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Testing noncommutative spacetimes and violations of the Pauli Exclusion Principle through underground experiments

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Abstract: We propose to deploy limits that arise from different tests of the Pauli Exclusion Principle: i) to provide theories of quantum gravity with experimental guidance; ii) to distinguish, among the plethora of possible models, the ones that are already ruled out by current data; iii) to direct future attempts to be in accordance with experimental constraints. We first review experimental bounds on nuclear processes forbidden by the Pauli Exclusion Principle, which have been derived by several experimental collaborations making use of various detector materials. Distinct features of the experimental devices entail sensitivities on the constraints hitherto achieved that may differ from one another by several orders of magnitude. We show that with choices of these limits, well-known examples of flat noncommutative space-time instantiations of quantum gravity can be heavily constrained, and eventually ruled out. We devote particular attention to the analysis of the κ-Minkowski and θ-Minkowski noncommutative spacetimes. These are deeply connected to some scenarios in string theory, loop quantum gravity, and noncommutative geometry. We emphasize that the severe constraints on these quantum spacetimes, although they cannot rule out theories of top-down quantum gravity to which they are connected in various ways, provide a powerful limitation for those models. Focus on this will be necessary in the future.

Keywords: non-commutative space-times, tests of violations of the Pauli Exclusion Principle, theta-Minkowski, kappa-Minkowski

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1   Introduction

The Pauli Exclusion Principle (PEP) is a direct implication of the Spin Statistics Theorem (SST) stated by Pauli in Ref. [1]. The PEP automatically arises from the anti-commutation properties of fermionic creation and annihilation operators in the construction of the Fock space of the theory. In turn, the SST is proven by assuming Lorentz invariance. This certainly implies that the PEP is closely connected to the structure of space-time itself. The PEP is indeed a successful fundamental principle not only when addressed from theoretical quantum field theory considerations, but is also in high-precision agreement with all atomic, nuclear, and particle physics experimental data. In other words, if the PEP is violated, the violating channels must be parametrized by very tiny coupling constants in front of the PEP-violating operators. This possibility was suggested within an effective field theory approach in Refs. [2–10].

The possibility of renormalizable PEP-violating operators can be seen as "un-aesthetic" and un-natural. However, the possibility of non-renormalizable effective operators induced by a PEP-violating new physics scale is still an open and natural possibility, which is predicted by many possible models of quantum gravity realizing an ultraviolet completion. A possible way to violate the PEP is, of course, to relax the main hypothesis on the basis of the Spin Statistics Theorem. For example, as mentioned above, the theorem in its standard enunciation — namely in terms of the commutation relation for bosonic ladder operators, and anticommutation relation for fermionic ladder operators — is no longer valid if Lorentz invariance is relaxed. Lorentz symmetry is one of the bases of the Standard Model of particle physics:

its explicit violation must allow any possible Lorentz Violating and CPT violating renormalizable operators. Even fine-tuned to very small couplings, the latter operators will introduce new UV divergent diagrams in the Standard Model sector, affecting the basic requirement of unitarity of the theory. This is why the Spin Statistics Theorem, as a companion of Lorentz symmetry, is considered a milestone of the Standard Model. Notice furthermore, as pointed out in Ref. [11], that Lorentz violating effects — for instance, those induced by the Planck scale in quantum gravity — might manifest themselves in the propagation of low-energy particles with a sizable magnitude that in some cases is already ruled out by experimental data[1].

Nevertheless, the eventuality that the Lorentz Symmetry is dynamically or spontaneously broken at a very high energy scale $\Lambda_{\rm UV}$ is still open, and this must turn into the generation of non-renormalizable operators suppressed as inverse powers of $\Lambda_{\rm UV}$. For example, many quantum gravity theories predict a noncommutative space-time geometry close to the Planck length scale. The idea that space-time can be noncommutative was accredited to W. Heisenberg in Ref. [12] and elaborated later in Refs. [13, 14]. After a few decades, it was realized that the noncommutativity of space-time can be rediscovered within the context of both[2] string theory [16–20] and loop quantum gravity [21–27]. In addition to these two frameworks, many other studies have shed light on the emergence of deformed symmetries as a feature of effective theories that can be derived from non-perturbative models of quantum geometry — see *e.g.* Ref. [28].

