0	0.431	3.79
γ=0.02	3.49	0.03	>0	<0	0.236	1.14
$\tilde{L}$=0.17	4.58	0.035	>0	<0	0.342	2.47
$\tilde{L}$=0.3	2.71	0.035	>0	<0	0.314	1.88
$1/\alpha$=1.1	3.40	0.035	>0	<0	0.378	3.33
$1/\alpha$=1.37	3.54	0.035	>0	<0	0.236	0.89
r=0.04	3.77	0.005	<0	<0	-	-
r=0.10	3.30	0.065	>0	<0	0.433	4.34
σ=1.1	1.25	0.0508	>0	<0	0.299	0.16
σ=3	5.91	0.0175	>0	<0	0.360	5.39
p=0.1	3.66	0.035	>0	<0	0.557	4.81
p=0.02	3.25	0.035	>0	<0	0.142	0.68

In order to check that the parameter values for χ consistent with welfare improving capital taxation are reasonable, I calculate the corresponding Frisch elasticity of labor supply and compare the results with the available estimates. With the specification of the utility function 37, the Frisch elasticity of labor supply in BGP is given by 43. The values of Frisch elasticity of labor supply consistent with optimal taxation are located between 1 to 2, with 1.15 the lowest and 2.59 the highest.[13] These values are consistent with the estimates of the Frisch elasticity found in the literature.

So we see that for a wide region of the reasonable parameters space, a tax on capital used to subsidize labor will increase welfare, even if it will decrease growth. Interestingly, comparing to the numerical results in Chapter 3, hereby the capital income tax cum wage subsidy can have bigger potential to improve welfare. It is because, in addition to the mechanism described in Chapter 3, there is another mechanism taking effect in this case: the endogenous innovation success rate is reduced by the taxation, therefore, the negative externality caused by obsolescence from innovations, or, the business stealing effect, will be dampened, and the overall welfare would thus be improved even if the balanced growth rate decreases with the taxation.

19 Comparison between the market economy and the social planner's economy

In this section we study the social planner's problem and compare the social planner's equilibrium with the market equilibrium in order to analyse whether the welfare is improved while the growth rate is reduced is due to the fact that the BGP growth rate in market economy is unoptimally higher than the socially optimal growth rate. This concern comes from the literature that since the incompletely competitive economy may run a higher growth rate than the Pareto optimality, exertion of a tax may pull down the too high growth rate to mimic the Pareto optimality, so as to improve welfare.

Let $X_s \equiv \int_0^N X_s(j)dj$, where $X_s(j)$ is the quantities employed of the leading-edge intermediates in each sector, and X_s is the total quantities of the intermediate goods in all sectors. Then the final output in equilibrium can be expressed as

$$Y_s = AL_s^{1-\alpha} \int_0^N \left(q^{k_j} X_s(j)\right)^\alpha dj. \tag{172}$$

The planner's problem is also constrained by the R&D technology. The probability $p(k_j)$ is assumed again to be given from equations 70 and 71 (notice that in the planner's economy we can use the aggregate amount of R&D effort $Z(k_j)$ to substitute for the individual amount $z(k_j)$) by

$$p(k_j) = \frac{Z(k_j)}{\zeta} q^{-\frac{\alpha}{1-\alpha}(k_j+1)}. \tag{173}$$

It is convenient first to work out the planner's choice of intermediate quantities (a static problem) and then use the result to write out a simplified Hamiltonian

[13] The values of the Frisch elasticity of labor supply associated with the before-tax parameter spaces are mainly located between 1 and 2, with 1.27 the lowest and 2.74 the highest.

expression. It is straightforward to show that the first-order condition for maximizing U with respect to the choice of $X_s(j)$ implies

$$X_s(j) = L_s A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} q^{\frac{\alpha}{1-\alpha} k_j}, \tag{174}$$

so the total quantities of all the intermediate goods are

$$X_s = Q_s L_s A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \tag{175}$$

where Q_s is the same aggregate quality index that is defined in equation 61 for the decentralized economy. Substitution for $X_s(j)$ from equation 174 into equation 172 gives an expression for aggregate output:

$$Y_s = Q_s L_s A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}}. \tag{176}$$

From equation 83, the expected change in Q_s per unit of time is given by

$$E(\Delta Q_s) = \int_0^N p(k_j) \left[\left(q^{\frac{\alpha}{1-\alpha}(k_j+1)} - q^{\frac{\alpha}{1-\alpha} k_j} \right) \right] dj.$$

Substitution for $p(k_j)$ from equation 173 leads to

$$E(\Delta Q_s) = \frac{Z_s}{\zeta} \left(1 - q^{-\frac{\alpha}{1-\alpha}} \right). \tag{177}$$

We again assume that the number of sectors is large enough so that we can treat Q_s as differentiable; hence we can use the equation 177 to represent the actual change, $\dot{Q}_s$, in the quality index.

