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Weibull distribution to describe grading evolution of materials with crushable grains

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Abstract

This work presents a study on the evolution of the Grain Size Distribution (GSD) of an artificial granular material with crushable grains and its relation with the observed mechanical behaviour. The main aim of the work is to find a relationship between the initial GSD, its evolution and the mechanical behaviour. The artificial material used is a Light Expanded Clay Aggregate (LECA) subject to one-dimensional compression at various stress levels with eight initial GSDs characterised by four coefficients of uniformity and two mean diameters. The evolution of the GSD is characterised by a double mechanism depending on the initial GSD and on the applied stress level. A link between the evolution of the GSD, breakage and the compressibility curves is observed. A bimodal Weibull distribution function is proposed to describe the GSD before and after testing. The long term objective of the work is to link the evolution of breakage, from moderate to high stress, with the mechanical behaviour and to formulate a constitutive model able to describe the observed behaviour.

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Keywords: Weibull distribution; crushing; grain size distribution; compressibility

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1. Introduction

Particles break when the applied stress exceeds their strength. In the usual geotechnical applications, grain crushing occurs especially in areas of high stress concentration (*e.g.* shear bands, soil-structure interfaces, *etc.*) and significant amounts of crushing are observed in nature in faults and glacial deposits, characterised by high stress concentrations along particular planes. Breakage causes variations of the aggregate Grain Size Distribution (GSD) and of the particles shape. The percentage of finer particles increases and the particles resulting from fragmentation are usually more angular and irregular than the original grains. It follows that packing density, permeability, and mechanical properties of the granular aggregate are affected by breakage. Grain crushing depends on many factors such as, *e.g.*: particle size, shape, strength, mineral composition, stress path, and porosity. The experimental work was carried out on samples of a Light Expanded Clay Aggregate (LECA), an artificial granular material with crushable grains, which has already been studied in previous works [1,2]. The testing programme consisted of 24 one-dimensional compression tests performed under controlled displacement rate with eight different initial GSDs, characterised by four coefficients of uniformity, U (= 3.5, 7, 14, and 28) and two mean diameters, d_{50} (= 0.5 and 1.0 mm) (Fig. 1). To understand the effects of breakage and its physical effects on the particles assembly, the evolving GSDs were fitted using a Weibull statistical distribution, whose parameters were found to depend on the initial GSD and on the stress level applied. The main aim of this work is to study the evolution of the grading of LECA at high stress level in order to get the ultimate GSD and link it to the observed mechanical behaviour.

2. Experimental work

Systematic experimental investigations of grain crushing occurring in natural materials are often difficult due to the relatively high stress required to crush the grains and the variability and heterogeneity of natural deposits, which makes it difficult to obtain repeatable results. For these reasons, the experimental work was carried on an artificial granular material consisting of crushed expanded clay pellet, whose grains break at relatively low stress. The expanded clay pellets are commercially available under the acronym LECA (Light Expanded Clay Aggregate), both as intact or crushed, in different grain sizes. The main physical characteristic of the material is its very low apparent unit weight. This is due to the existence of a double order of porosity: "inter-granular", or voids existing between particles, and "intra-granular", or closed voids existing within individual particles. The apparent specific unit weight of the particles, γ_s, is a function of particle diameter d [3]. The value of γ_s increases significantly with decreasing grain size and tends to the unit weight of the constituent clay, $\gamma_a \cong 26.5$ kN/m^3, for the smallest particles. In the same way, the intra-porosity, calculated as $n_i = 1 - \gamma_s/\gamma_a$, increases with the particle size and tends, for the bigger particles, to a value of 0.6 [2].

Table 1. Experimental program and stress, deformation at the end of the tests.

				Series 1		Series 2		Series 3	
				D = 100 mm; H = 40 mm		D = 100 mm; H = 40 mm		D = 50 mm; H = 80 mm	
d_{50} [mm]	U [-]	d_{max} [mm]	β [-]	ε_a [-]	σ_v [MPa]	ε_a [-]	σ_v [MPa]	ε_a [-]	σ_v [MPa]
0.5	3.5	0.812	1.430	0.33	3.4	0.50	12.6	0.57	50.0
	7.0	1.061	0.921	0.33	6.1	0.44	12.6	0.57	50.0
	14.0	1.388	0.679	0.33	6.1	0.42	12.6	0.50	50.0
	28.0	1.815	0.538	0.33	3.9	0.41	12.6	0.49	50.0
1.0	3.5	1.624	1.430	0.33	1.9	0.56	11.8	0.59	50.0
	7.0	2.123	0.921	0.33	3.2	0.52	12.6	0.58	50.0
	14.0	2.776	0.679	0.33	5.0	0.44	12.6	0.51	50.0
	28.0	3.629	0.538	0.33	7.3	0.40	12.6	0.50	50.0

