

Either or Both Competition: A “Two-sided” Theory of Advertising with Overlapping Viewerships*

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Abstract

In media markets, consumers spread their attention to several outlets, increasingly so as consumption migrates online. The traditional framework for competition among media outlets rules out this behavior by assumption. We propose a new model that allows consumers to choose multiple outlets and use it to study the effects on advertising levels and the impact of entry and mergers. We identify novel forces which reflect outlets’ incentives to control the composition of their customer base. We link consumer preferences and advertising technologies to market outcomes. The model can explain several empirical regularities that are difficult to reconcile with existing models.

Keywords: Media Competition, Two-Sided Markets, Multi-Homing, Viewer Composition, Viewer Preference Correlation

JEL-Classification: D43, L13, L82, M37

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1 Introduction

A central question in the ongoing debate about the changing media landscape is how competitive forces shape advertising levels and revenues and hence assist in achieving a number of long-standing public interest goals such as enhancing entry and diversity of content. In media markets, outlets fight for consumer attention and for the accompanying stream of advertising revenues. Online advertising networks, such as the Google and Yahoo ad-networks, and traditional broadcasting stations, such as CNN and Fox News, are among the most prominent examples.

The traditional approach in media economics posits that consumers stick to the outlet they like best (for example, Anderson and Coate, 2005). So, if anything, consumers choose *either* one outlet *or* some other. Competition is for *exclusive* consumers as all outlets are restricted a priori to be perfect substitutes at the individual level. While compelling, this approach fails to account for the fact that many consumers satisfy their content needs on *multiple* outlets. This is increasingly so as content moves from paper and TV towards the Internet. In fact, many contend that a distinguishing feature of online consumption is the users' increased tendency to spread their attention across a wide array of outlets. Table 1 shows the reach of the six largest online advertising networks, that is, the fraction of the U.S. Internet users who, over the course of December 2012, visited a website belonging to a given network. This table shows that while Google can potentially deliver an advertising message to 93.9% of all Internet users, even the smallest of the six networks (run by Yahoo!), can deliver a whopping 83.3%. The table highlights a key feature of these markets: different outlets provide advertisers alternate means of reaching the same users.

| Rank | Property | Unique Visitors (000) | % Reach |
|------|-------------------------------------|-----------------------|---------|
| | Total Internet : Total Audience | 221,486 | 100.0 |
| 1 | Google Ad Network** | 208,074 | 93.9 |
| 2 | Specific Media** | 198,119 | 89.5 |
| 3 | Federated Media Publisher Network** | 193,453 | 87.3 |
| 4 | AOL Advertising** | 186,595 | 84.2 |
| 5 | AT&T AdWorks** | 185,757 | 83.9 |
| 6 | Genome from Yahoo!** | 184,556 | 83.3 |

Table 1. Top 6 Online Ad Networks by reach.
(Source: Comscore press release: “ComScore MMX Ranks Top 50 U.S. Web Properties for December 2012,” 28th January 2013)

Motivated by these observations, the paper has two goals. First, we propose an alternative model of competition that replaces the assumption of perfect substitutability by allowing consumers to access content on *multiple* outlets. So, with two outlets, consumers can choose *either* one outlet or *both* (or none). Specifically, we work under the (extreme) assumption that consumer demand for one outlet does not affect the demand for another outlet. This is what we call *either or both* competition in contrast with the standard framework discussed above. We claim that this model of competition is an appealing alternative to existing ones for several reasons. It is a good approximation of reality in some non-trivial contexts where substitutability is limited. For example, choosing Facebook.com for online social networking services is arguably orthogonal to choosing Yelp.com as one’s supplier of restaurant reviews.¹ Moreover, and somewhat surprisingly, Gentzkow, Shapiro and Sinkinson (2014) document

¹According to the source supra cited, Facebook and Yelp are among the top 10 most-visited U.S. websites and in fact

limited substitutability even in traditional media markets such as that of U.S. newspapers. They show that on average 86% of an entrant’s circulation comes from households reading multiple newspapers or households who previously did not read any newspaper.

Second, we apply the model to study the market provision of advertising opportunities, which has been the focus of a large literature in media markets. For instance, does increased competition between media outlets reduce the amount of ads? Is competition weakened if outlets supply diverse content? For example, should we expect the impact of entry of Fox News on MSNBC’s choices to be different than that of Fox Sports on ESPN’s choices?² We propose a characterization of the incentives to provide advertising opportunities in duopoly and draw implications for the equilibrium advertising levels and for the impacts of entry and mergers. We also link consumer preferences and advertising technologies to market outcomes.

In particular, we consider competition between two ad-financed outlets that receive demand from consumers and advertisers. Consumers dislike ads whereas advertisers want to reach as many consumers as possible. Outlets choose the overall quantity of ads and then sell it to advertisers. In doing so, they trade off reduced consumer demand with higher ad-revenues per consumer.

A key component of our model is that consumers who cannot be reached through the rival outlet are more valuable to an outlet than those who can. Intuitively, the rents associated to the latter consumers are competed away. So ‘shared’ or ‘overlapping’ business is worth relatively less. This implies in our model that outlets do not only care about the overall consumer demand *level*, as in existing models, but also about its *composition*, i.e., the fraction of exclusive versus overlapping consumers.³ Indeed, it is common for ad networks to assess the extent of overlap and for advertisers to take into account the extent of duplication in large cross-outlet campaigns.⁴

Characterizing the equilibrium choices of the outlets, we find that accounting for multi-homing changes the nature of competition substantially. In particular, we show that two novel forces come into play when some consumers are shared. In duopoly, multi-homers receive advertising messages from two different sources. With diminishing returns from advertising, this implies that ads are less valuable than they are for a monopolist. This *duplication effect* induces competing outlets to supply *fewer* ads. Second, the fact that multi-homing consumers are of lower value implies that a reduction in the advertising level is less beneficial for a competing outlet than for a monopoly outlet. This occurs because such a reduction leads to the acquisition of some less valuable multi-homing consumers, whereas for a monopoly outlet all consumers are exclusive. As a result, duopolists are less wary of increases in advertising level. This *business-sharing effect* induces competing outlets to raise advertising levels. We provide a full

belong to different advertising networks.

²Fox News has, arguably, a conservative bias, so it is unlikely to appeal to MSNBC’s core liberal viewers. In contrast, Fox Sports, arguably, caters to the same preferences as ESPN.

³This is consistent with the well-documented fact in the television industry that the per-viewer fee of an advertisement on programs with more viewers is larger. In the U.S., for instance, Fisher, McGowan and Evans (1980) find this regularity. Our model accounts for it since reaching the same number of eyeball pairs through broadcasting a commercial to a large audience implies reaching more viewers than reaching the same number of eyeball pairs through a series of commercials to smaller audiences, because the latter audiences might have some viewers in common. See Ozga (1960) for an early observation of this fact.

⁴For example, Google assesses the effect on an advertising campaign for auto insurance on its Display Network (GDN) by emphasizing that GDN “exclusively reaches 30% of the auto-insurance seekers” that do not visit Yahoo, 36% that do not visit Youtube and so on. See “Google Display Network vs. Portal Takeovers for Auto Insurance seekers” available at <http://www.google.com/think/research-studies/>.

characterization of how the novel effects interact and shape equilibrium outcomes.

To understand under what conditions either effect prevails, we trace out the impact of competition to two sources: a preference-driven and a technology-driven source. We first determine how the correlation of consumer preferences affects the equilibrium advertising level. A key observation is that a positive correlation implies that there are relatively many overlapping and only few exclusive consumers. In particular, if preferences are positively correlated, a consumer with a high (low) value for outlet 1 is also likely to have a high (low) value for outlet 2. Therefore, he joins either both or no outlet with a high probability. By reducing its advertising level, an outlet attracts more consumers, both single-homers and multi-homers. Compared to its customer base, these marginal consumers are comprised of a larger portion of single-homers. Since single-homers are more valuable, there is high incentive to reduce the advertising level. The duplication effect dominates the business-sharing effect, leading to low advertising levels in equilibrium. Conversely, if the consumer preference correlation is negative, advertising levels increase with entry.

We provide a first empirical pass that provides suggestive evidence for these results, using data from the U.S. cable TV industry. In this industry, broadcasters and advertisers meet on a seasonal basis at an “upfront” event to sell commercials on the networks’ upcoming programs. Since at this stage the networks’ supply of commercial breaks is already determined, channels compete (among other things) in advertising levels. We exploit the changes in competition brought about by entry of TV channels in the 1980s and 1990s. While we find in aggregate an increase in the advertising levels after entry, this increase was smaller (or even negative) for channels in segments such as sports or movies & series, where viewer preference correlation can be expected to be positive.⁵

The interplay between the duplication and the business-sharing effect has also implications for content choice. Increasing the extent of differentiation (that is, choosing a content that is more negatively correlated with the one of competitors) attracts more exclusive viewers. At the same time, this induces the rival to compete more aggressively by increasing its supply of ads. As a consequence, choosing an intermediate level of correlation might be optimal.

On the technology side, we show that differences in the returns from advertising to overlapping and exclusive consumers determine if advertising levels increase or decrease after entry. To build intuition, suppose that multi-homing consumers are relatively *more* difficult to inform through advertising (for instance, because they spread their attention thin; hence, spending a smaller amount of it on each outlet than exclusive ones). This enhances the business-sharing effect because it further lowers the opportunity cost of losing shared business. Other things held constant, this leads to increased ad levels in duopoly. On the other hand, the strength of the duplication effect increases as well if marginal returns from an extra ad to overlapping consumers diminish compared to the returns from an extra ad to exclusive consumers. This tends to reduce advertising levels in duopoly. Due to these countervailing forces, the result is ambiguous. However, we show that the business-sharing effect dominates if the advertising technology takes the widely-used exponential form.⁶

On the normative side, we show that the conventional wisdom that there is an inefficiently amount

⁵The finding that advertising levels rise with competition is also in line with the so-called “Fox News Puzzle,” which is that the wave of channel entry during the 1990s in the U.S. cable TV industry coincided with an *increase* in advertising levels on many channels.

⁶If multi-homing consumers are easier to inform through advertising, both effects are also present but with opposing implications.

of advertising holds in our setting. Ad-financed platforms, the argument goes, do not directly internalize viewer welfare but only do so to the extent that more viewers increase ad revenues. We add to this debate by showing that mergers can actually mitigate the bias.

The rest of the paper is organized as follows: Section 2 discusses related literature. Section 3 introduces the model and Section 4 presents some preliminary analysis. Section 5 analyzes outlet competition and presents the main trade-offs of our model. Section 6 considers the effects of viewer preference correlation and Section 7 explores the advertising technology. Section 8 considers outlet mergers and provides a welfare analysis. Section 9 concludes. All proofs can be found in the Appendix.

2 Literature Review

Classic contributions in media economics, for example, Spence and Owen (1977) or Wildman and Owen (1985), impose perfect substitutability and do not allow for endogenous advertising levels or two-sided externalities between viewers and advertisers. More recently, the seminal contribution of Anderson and Coate (2005) explicitly accounts for these externalities.⁷ In their model, viewers are distributed on a Hotelling line with platforms located at the endpoints. Similar to early works, viewers watch only one channel while advertisers can buy commercials on both channels.⁸ In this framework, Anderson and Coate (2005) show, among several other results, that the number of entering stations can either be too high or too low compared to the socially optimal number, or that the advertising level can be above or below the efficient level.

The framework with single-homing viewers has been used to tackle a wide array of questions. Gabszewicz, Laussel and Sonnac (2004) allow viewers to mix their time between channels, Peitz and Valletti (2008) analyze optimal locations of stations, and Reisinger (2012) considers single-homing of advertisers. Dukes and Gal-Or (2003) explicitly consider product market competition between advertisers and allow for price negotiations between platforms and advertisers, while Crampes, Haritchabalet and Jullien (2009) consider the effects of free entry of platforms. Finally, Anderson and Peitz (2012) allow advertising congestion and show that it can also lead to increased advertising rates after entry of new platforms. These papers do not allow viewers to watch more than one station, i.e., they assume either/or competition, and usually consider a spatial framework for viewer demand.⁹

There are a few recent studies which also consider multi-homing viewers.¹⁰ Anderson, Foros and Kind (2015) consider a model similar in spirit to ours, i.e., both papers share the insight that overlapping viewers are less valuable in equilibrium. In contrast to our paper, they consider a different contracting environment (per-unit pricing) and use a different equilibrium concept (rational instead of adaptive expectations). The latter implies that platforms cannot attract consumers via lower ad-levels.¹¹ In

⁷For different applications of two-sided market models, see e.g., Rochet and Tirole (2003, 2006) and Armstrong (2006).

⁸In Section 5 of their paper Anderson and Coate (2005) extend the model by allowing a fraction of viewers to switch between channels, that is, to multi-home.

⁹A different framework to model competition in media markets is to use a representative viewer who watches more than one program. This approach is developed by Kind, Nilssen and Sørsgard (2007) and is used by Godes, Ofek and Savary (2009) and Kind, Nilssen and Sørsgard (2009). These papers analyze the efficiency of the market equilibrium with respect to the advertising level and allow for viewer payments. Due to the representative viewer framework, they are not concerned with overlapping viewers or viewer preference correlation.

¹⁰For a demand structure, which allows for consumer overlapping in a bundling model, see Armstrong (2013).

¹¹Anderson, Foros and Kind (2015) provide a justification for doing so, based on the inability of consumers to observe the

addition, the questions addressed in Anderson, Foros and Kind (2015) are different and complementary to those investigated in our paper. They mainly analyze public broadcasting and genre selection, and show e.g., that the well-known problem of content duplication is ameliorated with multi-homing viewers.¹² By contrast, our analysis provides implications of viewer composition on market outcomes and characterizes the marginal incentive of a media platform to supply advertising opportunities.

Athey, Calvano and Gans (2014) and Bergemann and Bonatti (2011, in Sections 5 and 6) also consider multi-homing viewers but are mainly concerned with different tracking/targeting technologies and do not allow for advertisements generating (negative) externalities on viewers, which is at the core of our model. Specifically, in Athey, Calvano and Gans (2014) the effectiveness of advertising can differ between users who switch between platforms and those who stick to one platform because of imperfect tracking of users, whereas Bergemann and Bonatti (2011) explicitly analyze the interplay between perfect advertising message targeting in online media markets and imperfect targeting in traditional media.

Armstrong and Wright (2007) also allow for multi-homing of agents. They use a Hotelling framework and analyze under which conditions the market structure of single-homing agents on one side and multi-homing agents on the other side arises endogenously. In their model, all agents of a given side either single-home or multi-home, while we allow agents on the same side to differ in their homing behavior. White and Weyl (2015) also consider multi-homing agents. In contrast to our paper, they explore questions regarding equilibrium uniqueness. In particular, they show that the concept of insulated equilibrium, developed by Weyl (2010), helps to overcome equilibrium multiplicity problems in two-sided markets.

On the empirical side, Gentzkow, Shapiro and Sinkinson (2014) develop a structural model of the newspaper industry that embeds the key prediction found here that advertising-market competition depends on the extent of overlap in readership. They find that competition increases diversity significantly, offsetting the incentive to cater to the tastes of majority consumers (George and Waldfogel, 2003).