We can use the results to write the social planner's Hamiltonian expression as:

$$J = \frac{C^{1-\sigma}}{1-\sigma} h(L_s) e^{-\rho t} + \kappa \left(Q_s L_s A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) - C_s - Z_s \right) + \mu \frac{Z_s}{\zeta} \left(1 - q^{-\frac{\alpha}{1-\alpha}} \right) \tag{178}$$

where κ is the Lagrangian multiplier applies to the resource constraint, $Y_s = C_s + X_s + Z_s$, and μ is the shadow price attaches to the expression for $\dot{Q}_s$ from equation 177. The social planner decides on the optimal path of the control variables L_s and C_s, and that of the state variable Z_s. The key optimality conditions are:

$$C_s = \frac{(\sigma - 1) h(L_s)}{h'(L_s)} A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) Q_s, \tag{179}$$

$$\kappa = \frac{\mu}{\zeta} \left(1 - q^{-\frac{\alpha}{1-\alpha}} \right),$$

$$\dot{\mu} = -\kappa L_s A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha),$$

and

$$-\sigma\frac{\dot{C}_s}{C_s}+\frac{h'(L_s)}{h(L_s)}\dot{L}_s-\rho=\frac{\dot{\kappa}}{\kappa}=\frac{\dot{\mu}}{\mu}=-\left(1-q^{-\frac{\alpha}{1-\alpha}}\right)\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}L_s. \quad (180)$$

In the balanced growth path, L_s is constant so $\dot{L}_s = 0$. From 180 we get

$$\frac{\dot{C}_s}{C_s}=\frac{\left(1-q^{-\frac{\alpha}{1-\alpha}}\right)\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}L_s-\rho}{\sigma}. \quad (181)$$

From equation 181, we can define: in equilibrium, the centralized economy's interest rate r_s is

$$r_s=\left(1-q^{-\frac{\alpha}{1-\alpha}}\right)\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}L_s. \quad (182)$$

By using the equations 175, 176, 177 and 181, the growth rate of Q_s can be expressed as

$$\frac{\dot{Q}_s}{Q_s}=\frac{1-q^{-\frac{\alpha}{1-\alpha}}}{Q_s\zeta}(Y_s-C_s-X_s)=\left(1-q^{-\frac{\alpha}{1-\alpha}}\right)\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}L_s\left(1-\frac{(\sigma-1)h(L_s)}{h'(L_s)L_s}\right). \quad (183)$$

We use g to denote the BGP growth rate in the centralized economy. In the BGP,

$$\frac{\dot{C}}{C}=\frac{\dot{Q}}{Q}=g.$$

Since the transversality condition requires $0<g<r_s$, from 182 and 183 we can see that it is equivalent to

$$0<\frac{(\sigma-1)h(L_s)}{h'(L_s)L_s}<1$$

(i.e. condition 135). Considering the fact that the investment should be positive for a growing economy, and by 175, 176 and 179, the economic intuition of condition 135 is exactly $C_s < Y_s - X_s$.

Equalizing 181 and 183 in the BGP and noting 182, we get

$$r_s=\frac{\rho}{1-\sigma\left(1-\frac{(\sigma-1)h(L_s)}{h'(L_s)L_s}\right)}$$

(the same as 136) and

$$L_s\left(1-\sigma+\sigma\frac{(\sigma-1)h(L_s)}{h'(L_s)L_s}\right)=\frac{\rho}{\left(1-q^{-\frac{\alpha}{1-\alpha}}\right)\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}}$$

which under our specification of the disutility function of labor leads to

$$L_s=\left(\sigma+\frac{\rho\,(\chi-1)}{(1-\sigma)\left(1-q^{-\frac{\alpha}{1-\alpha}}\right)\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}}\right)/(\chi+\sigma-1). \quad (184)$$

To ensure a positive r_s, there should be

$$\frac{(\sigma-1)h(L_s)}{h'(L_s)L_s} > 1 - \frac{1}{\sigma}$$

(i.e. condition 138). With $\sigma > 1$, combining 135 and 138 we have

$$1 - \frac{1}{\sigma} < \frac{(\sigma-1)h(L_s)}{h'(L_s)L_s} < 1$$

(i.e. condition 139).

