Note on the EPR–chameleon experiment

Luigi Accardi, Kentaro Imafuku, Massimo Regoli

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Centro Vito Volterra, Università di Roma "Tor Vergata"
email: accardi@volterra.mat.uniroma2.it,
WEB page: http://volterra.mat.uniroma2.it

Abstract

In the past 20 years quantum probability has challanged the widespread belief that classical macroscopic systems cannot, by local independent choices, produce sequences of data whose correlations violate Bell's inequality. The possibility of such a violation is not a matter of interpretation, but of fact: "local independent choices" means that two separated and non communicating experimenters make measurements but one does not know what the other measures (or even if the other one measures something); correlations are evaluated by means of standard procedures. The present experiment shows that this is not the case: in no way the EPR correlations and related experiments can be considered as a support of the incompatibility of quantum theory with local realistic theories, in particular relativity.

1 Bell's argument

Bell's inequality was proved in the paper [Be64]. In this paper, while the thesis, i.e. the inequality itself, is clearly stated and correctly proved, the mathematical assumptions from which the thesis follows (and without which the thesis cannot be proved) were not formulated.

This opened a debate whose goal was to try and establish which these assumptions effectively were. The reader interested in having an idea of the arguments used before quantum probability may consult the famous [Wig70] or [Stap82] or, for the connections with probability [SuZa76].

The mathematical formulation, now commonly adopted, of the Bell inequality was first given in [Ac81]. The main result of this paper consists in having realized that the mathematical assumptions on which the validity of the inequality depends are only the following two ones:

(i) that the random variables take values in the interval $[-1, 1]$ (originally Bell considered only the set $\{-1, 1\}$ but shortly after he extended his result to the full interval)

(ii) that all the random variables are defined on a single probability space.

More precisely:

Lemma (1) Let A, B, C be random variables defined on the same probability space $(\Omega, \mathcal{F}, P)$ and with values in the interval $[-1, 1]$. Denote

$$\langle AB \rangle := \int_\Omega A(\omega)B(\omega)P(d\omega)$$

the correlations (mean zero can be assumed without loss of generality). Then the following inequality holds:

$$|\langle AB \rangle - \langle CB \rangle| \le 1 - \langle AC \rangle \tag{1}$$

The following corollary of (**??**) (which is an equivalence for ± 1-values observables), due to Clauser, Horne, Shimony, Holt (CHSH), even if does not add anything to Bell's argument, is widely used in the literature.

Corollary (2) Let A, B, A', B' be random variables defined on the same probability space $(\Omega, \mathcal{F}, P)$ and with values in the interval $[-1, 1]$. Then the following inequality holds:

$$|\langle AB \rangle - \langle A'B \rangle + \langle AB' \rangle + \langle A'B' \rangle| \le 2 \tag{2}$$

Proof. With the replacements $B \to B'$, $C \to -C$, (**??**) becomes

$$|\langle AB' \rangle + \langle CB' \rangle| = 1 + \langle AC \rangle \tag{3}$$

Adding (**??**) to (**??**) and replacing C by A' we get

$$|\langle AB \rangle - \langle A'B \rangle| + |\langle AB' \rangle + \langle A'B' \rangle| \le 2$$

which implies (**??**).

The following Theorem is used to establish a connection between Bell's inequality and the predictions of quantum mechanics (cf. [AcRe00b] for a comparison of the various proofs).

Theorem (4). There cannot exist a stochastic process

$$S_a^{(1)} \ , \ S_b^{(2)} \qquad a, b \in [0, 2\pi]$$

defined on a probability space $(\Omega, \mathcal{F}, P)$ and with values in the set $\{\pm 1\}$, whose correlations are given by:

$$\langle S_a^{(1)} S_b^{(2)} \rangle = -\cos(a - b) \qquad ; \qquad a, b \in [0, 2\pi] \tag{4}$$

Remark. According to quantum theory the expression in the right-hand side of (**??**) is the correlation of two spin or polarization observables, along directions a, b, of two quantum particles in singlet state. These correlations have been experimentally confirmed by many experiments since the early days of quantum mechanics and this confirmation has been interpreted as experimental evidence that quantum theory is incompatible with any local realistic classical theory. While nobody doubts that the validity of the correlations (**??**) is a well established experimental fact, the claim that the experimental validity of the correlation (**??**) is incompatible with a local realistic interpretation of quantum mechanics, is definitively unwarranted both for theoretical (cf. sections 2, 3, 4) and for experimental reasons.

