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ELEMENTARY PARTICLE MASSES FROM A NON–PERTURBATIVE ANOMALY

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Abstract

A novel dynamical mechanism of elementary particle mass generation has recently been conjectured and numerically demonstrated by lattice simulations in a simple SU(3) gauge model where a SU(2) doublet of strongly interacting fermions is coupled to a complex scalar field doublet through a Yukawa and a Wilson-like term. As a first step towards building a natural (à la 't-Hooft) extension of the Standard Model, we argue that in the presence of weak gauge interactions the mechanism above, acting as a kind of non-perturbative anomaly, yields for both elementary fermions and weak gauge bosons effective masses proportional to the Λ–parameter of the theory, with particle–specific gauge coupling dependent prefactors.

1 Introduction

The Standard Model (SM) of elementary particles, in spite of its very impressive successes, is widely believed to be only an effective low energy theory because it can not account for quantum gravity and dark matter and has not enough CP-violation for baryogenesis. Moreover, by construction the SM is unable to shed light on the puzzling problems of EW scale naturalness [1] and fermion mass hierarchy [2]. Apart from these open problems, it has been noted [3] that, if a dynamical mechanism based on non-SM interactions gives rise to the mass of the known elementary fermions, one also obtains massive $W^{\pm}$, Z^0 gauge bosons and a composite Higgs particle in the W^+W^-, Z^0Z^0, and/or $t\bar{t}$ channel.

Here we consider a new non-perturbative (NP) mechanism for the dynamical generation of elementary fermion masses [4]. This mechanism is conjectured to be at work in non-Abelian gauge models with fermions and scalars where A) (as usual) chiral transformations acting on fermions and scalars are exact symmetries, but B) (deviating from common assumptions) purely fermionic chiral symmetries are explicitly broken by the UV regularization. We focus on the "natural" model where the bare parameters are tuned so as to minimize the breaking of fermionic chiral symmetries. In its quantum effective Lagrangian (EL) [5] operators of NP origin violating fermionic chiral symmetries, among which a fermion mass term, are expected to appear, if the scalar potential is such that the theory lives in its Nambu–Goldstone (NG) phase. Recently lattice simulations have provided good evidence in favor of this phenomenon, which (for reasons we explain below) is referred to as a "NP anomaly" of fermionic chiral symmetries [6].

2 The simplest gauge model with NP fermion mass generation

We start by reviewing the renormalizable $d = 4$ toy (yet highly non-trivial) model where the mechanism of interest has been numerically demonstrated – lacking analytical methods – by first principle simulations. The classical Lagrangian is a gauge–invariant ultraviolet (UV) regularization of

$$\mathcal{L}_{\rm toy} = \mathcal{L}_k(Q, A, \Phi) + \mathcal{V}(\Phi) + \mathcal{L}_{Wil}(Q, A, \Phi) + \mathcal{L}_{Yuk}(Q, \Phi) \tag{1}$$

with $\mathcal{L}_k(Q, A, \Phi)$ and $\mathcal{V}(\Phi)$ standing for the standard kinetic terms and scalar potential (with quartic coupling λ_0 and subtracted scalar mass μ_Φ^2). $\mathcal{L}_{\rm toy}$ includes an SU(3) gauge field, A_μ, with bare coupling g_0, a Dirac doublet, $Q = (u, d)^T$, transforming as a triplet under SU(3) and a complex scalar doublet, $\varphi = (\varphi_0 + i\varphi_3, -\varphi_2 + i\varphi_1)^T$, invariant under SU(3). For the latter we use the 2×2 matrix notation $\Phi = [\,\varphi\,|-i\tau^2\varphi^*]$. The model has a hard UV cutoff $\Lambda_{UV} \sim b^{-1}$ and its Lagrangian contains a Yukawa term, $\mathcal{L}_{Yuk}(Q, \Phi) = \eta\big(\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi^\dagger Q_L\big)$, as well as a non-standard (so called "Wilson–like") term

$$\mathcal{L}_{Wil}(Q,A,\Phi) = \frac{b^2}{2}\rho\big(\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + \bar{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \mathcal{D}_\mu Q_L\big)\,. \tag{2}$$