Figure 1 shows the eight initial GSDs by weight used to perform the one-dimensional compression tests. These are characterised by two mean diameters d_{50} = 0.5 mm and 1.0 mm, and four values of the coefficient of uniformity $U = d_{60}/d_{10}$ = 3.5, 7, 14, 28. The cumulative GSDs by weight have a fractal shape that can be described by the following equation: $P=(d/d_{max})^{\beta}$ where d_{max} is the maximum diameter, $\beta=3-\alpha$ and α is the fractal dimension. Exponent β can be computed from the following two equations: $d_{max} = d_{50}/0.5^{1/\beta}$ and $\beta = log_U 6$. Table 1 reports the values of β and d_{max} corresponding to the initial GSDs. The experimental programme consisted of 3 series of one-dimensional compression tests for each of the eight initial GSDs. The samples used for the first and second series of tests had a diameter D = 100 mm and a height h = 40 mm. The samples were loaded under controlled rate of vertical displacement (ν = 1 mm/min) up to maximum target displacements of 1/3h and 2/3h. However, the capacity of the loading frame of 100 kN (12.6 MPa) was always reached for displacements d < 2/3h. In order to reach higher stress levels, smaller samples (D = 50 mm and H = 800 mm), were adopted. In this case, the area of the cross section of the samples was $A = \pi\ D^2/4$ = 1953.5 mm^2 and the capacity of the loading frame corresponds to a maximum vertical stress of 50 MPa. Table 1 summarises the testing programme and reports the level of deformation imposed and measured stress. For the samples of Series 1, at the imposed value of ε_a = 0.33, the measured vertical stress increased with the initial coefficient of uniformity U, with the only exception of the GSD with d_{50} = 0.5 mm and U = 28. For Series 2, at σ_v = 12.6 MPa, the vertical deformation decreased with increasing coefficient of uniformity, the same trend was observed for Series 3. The observed behaviour indicates that samples initially well graded (*i.e.,* characterised by a higher coefficient of uniformity) are stiffer and less prone to grain crushing. This may be explained by the fact that bigger particles are surrounded by smaller ones, which increases the coordination number of the assembly and reduces the average contact stress.

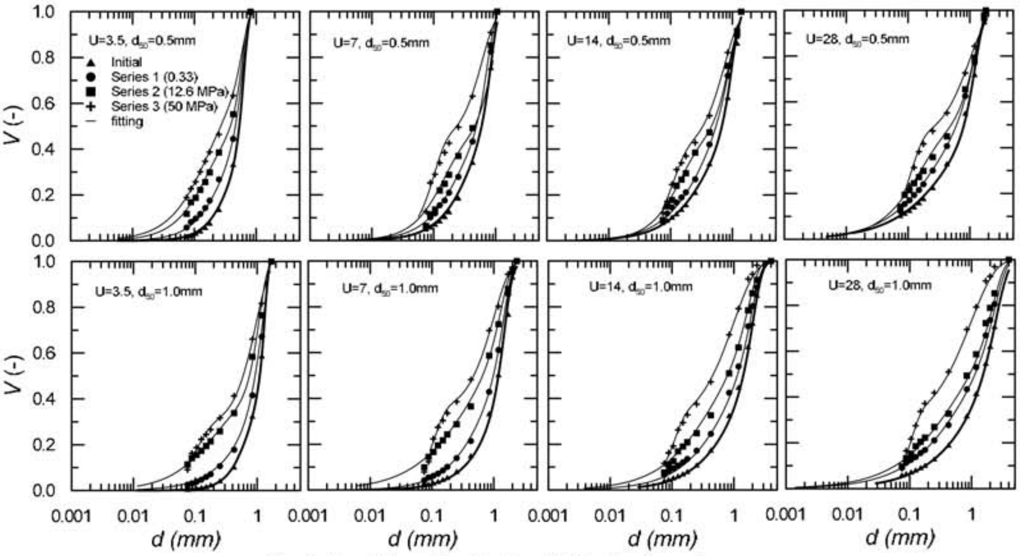

Fig. 1. Evolution of grain size distribution by volume.

