Designing Digital Rollovers: Managing Obsolescence through Release Times

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When releasing a new version of a durable product, a firm aims to attract new customers as well as persuade its existing customer base to upgrade. This is commonly achieved through a *rollover strategy*, which comprises the price of the new product as well as the decision to discontinue the sale of the existing product (*solo rollover*) or to sell the existing product at a discounted price (*dual rollover*). In this paper, we argue that the timing of the new product release is an important—but commonly overlooked—third lever in the design of a successful rollover strategy. The release timing influences the consumers' perception of obsolescence, by which an existing product is considered obsolete merely by reference to a new product. This reinforces the upgrading behavior of existing customers, but it also necessitates deep discounts of the existing product to keep its sale viable in a dual rollover. We analyze the impact of the release timing on solo and dual rollovers in markets composed of myopic and strategic consumers. We show that in both markets, the endogenization of the release time enables the firm to induce sufficiently large parts of its existing customer base to upgrade so that a solo rollover is optimal in commonly encountered market settings. We also characterize the resulting market segmentation, and we offer managerial as well as policy advice.

Key words: Product Rollover, Version Management, Perceived Obsolescence, Release Time

1. Introduction

We study the optimal release strategy of a firm that sells different versions of a durable digital good. Ideally, the new releases "drive people to upgrade" (Apple's CEO Tim Cook, as cited by Gibbs 2016) and at the same time attract new customers from the remainder of the market. To this end, firms often focus on high-value consumers in early product versions and subsequently persuade them to purchase the new releases at comparatively lower prices (Dhebar 1994, Kornish 2001). The recent pricing strategy of Apple's iPhone product line, for example, charged lower premiums for the new products relative to their bills of material costs (Kelly 2018). This strategy requires the new product versions to be perceived as sufficiently novel, for otherwise the existing consumer base may not upgrade to the new releases.[1]

[1] According to Hartmans (2018), when the iPhone X was launched, only 16% of the existing iPhone owners planned to upgrade, with the most commonly cited reason for not doing so being that "my iPhone works just fine".

Apart from the pricing of the new versions, an essential aspect of the release strategy concerns the decision whether to discontinue the existing product versions (*solo rollover*) or to keep them in the market at discounted prices (*dual rollover*), see Billington et al. (1998) and Erhun et al. (2007). Dual rollovers, which are often employed in the sale of mobile phones (*e.g.*, the iPhone and Samsung's Galaxy lines), television sets and other high tech consumer goods, allow to complement the sales of the new product versions with an additional revenue stream from low-value consumers that are attracted by the discounted prices of the older versions. Dual rollovers are considered to be particularly effective when the consumers do not perceive the new product versions as sufficiently novel (Billington et al. 1998). On the other hand, the continued discounted sale of the existing versions may cannibalize the sale of the new versions. For this reason, a firm may prefer a solo rollover, as frequently observed in the markets for specialized tablets (*e.g.*, Wacom's Intuos tablets) and scientific software (*e.g.*, Wolfram Research's Mathematica). Other factors that influence the choice of a solo versus a dual rollover include the presence of existing product inventories as well as product line complexity considerations.

We argue in this paper that, in addition to the product pricing and the rollover type, the timing of new product releases provides the firm with an important third lever to design its release strategy. A careful timing of the new releases allows the company to control the perceived obsolescence of its product range and thus "drive people to upgrade" without incurring the risks associated with a dual rollover. For example, the market typically perceives existing versions of EA Inc's FIFA sports games as obsolete once a new version has been released, even though the previous versions are not outdated in objective terms (Cooper 2004, Nahm 2004, Koenigsberg et al. 2011). Likewise, in the well-documented "slow iPhone phenomenon", users experienced a perceived slowdown of old iPhone versions precisely when a new version was released (Richter 2016).

The phenomenon of perceived obsolescence is well recognized in the consumer research literature. Cooper (2004) and Slade (2009) suggest that perceived obsolescence (*i.e.*, obsolescence in reference to a new product version) is the primary factor determining the novelty of a product, whereas absolute obsolescence (*i.e.*, physical deterioration) is only the secondary aspect. An old version of a product can be perceived as obsolete because *(i)* it no longer employs the most recent technology (Nair and Hopp 1992, Nair et al. 2004, Slade 2009); *(ii)* it no longer complies with the most recent style trends (Waldman 1996, Slade 2009, Hellmann and Luedicke 2018); or *(iii)* it is less suited for new standards, *e.g.*, due to higher capacity, memory or bandwidth requirements (Cooper 2004). The consumer research literature also recognizes the impact of release cycles on the perception of obsolescence. Clearly, each of the aforementioned three contributing factors to perceived obsolescence tends to be positively correlated with the length of the release cycles. Additionally, Bulow (1982), Dhebar (1994) and Schifferstein and Zwartkruis-Pelgrim (2008) argue that the perceived obsolescence increases with longer release cycles because the consumers' attachment to the earlier versions along with the perception of irreplaceability or indispensability decreases over time. Finally, consumers may "psychologically justify" their perception of obsolescence by acting more recklessly with the old product version to justify an upgrade without appearing wasteful (Bellezza et al. 2017). This link between the release timing and the perceived novelty gap of different versions may provide the firm with a lever to control the consumers' perception of obsolescence.

In this paper, we analyze the impact of the consumers' perceived obsolescence on the optimal rollover strategy of a firm, as well as the resulting market segmentation and welfare distribution. To study the effect of perceived obsolescence in isolation, we assume that the firm is a monopoly that sells two versions of a single, purely digital good (*i.e.*, with negligible reproduction and inventory costs), see, *e.g.*, Shapiro et al. (1998) and Varian (2001). We assume that the existing version of the good is to be replaced or complemented with a new version that can be released along a continuous timeline. We assume that the market is composed of two types of consumers: myopic consumers only consider the product offering at the present time, whereas strategic consumers anticipate the future actions of the firm when taking their decision.

A critical finding from our model is that once we incorporate the perception of obsolescence in the rollover design, the firm should discontinue the sale of the old product version when the new version is released. In fact, even when the market is reluctant to regard the new version as *objectively* novel, the firm can delay its release to create a sufficiently large *perceived* novelty gap. In that case, inducing the existing customer base to upgrade provides higher revenues than attracting new low-value customers with discounted prices for the old version. Our findings are in line with the solo rollover practices commonly observed for purely digital goods as well as some semi-digital products whose reproduction costs only increase gradually over time (Torres 2017).

Our analysis shows that the optimal release interval depends on both the consumers' perception of obsolescence as well as their ability to foresee the firm's future actions. If the market consists solely of myopic consumers, the optimal release interval decreases with the market's perception of the obsolescence. If the perceived obsolescence is high, this strategy incentivizes the consumers to purchase both versions, whereas it allows the company to price discriminate between consumers in the case of a low perceived obsolescence by offering a deep discount on the second version. If the market solely consists of strategic consumers, the firm establishes longer release intervals and decreases the price for the first product. That way, the firm creates a customer base that is willing to buy both product versions since the high-value consumers find that the early version has a reasonable price and sufficiently long life cycle prior to its obsolescence, while the perceived novelty gap between the versions is large enough to justify an upgrade. If the market is composed of both myopic and strategic consumers, finally, the firm's release strategy is driven by the dominant consumer group. The threshold that determines the dominant consumer group again depends on the market's perception of obsolescence: The higher the perceived obsolescence, the less weight the firm places on strategic consumers.

From a welfare perspective, we show that both the firm and the strategic consumers benefit from (some level of) obsolescence. In contrast, the surplus of myopic consumers decreases with the obsolescence as they regret their earlier purchase when the new version is released. Thus, a policy maker interested in increasing consumer surplus should ensure that the consumers are educated about the firm's future actions, for example by inducing the firm to communicate its release strategy to the consumers in a credible way.

The contributions of this paper may be summarized as follows.

1. We propose a product rollover model for purely digital goods that incorporates product pricing, the choice of the rollover type (solo/dual) as well as the release timing to capture the effects of perceived obsolescence.
2. We derive the optimal release strategy of the firm in markets composed of myopic and strategic consumers, and we characterize the resulting market segmentation and welfare implications.
3. We extend our analysis to goods where the reproduction costs are non-negligible.

The optimal release of different versions of a product has been studied in both the version management and the product rollover literature. While the version management literature tends to focus on the optimal pricing and release timing as well as the resulting market segmentation, the product rollover literature emphasizes supply-related issues such as reproduction costs and capacity constraints as well as the optimal choice of the rollover type (solo versus dual).

The version management literature typically assumes that the quality difference between the product versions is exogenous (*e.g.*, caused by a technological innovation) and that the market solely consists of strategic consumers. Moorthy and Png (1992) study the effects of price pre-announcements on the optimal release schedule in a two-stage model where consumers only buy one version of the product. They find that if the firm pre-announces the second-period price, a sequential release is preferred if and only if the consumers are less patient than the firm. If the firm does not pre-announce the second-period price, on the other hand, then a simultaneous release is always preferred. Employing a similar two-stage setting and assuming that the firm conducts a sequential release and does not pre-announce the second-period price, Kornish (2001) characterizes the optimal pricing when the consumers may buy both versions of the product. We refer to Chen and Chen (2015) for a review of the literature on contingent and pre-announced pricing of product versions. Calzada and Valletti (2012) study a model where the release time of the second product version can be chosen from a continuous timeline. They show that as the consumers become less patient, a sequential release outperforms a simultaneous release and the optimal release interval increases. In a similar setting, August et al. (2015) find that the optimal release interval depends on the absolute quality of the first version as well as the quality difference between the versions. In contrast to the previous contributions, Lobel et al. (2015) assume that the quality differences between the products depend on the release intervals. The authors show that when the firm faces strategic consumers and settles for solo rollovers, it selects larger release intervals and hence higher novelty gaps between the product versions when the consumers are less patient, and they confirm that price pre-announcements can significantly increase profits.

Our work complements the version management literature by studying the impact of perceived obsolescence on the firm's optimal release strategy for digital products. We show that the release timing remains an important lever for the firm even when the consumers are myopic, and we confirm that a solo rollover, which is typically imposed by assumption, is indeed optimal even in an unconstrained setting where the firm can choose between a solo and a dual rollover.

The product rollover literature typically assumes that the release intervals are fixed and that the market consists solely of myopic consumers. Instead of studying the choice of individual consumers, the literature focuses on aggregate demand functions that depend on exogenous price sensitivity parameters or consumer arrival rates. Ferguson and Koenigsberg (2007) and Koca et al. (2010) show that under a high perceived obsolescence or a high innovation rate, a solo rollover is preferred in the presence of capacity constraints. In contrast, Liang et al. (2014) show that a dual rollover is optimal if the innovation rate is high, provided that either the production costs are high or the market is dominated by myopic consumers. Liang et al. (2018) endogenize the quality of the second product version, and they show that the firm prefers to invest into a higher product quality when consumers are strategic and the salvage value of the first product version is low. Lim and Tang (2006) and Arslan et al. (2009) endogenize the release timing by assuming that the length of the release interval impacts the demand for the product versions. In contrast to the version management literature, both studies find that a dual rollover is preferable when the marginal costs of the two product versions are similar and demand is modeled at an aggregate level. For reviews of the product rollover literature, we refer the reader to Liu et al. (2018) and Wei and Zhang (2018).

Our contribution to the product rollover literature lies in the endogenization of the release times, which in turn impact the perceived obsolescence of the first product version. Moreover, we study the choice of individual consumers, which allows us to analyze markets composed of both myopic and strategic consumers as well as the resulting market segmentation and welfare distribution. To study the ramifications of these changes in isolation, we disregard supply-related concerns by focusing on digital goods where the reproduction costs and capacity constraints are negligible.

The remainder of this paper is organized as follows. Section 2 introduces our rollover model. Sections 3 and 4 derive the optimal release strategy as well as the resulting market segmentation for markets of purely myopic and purely strategic consumers, respectively, whereas Section 5 generalizes our findings to hybrid markets composed of both myopic and strategic consumers. We discuss extensions and welfare implications in Section 6, and we conclude in Section 7. All proofs as well as supporting material for our extensions and welfare analysis are relegated to the appendix.

Notation. We denote by $[x]_+ = \max\{x, 0\}$ the non-negative part of a scalar $x \in \mathbb{R}$. For $x \in \mathbb{R}$, we denote by x^+ and x^- quantities that are arbitrarily close to x but strictly larger or smaller than x, respectively.

2. Model

We consider a monopolist that launches a digital product with price p_1 at time $t = 0$ and plans to introduce a new version of the product with price p_2 at time $t_2 \in (0, \infty)$. The assumption of a digital good implies that the marginal production costs as well as potential capacity constraints are negligible (Mussa and Rosen 1978, Shapiro et al. 1998, Bhargava and Choudhary 2001, Johnson and Myatt 2003, Haruvy et al. 2013). We assume that the decision to develop the second version has already been taken, which implies that fixed costs (*e.g.*, product development costs) are sunk and do not affect the rollover strategy. The firm can conduct a solo rollover and discontinue the sale of the first product at time t_2, or it can execute a dual rollover and continue to sell the first product at a discounted price $\beta \cdot p_1$, $\beta \in (0, 1)$, from time t_2 onwards. The firm aims to maximize its expected future profit, which is discounted at a rate $\delta_s \in (0, 1)$.

At its respective launch, each version of the product provides a utility rate of $v \in \mathbb{R}_+$ per unit of time. Once the second version is introduced, however, the utility rate of the first product drops to $\alpha^{t_2} \cdot v$, where the decay factor $\alpha \in (0, 1)$ is dictated by the market and the introduction time t_2 is controlled by the firm. Markets that perceive products as less durable (such as soccer video games that are no longer aligned with the current football league compositions) correspond to lower values of α, whereas markets with a smaller perception of obsolescence (such as car racing games) correspond to higher values of α. Note that the drop in utility rate increases with the length of the release interval, which is in line with the findings of the consumer research literature (Bulow 1982, Dhebar 1994, Cooper 2004). This differentiates our work from earlier papers that have considered a fixed, exogenous obsolescence rate (Desai and Purohit 1998, Agrawal et al. 2015, Ferguson and Koenigsberg 2007). To concentrate on this aspect of perceived obsolescence, we assume that the new product does not add any utility compared to the old version when it is first introduced. It is straightforward to extend our model to account for different utility rates for both products.

Following Mussa and Rosen (1978), we assume that the consumers agree in their preference ordering, but they are heterogeneous in their appreciation of quality. This heterogeneity is reflected by the consumer type θ, which is assumed to be uniformly distributed over the interval $[0, 1]$. We assume that consumers use at most one product at a time, and they derive no salvage value from the old version when they upgrade to the new version. Thus, the versions are sufficiently close substitutes, and reselling an old version is not feasible. The absence of a second-hand market might be due to contractual restrictions (*e.g.*, personal software licenses), the impracticality of ownership transfer (*e.g.*, mobile phone apps), data and privacy concerns or the desire to keep the old version as a backup (*e.g.*, semi-digital goods such as smartphones, GSMA 2012).

We distinguish between myopic and strategic consumer foresight. A *myopic consumer* does not anticipate future releases or future price discounts, either since she is unaware of or not interested in obtaining this information. A myopic consumer therefore only considers the options immediately available to her. In contrast, a *strategic consumer* can predict the company's future actions, for instance through online channels (*e.g.*, rumor websites such as `www.macrumors.com` or consumer forums such as `www.techradar.com`) or company pre-announcements. A strategic consumer may therefore decide at time $t=0$ to wait until time $t=t_2$, either to buy the new version or to purchase the first version at a discount. We assume that independently of their foresight, consumers aim to maximize their lifetime utility, and that they discount this utility at a rate $\delta_c \in (0,1)$. Under our assumptions, both myopic and strategic consumers only make purchases at times $t \in \{0, t_2\}$.

3. Myopic Consumers

In this section, we discuss the optimal pricing and timing decisions of the firm, as well as the resulting market segmentation, when consumers are myopic and the firm employs a solo (Section 3.1) or a dual (Section 3.2) rollover. We subsequently compare both strategies in Section 3.2.

3.1. Solo Rollover

When the firm conducts a solo rollover, a myopic consumer of type $\theta \in [0,1]$ takes decisions at two time points: At time $t=0$ she buys the first product if and only if (iff)

$$\theta \int_0^\infty \delta_c^t v \,\mathrm{d}t \geq p_1 \quad \Longleftrightarrow \quad \theta \geq \frac{p_1}{u} =: \theta_E,$$

where $u := \int_0^\infty \delta_c^t v \,\mathrm{d}t$ denotes the lifetime utility of either product if no successor model was introduced. Assuming that the consumer has not bought the first product, she then buys the second product at time $t=t_2$ iff

$$\theta \int_0^\infty \delta_c^t v \,\mathrm{d}t \geq p_2 \quad \Longleftrightarrow \quad \theta \geq \frac{p_2}{u} =: \theta_L.$$

Alternatively, if she has already bought the first product at time $t=0$, she buys the second product at time $t=t_2$ iff the net utility of upgrading weakly exceeds the remaining utility of continuing to use the first product, that is, precisely when

$$\theta \int_0^\infty \delta_c^t v \,\mathrm{d}t - p_2 \geq \theta \int_0^\infty \delta_c^t \alpha^{t_2} v \,\mathrm{d}t \quad \Longleftrightarrow \quad \theta \geq \min\left\{ \frac{p_2}{(1-\alpha^{t_2})u}, 1 \right\} =: \theta_B.$$

Note that the subscripts E, L and B in the definitions of the threshold values are mnemonics for early product buyers, late product buyers, and both product buyers, respectively.

We assume that $p_1, p_2 \leq u$, that is, the firm chooses prices such that there are consumer types $\theta \in [0,1]$ that are willing to purchase either product. This is without loss of generality since the product development costs are sunk, and hence no profit maximizing firm would choose prices $p_1, p_2 > u$. As a consequence, all three threshold values θ_E, θ_L and θ_B lie between 0 and 1. Moreover, we have $\theta_B > \theta_L$ by construction, that is, both product buyers have a higher type than late product buyers. The relative ordering between θ_E and θ_L, on the other hand, depends on the pricing strategy of the firm. Note also that the threshold type θ_B characterizes the upgrading behavior of consumers that have already purchased the first product, and hence this threshold does not necessarily exceed θ_E.

We now characterize the market segmentation for a given solo rollover strategy.

THEOREM 1 **(Market Segmentation)**.