From condition 139 we can find that it is different from the counterpart condition 47 in the market equilibrium. Now we can compare the steady state labor supply in the social planner's economy and that in the decentralized economy. Notice that $\frac{(\sigma-1)h(L)}{h'(L)L}$ is a continuous and differentiable function of L so we can derive the first derivative of $\frac{(\sigma-1)h(L)}{h'(L)L}$ with respect to L. With respect to our specification of the disutility function of labor ($h(L) = (1-L)^{1-\chi}$), $\frac{(\sigma-1)h(L)}{h'(L)L}$ becomes $\frac{\sigma-1}{\chi-1}\frac{1-L}{L}$, which is a strictly decreasing function of L. since $\frac{(\sigma-1)h(L_s)}{h'(L_s)L_s} < 1 < \frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}$ (by 47 and 139) we deduce that the steady state labor supply in the social planner's economy is larger than in the market economy. In fact we can show this in general as we do in Section 12.

However, we cannot tell whether the socially optimal BGP growth rate is higher or not compared with the BGP growth rate in an initially taxless market economy. Under the specification of the disutility function of labor ($h(L) = (1-L)^{1-\chi}$), the BGP growth rate in social planner's economy is

$$g = \left(\left(1 - q^{-\frac{\alpha}{1-\alpha}}\right)\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}} - \rho\left(\frac{\chi-1}{\sigma-1}+1\right)\right)/(\chi+\sigma-1) \quad (185)$$

by substituting 184 for 181. However the BGP growth rate in decentralized economy is

$$\gamma = \left[\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}} - \rho\left(\frac{\chi-1}{\sigma-1}(1+\alpha)+1\right)\right] / \left[\alpha(\chi-1) + \left(\frac{\chi-1}{\sigma-1}+1\right)\frac{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{q^{\frac{\alpha}{1-\alpha}}-1}\right] \quad (186)$$

for which we have used

$$\gamma = \frac{q^{\frac{\alpha}{1-\alpha}}-1}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\left(\frac{\bar{\pi}}{\zeta}-\rho\right) = \frac{q^{\frac{\alpha}{1-\alpha}}-1}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\left(\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}}\tilde{L}-\rho\right)$$

(by 66 and 85) and

$$\tilde{L} = \left[1 - \frac{\chi-1}{\sigma-1}\frac{\rho q^{\frac{\alpha}{1-\alpha}}\frac{\alpha\zeta}{1-\alpha}A^{-\frac{1}{1-\alpha}}\alpha^{-\frac{1+\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right] / \left[1 + \frac{\chi-1}{\sigma-1}\left(1 + \frac{\alpha(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)\right]$$

(by 66 and 97). Comparison between 185 and 186 shows that: $-\rho\left(\frac{\chi-1}{\sigma-1}(1+\alpha)+1\right) < -\rho\left(\frac{\chi-1}{\sigma-1}+1\right)$ and

$$\alpha(\chi-1)+\left(\frac{\chi-1}{\sigma-1}+1\right)\frac{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{q^{\frac{\alpha}{1-\alpha}}-1}=\alpha(\chi-1)+\left(\frac{\chi-1}{\sigma-1}+1\right)\left(\frac{1}{q^{\frac{\alpha}{1-\alpha}}-1}+\sigma\right)$$

$$>\alpha(\chi-1)+\left(\frac{\chi-1}{\sigma-1}+1\right)\sigma=\left(\alpha+\frac{\sigma}{\sigma-1}\right)(\chi-1)+\sigma>\chi+\sigma-1$$

with $\sigma > 1$. Though we can not tell which one is bigger between $\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}}$ and $\left(1-q^{-\frac{\alpha}{1-\alpha}}\right)\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}$, we can use the pinned down values of the related parameters to get numerical values for these two terms and further obtain the numerical value of the socially optimal growth rate g (note that we can substitute C_2 for $\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}}$ so we can rewrite $\frac{1-\alpha}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}$ as $C_2\alpha^{-\frac{1}{1-\alpha}}$, and $\left(1-q^{-\frac{\alpha}{1-\alpha}}\right)$ equals $\frac{q^{\frac{\alpha}{1-\alpha}}-1}{q^{\frac{\alpha}{1-\alpha}}}$ which can be represented by the value of $\frac{\gamma}{p+\gamma}$ because $q^{\frac{\alpha}{1-\alpha}}-1=\frac{\gamma}{p}$). The numerical calculation shows that under reasonable parametrization space, the socially optimal growth rate g always exceeds the decentralized economy's growth rate γ.