3 The Model

The basic model features a unit mass of heterogeneous viewers, a unit mass of homogeneous advertisers¹³ and two outlets indexed by $i \in \{1, 2\}$.¹⁴

Viewer Demand

Viewers are parametrized by their reservation utilities $(q_1, q_2) \in \mathbb{R}^2$ for outlets 1 and 2, where (q_1, q_2) is distributed according to a bivariate probability distribution with smooth joint density denoted $h(q_1, q_2)$. A viewer of (q_1, q_2) -type joins outlet i if and only if $q_i - \gamma n_i \geq 0$, where n_i is the advertising level on outlet i and $\gamma > 0$ is a nuisance parameter. Given the advertising level on each outlet, we can back out the demand system:

contractual terms offered to the advertisers.

¹²For related ideas, see also Anderson, Foros, Kind and Peitz (2012).

¹³In an Online Appendix, we consider a model with heterogeneous advertisers and demonstrate that the effects identified with homogeneous advertisers are also at work then.

¹⁴We cast our model in terms of the television context. The model also applies to internet or radio, where the term viewers would be replaced by users or listeners.

$$\begin{aligned}
\text{Multi-homers: } & D_{12} := \text{Prob}\{q_1 - \gamma n_1 \geq 0; q_2 - \gamma n_2 \geq 0\}, \\
\text{Single-homers}_1: & D_1 := \text{Prob}\{q_1 - \gamma n_1 \geq 0; q_2 - \gamma n_2 < 0\}, \\
\text{Single-homers}_2: & D_2 := \text{Prob}\{q_1 - \gamma n_1 < 0; q_2 - \gamma n_2 \geq 0\}, \\
\text{Zero-homers: } & D_0 := 1 - D_1 - D_2 - D_{12}.
\end{aligned}$$

We make the necessary assumptions on $h(q_1, q_2)$ that ensure that the demand functions D_i , $i = 1, 2$, and D_{12} are well-behaved, that is, there is a unique equilibrium with interior solutions.¹⁵

A key property of the demand schedules is that if n_i changes but n_j is unchanged, the choice of whether to join outlet j remains unaffected. However, the composition of viewers on outlet j changes. That is, $D_i + D_{12}$ does not depend on n_j while D_{12}/D_i does. This property contrasts with settings where viewers choose one outlet *over* the other.¹⁶

Timing and Outlets' Choices

Outlets compete for viewers and for advertisers. They receive payments only from advertisers but not from viewers. To make the model as transparent as possible, we develop a four-stage game.

In stage 1, outlets simultaneously set the *total* advertising levels n_1 and n_2 . In stage 2, viewers observe n_1 and n_2 and choose which outlet(s) to join, if any. In stage 3, outlets simultaneously offer menus of contracts to advertisers. A contract offered by outlet i is a pair $(t_i, m_i) \in \mathbb{R}_+^2$, which specifies an advertising intensity $m_i \geq 0$ in exchange for a monetary transfer $t_i \geq 0$. Finally, in stage 4, advertisers simultaneously decide which contract(s), if any, to accept.¹⁷ Although outlets can offer different contracts to different advertisers, below we will show that each outlet only offers one contract in equilibrium, and this contract is accepted by all advertisers. This implies that, in equilibrium, $m_i = n_i$ for the unique advertising intensity m_i offered by outlet i .

To ensure that the announced advertising levels are consistent with the realized levels after stage 4, we assume that if total advertising levels accepted by advertisers at outlet i exceed n_i , then outlet i obtains a large negative payoff.¹⁸ Therefore, our game is similar to Kreps and Scheinkman (1983), i.e., in the first stage outlets choose an advertising level that puts an upper bound on the advertising intensities

¹⁵For example, sufficient (but not necessary) assumptions to obtain a unique interior solution are

$$\frac{\partial^2 D_i}{\partial n_i^2} \leq 0, \quad \frac{\partial^2 D_{12}}{\partial n_i^2} \leq 0 \quad \text{and} \quad \left| \frac{\partial^2 D_i}{\partial n_i^2} \right| \geq \left| \frac{\partial^2 D_i}{\partial n_i \partial n_j} \right|, \forall i = 1, 2 \text{ and } j = 3 - i.$$

See e.g., Vives (2000) for a detailed discussion of why these assumptions ensure concavity of the objective functions and uniqueness of the equilibrium.

¹⁶Conditional on the realization of the utility parameters (q_1, q_2) , a viewer's choice of whether to join outlet i does not depend on q_j . This 'demand independence' assumption should not be confused with nor does it imply *statistical* independence between q_i and q_j . For instance, the model allows preferences for i (say Facebook) and j (say Yelp) to be correlated to account for some underlying common covariate factor (say 'internet savviness'). In fact, the model nests those specifications which add structure to preferences by positing a positive or negative relationship between valuations of different outlets, such as Hotelling-type spatial models.

¹⁷We show in the Online Appendix that this game is equivalent (under some additional conditions on preferences) to a canonical two-stage model of platform competition à la Armstrong (2006) in which outlets simultaneously make offers and, upon observing the offers, all agents simultaneously make their choices. The role of stage 1 in our model is to relax the dependence of viewers' choices on advertisers' choices. Indeed, viewerships are fixed before outlets sell their advertising slots. The assumption that the aggregate advertising level is fixed at the contracting stage greatly simplifies the analysis.

¹⁸Our results would remain unchanged if we instead assume that actual advertising intensities are rationed proportionally for participating advertisers in case there is excess demand for an outlet's advertising intensities. We stick to the current formulation as it simplifies some of the arguments in the proofs.

they can sell subsequently.

We use subgame perfect Nash equilibrium (SPNE) as the solution concept.

Advertising Technology

Advertising in our model is informative. We normalize the return of informing a viewer about a product to 1.¹⁹ In line with the literature, e.g., Anderson and Coate (2005) or Crampes, Haritchabalet and Jullien (2009), we assume that advertisers can fully extract the value of being informed from the consumers.

The mass of informed viewers (also known as “reach”) is determined by the number of advertising messages (m_1, m_2) a particular advertiser purchases on each outlet. Without loss of generality, we decompose the total reach as the sum of the reach within the three different viewers’ subsets. We denote the probability with which a single-homing viewer on outlet i becomes informed of an advertiser’s product by $\phi_i(m_i)$. We assume that ϕ_i is smooth, strictly increasing and strictly concave, with $\phi_i(0) = 0$. That is, there are positive but diminishing returns to advertising.

Similarly, $\phi_{12}(m_1, m_2)$ equals the probability that a multi-homing viewer becomes informed on *some* outlet. In particular, $\phi_{12}(m_1, m_2)$ is one minus the probability that the viewer is not informed on either outlet, that is, $\phi_{12}(m_1, m_2) := 1 - (1 - \hat{\phi}_1(m_1))(1 - \hat{\phi}_2(m_2))$, where $\hat{\phi}_i(m_i)$ is the probability that an overlapping viewer becomes informed *on outlet i* , where $\hat{\phi}_i(m_i)$ is also smooth, strictly increasing and strictly concave.

Payoffs

An outlet’s payoff is equal to the total amount of transfers it receives (for simplicity, we assume that the marginal cost of ads is zero). An advertiser’s payoff, in case he is active on both outlets, is $u(n_1, n_2, m_1, m_2) - t_1 - t_2$, where

$$u(n_1, n_2, m_1, m_2) := D_1(n_1, n_2)\phi_1(m_1) + D_2(n_1, n_2)\phi_2(m_2) + D_{12}(n_1, n_2)\phi_{12}(m_1, m_2)$$

and t_1 and t_2 are the transfers to outlets 1 and 2, respectively. If he only joins outlet i , the payoff is $u(n_i, n_j, m_i, 0) - t_i = D_i(n_i, n_j)\phi_i(m_i) + D_{12}(n_i, n_j)\hat{\phi}_i(m_i) - t_i$ since the advertiser reaches viewers only via outlet i . Advertisers’ reservation utilities are normalized to zero.

4 Preliminaries: Contracting Stage

To identify the competitive forces, we proceed by contrasting the market outcome of the game just described, in which two outlets compete, with the monopoly case, in which only one outlet is present in the market. We first solve the contracting stage.

A key observation is that after any pair of first stage announcements (n_1, n_2) , in any continuation equilibrium, outlets spread their advertising level equally across all advertisers. This result follows due to diminishing returns from advertising. As there is a unit mass of advertisers, the number of advertising intensities offered to each advertiser by outlet i is equal to n_i . In turn, the equilibrium transfer is the incremental value that advertising intensity n_i on outlet i generates for an advertiser who already

¹⁹In the Online Appendix, we allow advertisers to be different with respect to this return.

advertises with intensity n_j on the other outlet.²⁰

Claim 1: *In any SPNE of a game with competing outlets, given any pair of first-stage choices (n_1, n_2) , each outlet i only offers one contract (t_i, m_i) . These contracts are accepted by all advertisers, and have the feature that $m_1 = n_1$, $m_2 = n_2$, $t_1 = u(n_1, n_2) - u(0, n_2)$ and $t_2 = u(n_1, n_2) - u(n_1, 0)$.*

The next claim establishes a parallel result for the single outlet case, whose proof we omit because it proceeds along the same lines as the proof of Claim 1. In particular, the monopolist offers a single contract that is accepted by all advertisers.

Claim 2: *In any SPNE of a game with a monopolistic outlet, given first-stage choice n_i , the monopolist offers a single contract (t, m_i) . This contract is accepted by all advertisers, and has the feature that $m_i = n_i$, and $t = u(n_i, 0)$.*

In what follows, we denote $D_i(n_1, n_2) + D_{12}(n_1, n_2)$ by $d_i(n_i)$, that is, $d_i(n_i) := \text{Prob}\{q_i - \gamma n_i \geq 0\}$. Claims 1 and 2 imply that, because in equilibrium viewers correctly anticipate the unique continuation play following stage 1, in any SPNE viewer demand on outlet i is $d_i(n_i)$, $i = 1, 2$. Furthermore, outlets' equilibrium profits in duopoly are lower than the equilibrium profit obtained by the monopolist. In duopoly, outlets can only demand the incremental value from an advertiser who is also active on the other outlet, whereas a monopolist can extract the whole surplus. Specifically, a monopolist outlet i obtains a profit of $d_i(n_i)\phi_i(n_i)$, since it has only exclusive viewers, whereas outlet i in duopoly only obtains $D_i(n_i)\phi_i(n_i) + D_{12}(n_1, n_2) \left(\phi_{12}(n_1, n_2) - \hat{\phi}_j(n_j) \right)$, because it *shares* some viewers with its rival.

5 Outlet Competition

We proceed by contrasting the choice of a monopolist²¹

$$n_i^m := \arg \max_{n_i} d_i(n_i)\phi_i(n_i), \quad (1)$$

with the duopoly outcome, that is, with the fixed point of the best reply correspondences

$$n_i^d := \arg \max_{n_i} D_i(n_i)\phi_i(n_i) + D_{12}(n_1, n_2) \left(\phi_{12}(n_1, n_2) - \hat{\phi}_j(n_j) \right) \quad i = 1, 2; \quad j = 3 - i. \quad (2)$$

It is useful to rewrite the duopolist's profit *as if* all viewers were exclusive plus a correction term that accounts for the fact that outlet i can only extract the incremental value from its shared viewers:

$$n_i^d := \arg \max_{n_i} d_i(n_i)\phi_i(n_i) - D_{12}(n_1, n_2) \left(\phi_i(n_i) + \hat{\phi}_j(n_j) - \phi_{12}(n_1, n_2) \right). \quad (3)$$

First consider problem (1). Its solution is characterized by the first-order condition

$$\frac{\partial \phi_i}{\partial n_i} d_i + \frac{\partial d_i}{\partial n_i} \phi_i = 0. \quad (4)$$

When increasing n_i , outlet i trades off profits on inframarginal viewers due to increased reach with profits on marginal viewers who switch off. If we introduce the advertising elasticities of the total demand d_i

²⁰With a slight abuse of notation, in what follows, we denote $u(n_i, n_j, n_i, n_j)$ by $u(n_i, n_j)$ and $u(n_i, n_j, n_i, 0)$ by $u(n_i, 0)$.

²¹Here we adopt the convention that i denotes the monopoly outlet.

and of the advertising function ϕ_i with respect to n_i ,

$$E_{d_i} := -\frac{\partial d_i}{\partial n_i} \frac{n_i}{d_i} \quad \text{and} \quad E_{\phi_i} := \frac{\partial \phi_i}{\partial n_i} \frac{n_i}{\phi_i},$$

the optimal advertising level is characterized by the simple and intuitive condition

$$E_{\phi_i} = E_{d_i}.$$

Consider now problem (3). In duopoly, condition (4) is augmented to account for the fact that some of the previously exclusive viewers are now shared. Using $\phi_i + \hat{\phi}_j - \phi_{12} = \hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}$, with $\Delta_{\phi_i} = \phi_i - \hat{\phi}_i$, the first-order condition can be written as

$$\frac{\partial \phi_i}{\partial n_i} d_i + \frac{\partial d_i}{\partial n_i} \phi_i - D_{12} \frac{\partial(\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i})}{\partial n_i} - \frac{\partial D_{12}}{\partial n_i} (\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}) = 0. \quad (5)$$

We can now provide a condition for advertising levels in duopoly being larger than in monopoly. Let

$$E_{D_{12}} := -\frac{\partial D_{12}}{\partial n_i} \frac{n_i}{D_{12}} \quad \text{and} \quad E_{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}} := \frac{\partial(\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i})}{\partial n_i} \frac{n_i}{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}}.$$

Proposition 1: *An incumbent monopolist's advertising level increases (decreases) upon entry of a competitor if and only if*

$$\frac{E_{D_{12}}}{E_{d_i}} > (<) \frac{E_{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}}}{E_{\phi_i}}, \quad (6)$$

where all functions are evaluated at $n_i = n_i^m$ and $n_j = n_j^d$.

To build intuition for the result, consider the simplest case in which the two outlets are symmetric, i.e., $d_i(n) = d_j(n)$, $\phi_i(n) = \phi_j(n)$, and $\hat{\phi}_i(n) = \hat{\phi}_j(n)$ for all n and suppose that competing outlets behave as the monopolist does, that is, $n_j^d = n_i^d = n_i^m$. Can these advertising levels constitute an equilibrium? First, overlapping viewers receive advertising messages from two outlets. If each outlet chooses the same advertising level as the monopolist, the amount of advertising that these viewers are exposed to *doubles* in duopoly. Other things held constant, decreasing marginal returns give an incentive to scale back advertising on outlet i , because its marginal contribution to the advertisers' surplus drops. This *duplication effect* gives an outlet the incentive to reduce its advertising level. However, there is a second, arguably more subtle, effect which goes in the opposite direction. For a duopolist, the total variation in demand due to a small increase in n_i decomposes to $\partial D_i / \partial n_i$ and $\partial D_{12} / \partial n_i$. The first term is the change in the mass of exclusive viewers and the second the change in the mass of overlapping viewers. Instead, a monopolist has only exclusive viewers and is therefore wary only of the total variation of $\partial d_i / \partial n_i$. Since exclusive viewers are more valuable than overlapping viewers, in duopoly the opportunity cost of losing shared business is lower than that of losing exclusive business. Other things held constant, this *business-sharing effect* gives the outlet an incentive to increase its advertising levels.

To see how this intuition is represented in the formula of the proposition, first note that the left-hand side of (6) is the ratio of the demand elasticity of overlapping viewers to the demand elasticity of viewers in monopoly. On the right-hand side, $\hat{\phi}_i \hat{\phi}_j$ is a measure of wasted (or duplicated) advertising. It is the probability that an overlapping viewer is informed twice. This term is adjusted by Δ_{ϕ_i} to account for

viewer heterogeneity, that is, an overlapping viewer may become informed with a different probability than an exclusive one. Loosely speaking, the numerator of the right-hand side of (6) is a measure of the elasticity of duplication. It represents which fraction of advertising messages gets wasted due to duplication across outlets following a one percentage point increase in the amount of messages sent.