1. *If $p_2 \geq (1-\alpha^{t_2})u$, none of the consumers buy both versions and*
   (a) *if $p_2 \geq p_1$, then the market consists solely of early buyers (region E);*
   (b) *if $p_2 < p_1$, then the market consists of early and late buyers (region EL).*
2. *If $p_2 < (1-\alpha^{t_2})u$, there are consumers that buy both versions and*
   (a) *if $p_2 \geq p_1$, then the market also comprises early buyers (region EB);*
   (b) *if $p_2 \in ([1-\alpha^{t_2}]p_1, p_1)$, then the market also comprises early and late buyers (region ELB);*
   (c) *if $p_2 \leq (1-\alpha^{t_2})p_1$, then the market also comprises late buyers (region LB).*

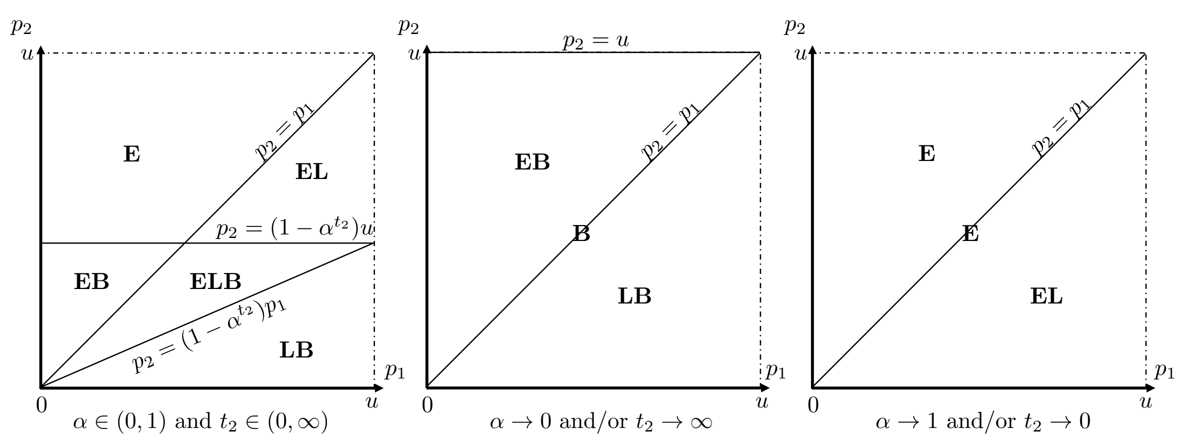

Figure 1 **Market segmentation for a solo rollover**

Figure 1 (left) visualizes the market segmentation as a function of the prices p_1 and p_2. In particular, we observe that *(i)* the late buyers segment disappears when $p_2 \geq p_1$, that is, if the second product is more expensive than the first one; *(ii)* the early buyers segment disappears if the price ratio p_2/p_1 drops below $1-\alpha^{t_2}$, that is, if the second product is inexpensive relative to the first product as well as its perceived novelty; and *(iii)* the both product buyers segment disappears if $p_2 \geq (1-\alpha^{t_2})u$, that is, if the second product is expensive relative to its perceived novelty.

It is instructive to consider the special cases of full (resp., absent) obsolescence and an indefinitely postponed (resp., immediate) release of the second product version.

Corollary 1 **(Market Segmentation: Special Cases)**.

1. *If $\alpha \to 0$ and/or $t_2 \to \infty$, there are consumers that buy both versions and*
   (a) *if $p_2 > p_1$, then the market also comprises early buyers (region EB);*
   (b) *if $p_2 = p_1$, then there are no other customers (region B);*
   (c) *if $p_2 < p_1$, then the market also comprises late buyers (region LB).*
2. *If $\alpha \to 1$ and/or $t_2 \to 0$, none of the consumers buy both versions and*
   (a) *if $p_2 \geq p_1$, then the market consists solely of early buyers (region E);*
   (b) *if $p_2 < p_1$, then the market consists of early and late buyers (region EL).*

Figure 1 (middle) visualizes the first case of Corollary 1: As $\alpha \to 0$ and/or $t_2 \to \infty$, we have $(1-\alpha^{t_2})p_1 \to p_1$ and thus the EL/ELB 'wedge' in Figure 1 (left) disappears. In this case, there are always both product buyers since there is a high perception of obsolescence of the first product once the second product is released. Likewise, Figure 1 (right) shows that as $\alpha \to 1$ and/or $t_2 \to 0$, we have $(1-\alpha^{t_2})u \to 0$ and thus the EB/ELB/LB 'bar' in Figure 1 (left) disappears. Here, there are no both product buyers since the consumers fail to see an added value in the second version.

We now consider the optimal solo rollover strategy of the firm. The firm's profits amount to

$$p_1[\theta_B - \theta_E]_+ + \delta_s^{t_2} p_2[\theta_E - \theta_L]_+ + (p_1 + \delta_s^{t_2} p_2)(1 - \max\{\theta_E, \theta_B\}), \tag{1}$$

where the first summand corresponds to the early buyers that buy the first product at time $t=0$ (since $\theta \geq \theta_E$) but do not upgrade at time $t=t_2$ (since $\theta < \theta_B$), the second summand refers to the late buyers that do not buy the first product at time $t=0$ (since $\theta < \theta_E$) but buy the second product at time $t=t_2$ (since $\theta \geq \theta_L$), and the third summand corresponds to the both product buyers that buy the first product at time $t=0$ (since $\theta \geq \theta_E$) and subsequently upgrade to the second product at time $t=t_2$ (since $\theta \geq \theta_B$).

Proposition 1. *For a fixed release time $t_2 \in (0,\infty)$, the optimal pricing strategy $(p_1^\star, p_2^\star)$ in a solo rollover satisfies $p_2^\star \in [(1-\alpha^{t_2})p_1^\star, p_1^\star]$.*

Proposition 1 shows that, independent of the release time t_2, it is always optimal for the firm to attract both early and late product buyers by choosing a sufficiently low price for the second version: Indeed, the optimal pricing will always fall into the EL/ELB 'wedge' in Figure 1 (left).

We now study the optimal solo rollover design.

Theorem 2 **(Optimal Solo Rollover Design)**. *There are threshold decay factors* $0 < \alpha_{LB} < \alpha_{EL} \leq 1$ *such that*

1. *The* optimal introduction time $t_2^\star$ *approaches* 0 *as* $\alpha \to 0$*, it is continuously increasing in* α *over* $\alpha \in (0, \alpha_{EL})$*, and it satisfies* $t_2^\star = 0^+$ *for* $\alpha \in [\alpha_{EL}, 1)$*.*
2. *The* optimal price $p_1^\star$ for the first product *approaches* $u/2$ *as* $\alpha \to 0$*, it is continuous and unimodal in* α *over* $\alpha \in (0, \alpha_{EL})$*, and it satisfies* $p_1^\star = 2u/3$ *for* $\alpha \in [\alpha_{EL}, 1)$*.*
3. *The* optimal price $p_2^\star$ for the second product *approaches* $u/2$ *as* $\alpha \to 0$*, it is continuously decreasing in* α *over* $\alpha \in (0, \alpha_{EL})$*, and it satisfies* $p_2^\star = u/3$ *for* $\alpha \in [\alpha_{EL}, 1)$*.*
4. *The* optimal expected profit *approaches* $u/2$ *as* $\alpha \to 0$*, it is continuously decreasing in* α *over* $\alpha \in (0, \alpha_{EL})$*, and it is equal to* $(u/3)^-$ *for* $\alpha \in [\alpha_{EL}, 1)$*.*

The optimal solo rollover design gives rise the following market segmentation.

Corollary 2 **(Optimal Market Segmentation)**. *For the threshold decay factors* $0 < \alpha_{LB} < \alpha_{EL} \leq 1$ *from Theorem 2, we have*

1. *For* $\alpha \in (0, \alpha_{LB}]$*, the firm serves late and both product buyers (region LB).*
2. *For* $\alpha \in (\alpha_{LB}, \alpha_{EL})$*, the firm serves early, late and both product buyers (region ELB).*
3. *If* $\alpha \in [\alpha_{EL}, 1]$*, the firm serves early and late product buyers (region EL).*

Figure 2 visualizes the results of Theorem 2 and Corollary 2 for a particular problem instance. If the consumers perceive the old version as completely obsolete when the new version is released, then the firm sells the same product to the same consumers at the same (monopoly) price in immediate succession twice. As the perceived obsolescence decreases, the firm counters the decreasing profits by delaying the release of the second product as well as lowering the second product's price, thus convincing all previous consumers to upgrade while also attracting late buyers. For $\alpha \in (\alpha_{LB}, \alpha_{EL})$ it is no longer optimal to induce every first-period buyer to upgrade to the second period. If the perceived obsolescence is very low, finally, the firm is no longer able to use release timing as a profit lever, and instead it introduces the second product instantaneously. Now, however, the consumers no longer upgrade, and the market is split between early and late buyers.

Corollary 3. *The threshold decay factors* α_{LB}, α_{EL} *increase with* δ_s *and are constant in* δ_c*.*

It is noteworthy that the discount factor of the consumers does not impact the threshold decay factors α_{LB} and α_{EL} that determine the optimal market segmentation of the company. This is due to the myopic outlook of the consumers, which implies that the consumers do not foresee the introduction of the second product when taking decisions at time $t = 0$.

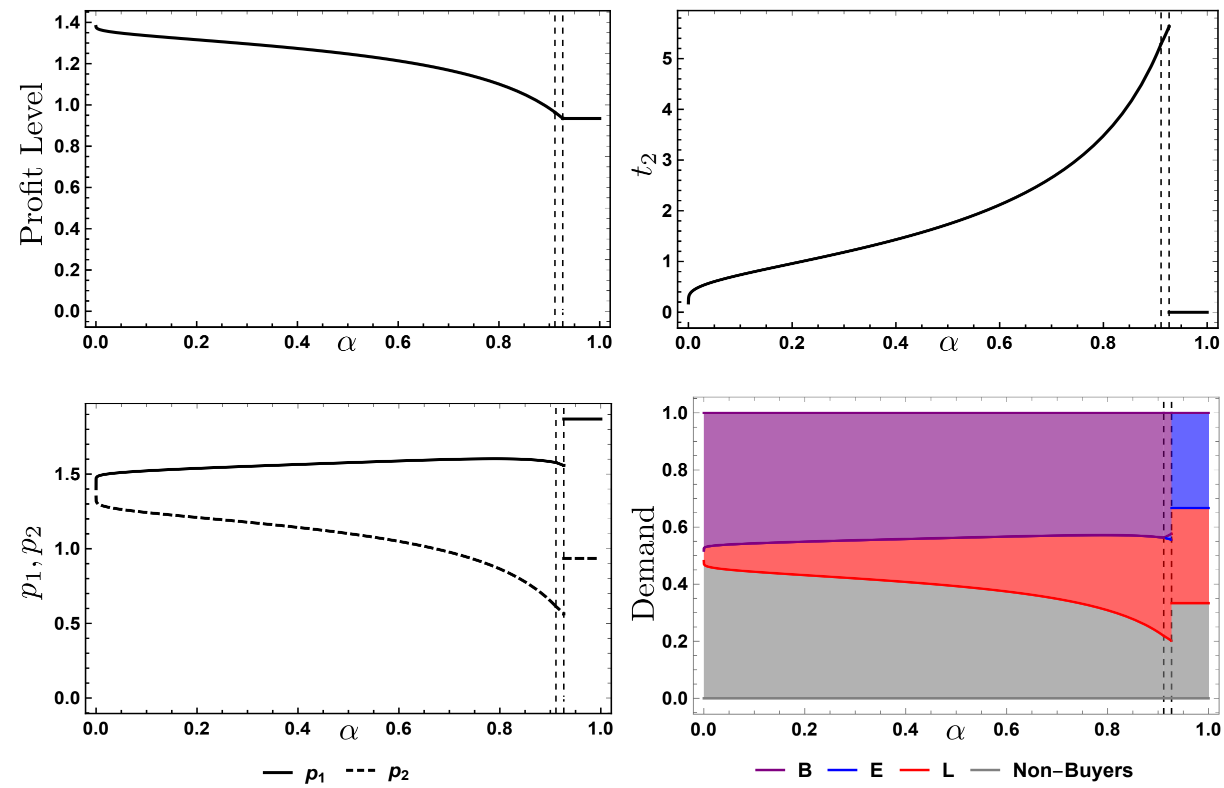

Figure 2 Optimal solo rollover strategy as a function of α.
In this example, we set $v = 1$, $\delta_c = 0.7$ (implying that $u = 2.8$) and $\delta_s = 0.9$.
In each graph, the vertical left (right) dashed line denotes α_{LB} (α_{EL}).

3.2. Dual Rollover

When the firm conducts a dual rollover, a myopic consumer of type $\theta \in [0,1]$ again takes decisions at two time points. As in the case of a solo rollover, she buys the first product at time $t = 0$ iff $\theta \geq \theta_E$. At time $t = t_2$, however, she faces several options:

1. If she has not bought the first product,
   (a) she prefers buying the first product at the price βp_1 over buying nothing iff
   $$\theta u \alpha^{t_2} \geq \beta p_1 \quad \Longleftrightarrow \quad \theta \geq \min\left\{\frac{\beta p_1}{u\alpha^{t_2}}, 1\right\} =: \theta_D;$$
   (b) she prefers buying the second product over buying nothing iff $\theta \geq \theta_L$;
   (c) she prefers buying the second product over buying the first product at the discounted price βp_1 iff
   $$\theta u - p_2 \geq \theta u \alpha^{t_2} - \beta p_1 \quad \Longleftrightarrow \quad \theta \geq \min\left\{\frac{p_2 - \beta p_1}{u(1-\alpha^{t_2})}, 1\right\} =: \theta_{DL}.$$
2. If she has already bought the first product, she buys the second product iff $\theta \geq \theta_B$.

As before, we assume that p_1, $p_2 \leq u$, which implies that all threshold values are between 0 and 1. By construction, we have $\theta_B > \theta_L$ as well as $\theta_B > \theta_{DL}$, whereas the ordering of the remaining threshold values depends on p_1, p_2, β, t_2 as well as α.

We first show that the firm has to offer a deep enough discount to attract discount buyers.

PROPOSITION 2. *There are discount buyers iff $\beta \in [0, \bar{\beta})$, where $\bar{\beta} = \alpha^{t_2} \cdot \min\left\{\frac{p_2}{p_1}, 1\right\}$.*

Proposition 2 implies that any dual rollover with a discount $\beta \geq \bar{\beta}$ is equivalent to a solo rollover since no consumer buys the discounted product at time $t = t_2$. Note that the required discount $\bar{\beta}$ for the first product is deeper when the second product is relatively cheaper and/or the perceived obsolescence of the first product is high. For the rest of this subsection, we assume that $\beta < \bar{\beta}$.

We now characterize the market segmentation for a given dual rollover strategy.

THEOREM 3 **(Market Segmentation)**.

1. *If $p_2 \geq (1-\alpha^{t_2})u$, none of the consumers buy both versions and*
   (a) *if $p_2 \geq (1-\alpha^{t_2}+\beta)p_1$, then the market consists of early and discount buyers (region ED);*
   (b) *if $p_2 < (1-\alpha^{t_2}+\beta)p_1$, then the market consists of early, late and discount buyers (region ELD).*
2. *If $p_2 < (1-\alpha^{t_2})u$, there are consumers that buy both versions and*
   (a) *if $p_2 \geq (1-\alpha^{t_2}+\beta)p_1$, then the market also comprises early and discount buyers (region EBD);*
   (b) *if $p_2 \in ([1-\alpha^{t_2}]p_1, [1-\alpha^{t_2}+\beta]p_1)$, then the market also comprises early, late and discount buyers (region ELBD);*
   (c) *if $p_2 \leq (1-\alpha^{t_2})p_1$, then the market also comprises late and discount buyers (region LBD).*

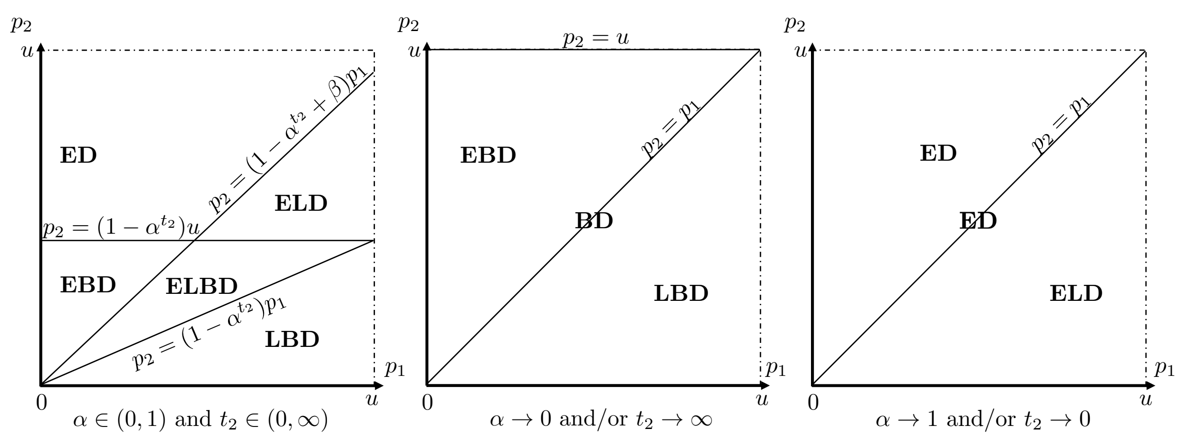

Figure 3 Market segmentation for a dual rollover

Figure 3 (left) visualizes the market segmentation as a function of the prices p_1 and p_2. We observe that *(i)* the late buyers segment disappears when $(p_2 - \beta p_1)/p_1 \geq 1 - \alpha^{t_2}$, that is, whenever the relative price premium of the second product over the discounted first product exceeds the perceived novelty of the second product; *(ii)* the early buyers segment disappears if the price ratio p_2/p_1 drops below $1-\alpha^{t_2}$, that is, if the second product is inexpensive relative to the first product as well as its perceived novelty; and *(iii)* the both product buyers segment disappears if $p_2 \geq (1-\alpha^{t_2})u$, that is, if the second product is expensive relative to its perceived novelty.

We again consider the special cases of full (resp., absent) obsolescence and an indefinitely postponed (resp., immediate) release of the second product version.

Corollary 4 **(Market Segmentation: Special Cases)**.

1. *If $\alpha \to 0$ and/or $t_2 \to \infty$, there are consumers that buy both versions and*
   (a) *if $p_2 > p_1$, then the market also comprises early and discount buyers (region EBD);*
   (b) *if $p_2 = p_1$, then the market also comprises discount buyers (region BD);*
   (c) *if $p_2 < p_1$, then the market also comprises late and discount buyers (region LBD).*
2. *If $\alpha \to 1$ and/or $t_2 \to 0$, none of the consumers buy both versions and*
   (a) *if $p_2 \geq p_1$, then the market consists of early and discount buyers (region ED);*
   (b) *if $p_2 < p_1$, then the market consists of early, late and discount buyers (region ELD).*

Figure 3 (middle) visualizes the first part of Corollary 4: When $\alpha \to 0$ and/or $t_2 \to \infty$, we have $(1-\alpha^{t_2})p_1 \to p_1$ as well as $(1-\alpha^{t_2}+\beta)p_1 \to p_1$ since $\bar{\beta} \to 0$ and $\beta < \bar{\beta}$ by assumption. Thus, the ELD/ELBD 'wedge' in Figure 3 (left) disappears, and there are always both product buyers since there is a high perception of obsolescence of the first product once the second product is released. Figure 3 (right) visualizes the second part of Corollary 4: When $\alpha \to 1$ and/or $t_2 \to 0$, we have $(1-\alpha^{t_2})u \to 0$ and thus the EBD/ELBD/LBD 'bar' in Figure 3 (left) disappears. Here, there are no both product buyers since the consumers fail to see an added value in the second version.