Several studies have hitherto been devoted to the analysis of the physical meaning of deformed symmetries in spacetime, as *e.g.* in Refs. [29–37]. Some possible phenomenological consequences have also been investigated in Refs [38–41], at least for those cases of noncommutativity that are the most manageable, namely those specific classes of models deforming the Lorentz symmetry to a noncocommutative space-time deformed symmetry group called κ-Poincaré and θ-Poincaré. Among these latter, there exists a specific class of models enjoying θ-Poincaré symmetries that can preserve the unitarity of the S-matrix in the Standard Model sector. This comes with a restriction [42] on the components of the space-time noncommutative matrix $\theta^{\mu\nu}$. Under these assumptions, noncommutative quantum field theories with the Groenewold-Moyal product will not lead to catastrophic violation of the probability conservation principle [43–47].

For these θ-deformed theories, and for the class of deformations that enjoy κ-Poincaré symmetries [48], it is possible to show that deformed versions of the CPT theorem exists, or anyway that a deformed notion of discrete C, P, and T symmetries can be recovered. This entails the introduction of a deformed SST, which encodes deformed commutation and anticommutation quantization rules, and thus deviation from the standard CPT theorem, which is nevertheless predicted to be small [49]. These deviations consequently lead to a violation of the standard PEP. Furthermore, it was shown in a series of work that CPT violation does not necessarily lead to violations of Lorentz invariance [50], and vice versa [50, 51], dismantling the bases of a much-celebrated no-go theorem that was instead predicting this link, based on standard relativistic quantum field theory. These results call for an investigation of PEP directly at the level of the Fock space of the theory, where the breakdown or deformation of the theory induces deviations from ordinary statistics. In such a large panorama of possibilities, an effective parametrization approach is highly anticipated to experimentally distinguish among different scenarios that are theoretically allowed. We will account for deviations from commutation/anti-commutation relations of the creation and annihilation operators, which act on the vacuum in the Fock space of the theories, and then show how the cases of the theories enjoying κ-Poincaré and θ-Poincaré symmetries lie in a specific class of parametrization that allows a phenomenological falsification of (standard) PEP violations.

2   Parametrization

Operatively, deviations from the PEP in the commutation/anti-commutation relations can be parametrized – see *e.g.* Refs. [8–10] – by introducing a deviation function $q(E)$, *i.e.*

$$a_i a_j^\dagger - q(E) a_j^\dagger a_i = \delta_{ij}, \tag{1}$$
$$q(E) = -1 + \beta^2(E), \quad \text{and finally} \quad \delta^2(E) = \beta^2(E)/2.$$

Among the possible parameterizations of the function $\delta^2(E)$, we propose a class corresponding to models that, depending on the order in the inverse powers of the energy scale of Lorentz violation, are classified at the k-th order as

$$M_k : \delta^2(E) = c_k \frac{E^k}{\Lambda^k} + O(E^{k+1}).$$

The phenomenological method we deploy here naturally takes into account, through an analytic expansion driven

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by dimensional analysis, the corrections to the standard statistics that may arise, in the infrared limit, from UV-complete quantum field theories. This parametrization can capture every possible first term of the power series expansions in E/Λ, for every possible deformation function $q(E)$ in Eq. (1). In other words, constraints on $\delta(E)$ can be translated into constraints on the new physics scale within the framework of the M_n parametrization.

3   Limits on PEP violating processes by underground experiments

To investigate the aforementioned models, we begin by referring to results obtained by underground experiments. Fig. 1 shows the most stringent limits on the relative strength (δ^2) for the studied non-Paulian transitions.