2 Criticising Bell's analysis

The quantum probabilistic approach offers to the physicists a way out from the "quantum muddle" by criticising Bell's analysis and proving that:

i) the contradiction, pointed out by Bell, arises only from his implicit postulate that 3 statistical correlations, coming from 3 mutually incompatible experiments, can be described within a single classical probabilistic model

ii) that this implicit postulate is by no means a consequence of locality and reality.

If the implicit postulate (i) is not assumed, then Bell's proof is at fault already in its first step because in the proof of (1) (and the same is true for (**??**)) one must user the apparently obvious identity

$$\langle AB \rangle - \langle CB \rangle = \langle AB - CB \rangle \tag{5}$$

However, by explicity writing this identity:

$$\int_\Omega A(\omega)B(\omega)P_{a,b}(d\omega) - \int_\Omega C(\omega)B(\omega)P_{c,b}(d\omega) = \int_\Omega \left(A(\omega)B(\omega) - C(\omega)B(\omega)\right) P_{a,c,b}(d\omega) \tag{6}$$

one immediately recognises that the left hand side is experimentally observable while the right hand side is not. In fact while the pair joint probabilities $P_{a,b}, P_{c,b}, ...$ are experimentally observable, there is no reason to postulate, as Bell implicitly does when using formula (**??**), that the, experimentally unobservable, triple joint probabilities $P_{a,c,b}$ exist.

It is well known from classical probability that there are constraints, i.e. compatibility conditions, which relate the pair with the triple joint probabilities and which are necessary conditions for the existence of the latter ones. Since the pair correlations are deduced from the pair probabilities and since, when using (**??**), Bell is postulating a priori the existence of these (experimentally unobservable) triple joint probabilities, the only rational conclusion he can draw from his argument is that the inequality (**??**) (Bell's inequality) is one of these necessary conditions. Consequently, the experimental violation of this inequality is simply an experimental proof of the fact that the triplet joint probabilities for 3 singlet correlations cannot exist.

This was the critique that, starting from 1981 [Ac81], quantum probability opposed to Bell's argument.

The chameleon effect

One might try to counter this critique by arguing that the existence of the triple probabilities is a consequence of the "realism" assumption.

For example suppose that in a box there are many pairs of balls whose color can be either green or brown. Moreover each ball is either made of glass or of wood and it weights either 10 or 20 grams. The rules of the game are such that you can only measure one observable at the time on each ball (color, weight, material). Thus on each pair we can simultaneously measure at most two observables and we can make an experimental analysis of the joint statistics of all possible pairs of observables ("color-material", "color-weight", ...).

Because of the rules of the game the triple joint probabilities "color-material-weight" are not accessible to experiment.

However the "realism assumption" tells us that any one of the possible triple combinations (color, material, weight) has a definite relative frequency in the box and therefore the pair statistics we observe, is a consequence of this triple statistics which, although unobservable "exists". Consequently, many authors have claimed, to postulate the existence of the triple probabilities for the singlet correlations simply amounts to postulate the objective existence of physical properties independently of the observer. This is a realism postulate. Hence, if we exclude superluminal communications (locality) the experimental proof of the non existence of the triple probabilities is equivalent to the experimental invalidation of the realism postulate.

Arguments of this kind are quite reasonable: for example they are at the basis of classical statistical mechanics and it is probable that Einstein had in mind something of this kind when speaking of "objective reality".

According to quantum probability there is a more subtle notion of "objective reality" which gives a better intuition of the behavior of quantum systems (but it it by no means restricted to them). We call the corresponding realism "chameleon realism" as opposed to the "ballot box realism" of classical statistical mechanics.

Suppose that, in the above example, you leave the rules of the game unaltered, but you replace the pairs of balls by pairs of chameleons and the observables (color, material, weight) by (color on the leaf, color on the wood, weight).

Is it still reasonable to believe that the pair statistics you observe, is a consequence of some (unobservable) triple statistics?

A little thought shows that the answer is: No!

According to quantum probability, quantum systems are much more similar to chameleons (adaptive: we measure the response to an interaction) than to balls (passive: we read what was in the box).

3 Mathematical formulation of the chameleon effect

The attempt to translate in a precise mathematical and physical language the intuitive difference between "ballot boxes" and "chameleons" leads to a natural generalization of von Neumann's measurement theory.

The generalization consists in introducing, in this theory, the notions of locality and causality.