This puts us in the position to provide a clear interpretation of (6). If the elasticity of overlapping viewers is large relative to the one of exclusive viewers (i.e., the left-hand side is large), the business-sharing effect prevails and advertising levels increase with entry. By contrast, if the elasticity of wasted impressions is large relative to the advertising elasticity (i.e., the right-hand side is large), the duplication effect dominates and advertising levels fall with entry.²²

We note that the business-sharing effect points in the *opposite* direction than the one brought about by competition in traditional single-homing setups. A key insight there is that competitive pressure induces competing outlets to put *more* emphasis on lost business than monopolists do. (See, for example, the discussion in Armstrong (2006), Section 4). Lost business on one side lowers revenues on the other side of the market, as consumers find the rival more attractive because of the indirect network effects. As a consequence, advertising levels, which act as a price for viewers, fall if competitive pressure increases.

An important merit of (6) is that it spells out the effect of applying competitive pressure in terms of empirical objects. However, its insight is limited without a theory that suggests when the condition should be satisfied. We address this issue in the next two sections. The condition asks if there are any systematic differences between the two pools of viewers that could tilt the trade-off one way or the other. The two sides of the inequality stress two different sources of dissimilarities between exclusive and overlapping viewers, both of which play a crucial role in duopoly. The left-hand side focuses on relative *preferences*, expressed by demand elasticities. The right-hand side focuses on potential differences in the advertising *technology*, expressed by elasticities of the advertising function. Given that these are two very different mechanisms, we tackle them separately. In Section 6, we add structure on the advertising functions in a way that guarantees that the right-hand side of (6) equals one. This shuts down the technological source. Results are then purely driven by systematic differences in preferences across types. In Section 7, we carry out the mirror exercise. We shut down the preferences source by using the insights gained in Section 6. As we shall see, it is possible to add structure to the joint distribution in a way that guarantees a left-hand side of (6) equal to one for all (n_1, n_2) . Our findings in Section 7 will therefore hinge solely on technological factors. This break-up is implemented for illustrative purposes only. In principle, we could carry out the two exercises simultaneously.

6 Viewer Preference Correlation

To isolate how relative preferences shape the effect of competition, in this section we assume that $\hat{\phi}_i(n_i) = \phi_i(n_i)$ for $i = 1, 2$. This amounts to considering the case where overlapping and exclusive viewers get informed with the same probability (on a single outlet). Using $\hat{\phi}_i(n_i) = \phi_i(n_i)$, one can verify that the right-hand side of (6) equals 1 for all (n_1, n_2) .²³ So condition (6) simplifies to:

$$\frac{E_{D_{12}}}{E_{d_i}} > (<) 1. \tag{7}$$

²²A similar intuition holds if we start from any number of incumbent outlets, not just a monopoly outlet. See the Online Appendix for a formal analysis of the case with two incumbent outlets.

²³See the proof of Proposition 2.

We now seek to identify features of the joint distribution of preferences that could lead to systematic differences in the relative elasticities of demand. A striking feature of (7) is that the effect of competition depends on the joint distribution of preferences through $E_{D_{12}}$ *only*. So any change in the joint distribution that results in a decrease of $E_{D_{12}}$ (for equal marginal distributions) yields downward competitive pressure on advertising levels. To bring out this effect, we add structure to the preferences. Specifically, we assume that (q_1, q_2) is drawn from a bivariate normal distribution with mean $(0, 0)$ and variance-covariance matrix

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

The parameter ρ is the coefficient of correlation between q_1 and q_2 and therefore captures content ‘likeness.’

We can now determine how a change in the correlation coefficient affects viewer composition.

Lemma 1 D_{12} is strictly increasing in ρ .

The lemma shows that the viewer composition changes monotonically with the correlation coefficient, i.e., a higher correlation coefficient, *ceteris paribus*, *is equivalent* to an increase in the extent of viewer overlap. Recall that the total demand of outlet i depends only on the marginal distribution, which is unchanged by an increase in ρ . Therefore, only the composition is affected by ρ .

At first thought, Lemma 1 suggests a *negative* relationship between ρ and the equilibrium advertising level. The higher the number of overlapping viewers, the stronger the duplication effect. However, a larger ρ could enhance the business-sharing effect as well. Indeed, a larger D_{12} may lead to a larger fraction of the variation coming from overlapping viewers. Other things held constant, this suggests a *positive* relationship. The resulting indeterminacy is reflected by the fact that what matters is how the *elasticity* of the demand D_{12} changes with correlation. The next lemma proves that a systematic relationship between $E_{D_{12}}$ and joint preferences as captured by ρ exists.

Lemma 2 $E_{D_{12}}$ decreases with ρ for all $n_1 = n_2 > 0$.

To main intuition behind this result is that the fraction of *marginal* consumers who are shared changes with ρ at a different rate than the fraction of the *total* consumers who are shared.

To see this in a simple way, consider, as an extreme example, the Hotelling model in which viewer preferences are perfectly negatively correlated. In this model (focusing on an interior equilibrium), single-homing viewers are located close to the respective outlet whereas multi-homing viewers are in the middle. A change in the advertising level of outlet i affects *only* the margin between multi-homers and single-homers on outlet j but not the margin between multi-homers and single-homers on outlet i . Therefore, although the inframarginal viewers consist of exclusive and overlapping viewers, all of the marginal viewers are overlapping ones. This implies that (in absolute terms) $\partial D_{12}/\partial n_i$ is large compared to D_{12} ; hence, $E_{D_{12}}$ is also large. As a consequence, the business-sharing effect dominates the duplication effect. Therefore, platform entry tends to increase advertising levels.

Our analysis generalizes this insight by showing that it applies for all $\rho < 0$. It also demonstrates that the insight reverses if viewer preferences are positively correlated. In that case, the total viewer demand is comprised of a large portion of overlapping viewers. Instead, the composition of marginal viewers consists of a relatively large portion of exclusive ones. This implies that $E_{D_{12}}$ is small because (in

absolute terms) $\partial D_{12}/\partial n_i$ is small relative to D_{12} . As a consequence, the duplication effect dominates the business-sharing effect. This leads to downward pressure on advertising levels.

Recall that d_i , and hence E_{d_i} , does not change with ρ . So we obtain a simple characterization:

Proposition 2 *An incumbent monopolist’s advertising level increases upon entry of a competitor if and only if ρ is negative. That is,*

$$\text{sign}(n_i^d - n_i^m) = \text{sign}(E_{D_{12}} - E_{d_i}) = -\text{sign}(\rho),$$

where all functions are evaluated at $n_i = n_i^m$ and $n_j = n_j^d$.

Before moving on, we use this result in two different ways. First, we discuss the implied strategic considerations of outlets when choosing which *kind* of content to produce. Second, we discuss how this result can be used as a first empirical test of the theory.

Implications for Content Choice

While content has been kept exogenous so far, a natural application of this model is content choice. In particular, it allows us to add to the ongoing debate on “competition and diversity” in the media, which is often spelled out as “ideological” diversity. The analysis relies on two premises: 1) potential entrants can affect the degree of correlation at a ‘content-production’ stage. This stage is akin to product positioning in standard models of product differentiation; 2) a decrease in ρ can be read as an increase in the supply of more diverse content. Our aim is not to provide a full-fledged model of differentiation, which would be largely outside the scope of the paper. Rather, we seek here to identify broad mechanisms that we do not expect to be sensitive to a particular model specification: 1) Would an entrant that caters to the same viewers as the incumbent be more or less profitable than an outlet that caters to those who find the incumbent unappealing? 2) In light of Proposition 2, do strategic considerations enhance or reduce the incentives to differentiate one’s content from the rival’s?

What we have in mind is a simple two-stage game. At stage 1, an entrant observes the content of the incumbent and chooses the extent of differentiation, measured by $-\rho$. The entrant incurs a quadratic investment cost $k(1 - \rho)^2$, where $k > 0$ is an arbitrary constant. This cost function captures the idea that duplication ($\rho = 1$) is costless while differentiation is increasingly costly. So the entrant maximizes $\pi_i^d(-\rho) - k(1 - \rho)^2$, where π_i^d is the equilibrium profit of outlet i in duopoly. At stage 2, competition takes place as described in Section 3. Given a well-behaved problem, in equilibrium ρ^* equates the marginal cost with the marginal benefit of differentiation. The latter is given by²⁴

$$\frac{\partial \pi_i^d}{\partial(-\rho)} = \left(\frac{\partial D_{12}}{\partial \rho} - \frac{\partial n_j}{\partial \rho} \left(\frac{\partial(\phi_{12} - \hat{\phi}_j)}{\partial n_j} \frac{D_{12}}{\phi_i + \hat{\phi}_j - \phi_{12}} - \frac{\partial D_{12}}{\partial n_j} \right) \right) (\phi_i + \hat{\phi}_j - \phi_{12}).$$

This term shows how the incentives to differentiate are shaped. Its decomposition is fairly straightforward in light of our previous results. The first term in the large bracket on the right-hand side ($\partial D_{12}/\partial \rho$) is positive.²⁵ It captures the basic insight that decreasing correlation leads to a demand that is comprised of relatively more exclusive viewers, which are more valuable to advertisers. Other things held constant, the entrant has an incentive to invest in ‘diverse’ content (or, diminish ρ). The second term accounts

²⁴We use here that $\partial D_{12}/\partial(-\rho) = -\partial D_{12}/\partial \rho$ and $\partial n_j/\partial(-\rho) = -\partial n_j/\partial \rho$.

²⁵Note that the second bracket $\phi_i + \hat{\phi}_j - \phi_{12}$ is also positive.

for the rival’s reaction ($\partial n_j / \partial \rho(\dots)$). As discussed, a lower ρ results in an equilibrium in which the rival outlet competes more aggressively for advertising dollars by increasing its supply of ads. This mechanism, which we conventionally refer to as the ‘strategic’ one, may point in the opposite direction as more ads from the rival reduce the extent of rent extraction from overlapping viewers. Interestingly, our ‘direct’ and ‘strategic’ effect can have opposite forces than in standard models of differentiation (see, for example, d’Aspremont, Gabszewicz and Thisse (1979) on horizontal differentiation or Shaked and Sutton (1982) on vertical differentiation). In these models, the strategic effect is that firms become more differentiated to ‘soften’ competition while the direct effect is that firms have a smaller secured demand (a smaller ‘hinterland’). In our case, the opposite could hold, escaping competition through less differentiation.

An interesting follow-up question is how consumer preferences shape ρ^* . In our setting, this is equivalent to the question how changes in the primitives shift $\partial \pi_i^d / \partial(-\rho)$.

A first parameter, considered in the literature on advertising and product design, is the dispersion of consumer valuations (e.g., Johnson and Myatt, 2006). It basically captures the extent of preference heterogeneity. A simple way to consider this issue in our setting is to parametrize the variance-covariance matrix by a variance parameter σ , symmetric across outlets:

$$\Sigma = \begin{bmatrix} \sigma & \rho \\ \rho & \sigma \end{bmatrix}.$$

A basic observation is that by moving mass towards the right tail of the marginal distributions, dispersion increases both d_i and D_{12} for all positive values of (n_1, n_2) . Having more shared business obviously increases the returns from differentiation. This gives the entering outlet an incentive to choose a lower ρ^* . To confirm this intuition we solved numerically for the optimal value of ρ for different values of σ . Figure 1-(a) shows the second-stage profits as a function of ρ for different values of σ . For illustrative purposes, we limit to two curves corresponding to $\sigma = 1.5$ (dashed) and $\sigma = 1$ (solid). The profit values on the left ordinate represent the profit values with $\sigma = 1.5$ and those on the right ordinate are the profit values with $\sigma = 1$. Profits *increase* with sigma. The reason is that dispersion increases the overall demand d_i and therefore brings in extra rents. It is evident from the figure that a larger dispersion shifts the peak to the left. Therefore, outlets choose a larger degree of differentiation, the more heterogeneous viewers are. The intuition is that, due to higher profits with increased dispersion, outlets gains more from a marginal increase in differentiation.

Figure 1-(b) refers to a change in the nuisance cost γ . We perform the same exercise and increase γ from 1 to 1.5. Doing so reduces profits, and this induces outlets to choose a lower extent of differentiation. Intuitively, if viewers dislike ads to a greater extent, demand falls, implying that there is less surplus to be extracted. This, in turn, reduces outlets’ incentives to invest in costly differentiation. Therefore, our model predicts that the extent of differentiation increases with factors that induce a larger viewer demand.

Empirical Analysis

We provide a brief empirical investigation of the link between entry and correlation on advertising levels. The analysis exploits variation in the extent of competitive pressure brought about by entry and exit of TV channels in the Basic Cable lineup in the 80s and 90s. As our data are limited, we regard our results as suggestive evidence.

The dataset is provided by Kagan-SNL, a highly regarded proprietary source for information on

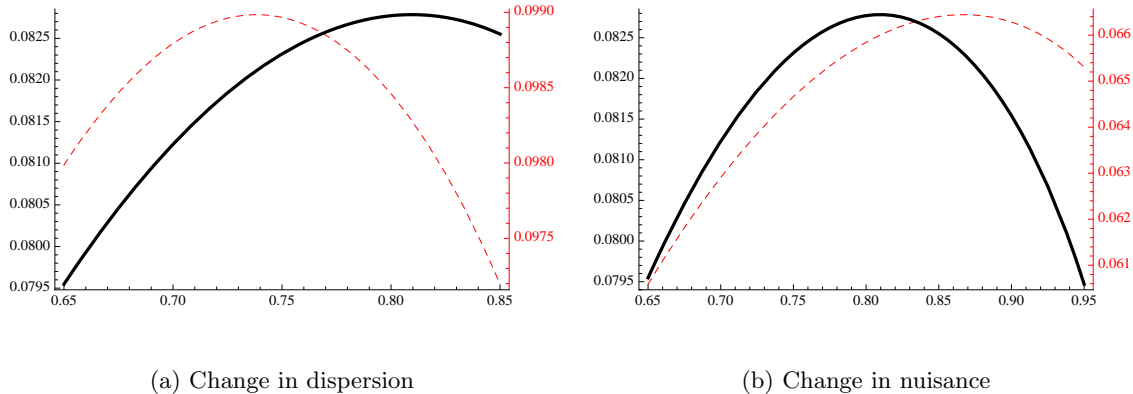


Figure 1: $\pi_i^d(-\rho, \sigma) - \frac{1}{10}(1 - \rho)^2$

broadcasting markets. It consists of an unbalanced panel data set of 68 basic cable channels from 1989 to 2002. The channels cover almost all cable industry advertising revenues. We know the date for each new network launch within our sample period (a total of 43 launches). In addition, for each network active in each year we have information on the average number of 30-second advertising slots per hour of programming (in jargon ‘avails’). We also have a good coverage for other network variables, such as subscribers, programming expenses and ratings.

We study the relationship between the avails broadcasted by each channel and the number of incumbents. As our model characterizes the effects of varying competition, we consider each channel within its *own* competitive environment. That is, we define a relevant market segment for each of the 68 channels. The hypothesis is that channels with content tailored to the same segment compete for viewers and advertisers. For this purpose, we divide channels in three segments: (i) sports channels (henceforth Sports), (ii) channels broadcasting mainly movies and TV series (henceforth Movies&Series), and (iii) all remaining channels, which is used as a reference group. To test whether viewer preference correlation affects the relationship between entry and advertising levels, we estimate separate parameters for the Sports and the Movies&Series segments. Our working hypothesis is that the viewers’ preferences within these segments are positively correlated. Our model predicts that avails would fall after entry in the Sports and Movies&Series segments relative to the reference group.^{26,27}

There are mainly two difficulties with the analysis. A first one is the issue of entry endogeneity on incumbent performance. In general, it is hard to instrument for entry (see e.g., Berry and Reiss, 2007) and we do not have exploitable variation for this purpose. A second one is that we do not directly observe consumers’ preferences and thus their correlation across different outlets. This said, we believe it is reasonable to assume that those who watch ESPN are more likely to watch ESPN2 or FoxSports.