Several methods of experimental investigations for testing PEP have been used. The VIP experiment [52] used a method of searching for PEP forbidden atomic transitions in copper; the limits on the probability that PEP is violated by electrons are reported in Fig. 1. The experimental method consists of the injection of "fresh" electrons into a copper strip, by means of a circulating current, and in the search for the X-rays following the possible PEP forbidden radiative transitions that occur if one of these electrons is captured by a copper atom and cascades down to the already-filled $1S$ state. In particular, the experiment searches for the K_α ($2P \rightarrow 1S$) transition. The energy of this PEP forbidden transition (7.729 keV) would differ from the normal K_α transition energy (8.040 keV) by a Δ term (approximately 300 eV) because of the presence of the other electrons in the already-filled shell. This energy shift can be detected by high-resolution CCD devices.

PEP forbidden radiative atomic transitions have also been searched for in iodine atoms deploying NaI(Tl) detectors, as performed using DAMA/LIBRA (DAMA(2009)A in Fig. 1) [54] and ELEGANTS V [53] experiments, and in germanium atoms in PPC HPGe detectors in the MALBEK experiment [55] (see Fig. 1). In such cases, when a PEP-violating electronic transition occurs, X-rays and Auger electrons are emitted by the transition itself and by the following rearrangements of the atomic shell. The detection efficiency of such radiation in the NaI(Tl) detectors of DAMA/LIBRA is $\simeq 1$ at the low-energy end of the process. Thus, all the ionization energy for the considered shell is detected, but it is actually shifted by a Δ term because of the presence of the other electrons in the already-filled shells. Generally, in this class of experiments, the K-shell is considered, as it provides the largest available energy in subsequent X-ray /Auger-electrons radiation emission. However, stringent limits (not reported in Fig. 1) have also been obtained by DAMA/NaI looking for transitions to the L-shell in iodine atoms [58], providing 4-5 keV radiation emission, thanks to the low energy thresholds of such NaI(Tl) detectors.

The most stringent constraint on this class of PEP violations in atomic transitions comes from the DAMA/LIBRA experiment, a 250 kg array of highly radiopure NaI(Tl) detectors hosted at the Gran Sasso National Laboratory. DAMA/LIBRA searched for PEP violating K-shell transitions in iodine using the data corresponding to 0.53 ton×yr; a lower limit on the transition lifetime of 4.7×10^{30} s has been set, giving $\delta^2 < 1.28 \times 10^{-47}$ at 90% C.L. [54]. This value is reported in Fig. 1.

A similar experiment, MALBEK, used a high-purity germanium (HPGe) detector with an energy threshold suitable for observing the transition from L- to K- shells in germanium. In this case, the energy of the transition was calculated to be 9.5 keV [55], once shifted down by the Δ term. The obtained limit on δ^2 is also reported in Fig. 1.

A different approach for studying PEP violating processes has been exploited by DAMA/LIBRA collaboration (DAMA(2009)B in Fig. 1) [54]. Specifically, PEP violating transitions in nuclear shells of ^{23}Na and ^{127}I have been investigated by studying possible protons emitted with $E_p \geqslant 10$ MeV. In such a case, events where only one detector fires, that is, each detector has all the others as veto, are considered to search for high-energy protons. The rate of emission of high-energy protons ($E_p \geqslant 10$ MeV) resulting from PEP violating transitions in ^{23}Na and ^{127}I was constrained to be $\lesssim 1.63 \times 10^{-33}$ s^{-1} (90% C.L.) [54]. This corresponds to a limit on the relative strength of the studied PEP violating transitions: $\delta^2 \lesssim 4 \times 10^{-55}$ at 90% C.L. (see Fig. 1).