The latter is a $\Lambda_{UV}^{-2} \times d = 6$ operator that leaves the model power-counting renormalizable [4], like it happens for the Wilson term in lattice QCD [7]. Among other symmetries, the Lagrangian (1) is invariant under the (global) chiral transformations involving fermions and scalars ($\Omega_{L/R} \in$ SU(2))

$$\chi_L \times \chi_R = [\tilde{\chi}_L \times (\Phi \to \Omega_L \Phi)] \times [\tilde{\chi}_R \times (\Phi \to \Phi\Omega_R^\dagger)]\,, \tag{3}$$

$$\tilde{\chi}_{L/R} : Q_{L/R} \to \Omega_{L/R} Q_{L/R}\,, \quad \bar{Q}_{L/R} \to \bar{Q}_{L/R}\Omega_{L/R}^\dagger\,. \tag{4}$$

No power divergent fermion mass can be generated by quantum corrections as a term like $\Lambda_{UV}(\bar{Q}_L Q_R + \bar{Q}_R Q_L)$ is not $\chi_L \times \chi_R$ invariant. For generic non-zero values of the bare parameters ρ and η neither $\mathcal{L}_{Wil}$ nor $\mathcal{L}_{Yuk}$ are invariant under the purely fermionic chiral SU(2) transformations, which we call $\tilde{\chi}_L \times \tilde{\chi}_R$.

The term $\mathcal{L}_{Wil}$ is a typical representative of the $d > 4$ terms in the UV regulated Lagrangian that yield $\tilde{\chi}_L \times \tilde{\chi}_R$ breaking. Whatever their form, one expects that their effects at momentum scales $\ll \Lambda_{UV}$ are equivalent to those of $\mathcal{L}_{Wil}$ with an appropriate value of ρ. This would end the discussion of $\tilde{\chi}_L \times \tilde{\chi}_R$ breaking for a Lagrangian with no $\mathcal{L}_{Yuk}$ term. In the presence of a $\mathcal{L}_{Yuk}$ term, which has $d = 4$, its coefficient η can be tuned to a *critical* value, $\eta_{cr} = \eta_{cr}(\rho, g_0^2, \lambda_0)$, where the quantum EL has a *vanishing* effective Yukawa term [4]. In such a *critical model* we investigate whether the quantum EL contains any $\tilde{\chi}_L \times \tilde{\chi}_R$ breaking operators with $d \leq 4$, describing $\tilde{\chi}$ breaking effects down to momentum scales $\ll \Lambda_{UV}$.

The answer to this question is obviously negative only in the phase where the exact $\chi_L \times \chi_R$ invariance is realized à la Wigner, i.e. when $\hat{\mu}_\phi^2 > 0$[1]. In the Wigner phase there is only one $\tilde{\chi}$ breaking, $d \leq 4$ operator allowed by the field content and symmetries of the model: the Yukawa term, which by definition of η_{cr} is absent in the EL of the critical model. Its $d \leq 4$ sector is thus given by

$$\Gamma_4^{Wig} \equiv \Gamma_{\hat{\mu}_\Phi^2 > 0} = \frac{1}{4}(FF) + \bar{Q}_L \not{\mathcal{D}}\, Q_L + \bar{Q}_R \not{\mathcal{D}}\, Q_R + \frac{1}{2}{\rm Tr}\big[\partial_\mu \Phi^\dagger \partial_\mu \Phi\big] + \mathcal{V}^{eff}_{\hat{\mu}_\Phi^2 > 0}(\Phi)\,. \tag{5}$$

[1] Due to the hard UV cutoff $\hat{\mu}_\Phi^2 = Z_{\mu_\Phi^2}\mu_\Phi^2 = Z_{\mu_\Phi^2}(m_{0,\Phi}^2 - \Lambda_{UV}^2\tau_{cr})$, with τ_{cr} a computable coefficient.

