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INTERPRETATION OF THE EXPERIMENTS ON BELL'S INEQUALITY

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The conceptual scheme of the optical polarization experiments on Bell's inequality is discussed. By invoking the distinction between the concepts of *state preparation* and *measurement* in quantum mechanics, it is argued that Bell's theorem is not applicable to this class of experiments in the way it is generally done. Consequently, by considering the specific features of the measurements performed hitherto, it is also shown that a local approach can yield the same theoretical prediction as the nonlocal quantum interpretation, even in the absence of other experimental loopholes.

Key words: quantum mechanics, Bell's inequality, nonlocality tests.

1.INTRODUCTION

The most precise experiments performed hitherto on Bell's inequalities [1] are those on polarization correlations of optical photons [2],[3]. The results obtained in these experiments are generally considered as giving strong evidence in favour of the non-local quantum interpretation, in contrast with the predictions of the local realistic theories (Bell's inequalities).

In reality, many local models have also been discussed, showing that the experimental results do not contradict locality. More generally,it has been shown [4] that the inequalities used to interpret the experiments are of two different types: "weak inequalities," deduced from local realism alone (i.e., basically Bell's original inequality), which have never been violated experimentally; and "strong inequal-

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ities," deduced by using additional assumptions regarding real detectors, which have been violated experimentally. It is then possible to devise local models that would obey Bell's inequality for ideal instruments (including polarizers, detectors, and directional correlation, etc), but which agree with quantum theory for the imperfect instruments presently available. These facts, although well-known and never criticized by any author explicitly, are usually neglected in most papers and books; probably because many physicists consider that such local models are very particular and rather contrived, and consider that the additional assumptions leading to the "strong inequalities" are reasonable in the interpretation of real experiments.

In the present paper, we suggest an approach different from the above ones: it is based on the conceptual distinction between "state preparation" and "measurement" in quantum mechanics, and it shows that optical polarization experiments cannot test the local realistic theories even if performed with truly ideal instrumentation.

For convenience, in Fig. 1 we recall the general scheme of an experiment on polarization correlations of optical photons, of the kind described in [2]. At one time the source S emits (with lifetime of a few nanoseconds) a pair of optical photons with total linear and angular momenta equal to zero. The photons of each pair propagate in opposite directions with parallel linear polarizations. Pairs of photons emitted at different times are uncorrelated, and may have different linear polarization in the $x-y$ plane. The detectors D_1 and D_2 record the arrival of photons, whose time coincidence can thus be monitored by the counter C_{12} as a function of the angle ϕ between the transmission axes of the polarizers A_1 and A_2, which are separated by a space-like interval.

The experimental results obtained up to-date are in very good agreement with the nonlocal quantum interpretation, which, for ideal detectors and for perfect correlation, and in the absence of accidental coincidences, predicts the counting rate measured by the coincidence counter to be $P = \frac{1}{2}\cos^2\phi$. However,as it has been recalled above, in the opinion of many physicists these results do not contradict locality.

Differently, as we have already said, we wish to test an alternative interpretation of the experiments. First, we shall argue that the Bell theorem is not applicable to optical polarization experiments, in the sense that the inequalities deduced from the theorem cannot be used to discuss the experimental results in the way it is generally done. Second, we shall show that the consequent analysis of the experimental scheme leads to the correct theoretical prediction. As far as we know, the present approach has not been discussed elsewhere.

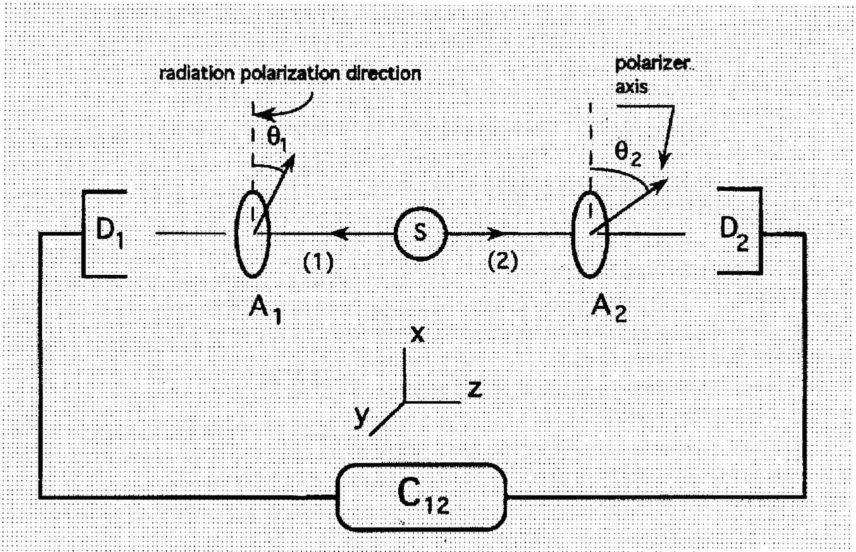

Fig. 1. General scheme of an experiment on polarization correlations of optical photons. The labels are explained in the text.