We use a panel analysis, that pools all channel-year observations from 1989-2002, so it relies on

²⁶Our data does not include viewer prices. This should not be a problem because their impact was not particularly important during our sample period (see for example, Strömberg, 2004). In addition, viewer prices were highly regulated in the 1990s.

²⁷We note that we intended to create a separate News segment as well, as this segment provides a natural counterpart to the others in that viewer preferences can be reasonably assumed to be negatively correlated. Unfortunately, the number of channels here is too small to obtain statistically meaningful results. The point estimates we obtain are nevertheless consistent with Proposition 2 (details are available from the authors).

within- and across-channel variation.²⁸ We estimate the following linear regression model:

$$\begin{aligned} \log(Avails_{it}) = & \beta * Outlets_{it} + \beta_M * Outlets_{it} * MoviesSeries_dummy \\ & + \beta_S * Outlets_{it} * Sports_dummy + \gamma * x_{it} + \alpha_i + \delta_t + \epsilon_{it}, \end{aligned}$$

where $Avails_{it}$ is the average number in year t of 30-second advertising slots per hour of programming by channel i , $Outlets_{it}$ is the number of channels in channel i 's segment at the end of year t , $Sports_dummy$ and $MoviesSeries_dummy$ are dummy variables equal to 1 when channel i belongs to the Sports and to the Movies&Series segments, respectively, (and zero otherwise), x_{it} is a vector of channel-time controls, α_i is a channel fixed effect and δ_t is a time fixed effect. Given that the dependent variable is transformed in logs, while the main explanatory variable is measured in units of channels, β has the following interpretation: when a new channel enters the control segment, the incumbents increase their 30-second advertising slots by $100\beta\%$. The coefficients β_M and β_S measure the *additional* effect that the number of channels has on the avails in the Sports and Movies&Series segments respectively.

Table 2 reports the estimation results. Starting from the single variable model in column (1), we progressively add controls and fixed effects: column (2) controls for the real GDP to capture the business cycle's effect on the advertising market. Starting from column (3), we report estimates for a fixed-effect model where the units of observations are the single channels. From column (4), we introduce time dummies, while in columns (5) and (6) we add channel-time controls: the channel's share of revenues in its segment and its rating. Since we only have US data, the real GDP control is dropped whenever time controls are included. All regressions are estimated with robust standard errors, which are clustered at the segment level.

Table 2 provides evidence that entry is associated with an increase in the advertising levels on incumbent channels. The coefficient is positive and significant across almost all specifications. This result is in line with the anecdotal evidence on the positive impact of entry of FOX News on the advertising levels supplied by MSNBC or CNN, which is often referred to as the "Fox News Puzzle". We are particularly interested in the coefficients β_S and β_M which we expect to be negative, given our theory. That is, we expect the effect of entry within the Sports and Movies&Series segment to be diminished compared to the average industry effect (and possibly negative overall). Indeed, the coefficients have the expected sign in all regressions: the effect of the number of channels on advertising levels in these two segments is significantly lower than in the reference group. This additional negative effect is particularly strong for Movies&Series where $|\beta_M| > |\beta|$ in almost all specifications.

7 Advertising Technology

Competition comes hand-in-hand with duplication: in duopoly multi-homers receive the same ads from two different sources. We now explore if this fact together with diminishing marginal returns injects downward pressure on advertising levels. To focus on the advertising technology, we shut down the preference channel and assume throughout this section that $E_{D_{12}} = E_{d_i}$. As shown in the last section, this assumption holds, for example, if the valuations for the two outlets are standard normally distributed and independent of each other (i.e., $\rho = 0$). Condition (6) tells us that in this particular case competition

²⁸In Appendix B, we demonstrate that using a model of entry episodes leads to similar conclusions as the panel analysis.

Table 2: Number of Incumbents and Avails - Effect by Segment

| | (1) | (2) | (3) | (4) | (5) | (6) |
|-----------------------|-------------------------|-----------------------|--------------------------|---------------------------|--------------------------|--------------------------|
| Outlets | 0.00615*** (0.00164) | 0.00139 (0.00496) | 0.00648** (0.00104) | 0.00672** (0.000878) | 0.00676*** (0.000517) | 0.00160* (0.000459) |
| Outlets×Movies&Series | -0.00613 (0.00379) | -0.0137* (0.00705) | -0.00902** (0.00164) | -0.00811** (0.00161) | -0.00788** (0.00177) | -0.0137*** (0.000325) |
| Outlets×Sports | -0.00261 (0.00600) | -0.00692 (0.00700) | -0.00687** (0.000913) | -0.00619*** (0.000377) | -0.00547 (0.00200) | -0.0154*** (0.000117) |
| Real GDP | | 0.00283 (0.00247) | 0.00188* (0.000620) | | | |
| MoviesSeries_dummy | 0.191*** (0.0595) | 0.257*** (0.0766) | | | | |
| Sports_dummy | 0.106 (0.0690) | 0.0493 (0.0866) | | | | |
| Rev Mkt Share | | | | | 0.156 (0.398) | |
| Rating | | | | | | -0.0904 (0.0542) |
| Constant | 2.908*** (0.0351) | 2.685*** (0.190) | 2.760*** (0.0469) | 3.242*** (0.0372) | 3.226*** (0.0621) | 3.273*** (0.0258) |
| Observations | 416 | 416 | 416 | 416 | 415 | 279 |
| R^2 | 0.048 | 0.050 | 0.284 | 0.307 | 0.311 | 0.299 |
| Channel FE | NO | NO | YES | YES | YES | YES |
| Time FE | NO | NO | NO | YES | YES | YES |
| No. of Outlets | 56 | 56 | 56 | 56 | 56 | 33 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

reduces advertising levels if and only if

$$1 < \frac{E_{\hat{\phi}_i \hat{\phi}_j + \Delta \phi_i}}{E_{\phi_i}}.$$

The next proposition shows that competition is shaped by the outlet's relative elasticities of informing different kinds of viewers.

Proposition 3 *An incumbent monopolist's advertising level increases upon entry of a competitor if and only if the elasticity to inform an overlapping viewer on a single outlet is higher than the elasticity to inform an exclusive viewer, that is,*

$$E_{\hat{\phi}_i} - E_{\phi_i} > 0, \quad (8)$$

where all functions are evaluated at $n_i = n_i^m$ and $n_j = n_j^d$.

If advertising levels increase or decrease with entry depends on the returns from advertising to

overlapping viewers being higher or lower than those to exclusive viewers. This difference in the effectiveness of advertising is measured by the difference in elasticities (i.e., $E_{\hat{\phi}_i} - E_{\phi_i}$). The intuition behind the result in Proposition 3 can again be traced back to the relative strength of the duplication and business-sharing effect. Suppose that overlapping viewers are harder to inform than exclusive viewers (i.e., $\partial\hat{\phi}_i/\partial n_i < \partial\phi_i/\partial n_i$). This increases the strength of the duplication effect because platforms in duopoly can extract only a small amount for their overlapping viewers. At the same time, the business-sharing effect is also more pronounced if overlapping viewers are harder to inform. Because $\hat{\phi}$ is smaller than ϕ_i , the demand reduction following an increase in the advertising level reduces profits of a duopoly platform only by a small amount compared to a monopoly platform. This implies that advertising levels tend to increase with entry. Therefore, if advertising to overlapping viewers is less effective, both effects are more pronounced, and the overall result is ambiguous. Similarly, if advertising to overlapping viewers is more effective, both effects get weaker in strength, leading again to an ambiguous result.

To gain further insights, consider the technology of the exponential form, that is, $\phi_i(n) = 1 - e^{-bn}$ and $\hat{\phi}_i(n) = 1 - e^{-\hat{b}n}$.²⁹ We allow for \hat{b} being larger or smaller than b , that is, overlapping viewers can become informed with a higher or lower probability than exclusives. If multi-homers spend a reduced amount of attention on a particular outlet, it is natural that $\hat{\phi}_i(n) < \phi_i(n)$, which equals $\hat{b} < b$. However, it is also conceivable that multi-homers react more to ads because they are exposed to more content, implying $\hat{b} > b$. In general, viewers may differ along dimensions other than (q_1, q_2) , which leads to inherent differences between overlapping and exclusive viewers that justifies either assumption. With the exponential technology, it is easy to show that the left-hand side of (8), $E_{\hat{\phi}_i} - E_{\phi_i}$, can be written as

$$(b - \hat{b})e^{-(b+\hat{b})n_i^m} + \hat{b}e^{-\hat{b}n_i^m} - be^{-bn_i^m}. \quad (9)$$

It is readily verified that (9) is strictly positive for $0 < \hat{b} < b$, strictly negative for $\hat{b} > b$, and zero for $\hat{b} = 0$ and $\hat{b} = b$. Interestingly, this shows that in the perhaps more natural case, in which exclusive viewers are more receptive to ads, advertising levels in duopoly are larger than in monopoly. Therefore, the business-sharing effect dominates the duplication effect. Conversely, if overlapping viewers are more receptive to ads, advertising levels in duopoly are lower. There are two cases (i.e., $\hat{b} = 0$ and $\hat{b} = b$), in which advertising levels in monopoly and duopoly are the same. In the latter case, $\hat{b} = b$, reaching exclusive and overlapping viewers (on a single outlet) is equally effective, leading to the same trade-off in monopoly and duopoly. In the other extreme case, $\hat{b} = 0$, overlapping viewers are of zero value. Therefore, an outlet in duopoly only cares about its exclusive viewers when choosing the advertising level. Since a monopolistic outlet has only exclusive viewers, the trade-off and the equilibrium advertising level in both scenarios are again equivalent.

8 Outlet Mergers and Welfare Analysis

The policy debate on mergers between media platforms features two opposite stances. A first view maintains that the merged entity would reduce the supply of ads and increase prices attempting to extract more surplus on the advertising side of the market. The opposite view maintains that competition

²⁹This functional form was firstly introduced in a seminal paper by Butters (1977) and has been widely used since then in applied work on advertising. It can be derived from natural primitive assumptions on the stochastic process that governs the allocation of messages to consumers.

for ad-averse viewers injects downward pressure on quantities.³⁰ Here we contribute to this debate by presenting a third argument that brings in the picture the impact of demand composition.

We contrast the duopoly outcome with the outcome that a hypothetical monopolist who controls *both* outlets would implement. An important insight in this comparison is that if advertising goes up on one outlet, previously shared business becomes exclusive on the other outlet. This potentially induces different incentives to set advertising levels because the two-platform monopolist owns the other outlet, whereas the duopolist does not. The relevant question turns out to be: is the opportunity cost of losing shared business smaller for a multi-outlet platform than for a single-outlet platform? We show that the answer to this question is given by a fairly straightforward condition.

Proposition 4: *The equilibrium advertising level in duopoly is strictly lower than under joint ownership (i.e., $n_i^d < n_i^{jo}$) if and only if*

$$\hat{\phi}_j < \phi_j.$$

The two advertising levels are equivalent if $\hat{\phi}_j = \phi_j$.

To build intuition, consider first the case $\hat{\phi}_i = \phi_i$. When marginally increasing n_i , a monopolistic owner controlling both outlets loses some multi-homing viewers who become single-homing viewers on outlet j . His per-viewer loss is therefore $\phi_{12} - \phi_j$. In duopoly, when outlet i increases n_i , it loses some multi-homing viewers who are worth $\phi_{12} - \phi_j$. But this implies that the trade-offs in both market structures are the same.³¹ As a consequence, we obtain that the ownership structure has no effect on advertising levels.

If instead $\hat{\phi}_i < \phi_i$, overlapping viewers can be reached with a lower probability by advertisers than single-homing viewers. Therefore, competing outlets can extract $(\phi_{12} - \hat{\phi}_j) > (\phi_{12} - \phi_j)$ from advertisers. This implies that the opportunity cost of losing an overlapping viewer is smaller for a joint owner. In other words, the business-sharing effect for a multi-outlet monopolist is larger than for competing outlets, which leads to lower advertising levels in duopoly. (The opposite results occurs if $\hat{\phi}_i > \phi_i$.)

Our result is different than the one obtained in previous work. For instance, in Anderson and Coate (2005) competition for viewers always reduces advertising levels relative to monopoly. In addition, the redistributive impact of a merger is very different. In our model, a joint owner can fully expropriate advertisers, whereas competing outlets cannot, implying that advertisers are hurt by a merger. In contrast, in Anderson and Coate (2005) a merger leads to an increase in the advertising level and a lower advertising price. Hence, advertisers are better off after a merger.

The result that advertising levels do not depend on the ownership structure for $\hat{\phi}_j = \phi_j$ is reminiscent of common agency models (e.g., Bernheim and Whinston, 1986), that predict equivalent allocations when firms compete and when they cooperate. However, common agency models feature a single agent who contracts with multiple principals instead of a continuum of agents, as in our framework. In particular, if there is only a single advertiser—or, equivalently, if all advertisers can coordinate their choices³²—even in

³⁰For example, in the merger between Facebook and WhatsApp an important question was if the policy of WhatsApp to present ad-free content will change after its acquisition by Facebook. See e.g., "Facebook plans to monetize WhatsApp: Ads or mobile payments?" on <http://marketrealist.com> or "Updated - Facebook buying WhatsApp - \$19B in cash and stock" on <http://www.forbes.com>.

³¹Note that in both cases increasing n_i also implies losing some single-homing viewers on outlet i . But the loss from these viewers is exactly the same for the monopolist and the duopolist.

³²For an analysis of consumer coordination in outlet competition in a setting with positive network externalities, see

models featuring single-homing of viewers, the equilibrium advertising levels are the same under duopoly and joint ownership.³³ In contrast to this, advertisers cannot coordinate their choices in our model. Therefore, the mechanism described above, which leads to the equivalence result in our model, is distinct to the one in common agency frameworks.

Finally, we analyze how the equilibrium advertising level relates to the socially optimal one. A common perception in media markets is that the market leads to an excessively high level of advertising.³⁴ This usually occurs because of the lack of prices on the consumer side, implying that platforms fail to internalize the dis-utility of nuisance borne by viewers. However, the literature on media markets obtains a more ambiguous result. Since competition for viewers takes place in advertising levels, fierce competition leads to very low advertising levels, which can result in under-provision of advertising opportunities.