Although the comparison between the value of g and γ is based on our example of disutility function of labor, the above analysis on the social planner's economy at least implies that it is not necessarily due to the unoptimally higher growth rate in the market economy that with exertion of capital income tax, the welfare can be increased even if the growth rate is decreased.

20 Economic intuition

Similarly as in Chapter 3, the total welfare can be raised under the capital income tax cum wage subsidy scheme though the growth rate will be decreased. It is because of the counteraction, as described in Chapter 3, between the gain in the static level of welfare induced by the positive substitution effect of the tax program, and the loss in equilibrium growth rate caused by the reduced rate of return to capital. Again, the results are sensitive to the parameters $\left\{\alpha,\sigma,\rho,\tilde{L}\right\}$, for which the economic intuitions are the same as explained in Chapter 3 and not repeated here. In this section, the externalities particular with the endogenous growth model of creative destruction will be mentioned and the influence of parameter p, the success rate of innovation, will be explained. Again, as stated in Chapter 2, the business stealing effect will give firms incentives to do more research than would be socially optimal. A higher value of the before-tax innovation success rate p implies a smaller value of the rung of quality ladder q. Other things given, smaller q indicates that the innovation, once taken by the entrant successfully, will bring to the innovator less profit (by equation 67) therefore the pace of invention of new patents will be decelerated. Consequently, the loss in growth rate caused by the capital income tax cum wage subsidy in a low-q economy will be smaller than in a high-q economy, as we can see from the comparison between row 12 and 13 in table 6.

Appendices

Appendix 10

We establish the factor exhaustion condition

$$Y - X = wL + \pi = wL + \frac{\bar{\pi}}{\zeta}V = wL + (r+p)V$$

as 102. Using 102 and substituting for C using equation 107 we can then write 145 as:

$$w\left(L + (1+t_w)\frac{(1-\sigma)h(L)}{h'(L)}\right) + \frac{\bar{\pi}}{\zeta}V - Z = 0. \tag{187}$$

With equations 78 and 79, and noticing that $\frac{\dot{V}}{V}$ should be the same as $\frac{\dot{Q}}{Q}$, by using 84 for $\frac{\dot{V}}{V}$, we can establish the relationship between V and Z as in 104:

$$Z = \dot{V} + pV.$$

Further substituting for Z from 104, applying 79 for V, and using the relation that $w = \frac{\bar{\pi}Q}{\alpha L}$ (by 65 and 66), equation 187 becomes

$$\frac{\dot{Q}}{Q} = \frac{\bar{\pi}}{\zeta}\frac{1}{\alpha}\left(1 + (1+t_w)\frac{(1-\sigma)h(L)}{h'(L)L}\right) + r. \tag{188}$$

We will now show how to deduce from these equilibrium conditions a differential equation for labor. Totally differentiating 107 we get:

$$\frac{\dot{C}}{C} = \frac{\dot{Q}}{Q} + \left(\frac{h'}{h} - \frac{h''}{h'}\right)\dot{L}.$$

Substituting this expression for $\frac{\dot{C}}{C}$ in 108 we get:

$$-\sigma\left[\frac{\dot{Q}}{Q} + \left(\frac{h'}{h} - \frac{h''}{h'}\right)\dot{L}\right] + \frac{h'}{h}\dot{L} = \rho - r(1-\tau_k^l).$$

Finally if we substitute in this expression for $\frac{\dot{Q}}{Q}$ given by 188 we obtain the dynamic of labor supply as 152 in the text.