For $\hat{\mu}_\Phi^2 < 0$ the $\chi_L \times \chi_R$ invariance is realized à la NG already at the classical level and three massless Goldstone bosons appear in the spectrum. Owing to the non-zero vacuum expectation value (v) of the scalar field, in the quantum EL the (effective) Φ field can now be written in polar form

$$\Phi = RU \,, \quad R = (v + \zeta_0) \,, \quad U = \exp[iv^{-1}\tau^k\zeta_k] \,, \quad \langle\Phi\rangle = v > 0 \,, \tag{6}$$

in terms of Goldstone ($\zeta_{1,2,3}$) and massive (ζ_0) scalars. The dimensionless field U transforms as $U \to \Omega_L U \Omega_R^\dagger$ under $\chi_L \times \chi_R$ and only makes sense if $v^2 > 0$, i.e. for $\hat{\mu}_\Phi^2 < 0$. In the NG phase the existence of U combined with the emergence (dimensional transmutation) of the intrinsic NP scale Λ_S allows for further $\tilde{\chi}$ breaking operators to appear in the quantum EL. For the critical model its $d \leq 4$ piece reads

$$\Gamma_4^{NG} = c_2\Lambda_S^2 \mathrm{Tr}[\partial_\mu U^\dagger \partial_\mu U] + c_1\Lambda_S[\bar{Q}_L U Q_R + \mathrm{h.c.}] + \tilde{c}\,\Lambda_S R\,\mathrm{Tr}[\partial_\mu U^\dagger \partial_\mu U] + \Gamma_{\hat{\mu}_\Phi^2<0} + \mathrm{O}(1/v^2) \,, \tag{7}$$

where $\Gamma_{\hat{\mu}_\Phi^2}$ is given in Eq. (5). The term $\propto c_1$ describes a kind of NP $\tilde{\chi}_L \times \tilde{\chi}_R$ anomaly in the quantum EL, as it was conjectured few years ago [4]. When U is expanded around the identity this terms yields

$$c_1\Lambda_S[\bar{Q}_L U Q_R + \bar{Q}_R U^\dagger Q_L] = c_1\Lambda_S \bar{Q}Q\,[1 + \mathrm{O}(\tau^k\zeta_k/v)] = M_Q^{eff}\bar{Q}Q\,[1 + \mathrm{O}(\tau^k\zeta^k/v)] \,, \tag{8}$$

thus a fermion mass term, $M_Q^{eff} = c_1\Lambda_S$, plus a host of complicated, non-polynomial $\bar{Q} - \zeta_{1,2,3}{}'s - Q$ effective vertices. One can argue that in the critical model NP corrections on top of the $\tilde{\chi}$ breaking terms in the correlators, which arise from residual $\mathrm{O}(v\,\Lambda_{UV}^{-2}\,\mathrm{momentum}^2)$ fermion bilinear Lagrangian terms, are responsible for all the NP $\tilde{\chi}$ breaking terms appearing in the quantum EL. In particular $c_1 = \mathrm{O}(g_0^4)$.

The critical model which we focused on is "natural" because it is defined by the criterion of maximally restoring at low energy the fermionic chiral symmetries ($\tilde{\chi}$) that are anyway broken in the far UV. The role of the other two terms involving Λ_S in Eq.(7) is clarified in Sect. 4.2.

3 Lattice evidence for NP fermion mass in the $\mathcal{L}_{\mathrm{toy}}$ model

Omitting technical details, our lattice study [6] of the model with classical Lagrangian (1) can be summarized as follows. In the Wigner phase by *setting to zero* a suitably chosen and normalized matrix element, called r_{AWI}, of the divergence of the Noether current $\tilde{J}_R^i - \tilde{J}_L^i \equiv \tilde{J}_A^i$ associated to the would-be $\tilde{\chi}_L \times \tilde{\chi}_R$ symmetries we *determine* η_{cr} (at nearly fixed renormalization conditions) for three different values of the UV cutoff b^{-1}. Data for r_{AWI} at different η are interpolated to find η_{cr} at each $\beta = 6/g_0^2$ value, as shown in Fig. 1a. Having made sure that the quantum EL can have no Yukawa term, we switch to the NG phase, where we take the continuum limit of the critical model at fixed renormalization conditions – now with a renormalized squared scalar mass $\hat{\mu}_\Phi^2 < 0$. We study the pseudoscalar meson mass (M_{PS}) and the ratio ($2m_{AWI}^R$) of the renormalized matrix elements of $(\partial \cdot \tilde{J}_A^i)$ and $P^i = \bar{Q}\gamma_5\frac{\tau^i}{2}Q$ between the vacuum and one pseudoscalar meson state. The results for M_{PS} and $2m_{AWI}^R$ (in a convenient hadronic scheme R) are shown in Fig. 1b,c in units of the Sommer scale [8] r_0 as a function of the squared lattice spacing[2] b^2. The continuum limit ($b \to 0$) results are non-zero within conservative error estimates.