To characterize how this tension plays out in our model, we determine social welfare. As mentioned, $q_i - \gamma n_i$ is the utility of a single-homing viewer of outlet i and $q_1 - \gamma n_1 + q_2 - \gamma n_2$ is the utility of a multi-homing viewer. Social welfare is given by

$$W = \int_{\gamma n_1}^{\infty} \int_0^{\gamma n_2} (q_1 - \gamma n_1) h(q_1, q_2) dq_2 dq_1 + \int_0^{\gamma n_1} \int_{\gamma n_2}^{\infty} (q_2 - \gamma n_2) h(q_1, q_2) dq_2 dq_1 \\ + \int_{\gamma n_1}^{\infty} \int_{\gamma n_2}^{\infty} (q_1 - \gamma n_1 + q_2 - \gamma n_2) h(q_1, q_2) dq_2 dq_1 + D_1 \phi_1 + D_2 \phi_2 + D_{12} \phi_{12}.$$

Comparing the equilibrium advertising level with the socially efficient advertising level we obtain the following result:

Proposition 5: *Equilibrium advertising levels are inefficiently high if $\hat{\phi}_i \geq \phi_i$.*

To provide intuition we build on Proposition 4 and consider the incentives of a joint platform owner. Under joint ownership, the merged entity fully internalizes the advertising surplus while it does not internalize viewer's welfare. More precisely, it only cares about viewers' utilities inasmuch as they contribute to advertising revenue, while the nuisance costs from advertising are not taken into account. From Proposition 4, we know that competing outlets implement the same advertising levels as long as $\hat{\phi}_i = \phi_i$ and even higher levels for $\hat{\phi}_i > \phi_i$. Therefore, if $\hat{\phi}_i \geq \phi_i$, the over-provision result follows.

The result implies that $\hat{\phi}_i < \phi_i$ is necessary for competition to improve welfare. However, even if $\hat{\phi}_i$ is smaller than ϕ_i , advertising levels in duopoly can be inefficiently high. First, by continuity, if $\hat{\phi}_i$ is close to ϕ_i , over-provision occurs. In addition, even if $\hat{\phi}_i$ is much lower than ϕ_i , the result also holds as long as viewer dis-utility from ads is sufficiently high. The reason is again that outlets do not take viewers' utilities directly into account in their advertising choice. The result can only be reversed if the

Ambrus and Argenziano (2009).

³³To see this, consider the case in which viewers join either outlet i or j , implying that $D_{12} = 0$. If there is only a single advertiser, the transfer that outlet i can charge to make the advertiser accept is the incremental value of the outlet, i.e., $u(n_1^d, n_2^d) - u(0, n_2^d)$. In the either/or framework, $u(n_1^d, n_2^d) = D_1(n_1^d, n_2^d)\phi_1(n_1) + D_2(n_1^d, n_2^d)\phi_2(n_2)$, while $u(0, n_2^d) = D_2(0, n_2^d)\phi_2(n_2)$. Hence,

$$\Pi_i^d = D_1(n_1^d, n_2^d)\phi_1(n_1) + D_2(n_1^d, n_2^d)\phi_2(n_2) - D_2(0, n_2^d)\phi_2(n_2).$$

The first two terms are equivalent to a monopolist's profit, while the last term is independent of n_i^d . Therefore, the first-order conditions for monopoly and duopoly coincide.

³⁴This perception triggered intervention by regulators in several countries. For examples, limiting the number of advertising minutes per hour on TV is common in many European countries.

dis-utility from ads is small and $\hat{\phi}_i \ll \phi_i$ because then advertising levels in duopoly are much lower than with joint ownership. As a consequence, the condition in Proposition 5 is sufficient, but not necessary.

We therefore obtain the result that competition may even lead to higher advertising levels (and to lower welfare) than joint monopoly. This result contrasts with the one of the standard competitive bottleneck model, where competition always lowers advertising (and improves welfare, as long as advertising levels are not excessively small). This demonstrates that the importance of demand composition brought about by either or both competition predicts an outcome that is closer to the common perception of excessive advertising in media markets.

We finally note that Proposition 5 should be interpreted with caution. The over-provision result hinges on the assumption that advertisers are homogeneous. If advertisers are heterogeneous with respect to their product valuations, an extensive margin comes into play in addition to the intensive margin considered so far. This extensive margin arises because, as in previous literature, an outlet owner trades off the marginal profit from an additional advertiser with the profits from infra-marginal advertisers. This effect would couple with ours, potentially diminishing its extent.

9 Conclusion

This paper presented a model of outlet competition with overlapping viewerships, allowing for fairly general viewer demand and advertising technologies. We emphasize the role that viewer composition plays for market outcomes, and identify novel competitive effects, such as the duplication and the business-sharing effect.

The generality of the framework allows the model to serve as a useful building block to tackle a variety of questions. For example, we took the quality of outlets to be exogenous in our analysis. Yet, competition in media markets (and in many other industries) often works through quality. Our model can be used to investigate whether markets in which users can be active on multiple outlets lead to higher or lower quality than those in which users are primarily active on a single outlet. Another interesting question pertains to pricing tools. We considered the case in which outlets offer contracts consisting of an advertising level and a transfer, but in some industries firms primarily charge linear prices. How then do our results depend on the contracting environment? Also, do linear prices lead to a more or less competitive outcome? We leave these questions for future research.

Our model is also not restricted to the media markets context. In particular, a characterizing feature of our model is that consumers are multi-stop shoppers, i.e., can patronize multiple firms, but that a firm's revenue is lower for a consumer who buys from several other firms. Hence our model contributes to understanding competition in settings where firms care not only about the overall demand but also about its composition. Such settings arise naturally when serving different types of customers yield different revenues (as in our model), as well as when there are consumption externalities among customers.

10 Appendix

Appendix A: Proof of Propositions

Proof of Claim 1:

First suppose that there is a non-singleton menu of contracts $(t_i^k, m_i^k)_{k=1}^K$ offered by outlet i such that each of these contracts is accepted by some advertisers. Then advertisers have to be indifferent between these contracts. Let $F(k)$ denote the cumulative density of advertisers accepting some contract $(t_i^{k'}, m_i^{k'})$ for some $k' \leq k$. Then, by strict concavity of ϕ_i and ϕ_{12} , if outlet i instead offered a single contract $(F(K)E(t_i^k), F(K)E(m_i^k))$, where the expectations are taken with respect to F , each advertiser would strictly prefer to accept the contract, resulting in the same total advertising level and profit for the outlet. But then outlet i could increase profits by offering a single contract $(F(K)E(t_i^k) + \varepsilon, F(K)E(m_i^k))$, for a small enough $\varepsilon > 0$, since such a contract would still guarantee acceptance from all advertisers. The same logic can be used to establish that it cannot be in equilibrium that a single contract (t_i, m_i) is offered but only a fraction of advertisers $F(1) < 1$ accept it, since offering $(F(1) \times t_i + \varepsilon, F(1) \times m_i)$ for small enough $\varepsilon > 0$ would guarantee acceptance by all advertisers and generate a higher profit for outlet i .

The above arguments establish that the total realized advertising level on outlet i is m_i , the intensity specified in the single contract offered by i . It cannot be that $m_i > n_i$, since then by assumption the outlet's payoff would be negative. Moreover, since ϕ_i and ϕ_{12} are strictly increasing, it cannot be that $m_i < n_i$, since then the outlet could switch to offering a contract $(t_i + \varepsilon, n_i)$, which for small enough $\varepsilon > 0$ would guarantee acceptance by all advertisers and generate a higher profit for outlet i . Thus $m_i = n_i$.

Finally, note that $t_1 < u(n_1, n_2) - u(0, n_2)$ implies that outlet 1 could charge a higher transfer and still guarantee the acceptance of all advertisers, while $t_1 > u(n_1, n_2) - u(0, n_2)$ would contradict that all advertisers accept both outlets' contracts. Hence, $t_1 = u(n_1, n_2) - u(0, n_2)$. A symmetric argument establishes that $t_2 = u(n_1, n_2) - u(n_1, 0)$.

The proof of Claim 2 proceeds exactly along the same lines and is therefore omitted. ■

Proof of Proposition 1:

We know that the equilibrium advertising level in case of duopoly is given by (5), while the equilibrium advertising level of a single outlet monopolist is given by (4). To check if advertising levels rise with entry, let us evaluate (5) at n_i^m and n_j^d . Since the first terms in equations (4) and (5) are the same, we have $n_i^d > n_i^m$ if and only if

$$-D_{12} \frac{\partial(\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i})}{\partial n_i} - \frac{\partial D_{12}}{\partial n_i} (\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}) > 0.$$

Due to the fact that the objective functions are single-peaked, it follows that the incumbent's equilibrium advertising level in duopoly is larger than the equilibrium advertising level in monopoly if the marginal profit evaluated at the pre-entry advertising level is positive, given that outlet j sets n_j^d . Rearranging this inequality gives

$$-\frac{\partial D_{12}}{\partial n_i} \frac{n_i}{D_{12}} > \left(\frac{\partial(\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i})}{\partial n_i} \right) \frac{n_i}{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}}.$$

Using our definitions

$$E_{D_{12}} := -\frac{\partial D_{12}}{\partial n_i} \frac{n_i}{D_{12}}$$

and

$$E_{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}} := \frac{\partial(\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i})}{\partial n_i} \frac{n_i}{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}},$$

we can rewrite (10) as $E_{D_{12}} > E_{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}}$. Dividing this expression by $E_{d_i} > 0$, we obtain $E_{D_{12}}/E_{d_i} > E_{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}}/E_{d_i}$. Finally, note that from (4) we have $E_{d_i} = E_{\phi_i}$, which yields

$$\frac{E_{D_{12}}}{E_{d_i}} > \frac{E_{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}}}{E_{\phi_i}}.$$

■

Proof of Lemma 1:

Our goal is to show that D_{12} is strictly increasing in ρ . D_{12} is defined as $\text{Prob}\{q_1 \geq \gamma n_1; q_2 \geq \gamma n_2\}$. To simplify the exposition, we set $\gamma = 1$. (We also do so in the proofs of Lemma 2 and Proposition 2). This implies $D_{12} := \text{Prob}\{q_1 \geq n_1; q_2 \geq n_2\}$. If (q_1, q_2) are drawn from a bivariate normal distribution with mean $(0,0)$ and variance $\Sigma = ((1, \rho), (\rho, 1))$, we can write

$$D_{12} = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{n_2}^{\infty} \int_{n_1}^{\infty} e^{-\frac{q_1^2 - 2\rho q_1 q_2 + q_2^2}{2(1-\rho^2)}} dq_1 dq_2. \quad (10)$$

To determine the sign of $\partial D_{12}/\partial \rho$, we first perform integration with respect to q_1 and then differentiate with respect to ρ . When integrating with respect to q_1 , we use the error function defined as $\text{erf}(n_1) = (2/\sqrt{\pi}) \int_0^{n_1} e^{-q_1^2} dq_1$ and exploit the facts that $\lim_{n_1 \rightarrow \infty} \text{erf}(n_1) \rightarrow 1$ and³⁵

$$\frac{1}{\sqrt{2}\sqrt{\pi}} \int_0^{n_1} e^{-q_1^2} dq_1 = \frac{1}{2} \left[1 - \text{erf}\left(\frac{n_1}{2}\right) \right].$$

After taking the derivative with respect to ρ we obtain

$$\frac{\partial D_{12}}{\partial \rho} = \frac{-1}{2\pi(1-\rho^2)^{3/2}} \int_{n_2}^{\infty} e^{-\frac{n_1^2 - 2\rho n_1 q_2 + q_2^2}{2(1-\rho^2)}} (q_2 - \rho n_1) dq_2. \quad (11)$$

We can integrate the right-hand side of (11) directly to obtain

$$\frac{\partial D_{12}}{\partial \rho} = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{n_1^2 - 2\rho n_1 n_2 + n_2^2}{2(1-\rho^2)}} > 0. \quad (12)$$

Because the integrand of the expression in (10) is well-behaved, we can also reverse the order of integration and differentiation to obtain the same result. As a consequence, we have $\partial D_{12}/\partial \rho > 0$ for all (n_1, n_2) . ■

³⁵See, for example, Greene (2007).

Proof of Lemma 2:

The elasticity $E_{D_{12}}$ is given by $-(\partial D_{12}/\partial n_i)(n_i/D_{12})$. Taking the derivative with respect to ρ yields

$$\frac{\partial E_{D_{12}}}{\partial \rho} = -\frac{\partial \left(\frac{\partial D_{12}}{\partial n_i} \right)}{\partial \rho} \frac{n_i}{D_{12}} + \frac{\partial D_{12}}{\partial n_i} \frac{n_i}{D_{12}^2} \frac{\partial D_{12}}{\partial \rho} = -\frac{n_i}{D_{12}} \left(\frac{\partial^2 D_{12}}{\partial n_i \partial \rho} - \frac{\partial D_{12}}{\partial n_i} \frac{1}{D_{12}} \frac{\partial D_{12}}{\partial \rho} \right). \quad (13)$$

Rearranging (13) and using $\partial D_{12}/\partial \rho > 0$ yields that $\partial E_{D_{12}}/\partial \rho < 0$ if and only if

$$\frac{\frac{\partial D_{12}}{\partial n_i}}{D_{12}} < \frac{\frac{\partial^2 D_{12}}{\partial n_i \partial \rho}}{\frac{\partial D_{12}}{\partial \rho}}. \quad (14)$$

We can write D_{12} as $D_{12} = \int_{n_2}^{\infty} \int_{n_1}^{\infty} h(q_1, q_2) dq_2 dq_1$, where $h(q_1, q_2)$ is the probability density function of the reservation values (q_1, q_2) . It is given by

$$\frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{q_1^2 - 2\rho q_1 q_2 + q_2^2}{2(1-\rho^2)}}.$$

Taking the derivative of $D_{12} = \int_{n_2}^{\infty} \int_{n_1}^{\infty} h(q_1, q_2) dq_2 dq_1$ with respect to n_1 and n_2 and using the Leibniz rule yields

$$\frac{\partial^2 D_{12}}{\partial n_1 \partial n_2} = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{n_1^2 - 2\rho n_1 n_2 + n_2^2}{2(1-\rho^2)}}.$$

The right-hand side of the last equation is $h(n_1, n_2)$. We therefore obtain that for the bivariate normal distribution, $\partial D_{12}/\partial \rho$ which is given by (12) equals the derivative of D_{12} with respect to n_1 and n_2 . Hence,

$$\frac{\partial D_{12}}{\partial \rho} = h(n_1, n_2).$$

We can therefore rewrite (14) as

$$-\frac{\int_{n_j}^{\infty} h(n_i, q_j) dq_j}{\int_{n_2}^{\infty} \int_{n_1}^{\infty} h(q_1, q_2) dq_2 dq_1} < \frac{\frac{\partial h(n_1, n_2)}{\partial n_i}}{h(n_1, n_2)}. \quad (15)$$

In the following, we show that the inequality is indeed fulfilled for $n_i = n_j$.