Moreover, the PEP has been tested with the Borexino detector [56], considering the nucleons in the ^{12}C nuclei. This research benefits from the extremely low background and the large mass (278 tons) of the Borexino detector. The exploited method is to look for γ, $\beta^\pm$, neutrons, and protons, emitted in a PEP violating transition of nucleons from the $1P_{3/2}$ shell to the filled $1S_{1/2}$ shell, and the following limits on the lifetimes for the different PEP violating transitions were set [56] (all the limits are at 90% C.L.): $\tau(^{12}\text{C} \rightarrow ^{12}\widetilde{C}+\gamma) \geqslant 5.0 \times 10^{31}$ yr, $\tau(^{12}\text{C} \rightarrow ^{12}\widetilde{N}+\text{e}^-+\bar{\nu}_e) \geqslant 3.1 \times 10^{30}$ yr, $\tau(^{12}\text{C} \rightarrow ^{12}\widetilde{B}+\text{e}^++\nu_e) \geqslant 2.1 \times 10^{30}$ yr $\tau(^{12}\text{C} \rightarrow ^{11}\widetilde{B}+\text{p}) \geqslant 8.9 \times 10^{29}$ yr, and $\tau(^{12}\text{C} \rightarrow ^{11}\widetilde{C}+\text{n}) \geqslant 3.4 \times 10^{30}$ yr.

These limits correspond to constraints on the relative strengths for the studied PEP violating electromagnetic, strong, and weak transitions: $\delta_\gamma^2 \leqslant 2.2 \times 10^{-57}$, $\delta_N^2 \leqslant 4.1 \times 10^{-60}$, and $\delta_\beta^2 \leqslant 2.1 \times 10^{-35}$ (see Fig. 1) [56].

Finally, we report here the results obtained by the large underground water Cherenkov detector, Kamiokande [57], where anomalous emission of γ rays
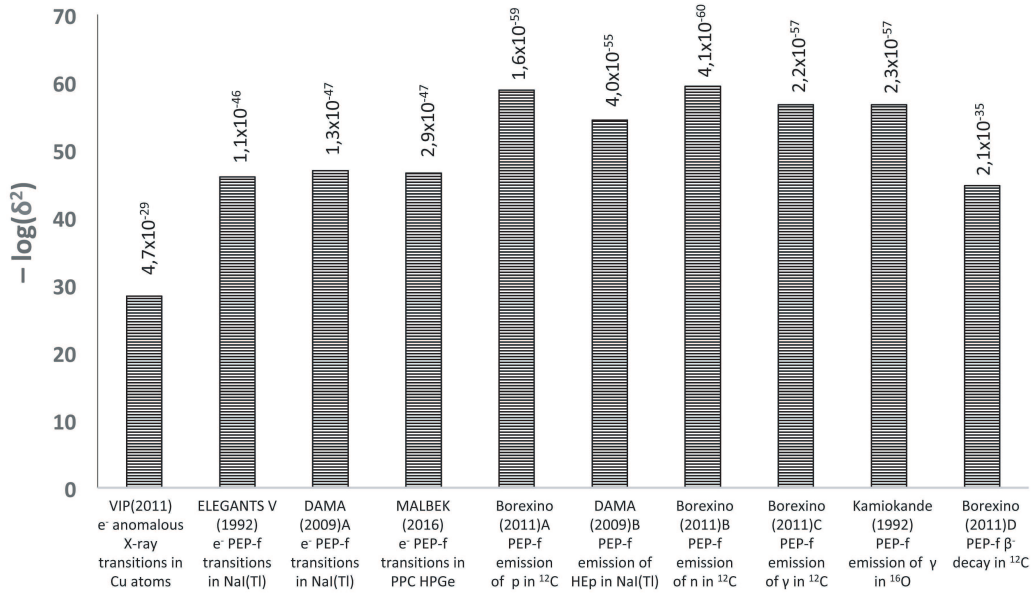
Fig. 1.   Limits at 90% C.L. on various PEP violation channels in logarithmic scale, displaying the $-\log\delta^2$ for different experimental collaborations: VIP(2011) [52]; ELEGANTS V (1992) [53]; DAMA(2009)A [54]; MALBEK(2016) [55]; Borexino(2011)A [56]; DAMA(2009)B [54]; Borexino(2011)B [56]; Borexino(2011)C [56]; Kamiokande(1992) [57]; Borexino(2011)D [56].