It is widely accepted, since von Neumann's original analysis, that a qualitative analysis of the measurement process should start from the joint (unitary) evolution $u_{S,A}$ (system, apparatus): for simplicity we consider discrete time. Thus, if ψ_o is the initial state of the system, its state at the time of measurement is

$$\psi := \psi_o \circ u_{S,A} \tag{7}$$

Now suppose that we want to measure the observable $S_a = S_a$ of the system (say: spin in direction a). Then the apparatus M must be prepared to measure

S_a (say: by orienting a magnetic field in direction a). Therefore the interaction Hamiltonian between system and apparatus, hence also the joint dynamics, will depend on a:

$$u_{S,A} := u_{S,A(a)} := u_a \tag{8}$$

In other words: the dynamics of a system depends on the observable we want to measure: this is the chameleon effect. As anybody can see, it is a simple corollary of the standard ideas on measurement theory.

Now suppose that the system, hence the apparatus, is made up of two spatially separated parts: $(1, M_1, 2, M_2)$ and that we measure independently

$$S_a^{(1)} = (S_a \otimes 1_2) \otimes 1_M \qquad (resp. S_b^{(2)} = (1_1 \otimes S_b) \otimes 1_M)\ , \quad M = (M_1, M_2)$$

on particle 1 (resp. particle 2). Then, according to the chameleon effect, we will have

$$u_{S,A} := u_{a,b} \tag{9}$$

and, according to quantum (or classical) mechanics, the pair correlations will be

$$\langle S_a^{(1)} S_b^{(2)} \rangle := \psi_o \circ u_{a,b} \left(S_a^{(1)} S_b^{(2)} \right) = \psi_{a,b} \left(S_a^{(1)} S_b^{(2)} \right) \tag{10}$$

This shows that the pair joint probability $P_{a,b}$, corresponding to these correlations, depends on a, b, hence the application of Bell's inequality is impossible. This dependence is called "contextuality".

However, by considering the mean value of a single particle observable, say $S_a^{(1)}$:

$$\psi(S_a^{(1)}) := \psi_o \left(u_{a,b}(S_a^{(1)}) \right) \tag{11}$$

we see that, for a general dynamics, the mean value of an observable of particle 1 will depend on the measurement we do on particle 2: this means that the EPR locality condition is not satisfied. Thus contextuality is not enough to guarantee locality.

If we want it to be satisfied, we have to restrict the class of allowed dynamics and also the class of allowed initial states.

The physical arguments which allow to define such restrictions have been discussed in previous papers of the authors (cf. [AcRe01a] for bibliography).

The EPR locality condition is mathematically expressed by the fact that the local dynamics of each particle is independent of the local dynamics of the other one, in formulae

$$u_{a,b} := u_a \otimes u_b \tag{12}$$

and the causality condition by the fact that the initial state of the particles is independent of the initial state of the apparatus (because the particles cannot know which measurement will be made on them). In formulae:

$$\psi_o := \psi_{1,2} \otimes \psi_{A_1} \otimes \psi_{A_2} \tag{13}$$

However both ψ_{A_1}, ψ_{A_2} may depend on state of the system 1 (resp. 2) because the local interaction of particle 1 (resp. 2) with the apparatus may depend on the state of the particle at the moment of interaction. With these restrictions one easily computes that the EPR locality condition is satisfied. However (**??**) and (**??**) show that the pair joint probabilities, corresponding to pair correlations, still depend on a, b, hence the application of Bell's inequality is still impossible.

This extension of the standard quantum theory of measurement was first proposed in [Ac93]. The experiment discussed in the present conference is a concrete realization of this abstract scheme.

4 Description of the dynamical model

In the present section we construct a dynamical system which simulates locally the EPR correlation (**??**).

In the idealized dynamical system considered in our experiment we consider only two time instants 0 (initial) and 1 (final) so, in our case, a "trajectory" consists of a single jump. We do not describe the space–time details of the trajectory because we are only interested in distinguishing 2 cases:

– at time 1 the particle is in the apparatus (and in this case it is detected with certainty)

– at time 1 the particle is not in the apparatus (and in this case it makes no sense to speak of detection)

Thus our "configuration space" for the single particle will be made of 3 points: s (source), 1 (inside apparatus), 0 (outside apparatus). Since at time 0 the "position" of both particles is always s, because of the chameleon effect, the position $q_{j,1}$ of particle $j = (1, 2)$ at time 1 will depend on the polarization a_j, on the initial state σ and on the state λ_j of the apparatus $M_j (j = 1, 2)$:

$$q_{j,1} = q_{j,1}(a_j, \sigma, \lambda_j) \quad ; \qquad j = 1, 2,$$

The local, deterministic dynamical law of this dependence is described as followed.