3. Results

For a material such as LECA, in which the intra-porosity causes the apparent unit weight of particle γ_s to depend to their size, it is necessary to distinguish between the GSD by weight and by volume. The grading by weight is determined experimentally using a discrete series of sieves. To calculate the grading by volume, the weight retained in each sieve has to be divided by the apparent unit weight γ_s associated with the range of diameters of the two subsequent sieves. Figure 1 shows the evolution of grading by volume with load. The breakage of particles increased with the imposed deformation for all the tested GSDs, with a general tendency of the GSD to move to the left and above the initial GSD. In particular, it is possible to define two different patterns of grading evolution on breakage. The first pattern, typically observed for the lower applied stress, consists in a clock-wise rotation around the point representing the maximum diameter, with an increase of U, and a progressively flatter GSD. The second pattern, typical of samples loaded at higher stress, is characterised by significant changes of the shape of the GSD. The maximum diameter reduces and a new grading emerges, resulting in a hump at smaller diameters. The new grading is characterized by lower values of the mean diameter and a very low coefficient of uniformity. These two types of behaviour have been observed for all the tested GSD but the second mechanism is more evident for initial GSDs characterised by higher coefficients of uniformity. At lower stress most of the large particles do not crush, as indicated by the fact that d_{max} remains constant, probably because the larger number of contacts outweighs the effect of the strength of individual particles reducing with increasing particle size [4]. As breakage continues however, a situation is reached where the coordination number for the large particles is maximum. A good approximation of this situation is an arrangement of the particles closely resembling the Apollonian fractal in which the voids between particles are progressively filled by smaller particles. With continuing deformation/loading, the GSD can only evolve changing behaviour and generating a new emerging grading.

4. Analysis of results

To quantify the patterns of evolution of the GSD described above, the experimental data before and after one-dimensional compression were fitted using a bimodal Weibull distribution. This is a flexible function to represent different breakage patterns, used by other authors [5,6,7] to study different features of breakage. The Weibull distribution is defined on positive and real numbers by the following repartition function: $F(X)=1-exp[-(X/\lambda)^k]$, where $F(X)$is the probability that any variable is less or equal to the variable X, $k>0$ is a shape parameter and $\lambda>0$ is a scale parameter of the distribution. The probability density function f is calculated from the derivative of the repartition function: $f(X)=k/\lambda^k\ X^{\ k-1} exp[-(X/\lambda)^k]$. Within the framework of statistic, particle breakage can be considered as a probabilistic event. When the stress or strain rate has reached a sufficient level, some particles may break, with a breakage probability, which depends by many complex factors, including stress level, average diameter, coordination number, component minerals of sample, particle shape, *etc*. In term of the GSD, if the quantity X represents the particle diameter, the Weibull distribution F represents the cumulative passing by volume. The scale parameter λ corresponds to diameter d_{63}. Fixing k and varying λ the Weibull distribution translates, to the right for higher values of λ and *vice versa*. The Weibull distribution has a typical S shape and the slope depends by the shape parameter k, as k increases the distribution becomes steeper, while lower values of k result in a flatter shape of the distribution. It is interesting to note that the derivative of F respect to $log(\mathrm{d})$, $\partial F/\partial log(d)=d^{\ k-1}/\lambda\ exp[-(d/\lambda)^k]\ \partial d/\partial log(d)$, yields a characteristic "bell" shape with the maximum for $d=\lambda$, and the height of the bell is proportional to the value of k through the coefficient of $k\ ln(10)=0.847\ k$. In this work the bimodal Weibull distribution was used to fit the GSD obtained before and after testing. The GSD is expressed as the sum of two Weibull distributions weighed by w_1 and w_2 (where $w_1+w_2=1$), $V(\mathrm{d})=w_1(1-exp[-(d/\lambda_1)^{k1}])+w_2(1-exp[-(d/\lambda_2)^{k2}])$, where V is the cumulative passing by volume, k_1 and k_2 the shape parameters, λ_1 and λ_2 the scale parameters and d the diameter. Figure 1 shows the experimental GSDs together their best fitted bimodal distribution. The five parameters were determined from the experimental data, minimizing the sum of standard deviation and imposing some restraints to the parameters.