in the energy range $19-50$ MeV has been searched for. No statistically significant excess was found above the background; this allows setting a limit on the lifetime of PEP violating transitions to $9.0\times10^{30}\times \mathrm{Br}(\gamma)$ yr per oxygen nucleus, where $\mathrm{Br}(\gamma)$ is the branching ratio of the $^{16}\mathrm{O}$ decay in the γ channel. In the case where the PEP violating transitions are due to the p-shell nucleons, the limit is $1.0\times10^{32}\times \mathrm{Br}(\gamma)$ yr. Thus, the limit at 90% C.L. of the relative strength for forbidden transitions to normal ones is $\delta^2 < 2.3\times10^{-57}$ [57], which is also shown in Fig. 1.

4   Implications for Planck-scale deformed symmetries

We can begin by considering a generic model, with the assignments M_k, for $k\in\mathbb{N}$. On these latter, using the DAMA/LIBRA results as an example, the following constraints can be derived:

$$\delta^2\leqslant4\times10^{-55}\longleftrightarrow\delta^2(E)=c_k\frac{E^k}{\Lambda^k}\leqslant4\times10^{-55}. \qquad (2)$$

We are interested in those cases that are mostly motivated by quantum gravity scenarios. This corresponds

to select $\Lambda = M_{Pl} \simeq 1.22\times10^{19}\,\mathrm{GeV}$. A straightforward estimate of k can be then achieved, which already has dramatic consequences for several models of quantum gravity. Because nuclear decays processes taking place in the detector have an energy whose order of magnitude is a few times 10^{-3} GeV, we may consider $E=10\pm1\,\mathrm{MeV}$. For a set of heuristic choices of $c_k=\{1,4,10\}$, this implies immediately that at 90% C.L., only $k>k^*$ power suppressions are still experimentally allowed, with respectively[1] $k^*=\{2.58,2.61,2.63\}\pm0.01$. The exclusion limits on the k–Λ plane are displayed in Fig. 2, in which we use the accurate values for E that pertain to the various experiments analyzed, and set the coefficients [2] $c_k=1$. The most stringent constraints on the k–Λ parameters' plane, obtained by the above-mentioned experimental limits on the relative strength for non-Paulian transitions, are provided by the Borexino [56], Kamiokande [57] and DAMA [54] collaborations.

A different scenario arises when working at a scale of energy $E\simeq10$ keV, which is induced by transitions from k-electronic shells. This provides the upper bound $\delta^2\simeq10^{-47}-10^{-48}$, which is less stringent than the former one. Nevertheless, it still entails rejection of PEP violat-

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1) The propagation of the error only affects the last digit of k^* and is effectively independent of these heuristic choices of c_k, which capture the range of values in the literature — see e.g. Refs. [29, 38, 47, 49, 63–65]. Conversely, the theoretical ambiguity in c_k may affect the estimate of k^* up to 2% of its value.

2) As noted in the previous footnote, this choice is motivated by the fact that in the literature on noncommutative spacetimes, the c_k coefficients are order 1.

ing terms that are suppressed at the second order in Λ, and at the same time are regulated by coefficients c_n of order one.
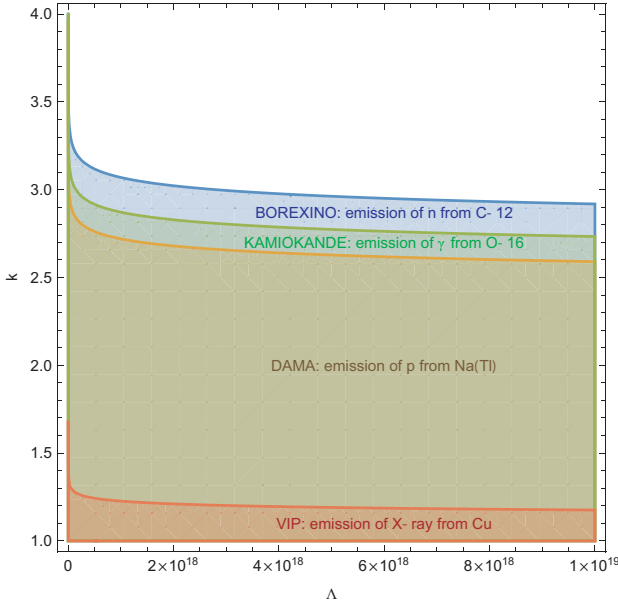