Appendix 11

In the BGP $B(\tilde{L}) = 0$. Taking total derivative to this equation with respect to $\tilde{L}$ and τ_k^l we have

$$\frac{d\tilde{L}}{d\tau_k^l} = -\frac{B'_{\tau_k^l}}{B'(\tilde{L})}.$$

We have also the partial derivative of $B(\tilde{L})$ with respect to τ_k^l as

$$B'_{\tau_k^l} = \frac{\sigma r\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\left(\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}-\frac{(\sigma-1)h}{h'\tilde{L}}\right),$$

for which we have used

$$\frac{\partial t_w}{\partial \tau_k^l} = \frac{\alpha\left(\frac{\rho\zeta}{\bar{\pi}}+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}{\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)^2} = \frac{\alpha\zeta r}{\bar{\pi}}\frac{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)} \tag{189}$$

(by 149 and 151) and

$$\frac{\partial r}{\partial \tau_k^l} = \frac{r}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)} \tag{190}$$

(by 149). Therefore we have

$$\begin{aligned}\frac{d\tilde{L}}{d\tau_k^l} &= \frac{\sigma r\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\left(\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)/B'(\tilde{L}) \\ &= \frac{r\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)}{\frac{\bar{\pi}}{\zeta\tilde{L}}\left[\begin{array}{c}(\sigma-1)(1-\tau_k^l)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)+\tau_k^l\frac{\zeta\rho}{\bar{\pi}}\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)+ \\ \frac{1}{\alpha}\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)(1+t_w)\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)\end{array}\right]}\end{aligned}$$

as 157 in the text.

The growth effect of τ_k^l can be deduced from 148 as:

$$\frac{d\gamma}{d\tau_k^l} = \frac{\partial\gamma}{\partial r}\frac{\partial r}{\partial\tilde{L}}\frac{d\tilde{L}}{d\tau_k^l}+\frac{\partial\gamma}{\partial r}\frac{\partial r}{\partial\tau_k^l}+\frac{\partial\gamma}{\partial\tau_k^l}$$

$$= \frac{r}{\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}\left[\begin{array}{c}\frac{\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}{\Gamma}\left(\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)+ \\ \frac{1}{\sigma}\left(\tau_k^l-\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\end{array}\right],$$

which can be rearranged into 159 as in the text.

Appendix 12

Using equation 107 and the relation $w=\frac{1}{\alpha L}\frac{\bar{\pi}}{\zeta}V$ (by 65, 66 and 79) we can obtain $C(0)=\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}\frac{\bar{\pi}}{\alpha\zeta}(1+t_w)V(0)$ where $V(0)$ is the total firm value at time 0. And note that along the BGP $\rho-\gamma(1-\sigma)=r(1-\tau_k^l)-\gamma$ (by using $\gamma=\frac{\dot{C}}{C}$ in the BGP and 148) while $r(1-\tau_k^l)-\gamma=\frac{\bar{\pi}}{\alpha\zeta}\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}(1+t_w)-1\right)-r\tau_k^l=$

$\left(\frac{\bar{\pi}}{\alpha\zeta}+r\tau_k^l\right)\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}-1\right)$ (by using $\gamma=\frac{\dot{Q}}{Q}$ in the BGP and 188 for $\frac{\dot{Q}}{Q}$, and noting 146). Transversality condition 9 is satisfied with $\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}>1$ (see also 47 and 160 for the suitable range of $\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}$) so that $\gamma<r(1-\tau_k^l)$ in an initial taxless economy. We can thus rewrite 31 as:

$$W=\frac{1}{1-\sigma}V(0)^{1-\sigma}\frac{\left(\frac{\sigma-1}{h'(\tilde{L})\tilde{L}}\frac{\bar{\pi}}{\alpha\zeta}(1+t_w)\right)^{1-\sigma}h(\tilde{L})^{2-\sigma}}{\left(\frac{\bar{\pi}}{\alpha\zeta}+r\tau_k^l\right)\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}-1\right)}.$$

The monotonically increasing transformation of W we take is thus:

$$\frac{\log\left[(1-\sigma)W\right]}{1-\sigma}=\frac{1}{1-\sigma}\left[\begin{array}{c}\log(V(0)^{1-\sigma})+(1-\sigma)\left(\log\left(\frac{\sigma-1}{h'(\tilde{L})\tilde{L}}\right)+\log\left(\frac{\bar{\pi}}{\alpha\zeta}\right)+\log(1+t_w)\right)\\+(2-\sigma)\log h(\tilde{L})-\log\left(\frac{\bar{\pi}}{\alpha\zeta}+r\tau_k^l\right)-\log\left(\frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})\tilde{L}}-1\right)\end{array}\right].$$