We start with the right-hand side. Using $h(n_1, n_2) = 1/(2\pi\sqrt{1-\rho^2})e^{-\frac{n_1^2 - 2\rho n_1 n_2 + n_2^2}{2(1-\rho^2)}}$, we obtain

$$\frac{\partial h(n_1, n_2)}{\partial n_i} = -\frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{n_1^2 - 2\rho n_1 n_2 + n_2^2}{2(1-\rho^2)}} \frac{n_i - \rho n_j}{1-\rho^2}.$$

We can therefore write the right hand-side of (15) as

$$-\frac{n_i - \rho n_j}{1-\rho^2}.$$

For $n_i = n_j$, we obtain

$$-n_j \frac{1-\rho}{1-\rho^2}.$$

Now we turn to the left-hand side of (15). The numerator is

$$\int_{n_j}^{\infty} h(n_i, q_j) dq_j = \int_{n_j}^{\infty} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{n_i^2 - 2\rho n_i q_j + q_j^2}{2(1-\rho^2)}} dq_j.$$

Because $\lim_{x \rightarrow \infty} e^{-x} = 0$, the right-hand side of the last expression can be written as

$$\left| \int_{n_j}^{\infty} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_j \right|_{q_i=n_i}^{q_i=\infty}.$$

To make the numerator and the denominator of the left-hand side comparable with each other, we modify the last expression to a double integral form. This gives

$$\int_{n_j}^{\infty} \int_{\infty}^{n_i} \frac{1}{2\pi\sqrt{1-\rho^2}} \frac{\partial e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}}}{\partial q_i} dq_i dq_j.$$

Taking the derivate of the integrand yields

$$= \int_{n_j}^{\infty} \int_{\infty}^{n_i} \frac{1}{2\pi\sqrt{1-\rho^2}} \frac{\rho q_j - q_i}{1-\rho^2} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_i dq_j.$$

Finally, we apply $\int_a^b x dx = \int_b^a (-x) dx$ to get

$$\int_{n_j}^{\infty} \int_{n_i}^{\infty} \frac{q_i - \rho q_j}{2\pi\sqrt{1-\rho^2}(1-\rho^2)} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_i dq_j. \quad (16)$$

The denominator is given by

$$\int_{n_j}^{\infty} \int_{n_i}^{\infty} h(q_1, q_2) dq_2 dq_1 = \int_{n_j}^{\infty} \int_{n_i}^{\infty} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_2 dq_1. \quad (17)$$

Dividing (16) by (17), we can write the left-hand side of (15) as

$$-\frac{\int_{n_j}^{\infty} \int_{n_i}^{\infty} (q_i - \rho q_j) e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_i dq_j}{\int_{n_j}^{\infty} \int_{n_i}^{\infty} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_j dq_i} \frac{1}{1-\rho^2}.$$

For $n_i = n_j$, the two integrals have the same length, implying that the left-hand side of (15) is given by

$$-\frac{\int_{n_j}^{\infty} \int_{n_j}^{\infty} q_j e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_i dq_j}{\int_{n_j}^{\infty} \int_{n_j}^{\infty} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_j dq_i} \frac{1-\rho}{1-\rho^2} = -E(q_j | q_j \geq n_j, q_i \geq n_j) \frac{1-\rho}{1-\rho^2}.$$

We are now in a position to compare the two sides of (15) with each other. Since $-E(q_j | q_j \geq n_j, q_i \geq n_j) < -n_j$, we have

$$-E(q_j | q_j \geq n_j, q_i \geq n_j) \frac{1-\rho}{1-\rho^2} < -n_j \frac{1-\rho}{1-\rho^2}.$$

Therefore, the inequality in (15) is always fulfilled, implying that $\partial E_{D_{12}}/\partial \rho < 0$. ■

Proof of Proposition 2:

From the last Lemma, we know that $E_{D_{12}}$ is strictly decreasing with ρ at $n_i = n_j$. Because ρ only affects the composition of viewers but not the total demand, E_{d_i} is unaffected by ρ . Hence, the left-hand side of (6) is strictly decreasing in ρ .

Now let us look at the case $\rho = 0$. The left-hand side of (6) is given by $E_{D_{12}}/E_{d_i}$. The denominator can be written as

$$E_{d_i} = \frac{e^{-\frac{n_i^2}{2}}}{\int_{n_i}^{\infty} e^{-\frac{q_i^2}{2}} dq_i} n_i, \quad (18)$$

while the numerator is

$$E_{D_{12}} = \frac{\int_{n_j}^{\infty} e^{-\frac{n_i^2 + q_j^2}{2}} dq_j}{\int_{n_j}^{\infty} \int_{n_i}^{\infty} e^{-\frac{q_i^2 + q_j^2}{2}} dq_i dq_j} n_i.$$

For $n_j = n_i$, the last equation can be written as

$$E_{D_{12}} = \frac{e^{-\frac{n_i^2}{2}} \int_{n_i}^{\infty} e^{-\frac{q_i^2}{2}} dq_i}{\left(\int_{n_i}^{\infty} e^{-\frac{q_i^2}{2}} dq_i \right)^2} n_i = \frac{e^{-\frac{n_i^2}{2}}}{\int_{n_i}^{\infty} e^{-\frac{q_i^2}{2}} dq_i} n_i. \quad (19)$$

Dividing (19) by (18), it is easy to see that this equals 1, which implies that $E_{D_{12}}/E_{d_i} = 1$ at $\rho = 0$.

We now turn to the right-hand side of (6). Since we are considering the case $\hat{\phi} = \phi$, we have $\Delta_{\phi_i} = 0$. The numerator of the right-hand side of (6) can therefore be written as

$$E_{\phi_i \phi_j} = \frac{\partial \phi_i \phi_j}{\partial n_i} \frac{n_i}{\phi_i \phi_j}.$$

Using that ϕ_j is independent of n_i (that is, $\partial \phi_j / \partial n_i = 0$), we can write the numerator of the right-hand side of the last expression as

$$\frac{\partial \phi_i}{\partial n_i} \frac{\phi_j n_i}{\phi_i \phi_j} = \frac{\partial \phi_i}{\partial n_i} \frac{n_i}{\phi_i}.$$

The denominator is

$$E_{\phi_i} = \frac{\partial \phi_i}{\partial n_i} \frac{n_i}{\phi_i}.$$

It follows that $E_{\phi_i \phi_j} = E_{\phi_i}$, implying that the right-hand side of (6) is equal to 1, independent of ρ . This result coupled with the fact that the left-hand side equals 1 at $\rho = 0$ and that it is strictly decreasing in ρ yields the result. ■

Proof of Proposition 3:

We know that $n_i^d > n_i^m$ if and only if

$$1 > \frac{E_{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}}}{E_{\phi_i}} = \frac{E_{\hat{\phi}_i \hat{\phi}_j + \phi_i - \hat{\phi}_i}}{E_{\phi_i}}.$$

or

$$E_{\phi_i} > E_{\hat{\phi}_i \hat{\phi}_j + \phi_i - \hat{\phi}_i}.$$

Writing out the respective expressions for the elasticities gives

$$\frac{\partial \phi_i}{\partial n_i} \frac{n_i}{\phi_i} > \frac{\partial \left(\hat{\phi}_i \hat{\phi}_j + \phi_i - \hat{\phi}_i \right)}{\partial n_i} \frac{n_i}{\hat{\phi}_i \hat{\phi}_j + \phi_i - \hat{\phi}_i}. \quad (20)$$

Since $\partial \hat{\phi}_j / \partial n_i = 0$, we have $\partial(\hat{\phi}_i \hat{\phi}_j) / \partial n_i = \hat{\phi}_j (\partial \hat{\phi}_i) / \partial n_i$. Inserting this into (20) and rearranging yields

$$\frac{\partial \phi_i}{\partial n_i} \left(\frac{n_i}{\phi_i} - \frac{n_i}{\hat{\phi}_i \hat{\phi}_j + \phi_i - \hat{\phi}_i} \right) > -(1 - \hat{\phi}_j) \frac{\partial \hat{\phi}_i}{\partial n_i} \frac{n_i}{\hat{\phi}_i \hat{\phi}_j + \phi_i - \hat{\phi}_i}. \quad (21)$$

Simplifying and dividing (21) by $\hat{\phi}_i(\hat{\phi}_j - 1) < 0$ yields

$$\frac{\partial \phi_i}{\partial n_i} \frac{n_i}{\phi_i} < \frac{\partial \hat{\phi}_i}{\partial n_i} \frac{n_i}{\hat{\phi}_i}$$

or

$$E_{\phi_i} < E_{\hat{\phi}_i}.$$

■

Proof of Proposition 4:

Consider first the case of competing outlets. From (2), we know that outlet i 's profit maximization problem is

$$\max_{n_i} \Pi_i^d = \left[D_i(n_i, n_j) \phi_i(n_i) + D_{12}(n_i, n_j) (\phi_{12}(n_i, n_j) - \hat{\phi}_j(n_j)) \right].$$

The equilibrium advertising levels are therefore characterized by the following system of first-order conditions (arguments omitted):

$$\frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i} (\phi_{12} - \hat{\phi}_j) + D_{12} \frac{\partial \phi_{12}}{\partial n_i} = 0, \quad i, j = 1, 2; \quad j = 3 - i. \quad (22)$$

Consider now the case of joint ownership. The joint monopolist's problem is

$$\max_{n_i, n_j} \Pi^{jo} = D_1 \phi_1 + D_2 \phi_2 + D_{12} \phi_{12}, \quad i, j = 1, 2; \quad j = 3 - i. \quad (23)$$

Taking the first-order condition of (23) with respect to n_i we obtain

$$\frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_j}{\partial n_i} \phi_j + \frac{\partial D_{12}}{\partial n_i} \phi_{12} + D_{12} \frac{\partial \phi_{12}}{\partial n_i} = 0, \quad i, j = 1, 2; \quad j = 3 - i. \quad (24)$$

We know that the total demand of outlet j , $d_j(n_j) = D_j(n_1, n_2) + D_{12}(n_1, n_2)$, is independent of n_i . This implies $\partial D_j / \partial n_i + \partial D_{12} / \partial n_i = 0$ or $\partial D_j / \partial n_i = -\partial D_{12} / \partial n_i$. Using this, we can rewrite (24) as

$$\frac{\partial D_i}{\partial n_i} \phi_i + D_i \frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i} (\phi_{12} - \phi_j) + D_{12} \frac{\partial \phi_{12}}{\partial n_i} = 0, \quad i, j = 1, 2; \quad j = 3 - i. \quad (25)$$

Comparing (25) with (22), it is evident that at $n_i = n_i^d$, (25) is positive if and only if $\phi_j > \hat{\phi}_j$. This implies that $n_i^{jo} > n_i^d$ if and only if $\phi_j > \hat{\phi}_j$ and $n_i^{jo} < n_i^d$ if and only if $\phi_j < \hat{\phi}_j$. If $\phi_j = \hat{\phi}_j$, the two advertising levels are equivalent (i.e., $n_i^{jo} = n_i^d$). ■

Proof of Proposition 5:

We first look at the last three terms in W , i.e., $D_1\phi_1 + D_2\phi_2 + D_{12}\phi_{12}$. Taking the derivative of these terms gives

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i\frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_j}{\partial n_i}\phi_j + \frac{\partial D_{12}}{\partial n_i}\phi_{12} + D_{12}\frac{\partial \phi_{12}}{\partial n_i}. \quad (26)$$

We can now substitute $\partial D_j/\partial n_i = -\partial D_{12}/\partial n_i$ into (26) to obtain

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i\frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_j) + D_{12}\frac{\partial \phi_{12}}{\partial n_i}.$$

It is evident that this expression is equivalent to (25). Therefore, at $n_i = n_i^{jo}$, this expression equals zero. Since $n_i^d = n_i^{jo}$ for $\phi_j = \hat{\phi}_j$, the last three terms of W are maximized at $n_i = n_i^d$.

However, the first terms in W are the utilities of the viewers which are strictly decreasing in n_i . As a consequence, the first-order condition of W with respect to n_i evaluated at $n_i = n_i^d$ is strictly negative, which implies that there is too much advertising in duopoly at $\phi_j = \hat{\phi}_j$. Because n_i^d is even larger than n_i^{jo} for $\phi_j < \hat{\phi}_j$, there is also too much advertising in this range. ■

Appendix B: Empirical Analysis with Entry Episodes Model

The panel analysis has the advantage of pooling data on different channels without taking a stance on the time it takes for entry to impact the incumbent choices. However, this strategy does not allow for accounting for within-channel omitted variables that vary over time. These variables may also operate at the segment level. To account for this, and as an alternative way to address the same issues as in the panel analysis, we also estimate a model for entry episodes, where our sample is now reduced to the periods when a given segment experiences the entry of a new channel. We estimate the following model:

$$\begin{aligned} \Delta \log(Avails_{it}) = & \beta + \beta_M * MoviesSeries_dummy + \beta_S * Sports_dummy \\ & + \gamma * x_{it} + \delta_t + \epsilon_{it} \end{aligned}$$

This model can be obtained by first differencing the previous model around the years when entry occurs. In fact, $\Delta \log(Avails_{it}) = \log(Avails_{it+1}) - \log(Avails_{it-1})$ and the effect of entry (changed number of incumbents) is captured by the constant terms. Channel fixed effects are now excluded (as they cancel out in taking first differences), but we keep time fixed effects and also add some channel controls. The constant β measures the effect of entry on the reference group, while β_S and β_M measure the additional effect for the Sports and Movies&Series segments, respectively. The estimates reported in Table 3 confirm our previous results: entry episodes are associated with an increase in the quantity of avails in the reference group, while the effect is lower in the Sports and Movies&Series segments. Since there are half as many observations in this setup, the point estimates are less precisely estimated than in Table 2. Furthermore, because here we are looking at the effect one year after entry (t+1), the magnitude of the parameters is notably bigger. The point estimate of the percent variation in avails due to an additional channel is on

the order of 5% in column (3). Notably, the interaction term that captures the differential impact of entry in sports is around 11% less than the industry average. The difference is statistically and economically significant.

Table 3: Entry Episodes - Average Effect and Effect by Segment

| | (1) | (2) | (3) | (4) |
|-----------------------|----------------------|--------------------------|-------------------------|------------------------|
| MoviesSeries dummy | | -0.0327*** (0.000919) | -0.0494** (0.00841) | -0.0314 (0.0124) |
| Sports dummy | | -0.0563*** (0.000457) | -0.110*** (0.00460) | -0.0172 (0.0222) |
| Δ GDP[t-1,t+1] | 0.00380 (0.00133) | 0.00328* (0.00103) | | |
| Rating | | | -0.0392*** (0.00128) | |
| Rev Mkt Share | | | | -0.171 (0.0987) |
| Constant | 0.00393 (0.0213) | 0.0178 (0.00901) | 0.0615** (0.00875) | 0.263*** (0.000141) |
| Observations | 219 | 219 | 158 | 219 |
| R^2 | 0.009 | 0.028 | 0.121 | 0.091 |
| Time FE | NO | NO | YES | YES |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

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ONLINE APPENDIX

for

Either or Both Competition:

A “Two-sided” Theory of Advertising with Overlapping Viewerships

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This Online Appendix provides supplemental material for the paper “Either or Both Competition: A “Two-sided” Theory of Advertising with Overlapping Viewerships”. Specifically, it presents an analysis of a two-stage game in which outlets simultaneously make offers and afterwards all agents simultaneously make their choices, and provides the conditions for outcome-equivalence to the four-stage game considered in the paper. It also demonstrates that the effects identified in the paper are also at work in a game with two incumbent outlets and one entrant. Finally, the Online Appendix presents an analysis of heterogeneous advertisers and relates it to the analysis of homogeneous advertisers presented in the paper.

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1 Two-stage game

Consider the following two-stage game: In stage 1, outlets simultaneously offer menus of contracts to advertisers of the form $(t_i, m_i) \in \mathbb{R}_+^2$. After observing these contracts, viewers and advertisers simultaneously choose which outlet(s) to join and which contract(s) to accept, respectively.

In addition, consider the following assumptions:

A1 Outlets are symmetric.

A2 For any $\alpha \in [0, 1]$, the following inequality holds

$$t_i^*(1 - \alpha) > \alpha \left\{ d_i(\alpha \tilde{n}_i) \phi(\tilde{n}_i) - d_i((1 - \alpha)n_i^*) \phi(n_i^*) \right\}, \quad (1)$$

where $d_i(\cdot) := D_i(\cdot) + D_{12}(\cdot)$, $\tilde{n}_i = \arg \max_{n_i} d_i(\alpha n_i) \phi(n_i)$, n_i^* is implicitly defined by equation (3) of the paper and t_i^* is given by $D_i(n_i^*, n_j^*) \phi_i(n_i^*) + D_{12}(n_i^*, n_j^*) (\phi_{12}(n_i^*, n_j^*) - \hat{\phi}_j(n_j^*))$.

