

(*Reflection Positivity and Remote Detectability*). In other words, von Neumann algebras have a secret $O(2)$ -symmetry.

What is the “ $O(2)$ -action” on algebras that von Neumann algebras are the fixed points of? $L^p(A)$ is not invertible as an A -bimodule, meaning only “part” of $O(2)$ acts; how to formulate “noninvertible $O(2)$ -actions”? The L^p -space “ $L^{2\pi/\theta}$ ” is only sensible when θ is convex (positive); what physics rules out concave angles? Are there other reflection-positive remote-detectable flat-space Lorentz-invariant 2D QFTs with boundary, or are von Neumann algebras the exact classification?

We have no new results or solutions to these puzzles. Our hope is that, by raising these puzzles, we will find a story that extends to higher dimensions. Perhaps, for example, there will be “von Neumann 2-algebras” (maybe bicommutant categories of [Hen17, HP23]?) that are selected by an $O(1)^3$ -symmetry but are secretly $O(3)$ -symmetric.

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Teichmüller modular forms and holomorphic vertex operator algebras

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(joint work with Giulio Codogni)

Classical modular forms are holomorphic functions on the upper half plane \mathbb{H} satisfying certain functional equations related to the action of the modular group $SL(2, \mathbb{Z})$ on \mathbb{H} [9]. They are deeply related to the geometry of the moduli space of genus one compact complex curves. Vertex operator algebras (VOAs) give a mathematical description of chiral two-dimensional conformal field theories (chiral CFTs) [3]. One of the most remarkable and intriguing feature of VOAs is their relationship with modular forms.

VOAs with trivial representation theory are called holomorphic. They give a special important class of (modular invariant) two-dimensional CFTs. Their genus one partition functions give rise to classical modular forms. A very important example of holomorphic VOA is given by the moonshine VOA V^\natural having central charge $c = 24$ and automorphism group isomorphic to the monster finite simple group \mathbb{M} , see [3, 5]. A central open problem in VOA theory is the conjecture on the uniqueness of the moonshine VOA [3] that can be stated in the following

way: if $V = \bigoplus_{n \in \mathbb{Z}_{\geq 0}} V_n$ is a holomorphic VOA with central charge $c = 24$ and weight-one subspace $V_1 = \{0\}$ then V is isomorphic to the moonshine V^\natural .

Teichmüller modular forms are higher genus generalizations of classical modular forms [7]. For genus $g > 1$ the upper half plane is replaced by the Teichmüller space T_g and the higher genus modular group Γ_g is the (pure) mapping class group. The quotient T_g/Γ_g is the moduli stack \mathcal{M}_g of genus g complex compact curves. Examples comes from the genus g theta series Siegel modular forms Θ_L associated to an unimodular even positive-definite lattice L .

In [1] we study the connection between holomorphic VOAs and Teichmüller modular forms and analyze various consequence of this connection. This is done by means of the concept of genus g partition functions. For a general two-dimensional CFT defined through functional integrals of fields living on Riemann surfaces, provided that the latter functional integrals make sense and have the right mathematical properties. In the influential paper [4] Friedan and Shenker argued that a two-dimensional CFT is completely determined by the collection $\mathcal{Z} = \{\mathcal{Z}_g : g \in \mathbb{Z}_{\geq 0}\}$ of all genus g partition functions and moreover, that this allows a description of the CFT in terms of the geometry of the moduli spaces \mathcal{M}_g .

In the case of a holomorphic VOA one can define all the genus g partition functions as formal power series without the need of functional integrals. A central result in [1] is the following: if V is a holomorphic VOA with central charge c then, for every non-negative integer g , the genus g partition function of V gives rise in a natural way to a non-zero Teichmüller modular form of weight $c/2$ unique up to a multiplication by a complex number. This can be seen as a generalization of the construction of Teichmüller modular forms from unimodular lattices, cf. [10]. Moreover, it gives strong constraints on the partition functions of holomorphic VOAs. As an example we prove that the validity of a weak form of the Harris-Morrison slope conjecture about the geometry of the moduli spaces of compact complex curves would imply that if V is a holomorphic VOA with central charge $c = 24$ and $V_1 = \{0\}$ then $\mathcal{Z}_V = \mathcal{Z}_{V^\natural}$.

Another result in [1] is the clarification of the relation between unitary holomorphic VOAs having the same genus g partition function for all g in connection with the conjecture in [4]. To this end, for any unitary holomorphic VOA V , we define unitary vertex operator subalgebra PV of V naturally associated with the collection $\mathcal{Z}_V = \{\mathcal{Z}_{V,g} : g \in \mathbb{Z}_{\geq 0}\}$ of all genus g partition functions of V . It turns out that $PV \subset V^{\text{Aut}(V)}$.

If V and U are unitary we show that $\mathcal{Z}_V = \mathcal{Z}_U$ if and only if there is a unitary operator $\Phi : V \rightarrow U$ restricting to a vertex operator algebra isomorphism $\phi : PV \rightarrow PU$ and such that $\Phi Y^V(a, z) = Y^U(\phi(a), z)\Phi$ for all $a \in PV$. In particular, if the PV -module V has a unique VOA structure then U and V must be isomorphic. These relations can be also understood in terms of certain subfactors arising from inclusions of unitary VOAs through their correspondence with conformal nets [2]. This is because, if V is strongly local in the sense of [2] then the inclusion $PV \subset V$ gives rise to an inclusion of conformal nets $\mathcal{A}_{PV} \subset \mathcal{A}_V$ and to a corresponding irreducible subfactor $\mathcal{A}_{PV}(I) \subset \mathcal{A}_V(I)$ for any interval $I \subset S^1$.

These results open new perspectives on the conjectured uniqueness of the moonshine VOA and relate it to other important conjectures in different areas of mathematics. Assume for example that PV^{\natural} coincide with the monster orbifold $V^{\natural M}$ and that the latter is strongly rational. Then, the uniqueness of V^{\natural} would follow from the weak Harris-Morrison slope conjecture together with the results in [8].

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A 1d gapped boundary of a chiral theory over a 2d invertible subalgebra

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We discuss a gapped quantum Hamiltonian in $2 + 1d$ with a gapped boundary where the anyon theory of the bulk is chiral without any nontrivial boson. The example crucially uses a subalgebra of local operators that is not an infinite tensor product of matrix algebras [1].

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