We now consider the optimal dual rollover strategy of the firm. The firm's profits amount to

$$\begin{aligned} p_1[\theta_B-\theta_E]_+ \; + \; \delta_s^{t_2} p_2[\theta_E - \max\{\theta_L, \theta_{DL}\}]_+ + (p_1 + \delta_s^{t_2} p_2)(1-\max\{\theta_E,\theta_B\}) \\ + \, \delta_s^{t_2} \beta p_1 [\min\{\theta_E, \theta_{DL}\} - \theta_D]_+, \end{aligned} \tag{2}$$

where the first summand corresponds to the early buyers that buy the first product at time $t=0$ (since $\theta \geq \theta_E$) but do not upgrade at time $t=t_2$ (since $\theta < \theta_B$), the second summand refers to the late buyers that do not buy the first product at time $t=0$ (since $\theta < \theta_E$) but prefer to buy the second product at time $t=t_2$ over both buying nothing (since $\theta \geq \theta_L$) and over buying the first product at a discounted price (since $\theta \geq \theta_{DL}$), the third summand corresponds to the both product buyers that buy the first product at time $t=0$ (since $\theta \geq \theta_E$) and subsequently upgrade to the second product at time $t=t_2$ (since $\theta \geq \theta_B$), and the fourth summand corresponds to the discount buyers that do not buy the first product at time $t=0$ (since $\theta < \theta_E$) but prefer to buy the first product at the discounted price βp_1 at time $t=t_2$ over buying nothing (since $\theta \geq \theta_D$) and over buying the second product (since $\theta < \theta_{DL}$).

Proposition 3. *For a fixed release time $t_2 \in (0,\infty)$ and a sufficiently deep discount $\beta < \bar{\beta}$, the optimal pricing strategy $(p_1^\star, p_2^\star)$ in a dual rollover satisfies $p_2^\star \in [(1-\alpha^{t_2})p_1^\star,\, (1-\alpha^{t_2}+\beta)p_1^\star]$.*

Similar to Proposition 1, Proposition 3 shows that for a sufficiently deep discount β, it is always optimal for the firm to attract new customers at times $t = 0$ and $t = t_2$, independently of the release time t_2, as the optimal pricing will always will always fall into the ELD/ELBD 'wedge' in Figure 3 (left). For markets comprising myopic consumers only, however, any dual rollover with discount $\beta \in (0, \bar{\beta})$ is dominated by the optimal solo rollover characterized in Theorem 2.

Theorem 4 **(Optimal Rollover)**. *If the market consists of myopic consumers, then a solo rollover is always optimal.*

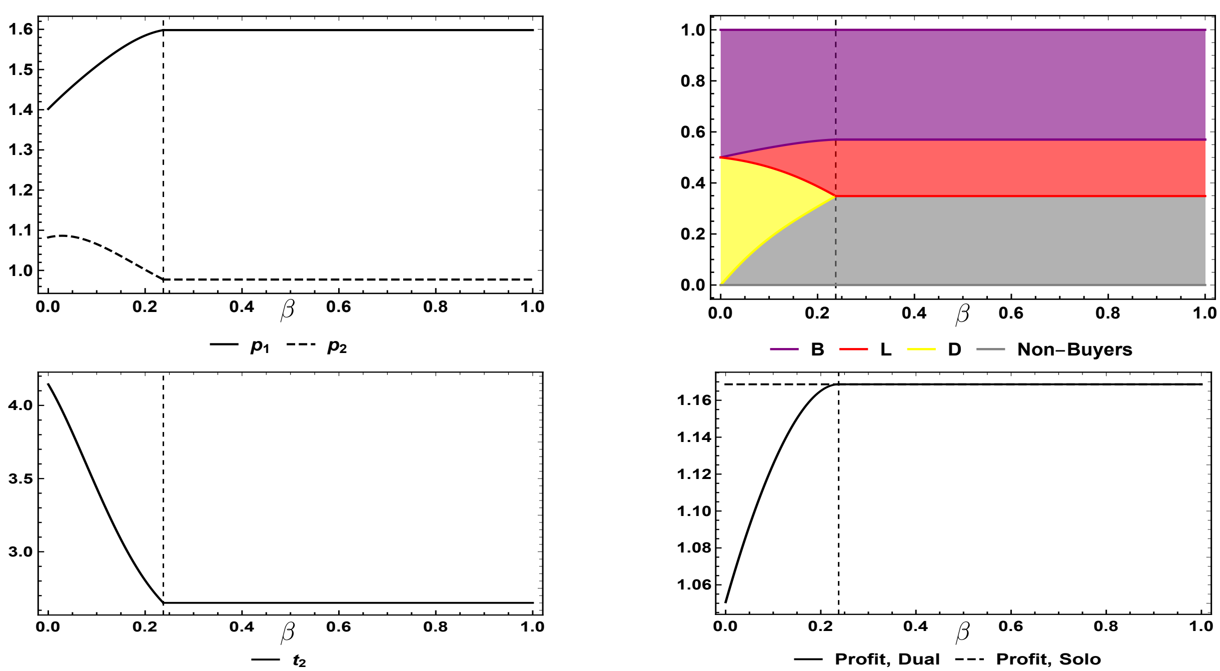

Figure 4 Optimal dual rollover strategy as a function of β.
In this example, we set $\alpha = 0.7$, $v = 1$, $\delta_c = 0.7$ (implying that $u = 2.8$) and $\delta_s = 0.9$.
In each graph, the vertical dashed line denotes $\bar{\beta}$.

Figure 4 conveys the intuition why a dual rollover is always inferior to a solo rollover, independent of the decay factor α and the discount rates δ_s and δ_c, in markets comprising myopic consumers only: For deep enough discounts $\beta \in (0, \bar{\beta})$, the company increases its market share by converting non-buyers to discount buyers. At the same time, however, deep discounts induce a large fraction of the late buyers to prefer purchasing the discounted first product over buying the second product at time t_2. Since the profit margin for the discounted product is low, this cannibalization effect overcompensates the gains of an increased market share. In conclusion, if the market consists of myopic consumers that do not base their decisions on the future actions of the firm, the company should conduct a solo rollover where the product prices, as well as the release time of the second product, are selected according to the perceived obsolescence α.

4. Strategic Consumers

We now discuss the optimal pricing and timing decisions of the firm, as well as the resulting market segmentation, when consumers are strategic and the firm employs a solo (Section 4.1) or a dual (Section 4.2) rollover. We again compare both strategies in Section 4.2. While many of the results in this section are qualitatively similar to those of Section 3, the strategic consumer behavior significantly complicates the analysis, and some of the results are less amenable to an intuitive interpretation. For ease of exposition, we therefore relegate some material to the appendix.

4.1. Solo Rollover

A strategic customer of type $\theta \in [0,1]$ decides on her entire purchasing strategy under full information of the firm's future decisions at time $t=0$. In particular, in the case of a solo rollover she compares *(i)* the utility 0 of purchasing neither product with *(ii)* the utility

$$\theta\left(\int_0^{t_2} \delta_c^t v \,\mathrm{d}t \,+\, \alpha^{t_2}\int_{t_2}^{\infty} \delta_c^t v \,\mathrm{d}t\right) - p_1 \;=\; \theta a(t_2)\, u - p_1$$

of purchasing the first product at time $t=0$ only, where $a(t_2) := 1-\delta_c^{t_2}(1-\alpha^{t_2})$ is the *obsolescence adjustment* of the early product (anticipating its obsolescence at time $t=t_2$), *(iii)* the utility

$$\theta\int_{t_2}^{\infty} \delta_c^t v \,\mathrm{d}t \,-\, \delta_c^{t_2} p_2 \;=\; \delta_c^{t_2}\,(\theta u - p_2)$$

of purchasing the second product at time $t=t_2$ only, as well as *(iv)* the utility

$$\theta\left(\int_0^{t_2} \delta_c^t v \,\mathrm{d}t \,+\, \int_{t_2}^{\infty} \delta_c^t v \,\mathrm{d}t\right) - p_1 - \delta_c^{t_2} p_2 \;=\; \theta u - p_1 - \delta_c^{t_2} p_2$$

of purchasing the first product at time $t=0$ and subsequently upgrading to the second product at time $t=t_2$. She then implements the purchasing strategy that maximizes her utility.

In the myopic consumer setting of Section 3, a late product buyer always has a lower type than an early product buyer. In fact, any myopic consumer that does (not) purchase the first product at time $t=0$ must have a type θ satisfying $\theta \geq \theta_E$ ($\theta < \theta_E$). Figure 5 illustrates that the consumer preferences are more involved in the strategic setting: The utility of purchasing the first product at time $t=0$ (solid blue curve) is composed of *(i)* the utility $\theta\int_0^{t_2} \delta_c^t v \,\mathrm{d}t$ of enjoying the product prior to its obsolescence (increasing dashed curve) and *(ii)* the utility $\alpha^{t_2}\int_{t_2}^{\infty} \delta_c^t v \,\mathrm{d}t$ of enjoying the product after its obsolescence (decreasing dashed curve). These two counteracting effects imply that the utility of purchasing the first product at time $t = 0$ first decreases and subsequently increases with the introduction time t_2 of the second product. In contrast, the utility associated with purchasing the second product is always a decreasing function of the introduction time t_2 (solid red curve).

We now show that the company can in fact influence the preference ordering of its customers by adjusting the introduction time t_2 of the second product.

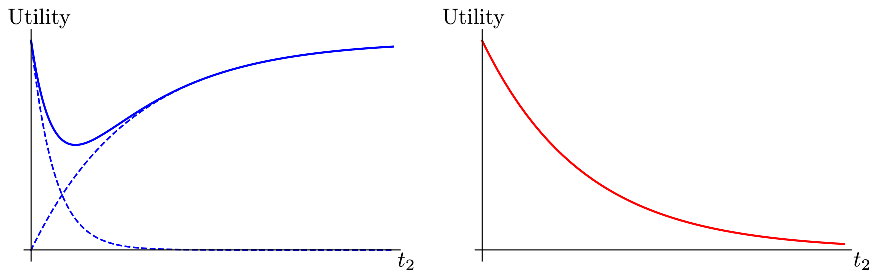

Figure 5 Utility derived from purchasing the first (left) and second (right) product.

Proposition 4. *For $\alpha, \delta_c \in (0,1)$, there is a threshold introduction time $\hat{\tau} \in [0, \infty)$ such that*

1. *If $t_2 > \hat{\tau}$, then late product buyers have* lower types *than early product buyers.*
2. *If $t_2 < \hat{\tau}$, then late product buyers have* higher types *than early product buyers.*

Moreover, we have $\hat{\tau} = 0$ if and only if $\alpha \in [\delta_c, 1)$.

Proposition 4 shows that, if faced with the choice of buying a single product, a high-value customer prefers to wait if the introduction time t_2 is sufficiently short, and she prefers to buy the first product otherwise. The opposite behavior can be observed for low-value customers. In the following, we say that the consumers exhibit a *myopic preference ordering* if $t_2 > \hat{\tau}$, which corresponds to the preference ordering in the myopic case, and we say that the consumers exhibit a *reversed preference ordering* otherwise. We define the *durability* of the first product as $d(t_2) = (1 - \delta_c^{t_2})/(1 - \alpha^{t_2})$. Note that $d(t_2) \geq 1$ if and only if $1 - \delta_c^{t_2} \geq 1 - \alpha^{t_2}$, which in turn is the case if and only if $\alpha \geq \delta_c$. According to Proposition 4, $d(t_2) \geq 1$ implies a myopic preference ordering. We also note that for a fixed discount factor δ_c, the durability $d(t_2)$ decreases with an increasing perceived obsolescence.

We now characterize the market segmentation for a given solo rollover strategy. Here and in the following, we focus on the myopic preference ordering. The corresponding results for the reversed preference ordering can be found in Appendix A.

Theorem 5 **(Market Segmentation)**. *Under a myopic preference ordering, we have:*

1. *If $p_2 \geq (1 - \alpha^{t_2})\, u$ or $p_1 \geq (1 - \delta_c^{t_2})\, u$, none of the consumers buy both versions and*
   (a) *if $p_2 \geq p_1 / a(t_2)$, then the market consists of early buyers (region E);*

(b) *if* $p_2 \in (p_1/d(t_2),\, p_1/a(t_2))$*, then the market consists of early and late buyers (region EL);*

(c) *if* $p_2 \leq p_1/d(t_2)$*, then the market consists of late buyers (region L).*

2. *If* $p_2 < (1-\alpha^{t_2})\,u$ *and* $p_1 < (1-\delta_c^{t_2})\,u$*, there are consumers that buy both versions and*

   (a) *if* $p_2 \geq p_1/a(t_2)$*, then the market also comprises early buyers (region EB);*

   (b) *if* $p_2 \in (p_1/d(t_2),\, p_1/a(t_2))$*, then the market also comprises early and late buyers (region ELB);*

   (c) *if* $p_2 \leq p_1/d(t_2)$*, then the market also comprises late buyers (region LB).*

Theorem 5 shows that, in order to encourage strategic consumers to buy both products, both prices p_1 and p_2 need to be sufficiently low. This is in contrast to the myopic setting studied in Theorem 1, where only the price p_2 of the second product has to be sufficiently low in order to encourage upgrading behavior. This result is intuitive as a strategic customer may decide to wait for the second product if the first product is too expensive, whereas a myopic customer lacks the foresight to anticipate the release of the second product. A similar reasoning shows that, in contrast to the myopic case, a company facing strategic customers may only encounter late product buyers if the price p_1 for the first product is high. We also observe that, in comparison to Theorem 1, the threshold purchase prices in the strategic case are adjusted by the obsolescence adjustment $a(t_2)$ as well as the durability $d(t_2)$ of the first product. This reflects the fact that a strategic consumer trades off the eventual obsolescence of the first product with the waiting time for the second product. We emphasize that the findings of Theorem 5 reduce to those of Theorem 1 when the consumers are very impatient, that is, when $\delta_c \to 0$.

We now consider the optimal solo rollover strategy of the firm.

Proposition 5. *Under a myopic preference ordering and for a fixed release time* $t_2 \in (\hat{\tau}, \infty)$*, the optimal pricing strategy* $(p_1^\star, p_2^\star)$ *in a solo rollover satisfies* $p_2^\star \in [p_1^\star/d(t_2),\, p_1^\star/a(t_2)]$.

In analogy to Proposition 1 for the myopic case, Proposition 5 shows that under a myopic preference ordering, it is always optimal for the firm to attract new customers at both times $t=0$ and $t=t_2$, the latter by choosing a sufficiently low price for the second version.

We now study the optimal solo rollover design when the company faces strategic consumers.

Theorem 6 **(Optimal Solo Rollover Design)**. *If* $\delta_c \geq \delta_s$*, then the firm only introduces the first product (*i.e., $t_2^\star \to \infty$*) at price* $p_1^\star = u/2$*, resulting in an expected profit of* $u/4$*.*

Otherwise, there are threshold decay factors $0 < \alpha_{LB} < \alpha_{EL} \leq 1$ *such that*

1. *The* optimal introduction time $t_2^\star$ *approaches* $t^{LB} > \hat{\tau}$ *as* $\alpha \to 0$*, it is continuously increasing in* α *over* $\alpha \in (0, \alpha_{EL})$*, it is continuously decreasing in* α *over* $\alpha \in [\alpha_{EL}, 1)$ *and approaches* $t^{EL} > \hat{\tau}$ *as* $\alpha \to 1$.

2. *The* optimal price for the first product $p_1^\star$ *approaches* $p_1^{LB} < u/2$ *as* $\alpha \to 0$*, it is continuous and unimodal in* α *over* $\alpha \in (0, \alpha_{EL})$*, it is continuously increasing in* α *over* $\alpha \in [\alpha_{EL}, 1)$ *and approaches* $p_1^{EL} < 2u/3$ *as* $\alpha \to 1$*.*
3. *The* optimal price for the second product $p_2^\star$ *approaches* $p_2^{LB} = u/2$ *as* $\alpha \to 0$*, it is continuously decreasing in* α *over* $\alpha \in (0, \alpha_{EL})$*, it is continuously increasing in* α *over* $\alpha \in [\alpha_{EL}, 1)$ *and approaches* $p_2^{EL} < u/3$ *as* $\alpha \to 1$*.*
4. *The* optimal expected profit *approaches* $\Pi^{LB} < u/2$ *as* $\alpha \to 0$*, it is continuously decreasing in* α *over* $\alpha \in (0, \alpha_{EL})$*, it is continuously increasing in* α *over* $\alpha \in [\alpha_{EL}, 1)$ *and approaches* $\Pi^{EL} < u/3$ *as* $\alpha \to 1$*.*

In contrast to the myopic setting, Theorem 6 shows that the release of the second product depends on the relative patience of the consumers and the firm. In particular, if the consumers are more patient than the firm, then their inclination to wait for the second product exceeds the firm's willingness to wait for the revenues from the sale of that product, and thus its release is delayed indefinitely. Theorem 6 also shows that the firm never releases the second product instantaneously. In fact, the firm always enforces a myopic preference ordering by choosing a release time t_2 that exceeds the threshold introduction time $\hat{\tau}$ from Proposition 4. While instantaneous product releases allow the firm to 'fool' their customers or conduct a price discrimination in the myopic case, strategic consumers anticipate the release of the second product and are hence not susceptible to such pricing strategies. It can be shown that for any fixed α, δ_c and δ_s, a firm facing strategic consumers *(i)* introduces the second product later, *(ii)* charges lower prices for both products and *(iii)* generates lower expected profits than in markets composed of myopic consumers. The lower prices and expected profits can be interpreted as concessions to more knowledgeable consumers, and the delayed release of the second product is required to reduce the fraction of consumers that only buy the second product. A similar reasoning explains why for a high degree of perceived obsolescence (*i.e.*, low values of α), the price for the second product ($p_2^{LB} = u/2$) exceeds that of the first one ($p_1^{LB} < u/2$): While myopic consumers would purchase the first product anyway, the firm needs to discourage strategic consumers from waiting for the release of the second version.

The optimal solo rollover design leads to the following market segmentation.