We provide a discussion of these assumptions after the proof of the following proposition. There we explain that A1 can be weakened while A2 is a relatively natural assumption in our framework.

Proposition *Suppose that A1 and A2 hold. Then, there is an equilibrium in the two-stage game with posted contracts, that is outcome-equivalent to the equilibrium of the game defined in Section 3 of the paper.*

Proof:

Suppose that in the two-stage game with posted contracts each outlet offers a contract with $n_i = n_i^*$, where n_i^* is implicitly defined by equation (3) of the paper, and a transfer

$$t_i^* = D_i(n_i^*, n_j^*) \phi_i(n_i^*) + D_{12}(n_i^*, n_j^*) (\phi_{12}(n_i^*, n_j^*) - \hat{\phi}_j(n_j^*)).$$

By the same argument as we used for the original model, these contracts will be accepted by all advertisers. As this is anticipated by viewers, viewerships are $D_i(n_i^*, n_j^*)$ and $D_{12}(n_i^*, n_j^*)$. Since advertising levels are the same as in the equilibrium of the original model, viewerships are also the same. Therefore, this candidate equilibrium is outcome-equivalent to the equilibrium of the original model.

Let us now consider if there exists a profitable deviation from this candidate equilibrium. We first show that there can be no profitable deviation contract of outlet i that still induces full advertiser participation on outlet j but a smaller participation on outlet i . Let x_i denote the fraction of advertisers who accept the offer of outlet i .

Consider a candidate contract (n_i, t_i) . Suppose that outlet i 's equilibrium profit from this contract is $t_i x_i$. Now consider the following alternative contract: $(x_i n_i, x_i t_i)$. Note that total advertising on outlet i is still equal to $x_i n_i$. So outlet i is at least as attractive as with the candidate equilibrium contract. Note moreover that because ϕ_i and ϕ_{12} are strictly concave in n_i , the incremental value of accepting offer $(x_i n_i, x_i t_i)$ must exceed $x_i t_i$ for all levels of advertiser participation. So all advertisers would accept $(x_i n_i, x_i t_i)$ regardless. It follows that outlet i can marginally increase $x_i t_i$ while still getting full participation. Therefore, profits would strictly increase. It follows that no offer inducing a level of participation $x_i < 1$ can be part of a best reply.

Now suppose outlet i deviates from the candidate equilibrium in such a way that it induces a fraction α of the advertisers to single-home on its outlet while the remaining fraction $1 - \alpha$ single-homes on outlet j . Using the definition $d_i(\cdot) := D_i(\cdot) + D_{12}(\cdot)$, the largest possible transfer that outlet i can ask is then

bounded above by

$$t_i^d = d_i(\alpha \tilde{n}_i) \phi_i(\tilde{n}_i) - u_{shj},$$

where \tilde{n}_i denotes the optimal deviation advertising level and u_{shj} denotes the payoff of an advertiser who chooses to reject the contract of outlet i and instead single-homes on outlet j . To determine u_{shj} we determine the advertiser's payoff when accepting only outlet j 's contract, which is the outlet's equilibrium contract after outlet i has deviated to induce a fraction α of advertisers to single-home on outlet i . We obtain

$$u_{shj} = d_j((1 - \alpha)n_j^*, \alpha \tilde{n}_i) \phi_j(n_j^*) - t_j^* = \\ d_j((1 - \alpha)n_j^*, \alpha \tilde{n}_i) \phi_j(n_j^*) - D_j(n_j^*, n_i^*) \phi_j(n_j^*) - D_{12}(n_i^*, n_j^*) \left(\phi_{12}(n_i^*, n_j^*) - \hat{\phi}_i(n_i^*) \right).$$

Outlet i 's profit is then αt_i . Hence, deviating is not profitable if

$$\alpha \left\{ d_i(\alpha \tilde{n}_i) \phi_i(\tilde{n}_i) - d_j((1 - \alpha)n_j^*) \phi_j(n_j^*) + D_j(n_j^*, n_i^*) \phi_j(n_j^*) + D_{12}(n_i^*, n_j^*) \left(\phi_{12}(n_i^*, n_j^*) - \hat{\phi}_i(n_i^*) \right) \right\} \\ < D_i(n_i^*, n_j^*) \phi_i(n_i^*) + D_{12}(n_i^*, n_j^*) \left(\phi_{12}(n_i^*, n_j^*) - \hat{\phi}_j(n_j^*) \right).$$

Now suppose that the two outlets are symmetric. Then the above condition reduces to

$$\alpha \left\{ d_i(\alpha \tilde{n}) \phi(\tilde{n}) - d_i((1 - \alpha)n^*) \phi(n^*) \right\} - (1 - \alpha) \left(D_i(n^*, n^*) \phi(n^*) + D_{12}(n^*, n^*) \left(\phi_{12}(n^*, n^*) - \hat{\phi}(n^*) \right) \right) < 0,$$

where $n_i^* = n_j^* = n^*$, $\tilde{n}_i = \tilde{n}^d$, $\hat{\phi}_i(\cdot) = \hat{\phi}_j(\cdot) = \hat{\phi}(\cdot)$, and $\phi_i(\cdot) = \phi_j(\cdot) = \phi(\cdot)$. This can be rewritten as

$$t_i^*(1 - \alpha) > \alpha \left\{ d_i(\alpha \tilde{n}_i) \phi(\tilde{n}_i) - d_i((1 - \alpha)n_i^*) \phi(n_i^*) \right\}.$$

which is fulfilled by A2. As a consequence, a deviation is not profitable. ■

We now shortly explain why the assumptions A1 and A2 are not very restrictive in our framework. First, consider A1. Since the game is continuous, A1 can be relaxed to some extent without affecting the result, implying that the proposition still holds if outlets are not too asymmetric. Now consider A2. It is evident from (1), that the assumption is fulfilled for α low enough. In this case the right-hand side is close to 0, while the left-hand side is strictly positive. Now consider the opposite case, i.e., $\alpha \rightarrow 1$. In that case the left-hand side goes to zero, while the right-hand side goes to $d_i(\tilde{n}_i) \phi(\tilde{n}_i) - d_i(0) \phi(n_i^*)$. Evidently, $d_i(0) > d_i(\tilde{n}_i)$. Hence, the right-hand side is negative if $\phi(\tilde{n}_i)$ is not much larger than $\phi(n_i^*)$. In general, n_i^* can be larger or smaller than \tilde{n}_i , implying that the difference can be either positive or negative. However, even in case $\tilde{n}_i > n_i^*$, if the slope of the advertising functions ϕ_i and ϕ_{12} is relatively small, the difference between n_i^* and \tilde{n}_i will be small, implying that the right-hand side is negative. Finally, consider intermediate values of α . Again, if the difference between n_i^* and \tilde{n}_i is relatively small, the term in the bracket on the right-hand side of (1) is close to zero. Since the left-hand side is strictly positive, A2 is then fulfilled as well.

2 Entry in case of two incumbent outlets

Consider the case of two incumbents and entry of a third outlet. After entry, the profit of outlet i is

$$\Pi_i(n_1, n_2, n_3) = D_i(n_1, n_2, n_3) \phi_i(n_i) + D_{ij}(n_1, n_2, n_3) \left(\phi_{ij}(n_i, n_j) - \hat{\phi}_j(n_j) \right)$$

$$+D_{ik}(n_1, n_2, n_3) \left(\phi_{ik}(n_i, n_k) - \hat{\phi}_k(n_k) \right) + D_{123}(n_1, n_2, n_3) (\phi_{ijk}(n_i, n_j, n_k) - \phi_{jk}(n_j, n_k))$$

As in the case of entry of a second outlet, we can rewrite this profit function as the profit without entry plus a negative correction term. This leads to (dropping arguments)

$$\begin{aligned} \Pi &= (D_i + D_{ik})\phi_i + (D_{ij} + D_{ijk})(\phi_{ij} - \hat{\phi}_j) \\ &\quad - D_{ik}(\phi_i + \hat{\phi}_k - \phi_{ik}) - D_{ijk} \left(\phi_{ij} - \hat{\phi}_j - (\phi_{ijk} - \phi_{jk}) \right). \end{aligned}$$

The first two terms are the profit in duopoly. Note that without entry D_{ik} did not exist since there was no outlet k and so outlet i could get ϕ_i for these viewers due to the fact that they were single-homing on outlet i . Similarly, D_{ijk} did not exist and these viewers were multi-homing in outlets i and j . The last two terms are the negative correction terms.

Taking the derivative with respect to n_i yields

$$\begin{aligned} \frac{\partial \Pi}{\partial n_i} &= \frac{\partial \Pi^d}{\partial n_i} + D_{ik}(\phi_i + \hat{\phi}_k - \phi_{ik}) \left[E_{D_{ik}} - E_{\phi_i + \hat{\phi}_k - \phi_{ik}} \right] \\ &\quad + D_{ijk} \left(\phi_{ij} - \hat{\phi}_j - (\phi_{ijk} - \phi_{jk}) \right) \left[E_{D_{ijk}} - E_{\phi_{ij} - \hat{\phi}_j - (\phi_{ijk} - \phi_{jk})} \right] = 0, \end{aligned}$$

where $\partial \Pi^d / \partial n_i$ is the derivative with respect to n_i of an outlet's profit in case of duopolistic competition. So we obtain that for $E_{D_{ik}} > E_{\phi_i + \hat{\phi}_k - \phi_{ik}}$ and $E_{D_{ijk}} > E_{\phi_{ij} - \hat{\phi}_j - (\phi_{ijk} - \phi_{jk})}$, the business-sharing effect dominates the duplication effect. The formula now consists of two additional terms since entry of a third outlet leads to changes in two viewer groups, namely, the exclusive ones and the overlapping ones before entry. Each term is multiplied by the absolute profits of the respective viewer group. In a similar vein, the analysis can be extended to any number of incumbent outlets.

3 Heterogeneous Advertisers

We discuss how the trade-off characterized in Proposition 1 extends to advertisers with heterogeneous product values, as in Anderson and Coate (2005). As we will show, the key insights obtained in the analysis with homogeneous advertisers carries through to heterogeneous advertisers. In particular, outlet competition is also characterized by the tension between the duplication and business-sharing effect. This holds although the analysis is more involved compared to homogeneous advertisers, as we need to characterize an entire contract schedule (i.e., the optimal screening contracts) offered by outlets, instead of only a single transfer-quantity pair.¹

Consider the following extension of our baseline model. The value of informing a viewer, ω , is distributed according to a smooth c.d.f. F with support $[\underline{\omega}, \bar{\omega}]$, $0 < \underline{\omega} \leq \bar{\omega}$, that satisfies the monotone hazard rate property. The value ω is private information to each advertiser. The timing of the game is the same as before. In the first stage, each outlet i announces its total advertising level n_i . Afterwards, consumers decide which outlet to join. Given these decisions, each outlet offers a menu of contracts

¹Our results also hold when outlets can perfectly discriminate between advertisers. In that case, the results for each type are the same as the ones in case of homogeneous advertisers.

consisting of a transfer schedule $t_i := [0, \bar{m}] \rightarrow \mathbb{R}$ defined over a compact set of advertising levels. $t_i(m)$ is the transfer an advertiser has to pay to get an advertising intensity m from outlet i . In the final stage, as before, advertisers decide which outlet to join. In what follows, we define $n = (n_1, n_2)$.

Let us start with the monopoly case. With an abuse of notation we still use $\omega u(m_i, n_i)$ to denote the surplus of advertiser type ω from advertising intensity m_i . The overall utility of an advertiser depends on the transfer schedule in addition to the surplus. If $m_i(\omega)$ denotes the optimal intensity chosen by type ω , then outlets i 's problem in case of monopoly is

$$\Pi = \max_{t_i(\cdot)} \int_{\omega}^{\bar{\omega}} t_i(m_i(\omega)) dF(\omega). \quad (2)$$

By choosing the optimal menu of contracts, the monopolist determines which advertiser types to exclude, that is, $m_i(\omega) = 0$ for these types, and which advertiser types will buy a positive intensity. We denote the marginal advertiser by ω_0^m . Problem (2) can be expressed as a standard screening problem:

$$\Pi = \max_{\omega_0^m, m_i(\omega)} \int_{\omega_0^m}^{\bar{\omega}} t_i(m_i(\omega)) dF(\omega)$$

$$\begin{aligned} \text{subject to } & m_i(\omega) = \arg \max_{m_i} v_i^m(m_i, \omega, n_i) - t_i(m_i), \\ & v_i^m(m_i(\omega), \omega, n_i) - t_i(m_i(\omega)) \geq 0 \text{ for all } \omega \geq \omega_0^m, \\ & \int_{\omega_0^m}^{\bar{\omega}} m_i(\omega) dF(\omega) \leq n_i, \end{aligned}$$

where $v_i^m(m_i, \omega, n_i) := \omega d_i(n_i) \phi_i(m_i)$ denotes the net value of advertising intensity m_i to type ω in the monopoly case. The first constraint is the incentive-compatibility constraint and the second one the participation constraint. The third one is the capacity constraint specifying that the aggregate advertising level cannot exceed the one specified by the outlet in the first stage. Provided that the function $v_i^m(m_i, \omega, n_i)$ satisfies the standard regularity conditions in the screening literature, we can apply the canonical screening methodology. Our assumptions on the viewer demand $d_i(n_i)$ and on the advertising technology $\phi_i(m_i)$ ensure that v_i^m is continuous and increasing in ω . It also has strictly increasing differences in (m, ω) .

Evidently, the capacity constraint will be binding at the optimal solution since it can never be optimal for the monopolist to announce a strictly larger advertising level than the one it uses. Applying the above-mentioned methodology, we can transform the maximization problem to get

$$\Pi = \max_{\omega_0^m, m_i(\omega)} \int_{\omega_0^m}^{\bar{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)} \right) d_i(n_i) \phi_i(m_i(\omega)) dF(\omega)$$

$$\text{subject to } n_i = \int_{\omega_0^m}^{\bar{\omega}} m_i(\omega) dF(\omega).$$

We show at the end of this section that the optimal advertising level n_i can be characterized by the following equation:

$$\int_{\omega_0^m}^{\bar{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)} \right) \left(\tilde{d}_i \frac{\partial \phi_i}{\partial m_i} + \frac{\partial d_i}{\partial n_i} \phi_i \right) dF(\omega) = 0, \quad (3)$$

with $\tilde{d}_i := (1 - F(\omega_0^m)) d_i$. We can compare this characterization with the one for homogeneous advertisers given by equation (4) of the paper. Due to the information rent that is required for incentive compatibility, the outlet can no longer extract the full rent from advertisers but only a fraction of it. This is expressed

by the first bracket in the integral. Inspecting the second bracket, the expression is analogous to the one with homogeneous advertisers. Note that in the latter case $m_i = n_i$ implies that the derivative was taken with respect to n_i in both terms. The above expression instead accounts for the fact that the optimal allocation $m_i(\omega)$ is heterogeneous across types. A second difference comes from the first term in the second bracket where we have \tilde{d}_i instead of d_i . When changing m_i , only those advertisers who participate are affected. This is only a mass of $1 - F(\omega_0^m)$. By contrast, with homogeneous advertisers all of them are active in equilibrium.

Therefore, with heterogeneous advertisers the equation characterizing n_i trades off the cost and benefits of increasing n_i over the whole mass of participating advertisers, implying that the average costs and benefits are important. However, the basic trade-off for homogeneous advertisers and heterogeneous advertisers is the same. In particular, the first term in the second bracket represents the average marginal profit from increased reach on infra-marginal consumers, whereas the second term represents the average loss from marginal consumers who switch off.