1. The state space of the composite system (particles, apparatus) is

$$\begin{aligned}\{\text{position space}\} \times \{\text{inner state space}\} \times \{\text{apparatus space}\} \\ = \{s, 0, 1\} \times [0, 2\pi]^2 \times [0, 1]^2\end{aligned}$$

2. The initial state is always of the form

$$(s, s, \sigma_1, \sigma_2, \lambda_1, \lambda_2) \in \{s\}^2 \times [0, 2\pi]^2 \times [0, 1]^2$$

   i.e. the initial position of both particles is always s.

3. To speak of correlations only makes sense if the deterministic trajectories of both particles end up in the detectors (pre-determination). This means that the statistics is conditioned to the subset

$$\{1\}^2 \times [0, 2\pi]^2 \times [0, 1]^2 \tag{14}$$

   of the state space.

4. Just by changing the order of the factors the state space can be realized as

$$\{(s_1, \sigma_1, \lambda_1; s_2, \sigma_2, \lambda_2)\} \in (\{s, 0, 1\} \times [0, 2\pi] \times [0, 1]) \times (\{s, 0, 1\} \times [0, 2\pi] \times [0, 1])$$

Therefore a local deterministic dynamics is uniquly determined by the assignment of two functions $T_{1,a}, T_{2,b}$:

$$(s_1, \sigma_1, \lambda_1; s_2, \sigma_2, \lambda_2) \to (T_{1,a}(s_1, \sigma_1, \lambda_1), T_{2,b}(s_2, \sigma_2, \lambda_2))$$
$$=: (q_{1,a}, s_{1,a}, m_{1,a}; q_{2,b}, s_{2,b}, m_{2,b})$$

where $(q_{1,a}, s_{1,a}, m_{1,a})$ (resp...) depends only on $(s_1, \sigma_1, \lambda_1)$ (resp....). Moreover it is convenient to identify the endpoints of both intervals $[0, 2\pi]$ and $[0, 1]$, i.e. to identify these intervals to circles so that the functions $q_{j,x}, s_{j,x}, m_{j,x} (j = 1, 2, x = a, b)$, as functions of the variables σ, λ can be extended by periodicity to the whole real line (period 2π in σ, period 1 in λ). This allows to give a meaning to formula (17) of [AcImRe01] in full generality, i.e. without appealing to special choice (11) of [AcImRe01].

5. With these convenctions, for every $a, b \in [0, 2\pi]$, a deterministic dynamics as follows

$$(s_1, \sigma_1, \lambda_1; s_2, \sigma_2, \lambda_2) \mapsto (q_{1,a}(s_1, \sigma_1, \lambda_1), \sigma_1, \lambda_1; q_{2,b}(s_2, \sigma_2, \lambda_2), \sigma_2, \lambda_2) \tag{15}$$

i.e. the inner state of the particle and of the apparatus do not vary under the evolution, but the position varies according to the law:

$$q_{1,a}(s, \sigma_1, \lambda_1) = \chi_{[0, p_{1,a}(\sigma_1)]}(\lambda_1) \ ; \ q_{2,b}(s, \sigma_2, \lambda_2) = \chi_{[0, p_{2b}(\sigma_2)]}(\lambda_1)$$
$$p_{1,a}(\sigma) = \frac{1}{4}|\cos(\sigma - a)| \quad , \quad p_{2,b}(\sigma) = 1 \tag{16}$$
$$(\chi(x) = 1 \text{ if } x \in I \ , \qquad I = 0 \text{ if } x \notin I)$$

Remember that the initial position of both particles is always s. Therefore it is sufficient to define the dynamics only in this case.

6. For every setting $(a, b) \in [0, 2\pi]^2$ of the apparata, the initial probability distribution $P_{a,b}$ of our deterministic dynamical system is given by:

$$\delta_{s,s_1}\delta_{s,s_2}\frac{1}{2\pi}\delta(\sigma_1 - \sigma_2)\delta(m_{1,a}(\sigma_1, \lambda_1) - m_a)\delta(m_{2,b}(\sigma_2, \lambda_2) - m_b)d\sigma_1 d\sigma_2 d\lambda_1 d\lambda_2 \tag{17}$$

where m_a, m_b are fixed numbers in $[0, 1]$ and

$$m_{1,a}(\sigma_1, \lambda_1) = \frac{4\lambda_1}{\sqrt{2\pi}|\cos(\sigma_1 - a)|}, \quad m_{2,b}(\sigma_2, \lambda_2) = \frac{\lambda_2}{\sqrt{2\pi}} \tag{18}$$

Finally the random variables $S_a^{(1)}, S_b^{(2)}$:

$$\{s\}^2 \times [0, 2\pi]^2 \times [0, 1]^2 \to \{\pm 1\}$$

are defined by

$$S_a^{(1)}(s, \sigma, \lambda) = \text{sgn}\,(\cos(\sigma - a))\,; S_a^{(2)} = -S_a^{(1)}$$

It is now a matter if simple calculations (cf. section (2) of [AcImRe01]) to verify that the correlations

$$\langle S_a^{(1)} S_b^{(2)} \rangle = \int_\Omega S_a^{(1)}(T_{1,a}(s, \sigma_1, \lambda_1)) S_b^{(2)}(T_{2,b}(s, \sigma_2, \lambda_2)) dP_{a,b}(\omega) \qquad (19)$$

($\omega = (s, \sigma_1, \lambda_1; s, \sigma_2, \lambda_2)$) are precisely the EPR correlations.