Corollary 5 **(Optimal Market Segmentation)**. *If* $\delta_c \geq \delta_s$*, then the firm only introduces the first product and serves early buyers (region E). Otherwise, the firm sequentially introduces both versions and induces a myopic preference ordering such that:*

1. *For* $\alpha \in (0, \alpha_{LB}]$*, the firm serves late and both product buyers (region LB).*
2. *For* $\alpha \in (\alpha_{LB}, \alpha_{EL})$*, the firm serves early, late and both product buyers (region ELB).*
3. *For* $\alpha \in [\alpha_{EL}, 1]$*, the firm serves early and late product buyers (region EL).*

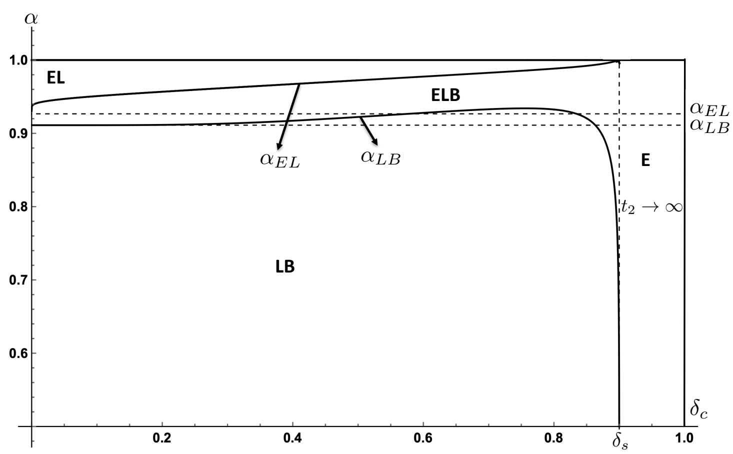

Figure 6 Optimal segmentation strategies.
In this example, we set $\delta_s = 0.9$ and $v = 1$.
The horizontal dashed lines correspond to the threshold decay factors of the myopic case.

Corollary 5 shows that as long as $\delta_c < \delta_s$, the market segmentation in the strategic consumer setting is qualitatively similar to that of the myopic setting. Figure 6 compares the threshold decay factors α_{LB} and α_{EL} for the two market settings. As discussed in Section 3.1, α_{LB} and α_{EL} do not depend on δ_c in the myopic case. In contrast, the inclination of a strategic consumer to purchase the early product decreases with her patience δ_c until δ_c approaches δ_s, in which case the company indefinitely delays the release of the second product.

4.2. Dual Rollover

In a dual rollover, a strategic consumer of type $\theta \in [0,1]$ has the additional option to wait and purchase the first product at the reduced price βp_1 at time $t = t_2$, resulting in the utility

$$\theta\alpha^{t_2} \int_{t_2}^{\infty} \delta_c^t v \, \mathrm{d}t - \delta_c^{t_2} \beta p_1 \; = \; \delta_c^{t_2} \cdot (\theta\alpha^{t_2} u - \beta p_1).$$

As in the myopic case, the firm has to offer a deep enough discount for this option to be attractive.

Proposition 6. *There are discount buyers iff $\beta \in [0, \bar{\beta})$, where $\bar{\beta} = \alpha^{t_2} \cdot \min\left\{\frac{p_2}{p_1}, \frac{1}{a(t_2)}\right\}$.*

In comparison to Proposition 2 for the myopic setting, the second term inside the minimum increases from 1 to $1/a(t_2)$, where $a(t_2) < 1$ by construction. Thus, the firm finds it easier to attract strategic consumers with discounts. Since $1/a(t_2)$ increases with δ_c and α, discounts are particularly attractive when the consumers are patient and/or the obsolescence is perceived as small.

Next, we discuss the market segmentation for a given dual rollover strategy. Here and in the remainder of this subsection, we assume that $\beta < \bar{\beta}$ and that the firm imposes a myopic preference ordering. The results for the reversed preference ordering are relegated to Appendix A.

Theorem 7 **(Market Segmentation)**. *Under a myopic preference ordering, we have:*

1. *If* $p_2 \geq (1-\alpha^{t_2})\,u$ *or* $p_1 \geq (1-\delta_c^{t_2})\,u$*, there are discount buyers but no both product buyers, and*
   (a) *if* $p_2 \geq b(t_2)p_1$*, then the market also comprises early buyers (region ED);*
   (b) *if* $p_2 \in (p_1/d(t_2),\, b(t_2)p_1)$*, then the market also comprises early and late buyers (region ELD);*
   (c) *if* $p_2 \leq p_1/d(t_2)$*, then the market also comprises late buyers (region LD).*
2. *If* $p_2 < (1-\alpha^{t_2})\,u$ *and* $p_1 < (1-\delta_c^{t_2})\,u$*, there are discount and both product buyers, and*
   (a) *if* $p_2 \geq b(t_2)p_1$*, then the market also comprises early buyers (region EBD);*
   (b) *if* $p_2 \in (p_1/d(t_2),\, b(t_2)p_1)$*, then the market also comprises early and late buyers (region ELBD);*
   (c) *if* $p_2 \leq p_1/d(t_2)$*, then the market also comprises late buyers (region LBD).*

Here, the quantity $b(t_2) = (1+\beta\delta_c^{t_2})/d(t_2) + \beta$ *increases with* δ_c *and* β*, and it decreases with* α*.*

The segmentation in Theorem 7 is qualitatively similar to those of Theorem 3 (myopic dual) and Theorem 5 (strategic solo). In comparison to Theorem 3, Theorem 7 replaces the expression $1-\alpha^{t_2}+\beta$ with $b(t_2)$. Both quantities exhibit a qualitatively similar behavior with respect to α and β, but the expression $b(t_2)$ additionally depends on δ_c. Compared to Theorem 5, Theorem 7 replaces the expression $1/a(t_2)$ with $b(t_2)$. Both quantities display a qualitatively similar behavior with respect to α and δ_c, but the expression $b(t_2)$ additionally depends on β. Moreover, Theorem 7 contains an additional segment LD (which does not occur in the myopic dual case), and for any pricing regime there are discount buyers (in contrast to the strategic solo setting).

We now study the optimal dual rollover strategy of the firm.

Proposition 7. *Under a myopic preference ordering, a fixed release time* $t_2 \in (\hat{\tau}, \infty)$ *and a sufficiently deep discount* $\beta \in [0, \bar{\beta})$*, the optimal pricing strategy* $(p_1^\star, p_2^\star)$ *in a dual rollover satisfies* $p_2^\star \in [p_1^\star/d(t_2),\, b(t_2)p_1^\star]$.

As in the previous three settings (myopic solo, myopic dual and strategic solo), it is always optimal for the company to price both products so as to attract both early and late buyers. We now show that under a myopic preference ordering, the additional revenues generated from discount buyers is outweighed by the loss of revenues due to product cannibalization.

Theorem 8 **(Optimal Rollover, Myopic Preference Ordering)**. *Under a myopic preference ordering, a solo rollover is always optimal.*

If, however, the firm decides to adopt a sufficiently short release cycle satisfying $t_2 < \hat{\tau}$ (*cf.* Proposition 4) and thus imposes a reversed preference ordering, then the additional revenues generated from discount buyers outweigh the losses incurred from product cannibalization.

Theorem 9 **(Optimal Rollover, Reversed Preference Ordering)**. *Under a reversed preference ordering, a dual rollover is always optimal.*

Assuming that the firm is free to choose its release cycle, it prefers a myopic preference ordering and hence implements a solo rollover.

Theorem 10 **(Optimal Rollover)**. *The optimal rollover strategy is a solo rollover.*

Theorem 10 shows that when facing strategic customers, the firm should adopt a longer release cycle so as to maximize the obsolescence effect and thereby reduce the lost revenues due to product cannibalization. In contrast, a dual rollover would require deep discounts β at time $t = t_2$ in order to attract low-value consumers, which results in a product cannibalization whose associated revenue losses outweigh the gains from the additional market share. Only if short release cycles are imposed exogenously, a dual rollover becomes an attractive option. Once the release times are endogenized, however, a solo rollover is optimal, regardless of the consumers' foresight.

5. Mixed Markets

There is ample empirical evidence that real-life markets comprise both myopic and strategic consumers, see, *e.g.*, Nair (2007), Li et al. (2014) and Osadchiy and Bendoly (2015). We therefore now consider a mixed market where a fraction $\gamma \in (0,1)$ of the consumers is strategic (*cf.* Section 4) and the remaining fraction $(1-\gamma)$ of the consumers is myopic (*cf.* Section 3). For simplicity, we assume that a consumer's quality appreciation θ is independent of her foresight (myopic vs. strategic).

We now characterize the optimal rollover strategy as well as the resulting market segmentation in markets where the firm faces both myopic and strategic consumers.

Theorem 11 **(Mixed Markets)**. *For any $\gamma \in (0,1)$, a solo rollover is optimal also in a mixed market. Moreover, the optimal market segmentation is as follows.*

1. ***Impatient Consumers.*** *For $\delta_c \in (0, \delta_s)$, there is $0 < \alpha_1 < \alpha_2 < \alpha_3 < 1$ such that*
   (a) *for $\alpha \in (0, \alpha_1]$, the firm serves late and both product buyers (region LB/LB);*
   (b) *for $\alpha \in (\alpha_1, \alpha_2]$, the firm serves early, late and both product myopic buyers as well as late and both product strategic buyers (region ELB/LB);*
   (c) *for $\alpha \in (\alpha_2, \alpha_3]$, the firm serves early, late and both product buyers (region ELB/ELB);*
   (d) *for $\alpha \in (\alpha_3, 1)$, the firm serves early and late product buyers (region EL/EL).*
2. ***Patient Consumers.*** *For $\delta_c \in [\delta_s, 1)$, there is $0 < \alpha_1 < \alpha_2 < 1$ and $0 < \gamma' < 1$ such that*
   (a) *for $\alpha \in (0, \alpha_1]$, as well as for $\alpha \in (\alpha_1, \alpha_2]$ and $\gamma \in [\gamma', 1)$, the firm serves late and both product buyers (region LB/LB);*
   (b) *for $\alpha \in (\alpha_1, \alpha_2]$ and $\gamma \in (0, \gamma')$, the firm serves early, late and both product myopic buyers as well as late and both product strategic buyers (region ELB/LB);*

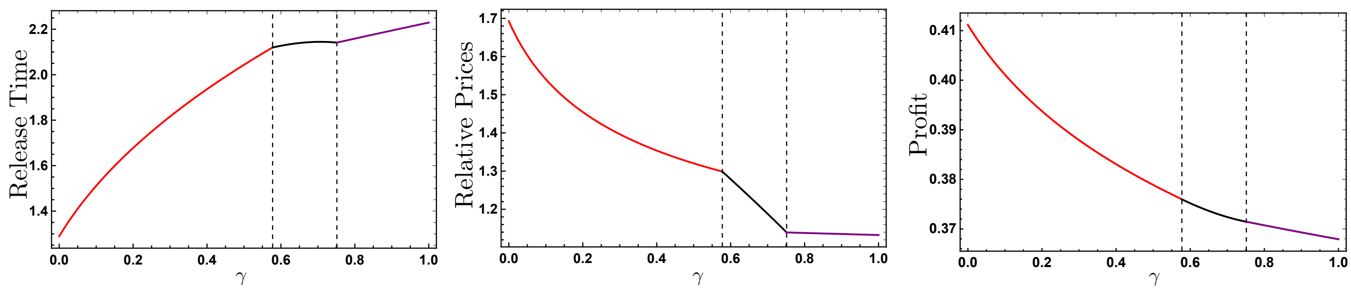

Figure 7 **Optimal rollover strategy as a function of the fraction γ of strategic consumers in the market. From left to right and separated by dashed lines, the regions in the three graphs correspond to the regions LB/LB (M), ELB/LB (C) and ELB/LB (S) in Figure 8. We use the parameter setting $\alpha = 0.5$, $u = 1$, $\delta_c = 0.37$ and $\delta_s = 0.79$.**

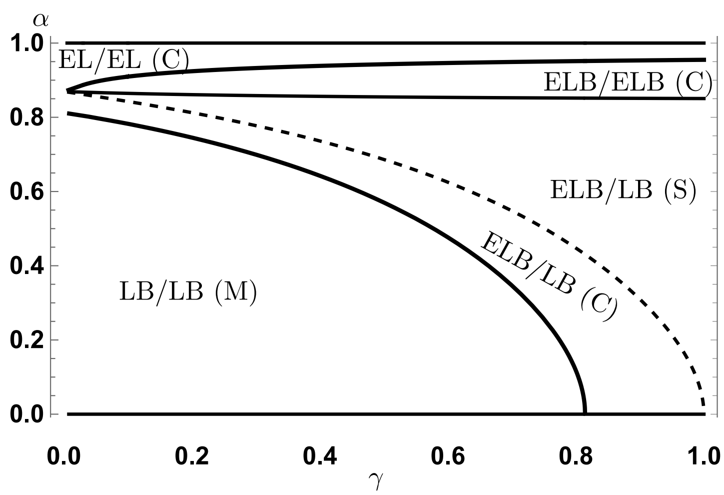

Figure 8 **Market segmentation resulting from the optimal rollover strategy. The names of the regions correspond to those of Theorem 11. We use the same parameter setting as in Figure 7.**

(c) *for $\alpha \in (\alpha_2, 1)$ and $\gamma \in (0, \gamma')$, the firm serves early and late product buyers (region EL/EL);*

(d) *for $\alpha \in (\alpha_2, 1)$ and $\gamma \in [\gamma', 1)$, the firm only serves early buyers by indefinitely delaying the release time for the second product.*

Figures 7 and 8 illustrate the optimal rollover design in a market with impatient consumers. In particular, Figure 7 (left) shows that the optimal release times $t_2^\star(\gamma)$, expressed as a function of the fraction γ of strategic consumers, are a (nonlinear) combination of the (earlier) optimal release time t_2^M from the purely myopic setting and the (later) optimal release time t_2^S from the purely strategic setting. In contrast, the relative prices $p_1^\star(t_2;\gamma)/p_2^\star(t_2;\gamma)$, expressed as a function of the release time t_2 as well as the fraction γ of strategic consumers, can be subdivided into different 'focus regimes' as illustrated in Figure 8. In the myopic focus regime denoted by '(M)', the relative price functionals $p_1^\star(t_2;\gamma)/p_2^\star(t_2;\gamma)$ coincide with the optimal relative price functionals $p_1^M(t_2)/p_2^M(t_2)$

of the myopic setting, whereas the relative price functionals $p_1^\star(t_2;\gamma)/p_2^\star(t_2;\gamma)$ coincide with the optimal relative price functionals $p_1^S(t_2)/p_2^S(t_2)$ of the strategic setting in the strategic focus regime denoted by '(S)'. Note, however, that the *actual* relative prices $p_1^\star(t_2^\star(\gamma);\gamma)/p_2^\star(t_2^\star(\gamma);\gamma)$ differ from both $p_1^M(t_2^M)/p_2^M(t_2^M)$ and $p_1^S(t_2^S)/p_2^S(t_2^S)$ since $t_2^\star(\gamma) \neq t_2^M, t_2^S$ in general. Finally, in intermediate market regimes denoted by '(C)', the firm adopts a (nonlinear) combination of the relative pricing strategies from the myopic and strategic settings. Note that as expected, the firm adopts a myopic (resp., strategic) focus regime for fractions γ close to zero (resp., one). The optimal rollover design for patient consumers, finally, closely mirrors that for impatient consumers. The key difference arises when the market is dominated by strategic consumers and the obsolescence is perceived to be small. In this case, it is optimal to indefinitely delay the release of the second product (*cf.* Theorem 6).

Our results show that even for markets that are 'almost homogeneous', that is, where $\gamma \to 0$ or $\gamma \to 1$, the firm's optimal rollover strategy will be informed by the presence of *both* the myopic and the strategic consumers. This is in contrast to earlier research, which found that when one consumer group is sufficiently "dominant", firms should focus exclusively on that group and exclude the others (Valletti and Szymanski 2006). The key difference to that work is that in our model, quality appreciation is identical for myopic and strategic consumers. Therefore, for any market composition γ, (fractions of) both consumer groups will make a purchase. Of course, the degree of consumer foresight plays a role in the intertemporal choice, and this is addressed by tilting the prices and release intervals towards the dominant group (*cf.* Theorem 11).

6. Extensions

In this section, we first broaden our analysis to semi-digital goods (Section 6.1), and we subsequently investigate the welfare implications of perceived obsolescence (Section 6.2). For brevity of exposition, the details of both of these extensions are relegated to Appendix B.

6.1. Semi-Digital Goods

So far, our analysis focused on purely digital goods (*e.g.*, software or e-books) with negligible reproduction costs and no inventory constraints. We now extend our discussion to semi-digital goods (*e.g.*, smartphones, tablets and smartwatches) where the reproduction costs are non-negligible, but where the inventory capacity can be disregarded. We denote by c_1 and c_2 the marginal (*i.e.*, per-unit) production costs of the first and second product, respectively. (Note that in the following, inventory holding costs can be included in the production costs c_1 and c_2.)

Proposition 8 **(Semi-Digital Goods)**. *For fixed $\alpha, \delta_c, \delta_s$ and v, there exist production cost threshold functions $c_2^M, c_2^S : \mathbb{R}_+ \mapsto \mathbb{R}_+$ such that a solo rollover is optimal whenever c_1 and c_2 satisfy*

1. $c_2 \le c_2^M(c_1)$ *if the firm faces* myopic consumers*;*
2. $c_2 \le c_2^S(c_1)$ *if the firm faces* strategic consumers*.*

Moreover, $c_1 < c_2^M(c_1) < c_2^S(c_1)$ *for all* $c_1 \in \mathbb{R}_+$*, and* c_2^M *and* c_2^S *are increasing functions of* c_1*.*

Proposition 8 shows that a solo rollover remains optimal whenever the marginal production costs of the second product are sufficiently low. This is the case especially when the marginal production costs do not increase between the product versions, as is often the case due to economies of scope. Indeed, for sufficiently low marginal production costs of the second product, the additional profits from the discount buyers are outweighed by the loss of profits from the sale of the second product. In contrast, if the second product is costly to produce, the optimal strategy of the firm is to release it shortly after the first product and to target high-value consumers. Proposition 8 also shows that the threshold value of the production costs is higher in markets consisting of strategic consumers, meaning that a solo rollover is more likely to be optimal with strategic than with myopic consumers, ceteris paribus. This can again be ascribed to the cannibalization effects that emerge when the first product is offered at a discounted price. In particular, strategic consumers are more likely to be discount buyers, due to their ability to anticipate the discounted price at time $t = t_2$. Finally, we emphasize that the conditions in Proposition 8 are sufficient but not necessary. Indeed, Appendix B.1 presents an example where a firm facing myopic consumers prefers a solo rollover even when $c_2 > c_2^M(c_1)$.

6.2. Welfare Implications

Recent years have witnessed an ongoing debate over the effects of managed obsolescence on consumer welfare. Perhaps most prominently, the forced battery slowdown of old Apple iPhone versions that accompanied the release of new versions has led to investigations in France (BBC *News* 2017) and the US (Mickle and McKinnon 2017). Our model allows us to cast some light on the welfare effects of managed obsolescence on different types of consumers.