Let us now turn to the optimal advertising levels in duopoly. The goal is to characterize the best-reply tariff $t_i(m_i)$ given outlet j 's choice $t_j(m_j)$. As in the monopoly case, it is possible to rewrite this problem as a standard screening problem. To this end, denote by $\omega u(m_1, m_2, n)$ the surplus of type ω from advertising intensities (m_1, m_2) . If $m_i(\omega)$ denotes the optimal quantity chosen by type ω , then outlets i 's optimization problem is

$$\Pi = \max_{\omega_0^i, m_i(\omega)} \int_{\omega_0^i}^{\bar{\omega}} t_i(m_i(\omega)) dF(\omega) \quad (4)$$

$$\begin{aligned} \text{subject to } m_i(\omega) &= \arg \max_{m_i} v_i^d(m_i, \omega, n) - t_i(m_i), \\ v_i^d(m_i(\omega), \omega, n) - t_i(m_i(\omega)) &\geq 0 \text{ for all } \omega \geq \omega_0^i, \\ \int_{\omega_0^i}^{\bar{\omega}} m_i(\omega) dF(\omega) &\leq n_i, \end{aligned}$$

where $v_i^d(m_i, \omega, n) := \max_y \omega u(m_i, y, n) - t_j(y) - \max_{y'} (\omega u(0, y', n) - t_j(y'))$, with $u(m_i, y, n) := D_i(n_1, n_2)\phi_i(m_i) + D_j(n_1, n_2)\phi_j(y) + D_{12}(n_1, n_2)\phi_{12}(m_i, y)$.

Note that the sole difference with respect to the monopoly case is that each advertiser's outside option accounts for the possibility of accepting the rival's offer. Hence, $v_i^d(m_i, \omega, n)$ is larger than $v_i^m(m_i, \omega, n_i)$. Again, our assumptions about the viewer demands $D_i(n_1, n_2)$ and $D_{12}(n_1, n_2)$ and about the advertising technology $\phi_i(m_i)$ and $\phi_{12}(m_1, m_2)$ ensure that v_i^d is continuous and increasing in ω . It also has strict increasing differences in (m, ω) .

In the derivation at the end of this section, we show by following the methodology of Martimort and Stole (2009) that it is possible to characterize the best-reply allocation as the solution to

$$\int_{\omega_0^i}^{\bar{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)} \right) \left(\tilde{d}_i \frac{\partial \phi_i}{\partial m_i} + \frac{\partial d_i}{\partial n_i} \phi_i + \tilde{D}_{12} \frac{\partial (\phi_{12} - \phi_i - \hat{\phi}_j)}{\partial m_i} + \frac{\partial D_{12}}{\partial n_i} (\phi_{12} - \phi_i - \hat{\phi}_j) \right) dF(\omega) + \kappa = 0, \quad (5)$$

with $\tilde{d}_i := (1 - F(\omega_0^i))d_i$, $\tilde{D}_{12} := (1 - F(\omega_0^i))D_{12}$, and κ defined in the derivation at the end of the section. Ignoring κ for the moment, it is evident that this optimal duopoly solution (5) is the analog of condition (5) of the paper accounting for the business sharing and duplication effect with heterogeneous advertisers.

Let us finally turn to κ . When changing the advertising intensity of type ω , outlet i has to take

into account that such a different intensity also affects the advertisers' demand from the rival outlet, m_j , given the posted schedule $t_j(\cdot)$. Intuitively, the higher the number of advertising messages on outlet i , the lower the utility from one additional ad on outlet j . This channel brings in new competitive forces that are absent with homogeneous advertisers. These forces are specific to the contracting environment considered and *in addition* to the ones discussed so far. To stress this, we note that if the rival outlet were to offer a single quantity-transfer pair (or, in other words, were to implement an incentive compatible allocation flat across all active types) then $\kappa = 0$.

Derivation of (3) and (5)

We first determine the solution to the more complicated duopoly problem. (Solving the monopoly problem proceeds along very similar lines and we will describe it very briefly towards the end.) The problem of a duopolist i is to maximize its profits $\int_{\omega}^{\bar{\omega}} t_i(m_i(\omega)) dF(\omega)$ with respect to the transfer schedule, given its rival's choice $t_j(m_j)$. From the main text, this problem can be rewritten as in (4). Denote by $m_j^*(m, \omega)$ the advertising intensity that type ω optimally buys from outlet j when buying intensity m from outlet i . Then, the net contracting surplus for type ω is

$$\begin{aligned} v_i^d(m, \omega, n) &= \max_y [\omega u(m, y, n) - t_j(y)] - (\max_{y'} [\omega u(0, y', n) - t_j(y')]) \\ &= \omega u(m, m_j^*(m, \omega), n) - t_j(m_j^*(m, \omega)) - \omega u(0, m_j^*(0, \omega), n) + t_j(m_j^*(0, \omega)) \end{aligned}$$

Incentive compatibility requires $m_i(\omega) = \arg \max_m v_i^d(m, \omega, n) - t_i(m)$, which implies

$$v_i^d(m_i(\omega), \omega, n) - t_i(m_i(\omega)) = \max_{y, y', m} \{ \omega u(m, y, n) - t_j(y) - (\omega u(0, y', n) - t_j(y')) - t_i(m) \}$$

By the envelope theorem the derivative of the above with respect to ω is

$$u(m, m_j^*(m_i(\omega), \omega), n) - u(0, m_j^*(0, \omega), n)$$

Since this pins down the growth rate of the advertiser's payoff, we find that $\max_{\omega_0^i, m_i(\cdot)} \int_{\omega_0^i}^{\bar{\omega}} t_i(m_i(\omega))$ subject to the first two constraints of (2) equals

$$\begin{aligned} &\max_{\omega_0^i, m_i(\cdot)} \int_{\omega_0^i}^{\bar{\omega}} \left\{ \omega u(m_i(\omega), m_j^*(m_i(\omega), \omega), n) - \omega u(0, m_j^*(0, \omega), n) - t_j(m_j^*(m_i(\omega), \omega)) + t_j(m_j^*(0, \omega)) \right. \\ &\quad \left. - \int_{\omega_0^i}^{\omega} [\omega u(m, m_j^*(m_i(z), z), n) - \omega u(0, m_j^*(0, z), n)] dz \right\} dF(\omega) \\ &= \max_{\omega_0^i, m_i(\cdot)} \int_{\omega_0^i}^{\bar{\omega}} \underbrace{\left\{ v_i^d(m_i, \omega, n) - \int_{\omega_0^i}^{\omega} [\omega u(m, m_j^*(m_i(z), z), n) - \omega u(0, m_j^*(0, z), n)] dz \right\}}_{\text{information rent}} dF(\omega), \end{aligned}$$

Integrating the double integral by parts gives

$$\begin{aligned} &\max_{m_i(\cdot), \omega_0^i} \int_{\omega_0^i}^{\bar{\omega}} \omega u(m_i(\omega), m_j^*(m_i(\omega), \omega), n) - \omega u(0, m_j^*(0, \omega), n) - t_j(m_j^*(m_i(\omega), \omega)) + t_j(m_j^*(0, \omega)) + \\ &\quad - \frac{1 - F(\omega)}{f(\omega)} (u(m, m_j^*(m_i(\omega), \omega), n) - u(0, m_j^*(0, \omega), n)) dF(\omega) \end{aligned}$$

The duopolist's best-reply allocation of advertising intensities $m_i^d(\omega)$ then solves

$$\begin{aligned} & \max_{m_i(\cdot), \omega_0^i} \int_{\omega_0^i}^{\bar{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)} \right) (u(m_i(\omega), m_j^*(m_i(\omega), \omega), n) - u(0, m_j^*(0, \omega), n)) \\ & \quad - (t_j(m_j^*(m_i(\omega), \omega)) - t_j(m_j^*(0, \omega))) dF(\omega), \\ & \text{subject to } \int_{\omega_0^i}^{\bar{\omega}} m_i(\omega') dF(\omega') \leq n_i. \end{aligned}$$

From now on we will denote the integrand function by $\Lambda^d(m_i(\omega), \omega, n)$. Recall that solving a canonical screening problem usually involves maximizing the integral over all served types, where the integrand is the utility of type ω minus his information rent, expressed as a function of the allocation. The utility here is the incremental value $u(m_i(\omega), m_j^*(m_i(\omega), \omega), n) - u(0, m_j^*(0, \omega), n)$, minus the difference in transfers.

The maximization problem in the first stage with respect to n_i can be written as

$$\max_{n_i} \left(\max_{m_i(\cdot), \omega_0} \int_{\omega_0^i}^{\bar{\omega}} \Lambda^d(m_i(\omega), \omega, n) dF(\omega) \quad \text{s.t.} \quad n_i = \int_{\omega_0^i}^{\bar{\omega}} m_i(\omega) dF(\omega) \right). \quad (6)$$

Let us first determine $u(m_i(\omega), m_j^*(m_i(\omega), \omega), n) - u(0, m_j^*(0, \omega), n)$. Abbreviating $m_j^*(m_i(\omega), \omega)$ by m_j^* and $m_j^*(0, \omega)$ by $(m_j^*)^*$ we can write

$$\begin{aligned} & u(m_i(\omega), m_j^*, n) - u(0, (m_j^*)^*, n) \\ &= D_i(n_1, n_2) \phi_i(m_i(\omega)) + D_j(n_1, n_2) \phi_j(m_j^*) + D_{12}(n_1, n_2) \phi_{12}(m_i(\omega), m_j^*) \\ & \quad - D_j(n_1, n_2) \phi_j((m_j^*)^*) - D_{12}(n_1, n_2) \phi_j((m_j^*)^*) \\ &= d_i(n_i) \phi_i(m_i(\omega)) + D_{12}(n_1, n_2) \left(\phi_{12}(m_i(\omega), m_j^*) - \phi_i(m_i(\omega)) - \hat{\phi}_j((m_j^*)^*) \right) + D_j(n_1, n_2) \left(\phi_j(m_j^*) - \hat{\phi}_j((m_j^*)^*) \right), \end{aligned}$$

where $\phi_{12}(m_i(\omega), m_j^*) = \hat{\phi}_i(m_i(\omega)) + \hat{\phi}_j(m_j^*) - \hat{\phi}_i(m_i(\omega)) \hat{\phi}_j((m_j^*)^*)$.

Adapting results from Martimort and Stole (2009), we know that at the optimal solution $m_i(\omega) = 0$ for all $\omega < \omega_0$ and that $m_i(\omega) = \arg \max_m \Lambda^d(m_i(\omega), \omega, n)$. By our assumptions about the demand and advertising function, the optimal solution involves a schedule $m_i(\omega)$ that is non-decreasing.

From (6), we can write the maximization problem with respect to the optimal allocation of advertising intensities, given n_i , as

$$\max_{m_i(\cdot), \lambda} \int_{\omega_0^i}^{\bar{\omega}} \Lambda^d(m_i(\omega), \omega, n) dF(\omega) + \lambda \left(n_i - \int_{\omega_0^i}^{\bar{\omega}} m_i(\omega) dF(\omega) \right).$$

Pointwise maximization with respect to $m_i(\cdot)$ yields

$$\begin{aligned} & \left(\omega - \frac{1 - F(\omega)}{f(\omega)} \right) \left[d_i(n_i) \frac{\partial \phi_i}{\partial m_i} + D_{12}(n_1, n_2) \left(\frac{\partial \left(\phi_{12}((m_i, m_j^*)) - \phi_i(m_i) \right)}{\partial m_i} \right) \right] \\ & \quad + [D_j(n_1, n_2) - D_{12}(n_1, n_2)] \frac{\partial \phi_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial m_i} - \frac{\partial t_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial m_i} = \lambda. \end{aligned} \quad (7)$$

Denoting the left-hand side of (7) by ψ , and integrating both sides from ω_0^i to $\bar{\omega}$, we obtain

$$\frac{\int_{\omega_0^i}^{\bar{\omega}} \psi dF(\omega)}{1 - F(\omega_0^i)} = \lambda.$$

The maximization problem of the first stage with respect to n_i is

$$\max_{m_i(\cdot), \lambda} \int_{\omega_0^i}^{\bar{\omega}} \Lambda_i^d(\omega, m_i(\omega)^*, n_i) dF(\omega) + \lambda \left(n_i - \int_{\omega_0^i}^{\bar{\omega}} m_i(\omega)^* dF(\omega) \right).$$

Differentiating with respect to n_i and using the Envelope Theorem yields

$$\begin{aligned} & \int_{\omega_0^i}^{\bar{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)} \right) \left[\frac{\partial d_i}{\partial n_i} \phi_i + \frac{\partial D_{12}}{\partial n_i} \left(\phi_{12}((m_i, m_j^*)) - \phi_i(m_i) - \hat{\phi}_j((m_j')^*) \right) \right. \\ & + D_{12} \left[\frac{\partial \phi_{12}}{\partial m_j^*} \frac{\partial m_j^*}{\partial n_i} - \frac{\partial \hat{\phi}_j}{\partial (m_j')^*} \frac{\partial (m_j')^*}{\partial n_i} \right] \left. + D_j \left[\frac{\partial \phi_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial n_i} - \frac{\partial \hat{\phi}_j}{\partial (m_j')^*} \frac{\partial (m_j')^*}{\partial n_i} \right] \right] dF(\omega) \\ & - \frac{\partial t_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial n_i} + \frac{\partial t_j}{\partial (m_j')^*} \frac{\partial (m_j')^*}{\partial n_i} = -\lambda. \end{aligned} \quad (8)$$

Combining (7) and (8) to get rid of λ yields expression (5) of the main text, where κ is defined as

$$\begin{aligned} \kappa \equiv & \int_{\omega_0}^{\bar{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)} \right) \left\{ \frac{1}{1 - F(\omega)} (D_j - D_{12}) \frac{\partial \phi_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial m_i} + D_{12} \left[\frac{\partial \phi_{12}}{\partial m_j^*} \frac{\partial m_j^*}{\partial n_i} - \frac{\partial \hat{\phi}_j}{\partial (m_j')^*} \frac{\partial (m_j')^*}{\partial n_i} \right] \right. \\ & + D_j \left[\frac{\partial \phi_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial n_i} - \frac{\partial \hat{\phi}_j}{\partial (m_j')^*} \frac{\partial (m_j')^*}{\partial n_i} \right] + \frac{\partial D_j}{\partial n_i} (\phi_j(m_j^*) - \phi_j((m_j')^*)) - \frac{\partial t_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial n_i} \left. \right\} dF(\omega) \\ & - \frac{\partial t_j}{\partial m_j^*} \frac{\partial m_j^*}{\partial n_i} + \frac{\partial t_j}{\partial (m_j')^*} \frac{\partial (m_j')^*}{\partial n_i}. \end{aligned}$$

It is evident that if outlet j offers a single transfer-intensity pair, then m_j^* equals $(m_j')^*$ and both are invariant to changes in $m_i(\cdot)$ and n_i . This implies that $\kappa = 0$.

Proceeding in the same way for the monopoly outlet, we obtain that its profit function is given by

$$\max_{n_i} \left(\max_{m_i(\cdot), \omega_0^m} \int_{\omega_0^m}^{\bar{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)} \right) d_i(n_i) \phi_i(m_i(\omega)) dF(\omega) \quad \text{s.t.} \quad n_i = \int_{\omega_0^m}^{\bar{\omega}} m_i(\omega) dF(\omega) \right).$$

The solution is then characterized by (3).

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