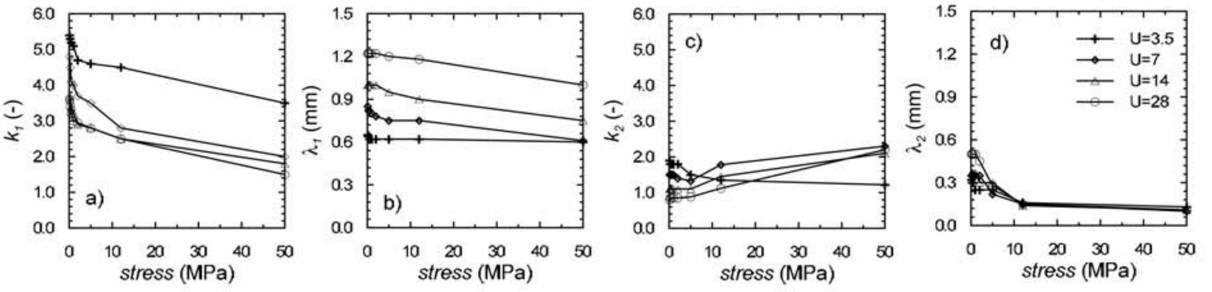

Fig. 2. Evolution of Weibull fitting parameters with stress for samples with d_{50}= 0.5 mm (a) k_1, (b) λ_1, (c) k_2, (d) λ_2.

The advantage to fit the GSD with a statistical function is to study how the statistical parameters evolve. Figure 2 shows the trends of λ_1, k_1, λ_2 and k_2 as a function of the applied stress for the samples with an initial $d_{50} = 0.5$ mm. The initial grading having $U = 3.5$ for example, presents the higher value of k_1 and the lowest value of λ_1 (Fig. 2a,b). This means that the slope of the distribution is steeper and the passing in volume at 0.63 is shifted to the left compared to the other distributions. On the other hand, the initial GSD with $U = 28$, presents the lowest value of k_1, and the maximum value of λ_1, therefore is flatter and shifted to the right. The parameters λ_1 and k_1 have, for all the samples, decreasing trends with stress, that is the upper part of the distribution tends to rotate to the left, as observed above. The lower part of the distribution is described by parameters λ_2 and k_2. The shape parameter k_2 (Fig. 2c) decreases for $U = 3.5$, and increases for all the other initial GSDs. This is due to the fact that the distribution having $U = 3.5$ has not yet reached the condition necessary to fully activate the second breakage mechanism. The first breakage mechanism is characterized by decreasing values of k_2, while k_2 increases in the second mechanism. From the data reported in Figure 1, it is evident that the predominant observed behaviour is of the first type for a strain of 0.33 (Series 1), while, for all the tested GSDs, the occurrence of the second mechanism is recognisable only at higher stress, with the only exception of the initial GSD characterised by $U = 3.5$ and $d_{50} = 0.5$ mm. The scale parameters λ_2 (Fig. 2d) decreases with increasing applied stress with a similar trend independently by the initial uniformity; all the data tend to λ_2=0.11 mm at 50 MPa. The asymptotic value of $\lambda_2 = 0.11$ mm highlights an interesting property of LECA: the new grading emerging from the second mechanism is the same for all the sample tested, characterized by a low value of $U \cong 2 \div 3$. Figure 3 shows all the GSDs obtained after compression at 50 *MPa* for $d_{50} = 0.5$ mm (a) and $d_{50} = 1$ mm (b). Regardless of the initial state in terms of uniformity and d_{50}, at high stress the samples seem to converge to a unique grain size distribution for the lower diameters. Figure 4 shows the particle size distribution *dV/dlog(d)* as function of diameter. In particular, being $(dV/dlog(d))_{max} = \lambda$ for each sub-grading, the initial curves are characterised by a single peak depending mostly by d_{max0} and U_0. As the load increases the peak value of the macro diameter decreases, while a progressive emergence of the second particle size distribution emerges with a peak value characterized by a $d = 0.11$ mm for all the distributions. This second peak is linked to the second mechanism of breakage.

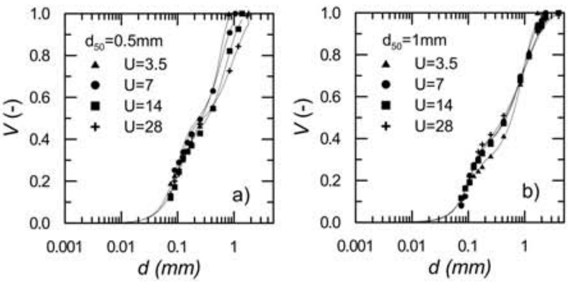

Fig. 3. Comparison between samples with $d_{50} = 0.5$ mm (a) and $d_{50} = 1.0$ mm (b) after testing at 50 MPa.