Taking partial derivative of $\frac{\log[(1-\sigma)W]}{1-\sigma}$ with respect to $\tilde{L}$ and noting 153, 154 and 155 we have:

$$\begin{aligned}\frac{\partial\left(\log\left[(1-\sigma)W\right]\right)}{(1-\sigma)\,\partial\tilde{L}} &= \frac{1}{1-\sigma}\left[(1-\sigma)\left(-\frac{h''}{h'}-\frac{1}{\tilde{L}}\right)+\frac{1-\sigma}{\tilde{L}}-\frac{(1-\sigma)\tau_k^l\frac{\alpha\zeta\rho}{\bar{\pi}\tilde{L}}}{(1+t_w)\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}\right]\\
&\quad+\frac{1}{1-\sigma}\left[(2-\sigma)\frac{h'}{h}-\frac{\frac{\bar{\pi}}{\zeta\tilde{L}}\left(\frac{1}{\alpha}+\frac{\tau_k^l\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)}{\frac{\bar{\pi}}{\alpha\zeta}+r\tau_k^l}-\frac{\frac{\sigma-1}{\tilde{L}}\left(1-\frac{hh''}{h'^2}-\frac{h}{h'\tilde{L}}\right)}{\frac{(\sigma-1)h}{h'\tilde{L}}-1}\right]\\
&= \frac{1}{1-\sigma}\left[\frac{h'}{h}\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)+\frac{(\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1+\frac{(\sigma-1)h}{h'\tilde{L}}-1}{\tilde{L}\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)}\right]\\
&\quad-\frac{1}{1-\sigma}\left[\frac{(1-\sigma)\tau_k^l\frac{\alpha\zeta\rho}{\bar{\pi}\tilde{L}}}{(1+t_w)\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}+\frac{\frac{\bar{\pi}}{\zeta\tilde{L}}\left(\frac{1}{\alpha}+\frac{\tau_k^l\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)}{\frac{\bar{\pi}}{\alpha\zeta}+\frac{\rho+\sigma\frac{\bar{\pi}}{\zeta}\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\tau_k^l}\right]\\
&= \frac{1}{1-\sigma}\left[\frac{\frac{\sigma}{\tilde{L}}-\frac{h'}{h}}{\frac{(\sigma-1)h}{h'\tilde{L}}-1}\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)+\frac{1}{\tilde{L}}-\frac{\frac{\bar{\pi}}{\zeta\tilde{L}}\left(\frac{1}{\alpha}+\frac{\tau_k^l\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)}{\frac{\bar{\pi}}{\alpha\zeta}+\frac{\rho+\sigma\frac{\bar{\pi}}{\zeta}\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\tau_k^l}\right]\\
&\quad-\frac{1}{1-\sigma}\left[\frac{(1-\sigma)\tau_k^l\frac{\alpha\zeta\rho}{\bar{\pi}\tilde{L}}}{(1+t_w)\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}\right].\end{aligned}$$

Noticing 151, this expression equals

$$\frac{1}{1-\sigma}\left[\frac{\frac{\sigma}{\tilde{L}}-\frac{h'}{h}}{\frac{(\sigma-1)h}{h'\tilde{L}}-1}\left((\sigma-1)\left(\frac{hh''}{h'^2}-1\right)+1\right)+\frac{\frac{\rho\tau_k^l/\tilde{L}}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}}{\frac{\bar{\pi}}{\alpha\zeta}+\frac{\rho+\sigma\frac{\bar{\pi}}{\zeta}\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\tau_k^l}\right]$$

$$-\frac{1}{1-\sigma}\left[\frac{(1-\sigma)\rho\tau_k^l/\tilde{L}}{\frac{\bar{\pi}}{\alpha\zeta}\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)+\left(\rho+\sigma\frac{\bar{\pi}}{\zeta}\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\tau_k^l}\right],$$

which by reordering is exactly 162 in the text.

Taking partial derivative of $\frac{\log[(1-\sigma)W]}{1-\sigma}$ with respect to τ_k^l and noting 189 and 190 we get:

$$\begin{aligned}\frac{\partial\left(\log\left[(1-\sigma)W\right]\right)}{(1-\sigma)\,\partial\tau_k^l} &= \frac{1}{1-\sigma}\left[\frac{(1-\sigma)\,\frac{\alpha\zeta r}{\bar{\pi}}}{1+t_w}\cdot\frac{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}-\frac{r+\frac{r\tau_k^l}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}}{\frac{\bar{\pi}}{\alpha\zeta}+r\tau_k^l}\right]\\ &= \frac{r}{1-\sigma}\cdot\frac{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\cdot\frac{-\sigma}{\frac{\bar{\pi}}{\alpha\zeta}+r\tau_k^l},\end{aligned}$$

for which we have used 146. Substituting 149 for r in the denominator of the last multiplier we obtain 163 in the text after reordering.