2. CONCEPTUAL SCHEME OF THE EXPERIMENTS

We recall the distinction between the concepts of *state-preparation* and *measurement* in quantum mechanics. This distinction was discussed exhaustively in old papers by W.E.Lamb Jr. [5], L.E.Ballentine [6], and H.Margenau [7]. More recently, the results obtained in these papers have been presented clearly in Ballentine's book on Quantum Mechanics [8]. Among these scholars, we quote the last author literally [6]: "*state preparation* refers to any procedure which will yield a statistically reproducible ensemble of systems. ... On the other hand, *measurement* of some quantity R for an individual system means an interaction between the system and a suitable apparatus, so that we may infer the value of R immediately before the interaction. ..... The essential distinction between the two concepts are that state preparation refers to the future, whereas measurement refers to the past; and equally important, that measurement involves *detection* of a particular system, whereas state preparation provides conditional information about a system *if* it passes through the apparatus". In particular, let a photon of arbitrary polarization fall upon a linear polarizer, converting the photon's original state ψ into ψ_θ, with θ as the transmission angle. It stems from the definitions given above that this act does not constitute a measurement, which on the contrary is usually done by observing the photon after passage through the polarizer, i.e., by allowing it to be absorbed in a detecting device yielding a numerical outcome.

Of course state preparation and measurement do occasionally coincide in many classical situations [7]. In quantal situations, there is the special case of a measurement of filtering type (for instance the Stern-Gerlach apparatus) for which measurement and state preparation are similar. However, these two concepts remain distinct one from the other [8].

By applying the concepts clearly discussed in [7] and [8] to optical polarization experiments, we can now stress the following point. If the detecting device is even omitted, the polarizer will in any case prepare the photon polarized state. Actually, in order to know the state preparation, at no time we need perform a measurement. Moreover, whatever combination of a rotatable linear-polarizer and a detector is thought (i.e., even with very small spatial separations one from the other), such a combination can noways be considered as in essence a unique measuring apparatus for photons of the specified polarization. For instance, if we even add the detector to the polarizer to make a single apparatus, yet the state preparation performed by the latter remains conceptually separated from the measurement performed by the former. The spatial separation of the polarizer from the detector is in fact totally irrelevant in the present context.

We believe that still nowadays the following statement by H. Margenau [7] remains true: "It is unfortunate that axiomatizations of quantum theory often either ignore the preparation concept or else refer to it only in a peripheral or tacit manner. Even worse, the idea of preparation is sometimes constructed to be identical with, or to be a form of, measurement."

On the ground of the above distinction between state preparation and measurement,it is readily understood that the rotation of the linear polarizers performed in optical polarization experiments is simply a procedure of state preparation; the measurement being on the contrary performed by the detectors and coincidence counter. In other words, from the knowledge of the relative rotation of the polarizer axes, we obtain only conditional information on the relative polarization of the photons *if* they pass through the polarizers.The true measurement of real simultaneous passage of photon pairs is not given by the polarizers, but it is given by the clicks of the detectors and by the counting of the coincidence counter.

If the above approach is correct, we can straightforwardly conclude that the Bell theorem is not applicable to polarization experiments in the way many authors do it, simply because the theorem requires to be used in relation to acts constituting a measurement. (The relation may be, for instance, the probability of the measurement outcome at the detectors). Therefore, this class of experiments does not seem suitable to test the local realistic theories.