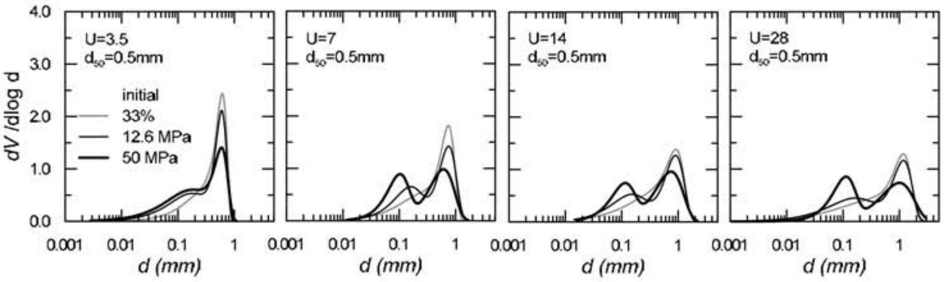

Fig. 4. Particle size distribution by volume.

The compressibility features are coupled with the two breakage mechanism observed for the GSD evolution. Figure 5 shows the compressibility curves obtained at the maximum stress level of 50 MPa: the two bands identify the range of stress corresponding to yielding (σ_p) and to the inflection point of the compressibility curve (σ_s). The yield stress σ_p takes values between 0.4 and 1 MPa, while the point of inflection of the compressibility curve corresponds to stress of the order of 5 to 11 MPa, for $d_{50} = 0.5$ mm and 6 to 12 MPa, for $d_{50}=1.0$ mm. These two ranges of values match the stress values corresponding to the change of slope of the plots of k_1 and k_2, respectively (Fig. 2a,b) As a first hypothesis, the first breakage mechanism could start at a stress value of σ_p while the second one at a stress the value of σ_s. Yielding at σ_p is due to particle comminution [8] while particle breakage at σ_p might be still negligible. At a stress of σ_s the first mechanism of breakage gets to its maximum capacity, after which to sustain further breakage a second mechanism must be activated.

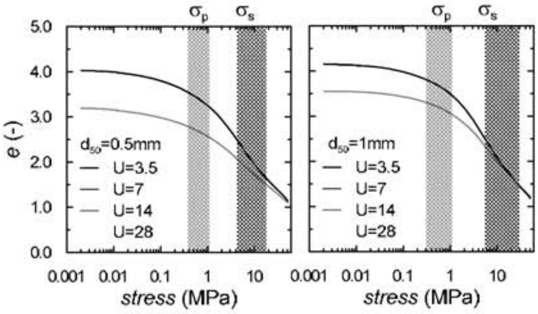

Fig. 5. Compressibility curves of the odometer tests at 50 MPa for $d_{50} = 0.5$ mm and $d_{50} = 1.0$ mm.

5. Conclusions

This paper illustrates the results of an extended experimental investigation of the evolution of grain size distribution of LECA material at high stress level. The main goal of the research is to study the existence of an ultimate distribution starting from different initial GSDs and the effects of particle breakage on the mechanical behaviour of the aggregate. One-dimensional compression tests under controlled displacement rate were performed using 8 different initial grain size distributions. The grading before and after test were fitted using a bimodal statistical Weibull distribution function. The evolution of distribution parameters (λ_1, k_1, λ_2, k_2) gave information about the breakage behaviour. This seemed to evolve from one pattern, characterised by a clockwise rotation around the maximum diameter and a decreasing trend of k_1, to a second pattern characterised by the emergence of a second grading at smaller diameters, characterised by an increasing trend of k_2. It is interesting that the new grading emerging from this second pattern of breakage is independent of the initial GSD and is centred around a diameter of about 0.1 mm, probably a characteristic length of the material, that may be related to some physical feature such as *e.g.*, a characteristic dimension of the intra-porosity. Comparing the observed evolution of the GSD and the compressibility curves, it appears that the mechanical properties of the aggregate are affected by breakage in different ways. The yield stress corresponds to the first breakage pattern, while the inflection of the compressibility curve is associated to the second pattern of breakage. The yield stress values seem to be the same for each grading and independent from d_{50} and U, besides the stress values related to the inflection point are bigger for $d_{50} = 1.0$ mm. This indicates that the second pattern of breakage appears previously for samples with bigger mean particle dimension.

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