Proposition 9 **(Welfare Implications)**. *For fixed* δ_s *and* δ_c*, the welfare of the market participants exhibits the following dependence on the decay factor* α*.*

1. ***Firm.*** *The profit of the firm* decreases *with* α*.*
2. ***Myopic Consumers.*** *The welfare of myopic consumers* increases *with* α*. Moreover, for sufficiently low but positive* α*, myopic consumers experience a negative surplus ex post.*
3. ***Strategic Consumers.*** *There is a threshold value of the consumer discount factor* $\delta_c' \in (0, \delta_s)$ *such that the welfare of strategic consumers is maximized at* $\alpha \to 0$ *if* $\delta_c \le \delta_c'$ *and at* $\alpha \in (\alpha_{LB}, \alpha_{EL})$ *if* $\delta_c > \delta_c'$*, where* α_{LB} *and* α_{EL} *are defined in Theorem 6.*

Proposition 9 shows that both the firm and the strategic consumers benefit from some obsolescence, whereas myopic consumers are always harmed by obsolescence. The key difference between myopic and strategic consumers is that they attach different values to the first product *prior* to the release of the second version. In particular, myopic consumers pay more than they would, had they known about the obsolescence. When they become aware of the obsolescence, however, their decision about the first product is already sunk. As a result, even if the second product is expensive, they might (ex post, optimally) prefer to upgrade since the alternative (remaining an early buyer) might be a worse option. In this sense, becoming aware of obsolescence creates regret and ultimately unhappy consumers. Hence, consumer welfare is maximized when they do not experience obsolescence, which happens if they obtain only one of the goods. For strategic consumers, instead, being a both product buyer is advantageous, as foresight limits the pricing power of the firm and can influence the release intervals. As a result, high obsolescence implies lower total prices and shorter release intervals, which in turn improve the welfare of strategic consumers.

As for the firm, the monopolist prefers full obsolescence, regardless of the consumer types, and is hence incentivized to accomplish a high obsolescence, if it can do so. A possible policy discussion that arises from this prediction is that, in order to protect the consumers, even when the degree of obsolescence is outside the policy-maker's control but in the hands of the firm, a policy-maker should ensure that consumers are informed about the upcoming releases. This can be achieved by educating the consumers about the firm's obsolescence management, so that myopic consumers are ultimately turned into strategic ones, for example by inducing the firm to pre-announce its release strategy in credible and unambiguous ways.

7. Conclusions

We analyzed a model for product rollovers that, in addition to the standard levers of the rollover type (solo vs. dual) and the pricing, endogenized the release time of the successor product. This allows the firm to exploit the perceived obsolescence, a phenomenon that has been widely studied in the consumer research literature but that has to date been largely neglected by the product rollover and version management communities. Equipped with this additional lever, a firm should always conduct a solo rollover and convince its customer base to upgrade through a careful management of the release times. In particular, the release intervals should be stretched out when the consumers are strategic and/or the market is confident about the durability of the old version. We also explored the extension to semi-digital goods, and we investigated how the welfare is distributed among the market participants when we account for perceived obsolescence. Of course there may be multiple reasons for the prevalence of dual rollovers in practice, but they seem to arise from additional constraints on the profit maximization problem of the firm, such as the existence of capacity constraints, longer-term pricing strategies or when the obsolescence is not actively managed.

One can envisage several fruitful extensions of our model. For example, the firm may be able to influence the perception of obsolescence through marketing campaigns that emphasize the advantages of the successor product (resulting in an endogenization of the parameter α). Alternatively, the firm may be able to credibly pre-announce their rollover policy and thus influence the composition of the mixed market in Section 5 (resulting in an endogenization of the parameter γ). More broadly, we believe that the manifold implications of perceived obsolescence warrant further investigation by the product rollover and version management communities.

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Appendix A: Proofs

Proof of Theorem 1. By definition of θ_L and θ_E, we have $p_2 \geq p_1$ if and only if $\theta_L = p_2/u \geq \theta_E = p_1/u$, which in turn holds if and only if there are no late buyers. We thus conclude that there are late buyers if and only if $p_2 < p_1$. Likewise, by definition of θ_E and θ_B and the assumption that $p_1 \leq u$, we have $p_2 \leq p_1(1-\alpha^{t_2})$ if and only if $\theta_E = p_1/u \geq \theta_B = p_2/[(1-\alpha^{t_2})u]$, which in turn holds if and only if there are no early buyers. Hence, we conclude that there are early buyers if and only if $p_2 > p_1(1-\alpha^{t_2})$. Finally, by definition of θ_B, we have $\theta_B = \min\{1, p_2/[(1-\alpha^{t_2})u]\} = 1$ if and only if $p_2 \geq (1-\alpha^{t_2})u$, which in turn holds if and only if there are no buyers that purchase both products. We thus conclude that there are both product buyers if and only if $p_2 < (1-\alpha^{t_2})u$. The statement of the theorem then follows from the various combinations of the previously established regions as well as the fact that $p_1, p_2 \in [0, u]$ and $(1-\alpha^{t_2}) \in (0, 1)$. □

Proof of Corollary 1. The cases 1 (a) and 1 (c) follow from Theorem 1 since $(1-\alpha^{t_2}) \to 1$ when $\alpha \to 0$ and/or $t_2 \to \infty$. In view of case 1 (b), we note that $\theta_B = \theta_L = \theta_E$ whenever $\alpha \to 1$ and/or $t_2 \to 0$, and hence there are only non-buyers and both product buyers. Similarly, the cases 2 (a), (b) and (c) follow from Theorem 1 since $(1-\alpha^{t_2}) \to 0$ when $\alpha \to 1$ and/or $t_2 \to 0$. □

Proof of Proposition 1. We prove the statement by contradiction. Assume first that the optimal prices would satisfy $p_2^\star > p_1^\star$. Since the optimal prices maximize the expected profits (1) over all $p_1, p_2 \in [0, u]$, they must *a fortiori* optimize (1) over all $p_1, p_2 \in [0, u]$ satisfying $p_2 \geq p_1$. For prices $p_2 \geq p_1$, it follows that $\theta_B \geq \theta_L \geq \theta_E$, and Theorem 1 implies that there are no late buyers. Hence, the expected profits (1) simplify to

$$\begin{aligned}
\Pi_{EB}(p_1, p_2) &= p_1[\theta_B - \theta_E]_+ + (p_1 + \delta_s^{t_2} p_2)(1 - \max\{\theta_E, \theta_B\}) \\
&= p_1(\theta_B - \theta_E) + (p_1 + \delta_s^{t_2} p_2)(1 - \theta_B) \\
&= p_1(\min\{p_2/[(1-\alpha^{t_2})u], 1\} - p_1/u) + (p_1 + \delta_s^{t_2} p_2)(1 - \min\{p_2/[(1-\alpha^{t_2})u], 1\}).
\end{aligned}$$

We note that for fixed p_1, the cases $p_2 = (1-\alpha^{t_2})u$ and $p_2 > (1-\alpha^{t_2})u$ generate the same expected profits since the second term in the last expression of $\Pi_{EB}(p_1, p_2)$ vanishes in both cases. We can thus restrict the prices to $p_1 \in [0, u]$ and $p_2 \in [0, (1-\alpha^{t_2})u]$, and the expected profits (1) further simplify to

$$\Pi_{EB}(p_1, p_2) = p_1(p_2/[(1-\alpha^{t_2})u] - p_1/u) + (p_1 + \delta_s^{t_2} p_2)(1 - p_2/[(1-\alpha^{t_2})u]).$$

For $(p_1^\star, p_2^\star)$ to optimize $\Pi_{EB}(p_1, p_2)$, it has to satisfy the KKT conditions. The unique KKT point for the problem $\max\{\Pi_{EB}(p_1, p_2) : p_2 \geq p_1,\ p_1 \in [0, u],\ p_2 \in [0, (1-\alpha^{t_2})u]\}$ is $p_1^\star = p_2^\star = \frac{(1+\delta_s^{t_2})(1-\alpha^{t_2})u}{2(1-\alpha^{t_2}+\delta_s^{t_2})}$ for $\alpha^{t_2} < \frac{1+\delta_s^{t_2}}{2}$ and $p_1^\star = p_2^\star = (1-\alpha^{t_2})u$ for $\alpha^{t_2} \geq \frac{1+\delta_s^{t_2}}{2}$. This is a contradiction to the initial assumption that $p_2^\star > p_1^\star$.

Assume now that the optimal prices would satisfy $p_2^\star < (1-\alpha^{t_2})p_1^\star$. Since the optimal prices maximize the expected profits (1) over all $p_1, p_2 \in [0, u]$, they must *a fortiori* optimize (1) over all $p_1, p_2 \in [0, u]$ satisfying $p_2 \leq (1-\alpha^{t_2})p_1$. For prices $p_2 \leq (1-\alpha^{t_2})p_1$, it follows that $1 \geq \theta_E \geq \theta_B \geq \theta_L$, and Theorem 1 implies that there are no early buyers. Hence, the expected profits (1) simplify to

$$\begin{aligned}
\Pi_{LB}(p_1, p_2) &= \delta_s^{t_2} p_2[\theta_E - \theta_L]_+ + (p_1 + \delta_s^{t_2} p_2)(1 - \max\{\theta_E, \theta_B\}) \\
&= \delta_s^{t_2} p_2(\theta_E - \theta_L) + (p_1 + \delta_s^{t_2} p_2)(1 - \theta_E) \\
&= \delta_s^{t_2} p_2(p_1/u - p_2/u) + (p_1 + \delta_s^{t_2} p_2)(1 - p_1/u).
\end{aligned}$$

The unique KKT point for the problem $\max\{\Pi_{LB}(p_1,p_2) : p_2 \leq (1-\alpha^{t_2})p_1,\ p_1,p_2 \in [0,u]\}$ is $p_2^\star = (1-\alpha^{t_2})p_1^\star = \frac{u(1-\alpha^{t_2})(1+(1-\alpha^{t_2})\delta_s^{t_2})}{2(1+(1-\alpha^{t_2})^2\delta_s^{t_2})}$. This is a contradiction to initial assumption that $p_2^\star < p_1^\star(1-\alpha^{t_2})$. □

The proofs of Theorem 2 as well as Corollaries 2 and 3 rely on the following auxiliary result, which we prove first.

LEMMA 1. *There are threshold decay factors* $0 < \alpha_{LB} < \alpha_{EL} \leq 1$ *such that for fixed* $\alpha, \delta_c, \delta_s \in (0,1)$ *and* v, *the release time* $t_2^\star$ *of the unique maximizer* $(p_1^\star, p_2^\star, t_2^\star)$ *of* (1) *satisfies the following properties:*

1. *for* $\alpha \in (0, \alpha_{LB}]$, *we have* $t_2^\star = \tau_{LB}(\alpha) \in \mathcal{T}_{LB}(\alpha)$;
2. *for* $\alpha \in (\alpha_{LB}, \alpha_{EL})$, *we have* $t_2^\star = \tau_{ELB}(\alpha) \in \mathcal{T}_{ELB}(\alpha)$;
3. *for* $\alpha \in [\alpha_{EL}, 1)$, *we have* $t_2^\star = 0^+$.

Here, the half-open intervals $\mathcal{T}_{ELB}(\alpha)$ *and* $\mathcal{T}_{LB}(\alpha)$ *satisfy* $\mathcal{T}_{ELB}(\alpha) = [0, \tau(\alpha))$ *and* $\mathcal{T}_{LB}(\alpha) = [\tau(\alpha), \infty)$ *for some* $\tau(\alpha) \in (0,\infty)$, *and* $\tau(\alpha)$, $\tau_{LB}(\alpha)$ *and* $\tau_{ELB}(\alpha)$ *are defined in the proof.*

Proof of Lemma 1. We set $\mathcal{T}_{ELB}(\alpha) = [0, \tau(\alpha))$ and $\mathcal{T}_{LB}(\alpha) = [\tau(\alpha), \infty)$, where $\tau(\alpha)$ is the root of the function $t_2 \mapsto h(t_2, \alpha) := -\alpha + \left(\frac{1+\delta_s^{t_2}}{2+\delta_s^{t_2}}\right)^{1/t_2}$. Note that $\tau(\alpha)$ exists since $t_2 \mapsto \left(\frac{1+\delta_s^{t_2}}{2+\delta_s^{t_2}}\right)^{1/t_2}$ maps $\mathbb{R}_+$ to $(0,1)$, and $\tau(\alpha)$ is unique since $h(t_2,\alpha)$ is strictly increasing in t_2 for every fixed $\alpha \in (0,1)$. We thus conclude that the intervals $\mathcal{T}_{ELB}(\alpha)$ and $\mathcal{T}_{LB}(\alpha)$ are well defined.

Proposition 1 shows that for any fixed introduction time t_2, the optimal prices satisfy $p_1^\star \geq p_2^\star \geq (1-\alpha^{t_2})p_1^\star$. Although the expected profits (1) are *not* jointly concave in p_1, p_2 and t_2, one can verify that they are jointly concave in p_1 and p_2 if t_2 is fixed. Therefore, we first determine the optimal prices for fixed t_2, and we subsequently find the optimal introduction time $t_2^\star(\alpha)$ of (1) with optimal prices.

Distinguishing the two cases $p_2 \leq (1-\alpha^{t_2})u$ and $p_2 \geq (1-\alpha^{t_2})u$, the profit function (1) can be written as

$$\Pi(t_2,\alpha) \;=\; \max\{\Pi_1(t_2,\alpha),\, \Pi_2(t_2,\alpha)\},$$

where

$$\Pi_1(t_2,\alpha) \;=\; \max\{\Pi(p_1,p_2,t_2,\alpha) \,:\, \min\{p_1, (1-\alpha^{t_2})u\} \geq p_2 \geq (1-\alpha^{t_2})p_1,\ \ p_1,p_2 \in [0,u]\}$$

and

$$\Pi_2(t_2,\alpha) \;=\; \max\{\Pi(p_1,p_2,t_2,\alpha) \,:\, p_1 \geq p_2 \geq (1-\alpha^{t_2})u,\ \ p_1,p_2 \in [0,u]\}.$$

In these equations, $\Pi(p_1,p_2,t_2,\alpha)$ denotes the expected profits (1). Let us first consider the profit maximization problem $\Pi_1(t_2,\alpha)$. Standard derivations show that its unique KKT point is

$$\begin{cases} p_2^\star(t_2,\alpha) \;=\; (2p_1^\star(t_2)-u)\delta_s^{-t_2} \;=\; \frac{3u(1-\alpha^{t_2})}{8-(4-\delta_s^{t_2})\alpha^{t_2}-\delta_s^{t_2}} & \text{if } t_2 \in \mathcal{T}_{ELB}(\alpha), \\ p_2^\star(t_2,\alpha) \;=\; p_1^\star(t_2)(1-\alpha^{t_2}) \;=\; \frac{u(1-\alpha^{t_2})(1+(1-\alpha^{t_2})\delta_s^{t_2})}{2(1+(1-\alpha^{t_2})^2\delta_s^{t_2})} & \text{if } t_2 \in \mathcal{T}_{LB}(\alpha), \end{cases}$$

and plugging this solution into problem $\Pi_1(t_2,\alpha)$ yields

$$\Pi_1(t_2,\alpha) \;=\; \begin{cases} \Pi_{ELB}(t_2,\alpha) \;:=\; \frac{u[2(1+\delta_s^{t_2})-\alpha^{t_2}(1+2\delta_s^{t_2})]}{8-(4-\delta_s^{t_2})\alpha^{t_2}-\delta_s^{t_2}} & \text{if } t_2 \in \mathcal{T}_{ELB}(\alpha), \\ \Pi_{LB}(t_2,\alpha) \;:=\; \frac{u\left(\delta_s^{t_2}\left(1-\alpha^{t_2}\right)+1\right)^2}{4\delta_s^{t_2}\left(1-\alpha^{t_2}\right)^2+4} & \text{if } t_2 \in \mathcal{T}_{LB}(\alpha). \end{cases}$$

We now study the introduction time $t_{21}^\star(\alpha)$ that maximizes $\Pi_1(t_2,\alpha)$. To this end, we show that *(a)* there exist unique first order optimality points $\tau_{ELB}(\alpha)$ and $\tau_{LB}(\alpha)$ associated with the auxiliary profit functions $\Pi_{ELB}(t_2,\alpha)$ and $\Pi_{LB}(t_2,\alpha)$ over $t_2>0$ respectively; *(b)* there exists a unique $\alpha_{LB}\in(0,1)$ such that $\tau_{ELB}(\alpha),\tau_{LB}(\alpha)\in\mathcal{T}_{LB}(\alpha)$ for $\alpha\in(0,\alpha_{LB}]$ and $\tau_{ELB}(\alpha),\tau_{LB}(\alpha)\in\mathcal{T}_{ELB}(\alpha)$ for $\alpha\in(\alpha_{LB},1)$; and *(c)* the previous two points imply that $t_{21}^\star(\alpha)=\tau_{LB}(\alpha)$ for $\alpha\in(0,\alpha_{LB}]$ and $t_{21}^\star(\alpha)=\tau_{ELB}(\alpha)$ for $\alpha\in(\alpha_{LB},1)$.

In view of *(a)*, we note that the profit functions $\Pi_{ELB}(t_2,\alpha)$ and $\Pi_{LB}(t_2,\alpha)$ are continuously differentiable over $t_2\in\mathbb{R}_+$ since they are quotients of polynomials with strictly positive denominators. We next observe that $\Pi_{ELB}(t_2,\alpha)$ and $\Pi_{LB}(t_2,\alpha)$ are optimized by finite introduction times since $\Pi_{ELB}(t_2,\alpha),\Pi_{LB}(t_2,\alpha)\to\frac{u}{4}$ for $t_2\to 0$ as well as $t_2\to\infty$, while at the same time $\Pi_{ELB}(\tau(\alpha),\alpha)=\Pi_{LB}(\tau(\alpha),\alpha)=\frac{u(1+\delta_s^{\tau(\alpha)})}{4+\delta_s^{\tau(\alpha)}}>\frac{u}{4}$. The previous arguments imply that the maximizers of $\Pi_{ELB}(t_2,\alpha)$ and $\Pi_{LB}(t_2,\alpha)$ must satisfy the first order unconstrained optimality conditions, and one can verify that these conditions are satisfied at unique points $\tau_{ELB}(\alpha)\in(0,\infty)$ and $\tau_{LB}(\alpha)\in(0,\infty)$, respectively.

As for *(b)*, let α_{ELB} and α_{LB} be the unique values of α at which $\alpha\mapsto h(\tau_{ELB}(\alpha),\alpha)$ and $\alpha\mapsto h(\tau_{LB}(\alpha),\alpha)$ vanish, respectively. The first order optimality conditions of Π_{ELB} and Π_{LB} imply that $\alpha_{ELB}=\alpha_{LB}$ as well as $\tau_{ELB}(\alpha_{LB})=\tau_{LB}(\alpha_{LB})=\tau(\alpha_{LB})$. Moreover, if $\alpha\le\alpha_{LB}$, then $\tau_{ELB}(\alpha),\tau_{LB}(\alpha)\ge\tau(\alpha)$ and hence $\tau_{ELB}(\alpha),\tau_{LB}(\alpha)\in\mathcal{T}_{LB}(\alpha)$ since h is strictly increasing in its first argument. For the same reason, we have $\tau_{ELB}(\alpha),\tau_{LB}(\alpha)<\tau(\alpha)$ and $\tau_{ELB}(\alpha),\tau_{LB}(\alpha)\in\mathcal{T}_{ELB}(\alpha)$ if $\alpha>\alpha_{LB}$.