So we derive the following:

$$\begin{aligned}\frac{d\left(\log\left[(1-\sigma)W\right]\right)}{(1-\sigma)\,d\tau_k^l} &= \left[\frac{\frac{\sigma}{\tilde{L}}-\frac{h'}{h}}{1-\sigma}\frac{\Psi}{\Phi}+\frac{\frac{\sigma}{\tilde{L}}\rho\tau_k^l}{(1-\sigma)\,\Theta}\right]\frac{r\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)}{\frac{\bar{\pi}}{\zeta\tilde{L}}\Gamma}\\ &\quad+\frac{\sigma}{\sigma-1}\frac{r\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}{\Theta}\end{aligned}$$

$$=\frac{\sigma r\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}{\sigma-1}\left[\frac{1}{\Theta}-\left[\left(1-\frac{h'\tilde{L}}{\sigma h}\right)\frac{\Psi}{\Phi}+\frac{\rho\tau_k^l}{\Theta}\right]\frac{\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}}{\frac{\bar{\pi}}{\zeta}\Gamma}\right].$$

We calculate the corresponding parts as follows:

$$\frac{1}{\Theta}\left[1-\frac{\rho\tau_k^l}{\frac{\bar{\pi}}{\zeta}\Gamma}\left(\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}\right)\right]$$

$$
\begin{aligned}
&= \frac{1}{\Theta\Gamma}\left[\frac{\Psi}{\alpha}\left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)(1+t_w)+(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\left(1-\tau_k^l-\frac{\tau_k^l\rho\zeta}{\bar{\pi}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}\right)\right] \\
&= \frac{1}{\Gamma}\left[\frac{\Psi}{\frac{\bar{\pi}}{\zeta}}+\frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{\Theta}\left(1-\tau_k^l-\frac{\tau_k^l\rho\zeta}{\bar{\pi}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}\right)\right],
\end{aligned}
$$

where the last equation is deduced with

$$
\begin{aligned}
\Theta &= \left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(\frac{\bar{\pi}}{\alpha\zeta}+r\tau_k^l\right) \\
&= \left(1-\tau_k^l+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\frac{\bar{\pi}}{\alpha\zeta}(1+t_w)
\end{aligned}
$$

by noting 146 and 149, and

$$
-\left(1-\frac{h'\tilde{L}}{\sigma h}\right)\frac{\Psi}{\Phi}\frac{\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}}{\frac{\bar{\pi}}{\zeta}\Gamma}=\frac{\frac{\sigma-1-\frac{h'\tilde{L}}{h}q^{\frac{\alpha}{1-\alpha}}}{\sigma\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}-\Phi}{\frac{\bar{\pi}}{\zeta}\Gamma}\frac{\Psi}{\Phi}
$$

$$
=\frac{\sigma-1-\frac{h'\tilde{L}}{h}q^{\frac{\alpha}{1-\alpha}}}{\sigma\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\frac{\bar{\pi}}{\zeta}\Gamma}\frac{\Psi}{\Phi}-\frac{\Psi}{\frac{\bar{\pi}}{\zeta}\Gamma}.
$$

Hence we have:

$$
\frac{1}{\Theta}-\left[\left(1-\frac{h'\tilde{L}}{\sigma h}\right)\frac{\Psi}{\Phi}+\frac{\rho\tau_k^l}{\Theta}\right]\frac{\frac{(\sigma-1)h}{h'\tilde{L}}-\frac{q^{\frac{\alpha}{1-\alpha}}}{1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}}{\frac{\bar{\pi}}{\zeta}\Gamma}
$$

$$
=\frac{1}{\Gamma}\left[\frac{(\sigma-1)\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{\Theta}\left(1-\tau_k^l-\frac{\tau_k^l\rho\zeta}{\bar{\pi}\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)}\right)+\frac{\sigma-1-\frac{h'\tilde{L}}{h}q^{\frac{\alpha}{1-\alpha}}}{\sigma\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\frac{\bar{\pi}}{\zeta}}\frac{\Psi}{\Phi}\right].
$$