Fig. 2.   (color online) Exclusion limits at 90% C.L. on the k–Λ parameters' plane. Most stringent constraints are provided by the Borexino [56], Kamiokande [57] and DAMA [54] collaborations.

Below, we focus on PEP violations that arise in the aforementioned models of noncommutative spacetime, with particular focus on models endowed with κ-Poincaré deformed symmetries and θ-Poincaré deformed symmetries. Notice that the latter can be recast in the language of the noncommutative geometries *à la* Connes [59], while for the former, the equivalence has been proven hitherto under certain restrictions [60, 61]. In addition to the links to noncommutative geometry, noncommutative spacetimes have also been derived in several frameworks of quantum gravity, most notably in string theory and loop quantum gravity. In the former scenario, the Groenewold-Moyal noncommutativity is induced by the expectation value of background B fields [20], while in the latter, several instantiations of κ-deformation have been derived so far, with mesoscale deviations from Lorentz invariance – see *e.g.* [22–24, 26]. Although these derivations do not yet decisively answer the question about the low energy limit of string theory and loop quantum gravity, the constraints that we are providing here will have the undoubtable advantage of providing guidance for the development of theoretical models of quantum gravity.

4.1   The case of the κ-Poincaré group

Small departures from locality — an essential requirement for micro-causality in standard quantum field the-

ory — may be kinematically or dynamically generated in some quantum gravity scenarios and have been shown in [62] to be connected to the emergence of deformed κ-Poincaré symmetries. This is a non-trivial Hopf-algebra of symmetries dual to the κ-Minkowski non-commutative spacetime. The latter is characterized by commutation relations among the spacetime point coordinates of the type

$$[x_i, x_j] = 0 \qquad \text{and} \qquad [x_i, x_0] = \frac{\imath}{\kappa} x_i,$$

where κ denotes a scale of energy assumed to be on the order of the Planck scale in effective quantum gravity frameworks. There exists at least a basis of the Hopf algebra in which the Lorentz sector is standard, but the action of the Lorentz generators on the translation subgroup is κ-deformed. For instance, in the *bicrossproduct basis*, there is one κ-deformed commutator, namely

$$[P_l, N_j] = -\imath \delta_{lj} \left(\frac{\kappa}{2} \left(1 - e^{-\frac{2P_0}{\kappa}} \right) \frac{1}{2\kappa} \boldsymbol{P}^2 \right) + \frac{\imath}{\kappa} P_l P_j,$$

where P_μ denotes spacetime translation generators and N_l denotes boost generators. Even in this basis, in which translation generators remain commutative, the co-product map Δ acquires a κ-deformation. This is a remarkable deviation from standard properties. When P_μ are represented as derivatives acting on "coordinates-ordered" exponentials [48], it is trivial to recognize that Δ generalizes the Leibniz rule of derivatives' action. The κ-deformed Leibniz rule can then be inferred by the only κ-deformed co-product, *i.e.*

$$\Delta(P_j) = P_j \otimes \mathbb{1} + e^{-\frac{P_0}{\kappa}} \otimes P_j.$$

The fate of discrete symmetries in the κ-Poincaré setting was addressed in Ref. [48], while a detailed analysis of the fate of the CPT theorem for κ-Poincaré symmetries and of its consequences is still missing. Nonetheless, a phenomenological analysis of deviations from the standard case is still possible. Moving from the parametrization in Eq. (1), by straightforward dimensional arguments, we can express

$$\delta^2(E) = c_1 \kappa E, \tag{3}$$

where it is assumed that $\kappa \simeq M_{Pl}^{-1}$. This implies automatically the rejection of every model available in the literature that predicts a c_1 that is non-vanishing and of order one.