In view of *(c)*, assume that $\alpha\in(0,\alpha_{LB}]$ but $t_{21}^\star(\alpha)\neq\tau_{LB}(\alpha)$. Since $\tau_{LB}(\alpha)$ maximizes $\Pi_1(\cdot,\alpha)$ over $t_2\in\mathcal{T}_{LB}(\alpha)$, we must then have $t_{21}^\star(\alpha)\in\mathcal{T}_{ELB}(\alpha)$. This, however, is impossible since $\tau_{ELB}(\alpha)\in\mathcal{T}_{LB}(\alpha)$ is the unique first order optimality point of $\Pi_{ELB}(\cdot,\alpha)$ over all $t_2\in\mathbb{R}_+$ and $\Pi_{ELB}(\tau(\alpha),\alpha)=\Pi_{LB}(\tau(\alpha),\alpha)$, which implies that $t_{21}^\star(\alpha)$ has to be an element of $\mathcal{T}_{LB}(\alpha)$. The case where $\alpha\in(\alpha_{LB},1)$ and $t_{21}^\star\neq\tau_{ELB}(\alpha)$ leads to a similar contradiction.

Next, we study the introduction time $t_{22}^\star(\alpha)$ that maximizes $\Pi_2(t_2,\alpha)$. Analogous steps as for the derivation of the introduction time $t_{21}^\star(\alpha)$ for problem $\Pi_1(t_2,\alpha)$ reveal that for any $\alpha\in(0,1)$, $\Pi_2(t_2,\alpha)$ is decreasing in t_2. We thus conclude that $t_{22}^\star(\alpha)=0^+$ for every $\alpha\in(0,1)$.

To conclude the proof, we compare the maximizer $t_{21}^\star(\alpha)$ of Π_1 with the maximizer $t_{22}^\star(\alpha)$ of Π_2. Assume first that $\alpha\in(0,\alpha_{LB}]$. Since $t_{22}^\star(\alpha)=0^+$, we have $\Pi_2(t_{22}^\star(\alpha),\alpha)=(\frac{u}{3})^-$. Moreover, we have $t_{21}^\star(\alpha)=\tau_{LB}(\alpha)\in\mathcal{T}_{LB}(\alpha)$, and hence $\Pi_1(t_{21}^\star(\alpha),\alpha)=\Pi_{LB}(\tau_{LB}(\alpha),\alpha)$. The closed-form expression of $\Pi_{LB}(\tau_{LB}(\alpha),\alpha)$ has been provided earlier, and a direct calculation reveals that this expression dominates $(\frac{u}{3})^-$. We thus conclude that $t_2^\star(\alpha)=t_{21}^\star(\alpha)=\tau_{LB}(\alpha)\in\mathcal{T}_{LB}(\alpha)$ for $\alpha\in(0,\alpha_{LB}]$. A similar reasoning shows that $t_2^\star(\alpha)=t_{21}^\star(\alpha)=\tau_{ELB}(\alpha)\in\mathcal{T}_{ELB}(\alpha)$ for $\alpha\in(\alpha_{LB},\alpha_{EL})$, where $\alpha_{EL}\in(\alpha_{LB},1)$, and $t_2^\star(\alpha)=t_{22}^\star(\alpha)=0^+$ for $\alpha\in[\alpha_{EL},1)$. □

Proof of Theorem 2. We begin with statement 1. Using the notation of Lemma 1, one can show that $\mathrm{d}\tau_{LB}(\alpha)/\mathrm{d}\alpha\to-\infty$ and thus $t_2^\star\to 0$ as $\alpha\to 0$. Moreover, we have $\mathrm{d}\tau_{LB}(\alpha)/\mathrm{d}\alpha>0$ and $\mathrm{d}\tau_{ELB}(\alpha)/\mathrm{d}\alpha>0$ for all $\alpha\in(0,1)$ and $\delta_s\in(0,1)$, which implies that $t_2^\star$ is indeed continuously increasing in α over $\alpha\in(0,\alpha_{EL})$. The fact that $t_2^\star=0^+$ for $\alpha\in[\alpha_{EL},1)$, finally, directly follows from the statement of Lemma 1.

In view of statement 2, the proof of Lemma 1 shows that the optimal price $p_1^\star$ satisfies $p_1^\star\to u/2$ as $\alpha\to 0$ and $p_1^\star=2u/3$ for $\alpha\in[\alpha_{EL},1)$. Moreover, combining the fact that $t_2^\star$ is continuously increasing in α over

$\alpha \in (0, \alpha_{EL})$ with the expression for $p_1^\star$ derived in the proof of Lemma 1 shows that $p_1^\star$ is indeed continuous and unimodal in α over $\alpha \in (0, \alpha_{EL})$. Analogous derivations for $p_2^\star$ prove statement 3 of the theorem.

As for statement 4, finally, the proof of Lemma 1 shows that $\Pi(t_2^\star(\alpha), \alpha) = \Pi_{LB}(\tau_{LB}(\alpha), \alpha)$ for $\alpha \in (0, \alpha_{LB}]$. The expressions derived for Π_{LB} and $\tau_{LB}(\alpha)$ then imply that the optimal expected profits approach $u/2$ as $\alpha \to 0$, and that the profits are continuously decreasing in α over $\alpha \in (0, \alpha_{EL})$. The proof of Lemma 1 further shows that $t_2^\star(\alpha) = 0^+$ for $\alpha \in [\alpha_{EL}, 1)$, which implies that $\Pi(t_2^\star(\alpha), \alpha) = u/3$ in that region. □

Proof of Corollary 2. The proof of Lemma 1 implies that $p_2^\star < (1 - \alpha^{t_2^\star})u$ as well as $p_2^\star = (1 - \alpha^{t_2^\star})p_1^\star$ for $\alpha \in (0, \alpha_{LB}]$. Thus, case 2 (c) of Theorem 1 shows that the firm serves late and both product buyers when $\alpha \in (0, \alpha_{LB}]$. Similarly, the proof of Lemma 1 implies that $p_2^\star < (1 - \alpha^{t_2^\star})u$ as well as $p_2^\star \in ([1 - \alpha^{t_2^\star}]p_1^\star, p_1^\star)$ for $\alpha \in (\alpha_{LB}, \alpha_{EL})$, and we conclude from case 2 (b) of Theorem 1 that the firm serves early, late and both product buyers when $\alpha \in (\alpha_{LB}, \alpha_{EL})$. Finally, the proof of Lemma 1 shows that $t_2^\star = 0^+$ for $\alpha \in [\alpha_{EL}, 1)$. In this case, however, we have $p_2^\star > (1 - \alpha^{t_2^\star})u$ as well as $p_2^\star < p_1^\star$, and thus case 1 (b) of Theorem 1 shows that the firm serves early and late buyers. □

Proof of Corollary 3. To see that α_{LB} is constant in δ_c, we note that α_{LB} was defined in the proof of Lemma 1 as the unique value of α at which $\alpha \mapsto h(\tau_{LB}(\alpha), \alpha)$ vanishes. The statement then follows from the fact that neither $\tau_{LB}(\alpha)$ nor h depend on δ_c. To see that α_{LB} increases with δ_s, we note that $\tau_{LB}(\alpha)$ increases with δ_s and $h(t_2, \alpha)$ increases with t_2 for any fixed α. We thus conclude that $h(\tau_{LB}(\alpha), \alpha)$ also increases in δ_s for fixed α, which implies by the definition of h that the root of $\alpha \mapsto h(\tau_{LB}(\alpha), \alpha)$ must also increase in δ_s.

A similar reasoning shows that α_{EL} increases with δ_s and is constant in δ_c. □

Proof of Proposition 2. By definition, the discount buyers must satisfy $\theta \geq \theta_D$ and $\theta < \theta_{DL}$. In other words, the market contains discount buyers if and only if $\beta \in [0, 1)$ satisfies

$$\theta_{DL} = \min\left\{\frac{p_2 - \beta p_1}{u(1 - \alpha^{t_2})}, 1\right\} > \min\left\{\frac{\beta p_1}{u\alpha^{t_2}}, 1\right\} = \theta_D \iff \frac{p_2 - \beta p_1}{u(1 - \alpha^{t_2})} > \frac{\beta p_1}{u\alpha^{t_2}} \text{ and } \frac{\beta p_1}{u\alpha^{t_2}} < 1$$
$$\iff \beta < \alpha^{t_2} \cdot \frac{p_2}{p_1} \text{ and } \beta < \alpha^{t_2} \cdot \frac{u}{p_1},$$

that is, if and only if $\beta \in [0, 1)$ satisfies $\beta < \alpha^{t_2} \cdot \frac{p_2}{p_1}$ and $\beta < \alpha^{t_2}$, where the second expressions follows since $p_1 \leq u$ by assumption. □

Proof of Theorem 3. Since $\beta < \bar{\beta}$, Proposition 2 implies that there are discount buyers. We have $p_2 \geq (1 - \alpha^{t_2} + \beta)p_1$ if and only if $p_1 \leq (p_2 - \beta p_1)/(1 - \alpha^{t_2})$, which in turn holds if and only if

$$\theta_E = \frac{p_1}{u} \leq \theta_{DL} = \min\left\{\frac{p_2 - \beta p_1}{u(1 - \alpha^{t_2})}, 1\right\},$$

that is, if and only if there are no late buyers. Similarly, we have $p_2 \leq (1 - \alpha^{t_2})p_1$ if and only if $p_1 \geq p_2/(1 - \alpha^{t_2})$, which in turn holds if and only if

$$\theta_E = \frac{p_1}{u} \geq \theta_B = \min\left\{\frac{p_2}{(1 - \alpha^{t_2})u}, 1\right\},$$

that is, if and only if there are no early buyers. Finally, we have $p_2 \geq (1-\alpha^{t_2})u$ if and only if $p_2/(1-\alpha^{t_2}) \geq u$, which in turn holds if and only if

$$\theta_B \;=\; \min\left\{\frac{p_2}{(1-\alpha^{t_2})u}, 1\right\} \;=\; 1,$$

that is, if and only if there are no both product buyers. The statement of the theorem follows from the various combinations of the previously established regions. □

Proof of Corollary 4. The cases 1 (a) and 1 (c) follow from Theorem 3 since $(1-\alpha^{t_2}) \to 1$ and $\bar{\beta} \to 0$ when $\alpha \to 0$ and/or $t_2 \to \infty$. In view of case 1 (b), we note that $\theta_B = \theta_L = \theta_E = \theta_{DL}$ whenever $\alpha \to 1$ and/or $t_2 \to 0$, and hence there are only non-buyers, discount buyers and both product buyers. The cases 2 (a) and (b) from Theorem 3 since $(1-\alpha^{t_2}) \to 0$ and $\bar{\beta} \to \min\left\{\frac{p_2}{p_1}, 1\right\}$ when $\alpha \to 1$ and/or $t_2 \to 0$. □

Proof of Proposition 3. Similar to the proof of Proposition 1, we show that statement by contradiction. Assume first that the optimal prices would satisfy $p_2^\star > (1-\alpha^{t_2}+\beta)p_1^\star$. Since the optimal prices maximize the expected profits (2) over all $p_1, p_2 \in [0,u]$, they must *a fortiori* optimize (2) over all $p_1, p_2 \in [0,u]$ satisfying $p_2 \geq (1-\alpha^{t_2}+\beta)p_1$. For prices $p_2 \geq (1-\alpha^{t_2}+\beta)p_1$ and discounts $\beta < \bar{\beta}$, it follows that $\theta_B \geq \theta_L \geq \theta_{DL} \geq \theta_E > \theta_D$, and Theorem 3 implies that there are no late buyers. Hence, the expected profits (2) simplify to

$$\begin{aligned}
\Pi_{EBD}(p_1,p_2) \;&=\; p_1[\theta_B - \theta_E]_+ \;+\; \delta_s^{t_2} p_2[\theta_E - \max\{\theta_L, \theta_{DL}\}]_+ \;+\; (p_1 + \delta_s^{t_2} p_2)(1-\max\{\theta_E, \theta_B\}) \\
&\quad + \delta_s^{t_2}\beta p_1[\min\{\theta_E, \theta_{DL}\} - \theta_D]_+ \\
&= p_1(\theta_B - \theta_E) + (p_1 + \delta_s^{t_2} p_2)(1-\theta_B) + \delta_s^{t_2}\beta p_1(\theta_E - \theta_D) \\
&= p_1\left(\min\left\{\frac{p_2}{(1-\alpha^{t_2})u}, 1\right\} - \frac{p_1}{u}\right) + (p_1 + \delta_s^{t_2} p_2)\left(1 - \min\left\{\frac{p_2}{(1-\alpha^{t_2})u}, 1\right\}\right) + \delta_s^{t_2}\beta p_1\left(\frac{p_1}{u} - \frac{\beta p_1}{\alpha^{t_2} u}\right).
\end{aligned}$$

We note that for fixed p_1, the cases $p_2 > (1-\alpha^{t_2})u$ and $p_2 = (1-\alpha^{t_2})u$ generate the same expected profits since the second term in the last expression of $\Pi_{EBD}(p_1,p_2)$ vanishes in both cases. We can thus restrict the prices to $p_1 \in [0,u]$ and $p_2 \in [0,(1-\alpha^{t_2})u]$, and the expected profits (2) further simplify to

$$\Pi_{EBD}(p_1,p_2) \;=\; p_1\left(\frac{p_2}{(1-\alpha^{t_2})u} - \frac{p_1}{u}\right) + (p_1 + \delta_s^{t_2} p_2)\left(1 - \frac{p_2}{(1-\alpha^{t_2})u}\right) + \delta_s^{t_2}\beta p_1\left(\frac{p_1}{u} - \frac{\beta p_1}{\alpha^{t_2} u}\right).$$

For $(p_1^\star, p_2^\star)$ to optimize $\Pi_{EBD}(p_1,p_2)$, it has to satisfy the KKT conditions. The unique KKT point for the problem $\max\{\Pi_{EBD}(p_1,p_2) : p_2 \geq (1-\alpha^{t_2}+\beta)p_1,\ p_1[0,u],\ p_2 \in [0,(1-\alpha_2^t)u]\}$ satisfies $p_2^\star = p_1^\star(1-\alpha^{t_2}+\beta^\star)$. This, however, contradicts our initial assumption that $p_2^\star > (1-\alpha^{t_2}+\beta)p_1^\star$.

Assume now that the optimal prices would satisfy $p_2^\star < (1-\alpha^{t_2})p_1^\star$. Since the optimal prices maximize the expected profits (2) over all $p_1, p_2 \in [0,u]$, they must *a fortiori* maximize (2) over all $p_1, p_2 \in [0,u]$ satisfying $p_2 \leq (1-\alpha^{t_2})p_1$. For prices $p_2 \leq (1-\alpha^{t_2})p_1$, it follows that $1 \geq \theta_E \geq \theta_B \geq \theta_{DL} > \theta_L > \theta_D$, and Theorem 3 implies that there are no early buyers. Hence, the expected profits (2) simplify to

$$\begin{aligned}
\Pi_{LBD}(p_1,p_2) \;&=\; p_1[\theta_B - \theta_E]_+ \;+\; \delta_s^{t_2} p_2[\theta_E - \max\{\theta_L, \theta_{DL}\}]_+ \;+\; (p_1 + \delta_s^{t_2} p_2)(1-\max\{\theta_E, \theta_B\}) \\
&\quad + \delta_s^{t_2}\beta p_1[\min\{\theta_E, \theta_{DL}\} - \theta_D]_+ \\
&= \delta_s^{t_2} p_2(\theta_E - \theta_{DL}) + (p_1 + \delta_s^{t_2} p_2)(1-\theta_E) + \delta_s^{t_2}\beta p_1(\theta_{DL} - \theta_D) \\
&= \delta_s^{t_2} p_2\left(\frac{p1}{u} - \frac{p_2 - \beta p_1}{(1-\alpha^{t_2})u}\right) + (p_1 + \delta_s^{t_2} p_2)\left(1 - \frac{p_1}{u}\right) + \delta_s^{t_2}\beta p_1\left(\frac{p_2 - \beta p_1}{(1-\alpha^{t_2})u} - \frac{\beta p_1}{\alpha^{t_2} u}\right).
\end{aligned}$$

The unique KKT point for the problem $\max\{\Pi_{LBD}(p_1,p_2) : p_2 \le (1-\alpha^{t_2})p_1,\ p_1,p_2 \in [0,u]\}$ satisfies $p_2^\star = (1-\alpha^{t_2})p_1^\star$. This is a contradiction to the initial assumption that $p_2^\star < (1-\alpha^{t_2})p_1^\star$. $\square$

Proof of Theorem 4. Proposition 3 shows that the optimal prices satisfy $(1-\alpha^{t_2}+\beta)p_1^\star \ge p_2^\star \ge (1-\alpha^{t_2})p_1^\star$, which in turn implies that the expected profits (2) simplify to

$$p_1(\theta_B - \theta_E) + \delta_s^{t_2} p_2(\theta_E - \theta_{DL}) + (p_1 + \delta_s^{t_2} p_2)(1-\theta_B) + \delta_s^{t_2} \beta p_1(\theta_{DL} - \theta_D).$$

Note that in this expression, θ_D and θ_{DL} depend on β. By distinguishing the cases $p_2 \le (1-\alpha^{t_2})u$ and $p_2 \ge (1-\alpha^{t_2})u$, the problem of maximizing the expected profits can be formulated as

$$\Pi(p_1,p_2,\beta,t_2) \;=\; \max\{\Pi_1(p_1,p_2,\beta,t_2),\, \Pi_2(p_1,p_2,\beta,t_2)\},$$

where

$$\begin{aligned}\Pi_1(p_1,p_2,\beta,t_2) \;=\; \max\Big\{\Pi(p_1,p_2,\beta,t_2) :\, & \min\{(1-\alpha^{t_2}+\beta)p_1,\,(1-\alpha^{t_2})u\} \ge p_2 \ge (1-\alpha^{t_2})p_1,\\ & \beta \le \bar{\beta},\ p_1,p_2 \in [0,u]\Big\}\end{aligned}$$

and

$$\Pi_2(p_1,p_2,\beta,t_2) \;=\; \max\Big\{\Pi(p_1,p_2,\beta,t_2) :\, (1-\alpha^{t_2}+\beta)p_1 \ge p_2 \ge (1-\alpha^{t_2})u,\ \beta \le \bar{\beta},\ p_1,p_2 \in [0,u]\Big\}.$$

The KKT conditions of Π_1 and Π_2 reveal that for any fixed p_1, p_2 and t_2, the optimal solutions to Π_1 and Π_2 all satisfy $\beta^\star = \bar{\beta}$, that is, the discount buyers segment disappears. We thus conclude that the optimal dual rollover does not attract any discount buyers and hence reduces to a solo rollover. $\square$

Proof of Proposition 4. We remind the reader that the value θ of an early buyer must satisfy $\theta a(t_2)u - p_1 \ge 0$, $\theta a(t_2)u - p_1 \ge \delta_c^{t_2}(\theta u - p_2)$ and $\theta a(t_2)u - p_1 \ge \theta u - p_1 - \delta_c^{t_2} p_2$. Likewise, the value θ of a late buyer must satisfy $\delta_c^{t_2}(\theta u - p_2) \ge 0$, $\delta_c^{t_2}(\theta u - p_2) \ge \theta a(t_2)u - p_1$ and $\delta_c^{t_2}(\theta u - p_2) \ge \theta u - p_1 - \delta_c^{t_2} p_2$. In particular, a necessary condition for a consumer to be an early buyer is $\theta a(t_2)u - p_1 \ge \delta_c^{t_2}(\theta u - p_2)$, and a necessary condition for a consumer to be a late buyer is $\delta_c^{t_2}(\theta u - p_2) \ge \theta a(t_2)u - p_1$. In the following, we define a threshold introduction time $\hat{\tau}$ such that for $t_2 > \hat{\tau}$ ($t_2 < \hat{\tau}$) these necessary conditions imply that late product buyers have lower types (higher types) than early product buyers, and that $\hat{\tau} = 0$ if and only if $\alpha \in [\delta_c, 1)$.