This lattice investigation, involving simultaneously gauge, fermion and scalar fields, was numerically quite challenging and thus carried out within the quenched (or valence fermion) approximation, which has been widely used in lattice QCD and is known to preserve locality and renormalizability of the model. Quenched results in the continuum limit are in fact enough to establish the presence of NP terms violating the would-be $\tilde{\chi}_L \times \tilde{\chi}_R$ symmetries in the quantum EL (7), even if quenching is likely to obscure their

[2] No $\mathrm{O}(b^{2n+1})$ cutoff effects occur in our model, as it follows from standard symmetry arguments [6].

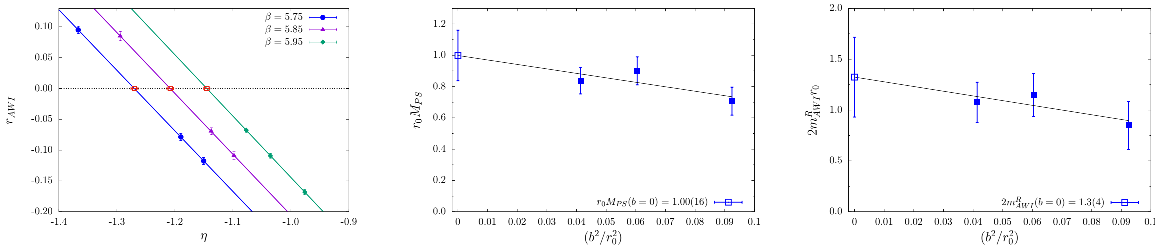

Figure 1: *a)* r_{AWI} *at various values of* η *for the lattice resolutions corresponding to* $\beta = 6/g_0^2 = (5.75, 5.85, 5.95)$ *and* $\rho = 1.96$*: straight lines show the linear interpolations in* η*; b) and c)* $M_{PS} r_0$ *and* $2m^R_{AWI} r_0$ *(renormalized in an hadronic scheme) versus* b^2*, with their linear extrapolation to* $b^2 = 0$*.*

universality properties (a point beyond the scope of the study). In particular, the non-vanishing result for $2m^R_{AWI}$ implies the occurrence in the quantum EL of the NP term $c_1 \Lambda_S [\bar{Q}_L U Q_R + \text{h.c.}]$, plus possible higher dimensional ones with equal quantum numbers. The non-zero result for the pseudoscalar meson mass M_{PS} nicely fits with $2m^R_{AWI} \neq 0$ in view of the (explicitly verified) spontaneous breaking of the would-be $\tilde{\chi}_L \times \tilde{\chi}_R$ symmetries owing to strong interaction dynamics – just as it happens in QCD.

4 Fermion and weak gauge boson NP mass generation

The toy model discussed above can be extended to encompass weak interactions by gauging its exact χ_L-symmetry. Besides a chiral weak $SU(2)_L$ gauge force, we consider two vector gauge interactions, which we call strong (gauge group $SU(3)_S$) and "Tera-strong" (gauge group $SU(3)_T$), together with two sets of Dirac fermions: quarks $q_R \in (1_T, 3_S, 1_L)$, $q_L \in (1_T, 3_S, 2_L)$ and "Tera-quarks" $Q_R \in (3_T, 3_S, 1_L)$, $Q_L \in (3_T, 3_S, 2_L)$. Ignoring leptons, possible Tera-leptons and hypercharge effects, we consider here the (yet unrealistic) model with basic classical Lagrangian

$$\mathcal{L}_{\text{basic}}(Q, q, G, A, \Phi, W) = \mathcal{L}_{kin}(Q, q, G, A, \Phi, W) + \mathcal{V}(\Phi) + \mathcal{L}_{Wil}(Q, q, G, A, \Phi, W) + \mathcal{L}_{Yuk}(Q, q, \Phi) \ , \quad (9)$$