We wish now to note that the state preparation obtained by means of the relative rotation ϕ is a function of $\cos^2\phi$. In polarization experiments the average number of pairs present at one time in the apparatus is always $n << 1$, and thus generally only one photon-pair is present at the same time, and of course each photon of the pair is transmitted through the polarizer with no change in energy and probability equal to 1/2, independently of the polarization direction of emission. Then, the two photon beams exiting the linear polarizers have relative polarization determined by the relative (variable) orientation of the polarizer axes. The classical intensities of these beams will be given by summing up the energy of a large number of single wave- trains propagating from the polarizers to the detectors. Let such intensities be I_1 and I_2. Any rotation ϕ (from the initial setting $\phi = 0$) of A_2 with respect to A_1, changes the number of wave-trains emerging from A_2 which are $polarization-correlated$ (and also time-correlated) with those emerging simultaneously from A_1. (Here and in the following, the expressions $polarization - correlated$ and $time - correlated$ have only the meaning that the photons of a pair are emitted with the same polarization and within the short time-interval determined by the life-time of the atomic-cascade emission). For a large number of wave-trains, such fraction can be calculated classically, and it is simply given by the component $I_2\cos^2\phi$ of I_2 in the polarization direction of I_1. We therefore obtain that the product of the two *prepared* (viz,after the interaction with the state-preparation device) and *correlated* (viz,with the same polarization) beam intensities is $I_1 \cdot I_2 \cos^2\phi$, or also that the normalized intensity correlation function of the experiment is $\cos^2\phi$.[1] Then, we can realize the following point. A rotation ϕ obviously does not change I_1 or I_2 (i.e.,the single detector D_1 or D_2 always detects

[1]Many authors calculate classically the normalized intensity correlation function of a polarization experiment as $P = < I_1(r_1)I_2(r_2) > = < cos^2(\alpha - \theta)cos^2(\beta - \theta) >$, averaged over all values of θ. The angles α and β give the polarization directions of A_1 and A_2 respectively, and θ is the angle of field polarization. For an isotropic distribution of the polarization vector in the $x - y$ plane, P can also be written as $P = \frac{1}{2\pi}\int_0^{2\pi}\cos^2\theta_1\cos^2(\theta_1 - \phi)d\theta_1 = \frac{1}{4} + \frac{1}{8}\cos(2\phi)$. The angle θ_1 and the axes xyz are defined in Fig.1, and $\theta_1 - \theta_2 = \phi$.

Such calculation is implicitly based on two assumptions: a) the polarizers act as measurement devices, so that the field distribution *behind* the polarizers has to be used; b) specific inter-beam temporal distributions are not considered, except the random (or constant) one. We do not agree with assumption (a) because we believe that the polarizers act as state- preparation devices; and with assumption (b) for the reasons discussed in section (2) of the text, which show that a specific temporal structure of the radiation source has indeed to be taken into account. We therefore think that such a calculation of the $time - independent$ intensity correlation function is inappropriate to be applied to real polarization experiments, and consequently it cannot but give incorrect predictions.

the same number of counts, independently of the rotation). Yet, by changing the intensity of that part of the beam emerging from A_2 which is time-correlated with the beam emerging from A_1, it changes the final fraction of time-correlated photons which can be recorded by the coincidence counter, thus determining the result of the measurement. In other words, the rates of time-correlated signals are determined by the state preparation, and monitored by the coincidence counter within its time-window.

Finally, we remark explicitly that the procedure of state preparation described above is an intrinsically local process, even for experiments performed at space-like separations. In fact the rotation of A_1 with respect to A_2 changes *locally* the number of wave-trains exiting A_1 at the same time of those exiting A_2, without influencing in any way the operation of A_2, independently of the separation of the latter from the former. Yet, as we have just discussed,the changes (as a function of ϕ) of such a local process of state preparation are recorded by the coincidence counter. In other words, in the present context, the ϕ-dependence of the experimental results is nowise evidence of nonlocal effects. Simply, such dependence is related to the changes of state preparation of the physical system as a function of ϕ; and it cannot therefore be invoked either to support or to contradict Bell's inequality, with which it has nothing to do.

3. THE RADIATION SOURCE

We shall now discuss some specific properties of the radiation sources generally used in optical polarization experiments of the kind described in [2]; in particular we shall examine their temporal properties.