To specify the threshold introduction time $\hat{\tau}$ for fixed α and δ_c, we define the function $f(t_2) = a(t_2) - \delta_c^{t_2}$ and let $\hat{\tau}$ the largest root of this function. We note that $f(t_2)$ is a continuous function that vanishes at $t_2 = 0$, which follows from the expressions of $a(t_2)$ and $\delta_c^{t_2}$. We further observe that $f(t_2)$ has a second, strictly positive (resp., negative) root if $\alpha < \delta_c$ (resp., if $\alpha > \delta_c$), whereas $t_2 = 0$ is the unique root if $\alpha = \delta_c$. As a result, we have $\hat{\tau} > 0$ when $\alpha < \delta_c$ and $\hat{\tau} = 0$ when $\alpha \ge \delta_c$.

Next, we note that for $t_2 > \hat{\tau}$, it follows that $f(t_2) > 0$ since $\hat{\tau}$ is the largest root and $\lim_{t_2 \to \infty} f(t_2) = 1$. This in turn implies that $\frac{p_1 - \delta_c^{t_2} p_2}{f(t_2)u} > 0$ and hence $\theta a(t_2)u - p_1 \ge \delta_c^{t_2}(\theta u - p_2)$ for $\theta < \frac{p_1 - \delta_c^{t_2} p_2}{f(t_2)u}$. A similar argument shows that when $t_2 < \hat{\tau}$, the late product buyers have higher types than early product buyers. The key difference is that $t_2 < \hat{\tau}$ is possible only when $\alpha < \delta_c$, since otherwise $\hat{\tau} = 0$. $\square$

Theorem 12 **(Market Segmentation; Reversed Preference Ordering)**. *Under a reversed preference ordering, we have the following market segmentation:*

1. *If* $p_2 \geq (1-\alpha^{t_2})\,u$ *or* $p_1 \geq (1-\delta_c^{t_2})\,u$*, none of the consumers buy both versions and*
   (a) *if* $p_2 \geq p_1/d(t_2)$*, then the market consists of early buyers (region E);*
   (b) *if* $p_2 \in (p_1/a(t_2),\, p_1/d(t_2))$*, then the market consists of early and late buyers (region EL);*
   (c) *if* $p_2 \leq p_1/a(t_2)$*, then the market consists of late buyers (region L).*
2. *If* $p_2 < (1-\alpha^{t_2})\,u$ *and* $p_1 < (1-\delta_c^{t_2})\,u$*, there are consumers that buy both versions and*
   (a) *if* $p_2 \geq p_1/d(t_2)$*, then the market also comprises early buyers (region EB);*
   (b) *if* $p_2 \in (p_1/a(t_2),\, p_1/d(t_2))$*, then the market also comprises early and late buyers (region ELB);*
   (c) *if* $p_2 \leq p_1/a(t_2)$*, then the market also comprises late buyers (region LB).*

Proofs of Theorems 5 and 12. Both proofs follow the same strategy as the proof of Theorem 1: We establish for which values of θ a consumer purchases *(i)* only the early product, *(ii)* only the late product, and *(ii)* both products. The statement of the theorem then follows from the various combinations of the previously established regions. The key difference is that, while the threshold θ_L for late buyers remains the same, the thresholds θ_E and θ_B for early buyers and both product buyers change due to the consumers' foresight. We omit the details for the sake of brevity. □

Proposition 10. *Under a reversed preference ordering and for a fixed release time* $t_2 \in (0,\hat{\tau})$*, the optimal pricing strategy* $(p_1^\star, p_2^\star)$ *in a solo rollover satisfies* $p_2^\star \in [p_1^\star/a(t_2), p_1^\star/d(t_2)]$*.*

Proof of Propositions 5 and 10. Both proofs follow the same strategy as the proof of Proposition 1: As for Proposition 5, we first establish how the firm's profit function simplifies if $p_2^\star < p_1^\star/d(t_2)$ or $p_2^\star > p_1^\star/a(t_2)$, and we subsequently show that the resulting optimal price tuples $(p_1^\star, p_2^\star)$ violate the KKT conditions of the emerging optimization problems. A similar reasoning establishes the proof of Proposition 10. We omit the details for the sake of brevity. □

Theorem 13 **(Optimal Solo Rollover Design; Reversed Preference Ordering)**. *Under a reversed preference ordering, which by Proposition 4 implies that* $\delta_c > \alpha$*, we have the following properties:*

1. *The* optimal introduction time $t_2^\star$ *satisfies* $t_2^\star = \hat{\tau}^-$ *for all* $\alpha \in (0,\delta_c)$*. In particular,* $t_2^\star$ *approaches* $-\frac{\log(2)}{\log(\delta_c)}$ *as* $\alpha \to 0$*, it is continuously decreasing in* α *over* $\alpha \in (0,\delta_c)$*, and it approaches* 0 *as* $\alpha \to \delta_c$*.*
2. *The* optimal price for the first product $p_1^\star$ *satisfies* $p_1^\star = [u(1-\delta_c^{\hat{\tau}})]/2$ *for all* $\alpha \in (0,\delta_c)$*. In particular,* $p_1^\star$ *approaches* $u/4 < u/2$ *as* $\alpha \to 0$*, it is continuously decreasing in* α *over* $\alpha \in (0,\delta_c)$*, and it approaches* 0 *as* $\alpha \to \delta_c$*.*
3. *The* optimal price for the second product $p_2^\star$ *satisfies* $p_2^\star = u/2$ *for all* $\alpha \in (0,\delta_c)$*.*
4. *The* optimal expected profit *is* $\frac{1}{4}u\left(1+\delta_s^{\hat{\tau}}-\delta_c^{\hat{\tau}}\right)$*. In particular, it approaches* $\frac{1}{4}u\left(\delta_s^{-\frac{\log(2)}{\log(\delta_c)}}+\frac{1}{2}\right)$ *as* $\alpha \to 0$*, it is continuously decreasing in* α *for* $\delta_c < \delta_s$ *and* $\alpha \in (0,\delta_c)$*, it is continuously increasing in* α *for* $\delta_c > \delta_s$ *and* $\alpha \in (0,\delta_c)$*, and it approaches* $u/4$ *as* $\alpha \to \delta_c$*.*

Proofs of Theorems 6 and 13. Both proofs follow the same strategy as the proof of Theorem 2. The key difference is that here, we restrict ourselves to introduction times $t_2 > \hat{\tau}$ to induce a myopic preference ordering (Theorem 6) or introduction times $t_2 < \hat{\tau}$ and consumer discount factors $\delta_c > \alpha$ to induce a reversed preference ordering (Theorem 13). We omit the details for the sake of brevity. □

Proof of Corollary 5. We note that under a myopic preference ordering, the statement immediately follows from Theorems 5 and 6. We now show that it is indeed optimal for the firm to select an introduction time $t_2^\star > \hat{\tau}$ and thus induce a myopic preference ordering.

To see that $t_2^\star > \hat{\tau}$, we compare the optimal expected profits from Theorems 6 and 13. Theorem 13 implies that the optimal introduction time $t_2^\star$ for the reversed preference ordering satisfies $t_2 = \hat{\tau}^-$. Moreover, one readily verifies that the expected profit of the myopic and the reversed preference orderings coincide at $t_2 = \hat{\tau}$. Finally, Theorem 6 implies that the optimal introduction time $t_2^\star$ of the myopic preference ordering satisfies $t_2^\star > \hat{\tau}$. The statement then follows from the fact that the optimal expected profit is continuous in $t_2^\star$. □

Proof of Proposition 6 The proof of Proposition 6 follows the same strategy as the proof of Proposition 2: we establish the conditions on β that imply the existence of discount buyers. The key difference is that the due to their strategic foresight, the consumers anticipate the price reduction at time $t = t_2$ when considering to purchase the early product at time $t = 0$. We omit the details for sake of brevity. □

Theorem 14 **(Market Segmentation; Reversed Preference Ordering)**. *Under a reversed preference ordering, we have:*

1. *If $p_2 \geq (1-\alpha^{t_2})\,u$ or $p_1 \geq (1-\delta_c^{t_2})\,u$, there are discount buyers but no both product buyers, and*
   (a) *if $p_2 \geq p_1/d(t_2)$, then the market also comprises early buyers (region ED);*
   (b) *if $p_2 \in (b(t_2)p_1,\, p_1/d(t_2))$, then the market also comprises early and late buyers (region ELD);*
   (c) *if $p_2 \leq b(t_2)p_1$, then the market also comprises late buyers (region LD).*
2. *If $p_2 < (1-\alpha^{t_2})\,u$ and $p_1 < (1-\delta_c^{t_2})\,u$, there are discount and both product buyers, and*
   (a) *if $p_2 \geq p_1/d(t_2)$, then the market also comprises early buyers (region EBD);*
   (b) *if $p_2 \in (b(t_2)p_1,\, p_1/d(t_2))$, then the market also comprises early and late buyers (region ELBD);*
   (c) *if $p_2 \leq b(t_2)p_1$, then the market also comprises late buyers (region LBD).*

Here, the quantity $b(t_2)$ is defined in Theorem 7.

Proofs of Theorems 7 and 14. Both proofs follow the same strategy as the proof of Theorem 3: We establish for which values of θ a consumer purchases *(i)* only the non-discounted first product, *(ii)* only the discounted first product, *(iii)* on the second product, and *(iv)* both products. The statements of the theorems then follow from the various combinations of the previously established regions. The key difference is that, while the threshold for the marginal consumer who is indifferent between late buying and discount buying remains the same, the threshold values for the marginal consumers who are indifferent between the other combinations of early buying, late buying and both product buying change due to the consumers' foresight. We omit the details for the sake of brevity. □

Proposition 11. *Under a reversed preference ordering, a fixed release time $t_2 \in (0, \hat{\tau})$ and a sufficiently deep discount $\beta \in [0, \bar{\beta})$, the optimal pricing strategy $(p_1^\star, p_2^\star)$ in a dual rollover satisfies $p_2^\star \in [p_1^\star / d(t_2), p_1^\star / a(t_2)]$.*

Proofs of Propositions 7 and 11. Both proofs follow the same strategy as the proofs of Propositions 1 and 3. As for Proposition 7, we first establish how the firm's profit function simplifies if $p_2^\star < p_1^\star / d(t_2)$ or $p_2^\star > p_1^\star b(t_2)$, and we subsequently show that the resulting optimal price tuples $(p_1^\star, p_2^\star)$ violate the KKT conditions of the emerging optimization problems. A similar reasoning established the proof of Proposition 11. We omit the details for the sake of brevity. □

Proof of Theorem 8. The proof follows the same strategy as the proof of Theorem 4: We optimize the expected profit function over $\beta \in (0, \bar{\beta}]$ and observe that the optimal discount satisfies $\beta^\star = \bar{\beta}$. The key difference is that the threshold values for the various consumer groups differ due to the consumers' foresight. We omit the details for the sake of brevity. □

Proof of Theorem 9. The proof follows the same strategy as the proof of Theorem 8: We optimize the expected profit function over $\beta \in (0, \bar{\beta}]$, but we now observe that the optimal discount satisfies $\beta^\star = \frac{\alpha^{t_2^\star}(1+\delta_c^{t_2\star})}{2[1-\delta_c^{t_2^\star}(1-\alpha^{t_2^\star})]}$, which implies that there are discount buyers and that a dual rollover is the optimal strategy for a reversed preference ordering. We omit the details for the sake of brevity. □

Proof of Theorem 10. Theorem 8 shows that a solo rollover is the optimal strategy for a myopic preference ordering, whereas Theorem 9 implies that a dual rollover is the optimal strategy for a reversed preference ordering. A similar reasoning as in the proof of Corollary 5 shows that the optimal introduction time $t_2^\star$ induces a myopic preference ordering, which implies that a solo rollover is the overall optimal strategy. We omit the details for the sake of brevity. □

Proof of Theorem 11. The proof follows the same strategy as the proof of Theorem 2: We first obtain the optimal prices for a fixed introduction t_2, and we subsequently optimize over the introduction time t_2 when fixing both prices to their optimal values (parametric in t_2). The key difference is that here, the parameter γ generates a combination of the two strategies studied in Theorems 2 and 6. We omit the details for the sake of brevity. □

Appendix B: Supplementary Material for Section 6

B.1. Semi-Digital Goods

The analysis proceeds as in Sections 3 and 4, with the key difference that now the production of a unit of either product gives rise to the costs c_1 and c_2, respectively. We begin our discussion with the myopic consumers. The expression for the expected profit becomes

$$\begin{aligned}\Pi(p_1,p_2,t_2,\beta) = &\ (p_1-c_1)[\theta_B-\theta_E]_+ + \delta_s^{t_2}(p_2-c_2)[\theta_E-\max\{\theta_L,\theta_{DL}\}]_+ \\ &+ (p_1+\delta_s^{t_2}p_2-c_1-\delta_s^{t_2}c_2)(1-\max\{\theta_E,\theta_B\}) \\ &+ \delta_s^{t_2}(\beta p_1-c_1)[\min\{\theta_E,\theta_{DL}\}-\theta_D]_+. \end{aligned} \tag{3}$$

The KKT conditions discussed in Theorem 4 imply that $p_2^\star \le p_1^\star(1-\alpha^{t_2}+\beta)$ is the necessary and sufficient condition for the optimality of the price tuple (p_1,p_2) in a solo rollover. We next investigate whether the optimal prices $(p_1^\star,p_2^\star)$ still satisfy $p_2^\star \le p_1^\star(1-\alpha^{t_2}+\beta)$ when $c_1,c_2\neq 0$. Assume, to the contrary, that $p_2^* > p_1^*(1-\alpha^{t_2}+\beta^\star)$ and that the firm employs a dual rollover strategy. Then, the expected profit simplifies to

$$\begin{aligned} &(p_1-c_1)(\theta_B-\theta_E)+(p_1-c_1+\delta_s^{t_2}[p_2-c_2])(1-\theta_B)+\delta_s^{t_2}(\beta p_1-c_1)[\theta_E-\theta_D]_+ \\ =&\ (p_1-c_1)\left(\frac{p_2}{(1-\alpha^{t_2})u}-\frac{p_1}{u}\right)+(p_1-c_1+\delta_s^{t_2}[p_2-c_2])\left(1-\frac{p_2}{(1-\alpha^{t_2})u}\right) \\ +&\ \delta_s^{t_2}(\beta p_1-c_1)\left[\frac{p_1}{u}-\frac{\beta p_1}{\alpha^{t_2}u}\right]_+. \end{aligned} \tag{4}$$

The optimal prices and discount that solve this problem are

$$p_1^\star = \frac{2(c_1+u)-c_1\delta_s^{t_2}}{4-\alpha^{t_2}\delta_s^{t_2}},\quad p_2^\star=\frac{1}{2}\left(c_2-u\alpha^{t_2}+u\right)\quad \text{and}\quad \beta^\star=\frac{(2+\alpha^{t_2}[1-\delta_s^{t_2}])+\alpha^{t_2}u}{2u+c_1(2-\delta_s^{t_2})}, \tag{5}$$

which in turn imply that $p_2^* \le p_1^*(1-\alpha^{t_2}+\beta)$ if $c_2 \le c_2^M(c_1) = c_1\frac{2(4-\alpha^{t_2}-\delta_s^{t_2})}{4-\alpha^{t_2}\delta_s^{t_2}}+\frac{\alpha^{t_2}(2+\delta_s^{t_2}[1-\alpha^{t_2}])}{4-\alpha^{t_2}\delta_s^{t_2}}$. In particular, it is immediate from these expressions that $c_2^M(c_1)>c_1$. Thus, we conclude that the firm never prefers a dual rollover when $c_2\le c_2^M(c_1)$, where $c_2^M(c_1)>c_1$.

We note that our condition is sufficient but not necessary for a dual rollover. To see this, we show two cases for $c_2>c_2^M(c_1)$ where a dual and a solo rollover emerge as optimal strategy, respectively. The two instances are identical ($v=1,\delta_c=0.7,\delta_s=0.79,\alpha=0.37,c_1=0.4$ and $c_2=1.6$) except for the value for α.

Example 1. For $\alpha=0.37$ we have $c_2^M(c_1)=1.4008<c_2$ at the introduction time $t_2^\star$ that maximizes the expected profit. If the firm employs a dual rollover by targeting early buyers, both product buyers and discount buyers, then it selects $p_1^\star\approx 1.64$, $p_2^\star\approx 1.69$, $\beta^\star\approx 0.30$ and $t_2^\star\approx 1.02$, resulting in an expected profit of approximately 0.5258. If the firm employs a solo rollover, on the other hand, then it selects $p_1^\star\approx 1.78$, $p_2^\star\approx 1.74$ and $t_2^\star\approx 3.80$, resulting in an inferior expected profit of approximately 0.5255. Hence, in this instance a dual rollover becomes optimal when c_2 is sufficiently large.

Example 2. For $\alpha=0.3$ we have $c_2^M(c_1)=1.40419<c_2$ at the introduction time $t_2^\star$ that maximizes the expected profit. If the firm employs a solo rollover, then it selects $p_1^\star\approx 1.80$, $p_2^\star\approx 1.77$ and $t_2^\star\approx 3.20$, resulting in an expected profit of approximately 0.5295. If the firm employs a dual rollover, on the other hand, then it selects $p_1^\star\approx 1.64$, $p_2^\star\approx 1.69$, $\beta^\star\approx 0.30$ and $t_2^\star\approx 0.84$, resulting in an inferior expected profit of approximately 0.5263. Hence, in this instance a solo rollover remains optimal despite c_2 being sufficiently large.

We now consider a market that consists solely of strategic consumers. Assume first that the firm induces a myopic preference ordering. In that case, Theorem 8 implies that for $p_1^\star/d(t_2) \leq p_2^\star \leq b(t_2)p_1^\star$, a solo rollover is the optimal strategy. One can further verify that $p_2^\star \leq b(t_2)p_1^\star$ is indeed the necessary and sufficient condition for the optimality of a solo rollover strategy. We now investigate whether $p_2^\star \leq b(t_2)p_1^\star$ holds when the marginal production costs c_1 and c_2 are nonzero. In analogy to the discussion for the myopic case, we obtain the optimal prices $(p_1^\star, p_2^\star, \beta^\star)$ assuming a dual rollover strategy. We then show that there exist production cost threshold functions $c_2^S \geq c_2^M$ such that $p_2^\star \leq b(t_2)p_1^\star$ for $c_2 \leq c_2^S(c_1)$. We thus conclude that for $c_2 \leq c_2^S(c_1)$, a solo rollover remains the optimal strategy under a myopic preference ordering. As the final step, we confirm that for $c_2 \leq c_2^S(c_1)$, the firm sets $t_2^\star > \hat{\tau}$ and hence never induces a reversed preference ordering. Figure 9 illustrates the cutoff values $c_2^M(c_1)$ and $c_2^S(c_1)$ in a numerical example.