where G_μ, A_μ and W_μ denote $SU(3)_T$, $SU(3)_S$ and the weak $SU(2)L$ gauge bosons and we have

$$\mathcal{L}_{kin} = \frac{1}{4} F^G \cdot F^G + \frac{1}{4} F^A \cdot F^A + \frac{1}{4} F^W \cdot F^W + \bar{Q}_L \gamma_\mu \mathcal{D}^{G,A,W}_\mu Q_L + \bar{Q}_R \gamma_\mu \mathcal{D}^{G,A}_\mu Q_R +$$
$$+ \bar{q}_L \gamma_\mu \mathcal{D}^{A,W}_\mu q_L + \bar{q}_R \gamma_\mu \mathcal{D}^{A}_\mu q_R + \kappa \frac{1}{2} \mathrm{Tr}\big[(\mathcal{D}^W_\mu \Phi)^\dagger \mathcal{D}^W_\mu \Phi\big] \quad (10)$$

$$\mathcal{L}_{Wil} = \frac{b^2}{2} \rho_Q \Big(\bar{Q}_L \overleftarrow{\mathcal{D}}^{G,A,W}_\mu \Phi \mathcal{D}^{G,A}_\mu Q_R + \text{h.c.} \Big) + \rho_q \Big(\bar{q}_L \overleftarrow{\mathcal{D}}^{A,W}_\mu \Phi \mathcal{D}^{A}_\mu q_R + \text{h.c.} \Big) \quad (11)$$

$$\mathcal{L}_{Yuk} = \eta_Q \Big(\bar{Q}_L \Phi Q_R + \text{h.c.} \Big) + \eta_q \Big(\bar{q}_L \Phi R + \text{h.c.} \Big) \ , \quad (12)$$

with standard gauge covariant derivatives, e.g. $\bar{Q}_L \overleftarrow{\mathcal{D}}^{G,A,W}_\mu = \bar{Q}_L (\overleftarrow{\partial}_\mu + i g_T \lambda^a G^a_\mu + i g_S \lambda^c A^c_\mu + i g_w \frac{\tau^i}{2} W^i_\mu)$.

As both quarks and Tera-quarks couple to gluons, while only Tera-quarks are coupled to Tera-gluons, the Tera-strong coupling g_T will have a significantly faster running than the strong coupling g_S. For three quark generations the ratio of the LO coefficients of the β functions is $\beta_0^T / \beta_0^S = 7/3$, while for the "toy" case of just one quark generation, which for simplicity is considered here, one has $\beta_0^T / \beta_0^S = 21/17$. The renormalization group invariant (RGI) dynamical scale of the theory is denoted by Λ_T, with the idea that if the two gauge couplings are similarly small at energy scales close to the UV cutoff, moving towards low energy g_T gets O(1) at a scale of order Λ_T where still $g_S \ll 1$.

The model, among other symmetries, such as the UV regulated version of translation and Lorentz invariance, CP, time-reversal and the $\mathrm{SU}(3)_T \times \mathrm{SU}(3)_S$ vector gauge symmetry, is invariant under a global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry group, which we call $\chi_L \times \chi_R$, with $\chi_{L,R} \equiv \tilde{\chi}_{L,R} \times \chi^{\Phi}_{L,R}$ and

$$\tilde{\chi}_L : \; Q[q]_L \to \Omega_L Q[q]_L, \; \bar{Q}[q]_L \to \bar{Q}[q]_L \Omega_L^\dagger, \; W_\mu \to \Omega_L W_\mu \Omega_L^\dagger, \qquad \chi^{\Phi}_L : \; \Phi \to \Omega_L \Phi, \quad \Omega_L \in \mathrm{SU}(2)_L \,, \tag{13}$$

$$\tilde{\chi}_R : \; Q[q]_R \to \Omega_R Q[q]_R, \; \bar{Q}[q]_R \to \bar{Q}[q]_R \Omega_R^\dagger, \qquad \chi^{\Phi}_R : \; \Phi \to \Phi \Omega_R^\dagger, \quad \Omega_R \in \mathrm{SU}(2)_R \,, \tag{14}$$

as well as under the corresponding local $\mathrm{SU}(2)_L$ gauge subgroup. The global $\chi_L \times \chi_R$ invariance is realized à la NG, i.e. spontaneously broken, already at the classical level if $\hat{\mu}^2_\Phi < 0$ in the scalar potential $\mathcal{V}(\Phi)$.