The source used in most experiments is a low-intensity Ca source, where, by means of a cascade decay, every excited atom radiates a polarization-correlated pair of wave-trains with a short decay time.The known value of the short lifetime of the intermediate state $^1P_1$ of the cascade is $\simeq 5\ ns$ and the estimated lifetime of the highest state $^1S_0$ of the cascade is $\sim 10\ ns$. For such a process, if the excitation rate of the highest level of the cascade is sufficiently small (as in fact it is in the experiments), the quantum theory of light predicts the observation of temporal correlation ("interbeam" intensity correlation) of the two photons produced in the same cascade-emission over time intervals shorter than τ_c [9]. Actually such a temporal correlation has been measured [10], showing up a time-delay coincidence spectrum with a very short peaking time and an exponential decay with 4.7 ns time constant. The observed spectrum is therefore analogous to that of a quasi-monochromatic thermal source with $\tau_c = 4.7\ ns$ [11]. In this sense, by assuming for simplicity the same average frequency for the two correlated wave trains, the low-intensity Ca source used in the experiments may be considered as a source emitting light in opposite directions with line-width determined by the radiative and collisional broadening of the intermediate level $^1P_1$. Analogous arguments apply to the experiments using Hg sources [9]; and also to more recent experiments with parametric downconversion sources [3], whose only difference (with reference to our discussion) from the experiments with atomic-cascade sources is the scale of the coherence time-intervals and time-delays (picoseconds rather than nanoseconds).

We shall therefore assume both that the source generates pairs of wave-trains with the same polarization, and that the time-delay coincidence spectrum of this emission is exponential. Merely in this sense, as we have already commented, the two beams will be considered as *correlated*.

4. CALCULATION OF THE COINCIDENCE RATE

(a) With the help of the arguments discussed in (1) and (2), we shall now apply some general concepts of the theory of radiation to the polarization experiments; in particular (and quite trivially) we shall calculate the normalized *temporal* intensity correlation function, which gives the theoretical prediction of the inter-beam coincidence rate. The use of the temporal intensity correlation function is justified (and dictated) by the above observation that the intensities of the two beams emitted by the radiation source have generally an *exponential* temporal correlation. Consequently, and contrary to the case of random time-delay coincidence spectrum (or, also, fixed time-delay, for instance equal to zero), the theoretical prediction of the coincidence rate is not obtainable by calculating the *time − independent* intensity correlation function, as many authors do obtaining incorrect results.[1] With elementary calculations we shall obtain, in the framework of a local model, the same prediction as the non-local quantum interpretation.

With conventional notation (see, for instance, Chap.3 of [9]), the normalized temporal intensity correlation function is defined as

$$g^{(2)}(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2; \mathbf{r}_2 t_2, \mathbf{r}_1 t_1) = \langle I_1(\mathbf{r}_1, t_1) I_2(\mathbf{r}_2, t_2)\rangle / \bar{I}_1 \bar{I}_2,$$

where $(\mathbf{r}_1 t_1)$ and $(\mathbf{r}_2 t_2)$ are the space-time points of the two detectors, and $\bar{I} \equiv \langle I(t)\rangle$ is the average value of a large number of measurements of the intensity $I(t)$ taken over a period of time much longer than the light coherence time τ_c. For plane-polarized parallel beams of light, only a single spatial coordinate, taken as the z-axis, is needed. Thus, with $\tau = t_2 - z_2/c - t_1 + z_1/c$, the definition can be written as: $g^{(2)}(z_1 t_1, z_2 t_2; z_2 t_2, z_1 t_1) = g^{(2)}(\tau)$. Then, using $|z_1| = |z_2|$, $\tau = t_1 - t_2$, and the definitions which follow below, we can readily write the normalized temporal intensity correlation function $g^{(2)}(\tau)$ at the coincidence counter of a polarization experiment as

$$g_p^{(2)}(\tau) = \frac{\langle I_1(z_1,t_1)\cdot I_2(z_2,t_2)\rangle}{\bar{I}_1\bar{I}_2} = \frac{I_1\cdot I_2\cos^2\phi\cdot\exp(-2\gamma\,|\tau|)}{I_1\cdot I_2}$$
$$= \cos^2\phi\cdot\exp(-2\gamma\,|\tau|), \quad (1)$$

where $I_1 = I_2$ is the intensity of the *prepared* beam (formed by summing up a large number of wave-trains) reaching the detector D_1 (or D_2) from the polarizer A_1 (or A_2). Assuming a constant rate of radiation emission over long time intervals, we have also that $\overline{I}_1 = \overline{I}_2$ is equal to I_1 (or I_2).

The factor $\cos^2\phi$ derives directly from the arguments presented in Sec. 1.