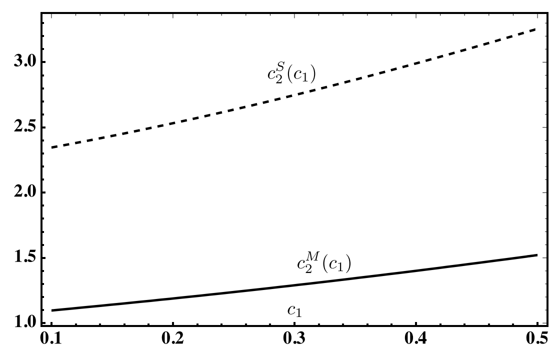

Figure 9 **Cutoff values $c_2^M(c_1)$ and $c_2^S(c_1)$.**

In this example, we set $v = 1$, $\delta_c = 0.7$, $\delta_s = 0.79$ and $\alpha = 0.37$.

B.2. Welfare Analysis

In this section, we discuss the welfare implications of the optimal rollover strategy, where welfare (or total surplus) is defined as the sum of the consumers' net surplus and the firm's profit. In particular, we state and prove the Lemmas 2 and 3 below, which jointly imply Proposition 9 from the main paper.

B.2.1. Myopic Consumers We being our analysis with myopic consumers. We first recall the implications of obsolescence on the firm's profit as implied by Theorem 2. As a result of myopia, consumers purchase goods that they would not have chosen at the given prices if they had taken future releases into account. Hence, the firm's pricing power improves with the obsolescence $(1-\alpha)$, that is, the discrepancy between the expectation and actual perception of the market. Thus, the firm's profit is maximized at $\alpha = 0$.

The actual welfare that the consumers experience at time t_2 is different to what they have contemplated at time $t = 0$. Myopic consumers believe that their net utility is $\theta u - p_1$ when they purchase the first product at time $t = 0$, while in reality their net utility will be $\theta[1 - \delta_c^{t_2}(1 - \alpha^{t_2})]u - p_1$ if they do not upgrade and $\theta u - p_1 - \delta_c^{t_2} p_2$ if they upgrade at time t_2. Therefore, the smaller the decay factor α, the lower the consumer surplus is *ex-post*. The consumer welfare is maximized at sufficiently large values of α. Lemma 2 summaries the welfare implications, and the left panel of Figure 10 illustrates the discussion with a numerical example.

Lemma 2 (**Welfare Implications; Myopic Consumers**). *For fixed δ_s and δ_c, the welfare of the market participants exhibits the following dependence on the decay factor α.*

1. ***Firm.*** *The welfare of the firm* decreases *with α.*
2. ***Consumers.*** *The welfare of myopic consumers* strictly increases *with α for $\alpha \in (0, \alpha_{EL})$, and it satisfies $CS(\alpha) = u/9^+$ for $\alpha \in [\alpha_{EL}, 1)$. Moreover, there exists $\hat{\alpha} \in (0, \alpha_{LB})$ such that for $\alpha \in (0, \hat{\alpha})$, the consumers experience a negative net welfare ex post.*

Proof of Lemma 2. The first statement follows directly from Theorem 2. As for second statement, we express the consumer surplus $CS(\alpha)$ as

$$\begin{cases} CS_{LB}(\alpha) = \int_{\theta_L^\star}^{\theta_B^\star} \delta_c^{t_2^\star}(\theta u - p_2^\star)\mathrm{d}\theta + \int_{\theta_B^\star}^{1}(\theta u - p_1^\star - \delta_c^{t_2^\star} p_2^\star)\mathrm{d}\theta & \text{if } \alpha \in (0, \alpha_{LB}], \\ CS_{ELB}(\alpha) = \int_{\theta_L^\star}^{\theta_E^\star} \delta_c^{t_2^\star}(\theta u - p_2^\star)\mathrm{d}\theta + \int_{\theta_E^\star}^{\theta_B^\star}\left(\theta u(1 - \delta_c^{t_2^\star}[1 - \alpha^{t_2^\star}]) - p_1^\star\right)\mathrm{d}\theta + \int_{\theta_B^\star}^{1}(\theta u - p_1^\star - \delta_c^{t_2^\star} p_2^\star)\mathrm{d}\theta & \text{if } \alpha \in (\alpha_{LB}, \alpha_{EL}), \\ CS_{EL}(\alpha) = \int_{\theta_L^\star}^{\theta_E^\star} \delta_c^{t_2^\star}(\theta u - p_2^\star)\mathrm{d}\theta + \int_{\theta_E^\star}^{1}\left(\theta u(1 - \delta_c^{t_2^\star}[1 - \alpha^{t_2^\star}]) - p_1^\star\right)\mathrm{d}\theta & \text{if } \alpha \in [\alpha_{EL}, 1], \end{cases}$$

where $\theta_B^\star = \frac{p_2^\star}{(1-\alpha^{t_2^\star})u}$, $\theta_L^\star = \frac{p_2^\star}{u}$ and $\theta_E^\star = \frac{p_1^\star}{u}$, and $(p_1^\star, p_2^\star, t_2^\star)$ are as characterized in Lemma 1 and Theorem 2.

We first consider the region $\alpha \in (0, \alpha_{EL})$. Lemma 1 implies that $CS_{LB}(\alpha)$ and $CS_{ELB}(\alpha)$ are continuously differentiable over $\alpha \in (0,1)$ and $CS_{LB}(\alpha) = CS_{ELB}(\alpha)$ at $\alpha = \alpha_{LB}$. Therefore, $CS(\alpha)$ is continuously differentiable over $\alpha \in (0, \alpha_{EL})$. The statement $\frac{\partial CS_{LB}}{\partial \alpha} > 0$ and $\frac{\partial CS_{ELB}}{\partial \alpha} > 0$ over $\alpha \in (0,1)$ follows from the expressions of $CS_{LB}(t_2, \alpha)$ and $CS_{ELB}(t_2, \alpha)$ as well as Theorem 2, which characterizes $\frac{\partial t_2^\star}{\partial \alpha}$, $\frac{\partial p_1^\star}{\partial \alpha}$ and $\frac{\partial p_2^\star}{\partial \alpha}$.

Next, we show that there exists a unique point $\hat{\alpha}$ in the region $\alpha \in (0, \alpha_{LB})$ such that $CS(\hat{\alpha}) = 0$. First, we eliminate the region $(\alpha_{LB}, \alpha_{EL})$ since $CS_{ELB}(\alpha) > 0$, which follows from the closed-form expressions of $CS_{ELB}(\alpha)$. However, we cannot obtain a closed-form expression for $CS_{LB}(\alpha)$ since we do not have a closed-form expression for $t_2^\star$ when $\alpha \in (0, \alpha_{LB})$. To show that $\hat{\alpha} \in (0, \alpha_{LB})$, we recall that $\frac{\partial CS_{LB}}{\partial \alpha} > 0$ and confirm that $CS_{LB}|_{\alpha \to 0} = u\left(1 - 2\delta_c^{t_2^\star}\right)/8 < 0$ since $\delta_c^{t_2^\star} > 1/2$ in this region. Thus, it follows that $CS_{LB} < 0$ if and only if $\alpha \in (0, \hat{\alpha})$, where $\hat{\alpha} \in (0, \alpha_{LB})$.

Finally, we consider the region $\alpha \in [\alpha_{EL}, 1)$. By Lemma 1, we have $t_2^\star = 0^+$ and hence $CS_{EL}(\alpha) = u/9^+$. Recall that $\frac{\partial CS_{ELB}}{\partial \alpha} > 0$ and note that $\lim_{\alpha \to 1} CS_{ELB} = (1 + \delta_s^{t_2})/8$. Hence, it follows that $CS_{EL}(\alpha_{EL}) > CS_{ELB}(\alpha_{EL})$, and we conclude that $CS(\alpha)$ attains its maximum when $\alpha \in (\alpha_{EL}, 1)$. This completes the proof of second statement. □

In addition to the consumers' myopia, the discount factor δ_c plays a crucial role for the consumer welfare. When consumers are very patient (high δ_c), the discrepancy between the contemplated and actual experience will be higher. As a result, $\hat{\alpha}$ increases with δ_c. That is, myopic consumers experience a negative welfare over a wider range of α when they are patient but do not foresee future releases. Corollary 6 summarizes the discussion, and the right panel of Figure 10 depicts $\hat{\alpha}$ over $\delta_c \in (0,1)$ for fixed values of v and δ_s.

Corollary 6. *The threshold decay factor $\hat{\alpha}$ in Lemma 2 increases with δ_c.*

Proof of Corollary 6. Using the expression of $CS_{LB}(t_2, \alpha, \delta_c)$, one can show that $CS_{LB}(t_2, \alpha, \delta_c)$ decreases with δ_c for fixed values of t_2 and α. In addition, Lemma 2 shows that $CS_{LB}(\alpha)$ increases with α and that $\hat{\alpha} \in (0, \alpha_{LB})$. These three findings jointly imply that the threshold decay factor $\hat{\alpha}$ increases with δ_c. □

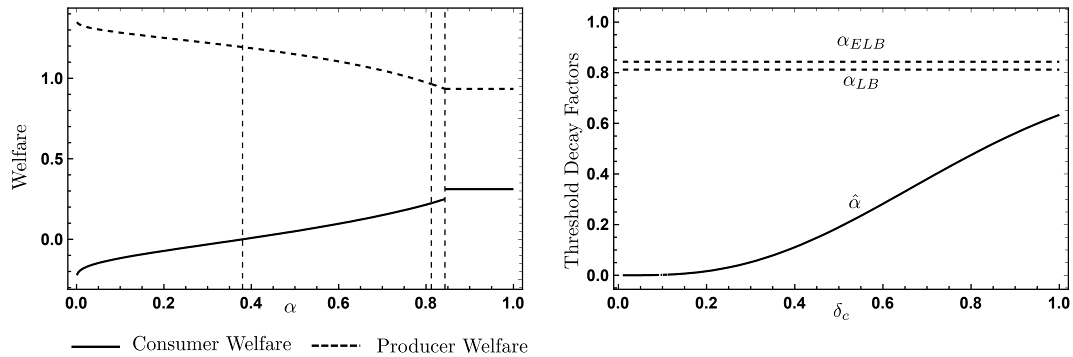

Figure 10 Welfare implications of obsolescence in the presence of myopic consumers. We use the parameters $v = 1$ and $\delta_s = 0.79$ for both graphs and $\delta_c = 0.7$ in the left graph. For the left graph, the vertical dashed lines denote $\hat{\alpha}$, α_{LB} and α_{ELB}, respectively.

B.2.2. Strategic Consumers We now discuss the welfare implications when the firm faces strategic consumers. Recall that the firm does not release the second product when $\delta_c \geq \delta_s$, in which case the perception of obsolescence does not matter. In this section, we therefore assume that $\delta_c < \delta_s$.

As discussed in the main text, the strategic consumer foresight leads to lower prices and longer release intervals than in a market composed of myopic consumers. We also showed that the higher the obsolescence (low α), the higher the pricing power of the firm at time t_2, since a strategic consumer has a higher incentive to wait for the second version, which in turn motivates the firm to set shorter release intervals to limit the obsolescence and a lower price for the first product to encourage consumers to purchase both goods. As a result, strategic consumers become both product buyers and enjoy higher quality overall by paying sufficiently low total prices $p_1 + \delta_c^{t_2} p_2$ and experiencing shorter release intervals. This thought experiment suggests that also strategic consumers, and not only the firm, might be better off with a high level of obsolescence.

There is a caveat to this scenario, however: If the consumers are sufficiently patient, then they feel the obsolescence strongly and the total prices they face are higher as they discount the prices at time t_2 less. As a result, a patient consumer might be better off with a low obsolescence. An interesting question here is whether the consumers prefer no obsolescence when they are patient. To answer this question, we revisit Theorem 6, which shows that the firm's pricing power increases when α approaches 1 (for $\alpha \in [\alpha_{EL}, 1)$) and none of the consumers buy both products. In other words, the price and the release time advantage, received as a result of being strategic, decreases. Therefore, we would expect a patient strategic consumer to prefer a low obsolescence to a no-obsolescence scenario. Lemma 3 formalizes this discussion.

Lemma 3 **(Welfare Implications; Strategic Consumers)**. *For $\delta_s > \delta_c$, the welfare of the market participants exhibits the following dependence on the decay factor α.*

1. ***Firm.*** *The welfare of the firm* decreases *with α.*
2. ***Consumers.*** *There is a threshold value of the consumer discount factor $\delta'_c \in (0, \delta_s)$ such that the welfare of strategic consumers is maximized at $\alpha \to 0$ if $\delta_c \leq \delta'_c$ and at $\alpha \in (\alpha_{LB}, \alpha_{EL})$ if $\delta_c > \delta'_c$.*

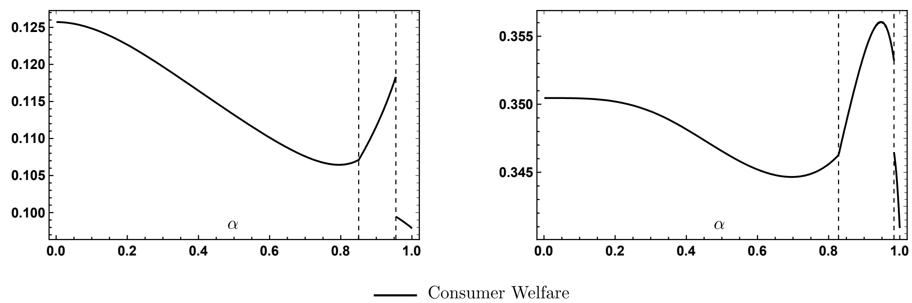

Figure 11 Welfare implications of obsolescence in the presence of strategic consumers. We use the parameters $v = 1$ and $\delta_s = 0.79$ for both graphs (implying $\delta_c' \approx 0.57$). We set $\delta_c = 0.37$ for the left graph and $\delta_c = 0.7$ for the right graph. The vertical dashed lines denote α_{LB} and α_{ELB}, respectively.

Figure 11 illustrates the two cases of Lemma 3: the left panel for $\delta_c \leq \delta_c'$ and the right panel for $\delta_c > \delta_c'$.

Proof of Lemma 3. The first statement directly follows from Theorem 2. As for the second statement, we evaluate the consumer surplus $CS(\alpha)$ in analogy to Lemma 2, except that the threshold values $\theta_B^\star$, $\theta_L^\star$ and $\theta_E^\star$ are derived using the expressions for $(p_1^\star, p_2^\star, t_2^\star)$ implied by Theorem 6. We thus obtain $\theta_B^\star = \max\{p_2^\star/[(1-\alpha^{t_2^\star})u], 1\}$, $\theta_L^\star = p_1^\star/u$ and $\theta_E^\star = (p_1^\star - \delta_c^{t_2^\star} p_2^\star)/(a(t_2^\star) - \delta_c^{t_2^\star})$.

We show the second statement by verifying that *(i)* $CS_{LB}(\alpha)$ is convex (with one local minimum) over $\alpha \in (0, \alpha_{LB}]$; *(ii)* $CS_{ELB}(\alpha)$ is increasing in α over $\alpha \in (\alpha_{LB}, \alpha_{EL})$ and concave (with one local maximum) over $[\alpha_{EL}, 1)$ for $\delta_c \leq \delta_c'$, and $CS_{ELB}(\alpha)$ is concave in α (with one local maximum) over $\alpha \in (\alpha_{LB}, \alpha_{EL})$ and decreasing in α over $\alpha \in [\alpha_{EL}, 1)$ for $\delta_c > \delta_c'$; *(iii)* $CS_{EL}(\alpha)$ decreases with α over $\alpha \in [\alpha_{EL}, 1)$; *(iv)* there exists a unique δ_c' such that $CS_{LB}(\alpha)|_{\alpha\to 0} > CS_{ELB}(\alpha_{EL})$ iff $\delta_c < \delta_c'$; and *(v)* $CS_{ELB}(\alpha_{EL}) > CS_{EL}(\alpha_{EL})$.

To prove the statements *(i)*, *(ii)* and *(iii)*, one can use the properties outlined in Theorem 6 about $\frac{\partial t_2^\star}{\partial \alpha}$, $\frac{\partial p_1^\star}{\partial \alpha}$ and $\frac{\partial p_2^\star}{\partial \alpha}$ as well as the expressions for $CS_{LB}(t_2, \alpha)$, $CS_{ELB}(t_2, \alpha)$ and $CS_{EL}(t_2, \alpha)$. We remind the reader that we can provide closed-form expressions for $p_1^\star$ and $p_2^\star$ but not for $t_2^\star$, and hence one needs to use implicit differentiation for $\frac{\partial t_2^\star}{\partial \alpha}$.

As for statement *(iv)*, we evaluate the function $CS_{LB}(t_2, \alpha)$ at the limit where $\alpha \to 0$, which in turn implies that $CS_{LB}(\alpha) \to u/8$. As for $CS_{ELB}(\alpha_{EL})$, we evaluate the function $CS_{ELB}(t_2, \alpha)$ at $\alpha \to 1$ and obtain $CS_{ELB}(\alpha) \to u(1 + 3\delta_c^{t_2^\star})/8$. By statement *(ii)*, we have $CS_{ELB}(\alpha_{EL}) > u(1 + 3\delta_c^{t_2^\star})/8$ and hence $CS_{ELB}(\alpha_{EL}) > CS_{LB}(\alpha)|_{\alpha\to 0}$ for $\delta_c > \delta_c'$. As for $\delta_c \leq \delta_c'$, we define a function $f(\delta_c) := u/8 - CS_{ELB}(\alpha_{EL})$, which satisfies $f(\delta_c)|_{\delta_c \to 0} > 0$ and $f(\delta_c)|_{\delta_c \to \delta_s} < 0$, and which is decreasing in δ_c. Therefore, there exists a unique point at which f vanishes, and by statement *(ii)* f vanishes at δ_c'. Thus, for $\delta_c \leq \delta_c'$, we have $f(\delta_c) > 0$, which implies the statement *(iv)*.

As for statement *(v)*, we do not have closed-form expressions for $CS_{ELB}(\alpha_{EL})$ and $CS_{EL}(\alpha_{EL})$. Therefore, we compare $CS_{ELB}(x, \alpha)$ with $CS_{EL}(y, \alpha)$ at the point where x and y satisfy the first-order optimality conditions for Π_{ELB} and Π_{EL}, respectively, and where $\alpha = \alpha_{EL}$. We show these conditions by plotting the functions over the whole domain. □