4.1 The critical model for $g_w > 0$

The critical model is again defined as the one where the $\tilde{\chi}_L \times \tilde{\chi}_R$ symmetries, which are explicitly broken in the generic UV regulated model (9), are maximally restored in the quantum EL. Noting that W_μ transforms in the adjoint representation of the $\mathrm{SU}(2)_L$ group, maximal restoring $\tilde{\chi}_L$ at low energy corresponds to eliminating from the $d = 4$ sector of the quantum EL the effective $\tilde{\chi}$ breaking terms, i.e.

$$\Gamma^{Wig\,/\,NG}_{4,\,\tilde{\chi}\,\mathrm{breaking}} = \kappa_{eff} \frac{1}{2} \mathrm{Tr}\big[(\mathcal{D}^W_\mu \Phi)^\dagger \mathcal{D}^W_\mu \Phi\big] + y_{Q,eff} \Big(\bar{Q}_L \Phi Q_R + \mathrm{h.c.}\Big) + y_{q,eff} \Big(\bar{q}_L \Phi q_R + \mathrm{h.c.}\Big) \,. \tag{15}$$

Taking into account the mixing of the Wilson–like terms with coefficients $\rho_{Q,q}$ in the Lagrangian (9) – which are the typical representatives for all $d > 4$ $\tilde{\chi}$ breaking operators – with the $d = 4$ Yukawa and scalar kinetic terms, one proves that critical values of the bare coefficients of the latter, namely $\eta_{Q,cr}$, $\eta_{q,cr}$ and κ_{cr}, exist for which the criticality conditions [3] on the effective $\tilde{\chi}$–violating couplings, v.i.z.

$$\kappa_{eff} \to 0^+ \,, \quad y_{Q,eff} = 0 \,, \quad y_{q,eff} = 0 \,, \tag{16}$$

are realized for each g_T, g_S, g_w, λ_0, ρ_Q and ρ_q independently of the squared scalar mass (μ^2_Φ) value.

4.2 $\tilde{\chi}$ violating universal NP terms in the quantum EL (NG phase)

In the NG phase of the *critical model* defined above NP corrections to $\tilde{\chi}$ breaking effects (due to $\mathcal{L}_{Wil,Yuk}$) are expected to produce a number of $\tilde{\chi}$–violating terms in the quantum EL, according to a mechanism closely analogous to the one we discussed in Sections 2 and 3. The quantum EL should thus read $\Gamma^{NG} = \Gamma_{d\le 4,\,\hat{\mu}^2_\Phi} + \Delta\Gamma^{NG}_{d\le 4,\,\hat{\mu}^2_\Phi} + \Gamma_{d>4,\,\hat{\mu}^2_\Phi}$, where, with U as in Eq. (6), the $d \le 4$ sector is given by

$$\Gamma_{d\le 4,\,\hat{\mu}^2_\Phi} = \frac{1}{4} \sum_{X=G,A,W} (F^X \cdot F^X) + \bar{Q}_L \, \not{\mathcal{D}}^W Q_L + \bar{Q}_R \, \not{\mathcal{D}} \, Q_R + \bar{q}_L \, \not{\mathcal{D}}^W q_L + \bar{q}_R \, \not{\mathcal{D}} \, q_R + \frac{\hat{\mu}^2_\Phi}{2} \, \mathrm{Tr}\left[\Phi^\dagger \Phi\right] + \frac{\hat{\lambda}}{4} \, \mathrm{Tr}\left[\Phi^\dagger \Phi\right]^2 \tag{17}$$

plus, noting that ζ_0 decoupling implies $(C_2 \Lambda_T^2 + \tilde{C} \Lambda_T R) \to C_2 \Lambda_T^2$ as $\kappa_{eff} \to 0^+$, the NP terms

$$\Delta\Gamma^{NG}_{d\le 4,\,\hat{\mu}^2_\Phi} = \theta(-\hat{\mu}^2_\Phi) \Big[\sum_{\psi=Q,q} C_{1,\psi} \Lambda_T (\bar{\psi}_L U \psi_R + \mathrm{h.c.}) + C_2 \Lambda_T^2 \frac{1}{2} \mathrm{Tr}[(\mathcal{D}^W_\mu U)^\dagger \mathcal{D}^W_\mu U] \Big] \,. \tag{18}$$