Therefore we obtain finally:

$$
\frac{d\left(\log\left[(1-\sigma)W\right]\right)}{(1-\sigma)\,d\tau_k^l}=\frac{\sigma r}{\Gamma}\left[\frac{\left(q^{\frac{\alpha}{1-\alpha}}-1\right)}{\Theta}\left(\left(1+\sigma\left(q^{\frac{\alpha}{1-\alpha}}-1\right)\right)\left(1-\tau_k^l\right)-\frac{\tau_k^l\rho\zeta}{\bar{\pi}}\right)+\frac{1-\frac{h'\tilde{L}}{(\sigma-1)h}q^{\frac{\alpha}{1-\alpha}}}{\sigma\frac{\bar{\pi}}{\zeta}}\frac{\Psi}{\Phi}\right],
$$

which is exactly 164 in the text.

Conclusion

By incorporating endogenous saving and labor-leisure choices into the R&D growth model of Barro and Sala-i-Martin (2004), this thesis showed that lump-sum taxes and capital income taxes can affect growth and welfare in the long-run. More specifically, this thesis studied the lump-sum taxes, whose proceeds are thrown to ocean, will increases equilibrium labor supply and long-run growth rate, and may improve welfare as well. And the capital income taxes, with the revenue returned as labor income subsidies, though the growth will be reduced, can be Pareto optimal. These results stand in sharp contrast with the conventional conclusion that in general all types of taxation have negative effects on long-run economic growth, especially when the tax revenues are not used in a productive way. This different conclusion is mainly reached by my use of the R&D growth model that emphasizes innovation while abstracting from other growth determinants often used in the literature of optimal taxation, like human capital investment or subsidies to R&D investment. And it need be mentioned that though my models belong to the family of endogenous growth model with scale effect, by normalizing the size of the labor force to unity any level effect caused by growing size of it is removed, as Zeng and Zhang (2007) do. This thesis also reveals that when labor-leisure choice is endogenous, lump-sum taxes cannot be taken as non-distortionary, contrary to that usually assumed in the literature. Capital income taxes, often proposed to be zero or negative in the long-run for its negative effect on growth, in my model with endogenous labor-leisure choice can be welfare-improving with the proceeds used as labor income subsidies, so this result complements those in the literature in that capital income taxes can be positive even if it reduces long-run growth. Numerical calculation shows that these counterintuitive effects can arise when choosing values for model parameters consistent with the micro and macro empirical evidence.

The mechanism through which the taxes take effect is that the taxes cause either income effect or substitution effect on the labor-leisure choice so labor supply is increased. Increased labor supply induces a higher demand for the intermediate goods. This in turn induces a higher demand for investment in R&D. In the case of lump-sum taxation, the interest rate is raised and long-run growth rate is thus increased, whereas as for the capital income taxation, the after-tax interest rate is smaller than the interest rate in a no-tax economy so the long-run growth rate decreases. However, in the dynamic monopoly economy, the direction of growth effect is not necessarily the same as the that of welfare change. In both cases, the instantaneous utility changes in opposite direction from the growth rate does, therefore, the one with bigger momentum will determine the sign of welfare effect. With the lump-sum taxation program instantaneous utility will decrease whereas the grow rate will rise, however, the former will increase while the latter will decrease with the capital income taxation program. So the beneficial welfare effect of both taxes cannot be obtained at the same time for the same economy. Furthermore, this thesis inserts that the tax priority cannot be proposed without consideration of the economic parametrization: for example, in an economy with bigger time discount rate, lower intertemporal elasticity of substitution in consumption, smaller initial equilibrium labor supply, lower markup for intermediate goods, larger (Frisch) elasticity in labor supply, or smaller size in innovation incremental, it will be more plausible to exert capital income tax than labor income tax, whereas the labor income taxes will be prior to capital income taxes in an economy with contrary parametric characteristics.

In a real economy, which is a composite of both innovation in variety expansion and quality improvement, characterized by certain degree of creative destruction, the potential of the taxes to be welfare-enhancing should be located in between those shown in the parellel models of variety expansion and creative destruction. Policy implication of this thesis is that we should take into account the competitiveness of the market, the consumers' tastes and time preferences, the pace of innovation, and the flexibility of labor market when designing optimal taxation policy.

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