Quantum theory, Chameleons and the statistics of adaptive systems

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The fact that the mathematical formalism, used to make predictions on quantum phenomena, is completely different from the mathematical formalism used in classical probability, was recognized since the early days of quantum theory in the late 1920's.

However only in the early 1980's it was realized that this difference of the mathematical formalisms reflects a deeper probabilistic phenomenon: namely the fact that, in general, the Bayes definition of "conditional probability" cannot be applied to the conditional probabilities that are considered in quantum theory

(cf. [Ac81a], [Ac84] and [Ac86]) for a more systematic exposition.

This discovery lead to a rethinking of the basic axioms of probability theory in the light of the new probabilistic ideas emerged from quantum physics.

The new elements that quantum theory bought into the probabilistic thought are essentially two:

i) the existence of incompatible events

ii) the "chameleon effect"

Classical probability is based on Boolean logic, where it is postulated that the "join" of two meaningful proposition ("and") is always a meaningful proposition.

However, if we replace the platonic notion of "proposition" by the empirical notion of "experimentally verifiable proposition", we see that the existence of incompatible events implies the existence of statements which, when considered individually, are empirically verifiable, but whose "joint event" is not. The most famous example, first pointed out by Heisenberg, is given by the two statements:

– the position of a particle at time t is in an interval I

– the momentum of a particle at time t is in an interval J

It is known that, if the intervals I and J are small enough, the joint event is not empirically verifiable because of the Heisenberg principle.

Notice that this principle concerns a single particle and no probability is involved.

A few years later H. Weyl formulated a statistical variant of the Heisenberg principle, expressing the fact that the product of the covariances of position and momentum in a given quantum state cannot be smaller than a certain quantity which is independent of the state considered.

The Heisenberg principle, in its original formulation, expresses a physical limitation, not a logical impossibility. There is no logical reason why we should not be able to measure with arbitrary precision, on the same particle and at the same time, position and momentum. In fact classical mechanics is a perfectly coherent theory from the logical point of view, but does not contemplate such an impossibility.

The development of quantum probability has brought to light new forms of incompatibility between pairs of events, which have a logic rather than physical root. This type of incompatibility is not peculiar to quantum theory.

Consider, for example, the response of an iron ball, rotating along fixed axis a to the action of a constant magnetic field, directed along a different axis a. For example one can imagine that a is the z–axis of a frame, that the rotating particle is "fired" in a direction perpendicular to the z–axis, and that one measures if it deviates on the left or on the right of the z–axis. This is a perfectly measurable event and the same is true if we replace the z–axis by another axis not parallel to z.

Let us denote A the former event and B the latter.

A little thought shows that the joint event $A \cap B$ cannot be experimentally realized because a single particle cannot be simultaneously subject to the "only action of the a–field" and to the "only action of the b–field": two different magnets cannot be put in the same space point.

Another example can be taken from medicine: suppose that two different medicines a and b are proposed as a cure of the same illness. We can separately experiment medicine a or medicine b on a patient, but again the joint event makes no sense because the same patient, at the same time, cannot be cured "only by medicine a " and " only by medicine b ".

A third example is given by the color of a chameleon: the chameleon is in a box and an experimentalists can measure either its color on a leaf (a) or its color on the wood (b). It is clear that the simultaneous joint event $a \cap b$ makes no sense because the same chameleon cannot be at the same time both "only on the leaf" and "only on the wood".

Notice the physical difference between the three examples described above and the following, more classical situation: in a box there are many balls whose color can be either green or brown. Moreover each ball is either made of glass or of wood. Clearly in this case we can make an experimental analysis of the joint statistics "color-material".

The difference between the two types of examples considered above is that, in the former case one measures the "response" to an action; in the latter one measures a property which is pre-existing to the measurement and independent of it.

Notice however that in all the above situations the results of the measurements are "pre-determined": the laws of classical electromagnetism allow (in principle) to predict exactly the deviation of the particle once the initial data (velocity, angular velocity, strength of the magnetic field, ...) are known; the laws of chemistry and biology allow a similar prediction for the medicines; the knowledge that the chameleon is a usual one and not a mutant which becomes brown on a leaf and green on a piece of wood, allows to predict deterministically its color; finally for balls the situation is even simpler.

However it is also clear that the word "predetermination" is used with a different meaning in the two contexts: in the case of balls "we measure what it was"; in the case of chameleons "we measure what it happens".

In the former case we will speak of "passive systems"; in the latter we will speak of "adaptive systems".

The difference between the two cases has rather deep consequences on the type of inductions that we can make in the two situations. Since statistical inference is a generalization of inductive inference, it is clear that these consequences will also have non trivial implications on the kind of statistical inference we can make on the two types of systems.

To illustrate the implications for statistics of the difference between active and passive systems let us consider the following variant of the above experiments: consider two boxes; in one there are pairs of balls; each pair contains 1 green ball and 1 brown ball, moreover one ball of the pair weights 10 grams and the other one 20 grams. The other box contains pairs of chameleons: exactly one, in each pair weights 10g and the other 20g, moreover, in each pair, exactly one is healthy (becomes green on a leaf, brown on a piece of wood) and the other one is a mutant (green on wood, brown on leaf). In both cases we do not know the statistics of the joint distributions color-weight.

Suppose you are interested in such a statistics at time t, but the rules of the game are such that you can do only one measurement at a time, both on balls or on chameleons (incompatibility). A reasonable strategy for balls would be the following: at time t you measure the color of one ball and the weight of the other one. Suppose you find: "brown" in ball 1 and "$10g$" in ball 2. Then you conclude that ball 1, at time t, is brown and weights $20g$ and ball 2, at time t, is green and weights $10g$.

Suppose now you want to apply the same strategy of measurement to chameleons. Can you draw the same conclusion? No of course! In fact suppose that, at time t, you measure the weight of chameleon 2 while he was in the box and you find $10g$. Suppose you measure, at time t, the color of chameleon 1 on the leaf and you find "brown". You can only be sure that "if you had measured the color of chameleon 2 on the leaf you would have found green". However nothing prevents the possibility that the chameleon in the box, i.e. at time t is brown. Since you are interested in the joint statistics color-weight at time t, you are not allowed to make, for chameleons, the same inference you made for balls.

Can we push this difference further? The answer is "yes". In my talk a simple experimental situation will be described in which, by exploiting the chameleon effect, one can reproduce exactly, by local independent binary choices of individuals who are situated far away of each other and do not communicate with each other, exactly the same empirical correlations that are experimentally obtained in the well known Einstein-Podolsky-Rosenexperiments. In the past 37 years the possibility of realizing such an experiment has been firmly denied by the entire community of physicists.

Without using the Chameleon effect, the possibility of such a reproduction is excluded by a mathematical constraint: an inequality among these correlations, discovered by Bell and violated by some quantum mechanical system. The probabilistic meaning of this inequality, first pointed out in [Ac81a] will be shortly reviewed.

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