At quantum level the would-be $\tilde{\chi}$ symmetries are thus broken by fermion and weak boson mass terms and further $\tilde{\chi}$ violating NP vertices that involve U but are independent of $v^2 \sim \hat{\mu}^2_\Phi / \hat{\lambda}$. The W boson mass is $M_W^{eff} = g_w \sqrt{C_2} \Lambda_T$, while the Tera-quark and quark masses read $M_Q^{eff} = C_{1,Q} \Lambda_T$ and $M_q^{eff} = C_{1,q} \Lambda_T$. One can show [9] that $\sqrt{C_2} = \mathrm{O}(g_T^4)$, $C_{1,Q} = \mathrm{O}(g_T^4)$, $C_{1,q} = \mathrm{O}(g_S^4)$ and, owing to renormalizability of the basic Lagrangian (9), ratios of masses (such as W boson, Tera-hadron or hadron masses) are expected to be independent of UV regularization details (universality). Elementary fermion and weak gauge boson masses hence arise as a kind of *NP anomaly*. Based on dynamical properties of the basic model one can also argue, and check by numerical simulations, that $C_2 \ll 1$, i.e. $M_W^{eff} \ll \Lambda_T$ (*little hierarchy*).

4.3 Mass interpretation of the $\tilde{\chi}$ violating NP terms

To make contact with the standard phenomenological description of elementary particle mass effects, one can imagine to describe the physics of the critical model (9) with NP-ly anomalous $\tilde{\chi}$ symmetries in terms of an effective Lagrangian where the UV regularization preserves the $\tilde{\chi}$ symmetries and explicit terms $m_Q \bar{Q}_L U Q_R$, $m_q \bar{q}_L U q_R$ and $\frac{F^2}{2}\mathrm{Tr}[(\mathcal{D}_\mu^W U)^\dagger \mathcal{D}_\mu^W U]$ are explicitly included. Owing to ζ_0 decoupling, the dimensionless Goldstone boson field $U = \exp(iF^{-1}\tau^k \zeta_k)$ is necessary to guarantee $\chi_L \times \chi_R$ invariance, which makes this effective Lagrangian description renormalizable only order by order in a $1/F$ expansion. The many finite low energy parameters associated with all the necessary UV counterterms are in principle fixed by using the info coming from Γ^{NG} of the basic model. Among these effective parameters we now find the *running masses* $\hat{m}_Q(\mu)$, $\hat{m}_q(\mu)$ and $\hat{m}_W(\mu)$, which at leading order are just m_Q, m_q and $g_w F$ and whose RG evolution is given by the anomalous dimension of the associated Lagrangian densities. It should also be noted that for particles (like q and possibly W) with effective mass much smaller than Λ_T the $d = 4$ soft mass terms are sufficient to describe the dominant effects of $\tilde{\chi}$ breaking, whereas for particles (like Q) with mass of order Λ_T all the $d \geq 4$ operators violating $\tilde{\chi}$ are equally important.

5 Outlook and conclusions

To proceed towards realistic models with "natural" elementary particle mass one must of course introduce hypercharge effects, leptons and possibly Tera-leptons (which can play a key role in gauge coupling unification [10]), while keeping the (gauged) $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry exact and maximally restoring the would-be fermionic chiral symmetries. From the discussion above it is clear that, if the observed top, $W^\pm$ and Z^0 masses have to be reproduced, a realistic model must include a new strong interaction with an intrinsic RGI scale Λ_T in the few TeV range and Tera-hadrons having masses of the same order, which is also crucial to pass electroweak precision tests. Owing to unitarity one can expect the low energy description that is valid for momenta well below Λ_T to be, even quantitatively, very similar to the SM if (as it is suggested by non-relativistic arguments [9]) the Higgs particle is given by a single bound state in the $WW + ZZ$ channel arising from the new strong interaction.

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