Following a constructive procedure, we can show that most part of the models hitherto addressed in the literature — see *e.g.* Refs. [29, 38, 63–65] — either reproduce the case where c_1 is of order one, or they fall in the class of a vanishing c_1, for which they cannot be falsified at the level of PEP violations, for instance, in Refs. [29, 38, 63] $c_1 = 1$; consequently, these models are ruled out. In Refs. [64, 65], where $c_k = 0$, for $k \in \mathbb{N}$, the commutation relations are unmodified. This scenario can then be

falsified up to the second order in the ratio E/M_{Pl}, but it is not distinguishable from the standard unmodified case.

4.2   The case of the Groenewold-Moyal plane

The algebra $\mathcal{A}(R^d)$ of commutative functions on a smooth d-dimensional space-time manifold can be mapped into that of noncommutative functions on the Groenewold-Moyal plane $\mathcal{A}_\theta(R^d)$, if the star-product is considered $(\alpha*\beta)(x) = \left(\alpha e^{\frac{\imath}{2}\overleftarrow{\partial_\mu}\theta^{\mu\nu}\overrightarrow{\partial_\nu}}\beta\right)(x)$, where $\theta^{\mu\nu} = -\theta^{\nu\mu}$ and $x = (x^0,...,x^{d-1})$. Accordingly, the Groenewold-Moyal (GM) multiplication map m_θ reads

$$m_\theta(\alpha\otimes\beta) = m_0\left(e^{-\frac{\imath}{2}\theta^{\mu\nu}(-\imath\partial_\mu)\otimes(-\imath\partial_\nu)}\cdot(\alpha\otimes\beta)\right), \quad (4)$$

where $m_0(\gamma\otimes\delta)(x) \equiv \gamma(x)\delta(x)$ stands for the standard point-wise multiplication rule.

Introducing the invertible element of the $\mathcal{R}$-matrix

$$\mathcal{F}_\theta = e^{-\frac{\imath}{2}(-\imath\partial_\mu)\theta^{\mu\nu}\otimes(-\imath\partial_\nu)},$$

the GM multiplication rule can be recast as $m_\theta(\alpha\otimes\beta) = m_0(\mathcal{F}_\theta\alpha\otimes\beta)$. The invertible element of the $\mathcal{R}$-matrix enters in a natural way the twisted deformation of the Fock space of scalar field theory, with spin zero, and thus the commutation relations of the ladder operators, i.e.

$$a(p)a^\dagger(q) = \tilde{\eta}'(p,q)\tilde{F}_\theta^{-2}(-q,p)a^\dagger(q)a(p) + 2p_0\delta^d(p-q), \quad (5)$$

where $\tilde{\eta}'$ approaches the constant $+1$ in the low-energy limit — this is formally equivalent to the commutative limit $\theta^{\mu\nu} \to 0$. Anti-commutation relations for free spinor fields are equal to the ones given in Eq. (5), provided that $\tilde{\eta}'$ approaches -1 in the low-energy limit.

We may expand Eq. (5) at the first order in $\theta^{\mu\nu}$, neglecting orders $O(\theta_{\mu\nu}\theta^{\mu\nu})$. This corresponds to a second-order expansion in Λ, as $\theta^{\mu\nu}$ has dimensions of length square. This immediately entails $c_1 \equiv 0$ and allows setting $c_2 = 1$, provided that $\theta^{12} = \theta^{13} = \theta^{23} = 1/(3\Lambda^2)$. Focusing on the data provided by DAMA (2009) B [54], and accounting for an isotropic distribution of the protons' momenta, we obtain

$$\delta_\theta^2 = \left(\frac{E}{\Lambda}\right)^2,$$

the exponent of which can be confronted with the values of k excluded in Fig. 2. Thus, this model seems to be already excluded by the present data. This is as transparent as it is surprising, as it was never pointed out in the wide literature devoted to non-commutative space-times.