Finally notice that the dynamics (**??**) is slightly simplified with respect to the one described in [AcImRe01]. However, due to the choice (11) of [AcImRe01] this simplification does not change the calculations in the specific case under consideration. For more general classes of models the simplification (**??**) is convenient because with this choice the state space is mapped into itself by the dynamics and no additional identifications are required.

There is no conceptual difficulty to include in our model the consideration of the space–time trajectory of the particle. This surely would improve the present model, however the main conclusion of our experiment, i.e. the reproducibility of the EPR correlations by a classical, deterministic, local dynamical system, will not change.

5 Comments on the experiment

The realization of the computer simulation of the local dynamical system constructed in the previous section has been described in detail in the paper [5] and will not be repeated here.

Our goal in this section is to briefly illustrate the conceptual meaning of this implementation.

Recall the basic idea of the chameleon effect: *the local dynamics influences the statistics and since the factorization of the dynamics (**??**), i.e.* $((1, M_1), (2, M_2))$*, is different from the factorization of the state (**??**), i.e.* $((1, 2), (M_1, M_2))$*, the result of the local interaction is a global dependence of the final state on the whole measurement setting, i.e.* (a, b).

Now, in any dynamical system, the statistics is determined by the number of trajectories that fall into a pre–assigned region of the state space.

Therefore, by definition, to say that the dynamics influences the statistics, means that the dynamics changes the trajectories of the single particles.

As explained above this change will be local because of the form (**??**) of the dynamics, but the influence on the statistics will be global because of the form (**??**) of the initial state.

Here the word "change" has to be interpreted with respect to the trajectories that the particles would have in absence of interaction with the apparatus.

Another important point is that, in all the EPR–type experiments, the two apparata must be spatially separated: if the two apparata were contiguous there might exist communications between them without violating the locality principle.

This obvious fact has an important conceptual consequence, namely that: *in all EPR–type experiments, the statistics is conditioned on those trajectories that lead both particles to interact with the apparatus*.

Now let us forget, for the moment, the quantum mechanical situation and let us concentrate our attention on a classical dynamical system composed, as in our experiment, of two particles and two apparata.

The state space of the particles will consist, as in our experiment, of their space position and of inner degrees of freedom.

The condition that the two apparata are spatially separated implies on a priori selection of the trajectories and this selection is the physical counterpart of the probabilistic operation of conditioning.

Combining this remark with what said before on the local deformation of the trajectories due to interaction with the apparatus, we see that this local deformation can manifest itself only in two ways:

(i) by altering the space trajectories

(ii) by altering the "trajectories" of the inner degrees of freedom.

They are both local effects and they can alter the statistics

(j) by changing the number of pairs of trajectories which end in the influence region of the apparatus

(jj) by altering the response of a single particle to the interaction with the apparatus.

Now, if we want to respect the singlet law, the response of each particle to the apparatus must be pre–determined. Therefore the simplest way to construct a dynamical system which respects the singlet law and realizes the chameleon effect (i.e. the alteration of statistics due to local interactions with the apparatus) is to construct the deterministic dynamics in such a way that the space dynamics of the particles is influenced by the local interaction, while its inner degree of freedom is not.

This is what we have realized with the dynamics (**??**).

Such a deformation is perfectly compatible with the assumption of an 100 per cent (ideal) efficiency of the detectors. In fact the efficiency is measured by the ratio of the number of detected particles over the number of particles which have interacted with the apparatus.

It would be totally meaningless to take into account, in the determination of the efficiency, also those particles whose space trajectory has brought them so far from the apparatus that no physical interaction between them is conceivable.

Moreover, and this is a possible difference between the classical and the quantum case, the very notion of "total number of pairs emitted by the source" is a totally platonic and in principle unobservable quantity in the quantum case (under the assumption of a neat space separation between the two apparata).

In some, but not all, classical sitautions this number might be observable, but in a quantum context, where you cannot follow the trajectory of single particles without altering it, this number is quite unobservable.

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