The factor $\exp(-2\gamma\,|\tau|)$, where $\gamma = 1/\tau_c$, arises, as it has been discussed in Sec. 2, from the nature of the time correlation of the two beams emitted by the atomic-cascade source; τ is the total time-delay between the two wave trains of a single pair.[2]

(b) So far we have adopted a model directly suggested by the examination of the conceptual scheme of the measurement. In this framework we have seen that the correct intensity correlation function for the two coherent beams is given by expression (1). However, in order to have a realistic prediction for the coincidence rate, it is necessary to perform an average of (1) to take into account both the finite response time of the detecting system and the coincidence time uncertainty associated to the finite lifetime τ_c of the intermediate state of the atomic cascade. In the experiments of the kind described in [2], the coincidence counter was started by the pulse arrival at detector 1, and stopped by the pulse arrival at detector 2 at any time within the time window, which was longer than the light coherence time τ_c ($\gamma\tau \simeq 4$). Only one coincidence occurring within this time interval could be measured, and the condition $r\,\tau << 1$ (r is the average rate of true coincidences) was always satisfied. Since the time resolution of the detectors ($\sim 1\ ns$) was shorter than the time-delay uncertainty of the wave-trains (which can be taken as equal to $\tau_c \simeq 5\ ns$), the average is to be taken over a time interval equal to the latter value. This additional average of (1) over τ_c gives for the true coincidence probability P:

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[2]We wish to note another consequence of the specific time structure of the source. The coincidence counter can record a count *only* from the arrival of the two correlated photons of a pair,except obviously accidental coincidences: in fact only one pair is generally present in the apparatus within its narrow time-window, the occurrence of coincidences from simultaneous emission of two or more pairs being negligible in these experiments. Consequently, polarization experiments can measure only *perfect* correlations.

$$P = \frac{1}{\tau_c} \int_0^{\infty} g^{(2)}(\tau) d\tau = \frac{1}{2} \cos^2 \phi, \tag{2}$$

where in the integral the upper limit τ_c has been replaced by ∞ to a good approximation, since $\exp(-2\gamma|\tau|)$ becomes very small for values of τ in excess of the coherence time of the light. An analogous calculation can be performed for the experiments with parametric down-conversion sources,the only difference being the shorter coherence time and the shorter timing scale of the measurement process.Of course the calculation of P would not change if the counts at each detector are registered and recorded by computers, thus allowing a later analysis of coincidences.

5. CONCLUDING REMARKS

The final result expressed by equation (2) has been obtained by elementary calculations concerning a simple and local model derived directly from the conceptual distinction between state-preparation and measurement. We may now summarize such model, basically formulated by the expression (1), as follows: a) the final state-preparation of the physical system analized by the coincidence counter in polarization experiments is produced by the polarizers. This assumption has been based on the distinction between state preparation and measurement, and on the consideration that any rotation ϕ changes locally the relative intensity of two time-correlated beams on which the final measurement is performed; b) in order to take into account the specific temporal properties of the radiation source, the temporal intensity correlation function, with the interbeam time-correlation given by $\exp(-2\gamma|\tau|)$, is used to obtain the coincidence probability; c) the experimental features of the detecting system require to performe an additional temporal average in order to obtain the final theoretical prediction.

No entanglement, no quantum-mechanical nonlocal correlations after emission have been used either explicitly or implicitly in the present model. It however leads straightforwardly to the theoretical prediction (2) of the coincidence rate P, which is identical with the nonlocal quantum interpretation, in which P is derived, for ideal detectors and for perfect correlation, from the calculation of the state vectors for one photon in each of two modes of a pair.[3]

[3] We may note that also for other experiments *classical and quantum electrodynamics give the same theoretical prediction* for the intensity correlation function of two photon beams (see, for instance,Chap.3 and 6 of [9]). We recall here the Hanbury-Brown and Twiss experiments which use quasi-monochromatic thermal sources with exponential time-delay spectrum. In the text we have remarked on

We wish to stress again that this result does not present any problem of compatibility with Bell's theorem. The reason is that the discussion above seems to exclude the possibility that polarization experiments (as those performed hitherto) may test inequalities deduced from the theorem: in fact the rotation ϕ is basically a state-preparation process which has nothing to do with such inequalities, which consequently cannot be used to discuss the experimental results. In other words, whatever application of Bell's theorem to the scheme of polarizer orientation is incorrect and misleading, since the theorem requires to be used only in relation to acts constituting a measurement.

In conclusion, on the ground of the discussion above, it may be fair to state that optical polarization experiments cannot test the local realistic theories. This result agrees with the local models showing that the polarization experiments performed hitherto do not contradict locality; yet it disagrees with these models in the prediction that polarization experiments with truly ideal instruments might test Bell's original inequality (or "weak inequalities"). Then, if the present conclusion is tenable, we may stress the great importance of proposals of new experiments (some are discussed, for instance, in [4]), and the great interest of new theoretical speculations [12].

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