4.3   Quantum gravity with lower energy scales

We can resort to the experimental bound in (2) to constrain departures from the standard spin-statistics

theorem within those theoretical frameworks that predict a lower energy scale of quantum gravity. Several models fit this scenario, notably the proposals that take into account the eventual role of large-scale extra dimensions in the resolution of the hierarchy problem — see e.g. Refs [66–68]. It is then straightforward to determine that any violation of PEP could arise up to the ninth order in the ratio E/Λ, within those proposals where the scale of quantum gravity is reduced down to the threshold hitherto achievable on terrestrial experiments, $\Lambda \simeq 10$ TeV. This rules out any reliable model of extra dimensions that would break Lorentz invariance and would predict violations of PEP.

5   Conclusions and outlooks

Although a direct connection between the deformation of space-time symmetries and quantum gravity has not yet been decisively proven, there are nevertheless many results in the literature that provide clear instantiations of space-time symmetry deformation or space-time symmetry breakdown regulated by the Planck scale. Making contact with those models that predict a deformation of the energy-momentum dispersion relations for one-particle states in particular, entailing a deformation of the Fock space states and of the SST, we developed a framework to falsify these scenarios accounting for possible PEP violations.

We emphasize that the phenomenological analysis we developed here differs from previous phenomenological investigations accounting for the one-particle Hilbert space structure of quantum field theories on non-commutative space-times. Constraints on energy-momentum dispersion relations apply only to certain classes of non-commutative space-times. For instance, quantum field theories endowed with κ-Poincaré symmetries, in which the algebra and the mass Casimir are deformed, provide an arena to test deformations of the energy-momentum dispersion relations. Alternatively, quantum field theories endowed with θ-Poincaré symmetries can be falsified only by looking at deformations of the Fock space structure, including eventual violations of the Pauli Exclusion Principle.

The tightest constraints on in-vacuo dispersion relations that are sensitive to the Planck scale, as discussed within the phenomenological models in Refs. [69–74], are provided for photons by the observation of TeV flares that originate from active galactic nuclei at redshifts smaller than 1 — see e.g. Refs. [74–79]. Taking into account deformation effects that are linear on the Planck length scale, the bounds can reach 1/10 of the Planck scale. Conversely, the best constraints on anomalous in-vacuo dispersion that are quadratic on the Planck length

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1) Indeed, for these Neutrinos, there is also the possibility to compare the arrival times with those of low-energy photons.

scale may be obtained from the detection of neutrinos[1] emitted by gamma ray bursts, with energies between 10^{14} and 10^{19} TeV — see e.g. Refs. [74, 80–86]. This clearly shows the relevance of our analysis with respect to the constraints previously discussed in the literature. Our analysis indeed provides either a restriction of the dimensionful parameters entering the UV-complete theories to be tested or a rejection/acceptance of their theoretical predictions. For instance, for string theory, we can only restrict the values of the parameters involved in the theoretical construction, while in the case of loop quantum gravity, the only dimensionful scale is the Planck scale, and all the order-one dimensionless parameters are fixed by the theory. Thus, with our analysis, we are able to provide for all these attempts a restriction of the universality classes that are allowed on the theoretical side

and rule-out values of the parameters that are either the most natural ones — from a theoretical perspective — to be considered, or the only ones that can be considered.

Dedicated measurements can be planned in forthcoming updates of DAMA/LIBRA and other experiments. This may provide the chance to constrain M_n with $n \geqslant 3$ contributions, which are suppressed by the n-th power of the energy scale Λ. In particular, an increase in the sensitivity of δ^2 would trigger the possibility of constraining third-order suppressed PEP violating terms. For completeness, we also mention the potentiality of a very interesting result on this topic from data collected by Super-Kamiokande.

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