QUANTUM DYNAMICS AND BEREZIN'S DEFORMATION QUANTIZATION

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Abstract. *Let Γ be a discrete subgroup $PSL(2,\mathbb{R})$. We describe a class of completely positive maps related to the von Neumann algebras in the Berezin's equivariant deformation quantization of the upper half plane, modulo the discrete subgroup Γ. When Γ is $PSL(2,\mathbb{Z})$ we find a non-trivial obstruction for the existence of a one parameter family of isomorphisms, having a generator that preserves a certain differential structure, between the von Neumann algebras in the deformation.*

The aim of this note is to analyze the structure of a class of completely positive maps between the von Neumann algebras in the (equivariant) Berezin's deformation quantization. It turns out that that the symbol map is generating a quantum dynamics (or more precisely) an evolution process $\Psi_{s,t}$ in which the maps have the property that they are completely positive and $\Psi_{s,t}\circ\Psi_{t,r}=\Psi_{s,r}$.

In the following definition we summarize some of the properties of the Berezin's deformation quantization.

Definition. *Let $(\mathcal{A}_s)_{s>s_0}$ be a family of semifinite von Neumann algebras with faithful, semifinite trace $\tau=\tau_{\mathcal{A}_s}$, indexed by the numbers s in an interval (s_0,∞). Let $(\Psi_{s,t})_{s\geq t>s_0}$ be a family of completely positive maps, $\Psi_{s,t}:\mathcal{A}_t\to\mathcal{A}_s$. We call $(\mathcal{A}_s,\Psi_{s,t})$ a formal deformation quantization if the following conditions hold:*

(i) *$\Psi_{s,t}$ are unital, injective with weakly dense range and trace preserving.*

(ii) *The (Markovianity) condition holds for all $s\geq t\geq r>s_0$: $\Psi_{s,t}\circ\Psi_{t,r}=\Psi_{s,r}$.*

(iii) *The functions $s\to\tau_{\mathcal{A}_s}(\Psi_{s,t}(a_1)...\Psi_{s,t}(a_n))$, for all n and all $a_1,...,a_n$ in a weakly dense subalgebra $\hat{\mathcal{A}}_t$ are differentiable at any point s.*

In the examples we consider the following condition also holds true:

(iv) *If $\Psi^*_{s,t}$ is the adjoint of $\Psi_{s,t}$ with respect to the standard scalar product on $L^2(\mathcal{A}_t,\tau_{\mathcal{A}_t})$, then $(\Psi^*_{s,t}\Psi_{s,t})_{s\geq t}$ is a commuting family of completely positive maps which converges point weakly to the identity when s approaches t.*

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To such a formal deformation quantization we are associating a cyclic cohomology 2-cocycle ([5]) which is very similar to the cocycles introduced in [7] (see also [12]). The cocycles have a positivity property that is reminiscent to the Lindblad condition ([11], [8], [15]). This is proved in the following

Proposition. *Let $(\mathcal{A}_s, \Psi_{s,t})$ be as in the preceding definition. Also we assume the (technical) condition that $\Psi_{s,t}^{-1}(\Psi_{s,t}(a)\Psi_{s,t}(b))$ is defined for a, b in a weakly dense subalgebra of $\hat{\mathcal{A}}_t$. Let η_t, θ_t be the cocycles on $\hat{\mathcal{A}}_t$ defined by*

$$\eta_t(a,b,c) = \frac{d}{ds}\{\tau_{\mathcal{A}_s}[[\Psi_{s,t}(a)\Psi_{s,t}(b) - \Psi_{s,t}(ab)]\Psi_{s,t}(c)]\}|_{s=t}, a, b, c \in \hat{\mathcal{A}}_t,$$

$$\theta_t(a,b,c) = \frac{d}{ds}\{\tau_{\mathcal{A}_s}[\Psi_{s,t}(a)\Psi_{s,t}(b)\Psi_{s,t}(c)] -$$

$$1/2\tau_{\mathcal{A}_s}[\Psi_{s,t}(ab)\Psi_{s,t}(c) + \Psi_{s,t}(bc)\Psi_{s,t}(a) + \Psi_{s,t}(ca)\Psi_{s,t}(b)]\}|_{s=t}, a, b, c \in \hat{\mathcal{A}}_t.$$

Recall that B, the coboundary operator in Connes' cyclic cohomology, is defined on a multilinear functional ϕ on $\hat{\mathcal{A}}_t$ by the formula

$$B\phi(a,b,c,d) = \phi(ab,c,d) - \phi(a,bc,d) + \phi(a,b,cd) - \phi(da,b,c), a, b, c, d \in \hat{\mathcal{A}}_t.$$

Then both $B\eta_t$ and $B\theta_t$ do vanish. Moreover θ_t is cyclic in the variables a, b, c. In addition η_t is positive in the sense that the matrix

$$(\eta_t(a_i^*, a_j, c_i^* c_j))_{i,j}$$

is positive for all $a_1, ..., a_n, c_1, ..., c_n$ in $\hat{\mathcal{A}}_t$.

Proof (Sketch) We consider the associative product $\circ_s$ defined on a dense subalgebra of $\hat{\mathcal{A}}_t$ by the formula $a \circ_s b = \Psi_{s,t}^{-1}(\Psi_{s,t}(a)\Psi_{s,t}(b))$. The properties $B\eta_t = 0$ and $B\theta_t = 0$ are obtained by taking the derivative of the associativity relation $a \circ_s (b \circ_s c) = (a \circ_s b) \circ_s c$.

The positivity condition on η_t is also a consequence of the fact that $\Psi_{s,t}$ are completely positive maps and hence the matrix $(a_i^* \circ_s a_j)_{i,j}$ is less (with respect to the order induced by the involutive product $\circ_s$ on a dense subalgebra of $\mathcal{A}_t$) that the matrix $(a_i^* \circ_t a_j)_{i,j}$.

Hence for all $a_1, ..., a_n, c_1, ..., c_n$ in $\hat{\mathcal{A}}_t$, the function

$$f(s) = \sum_{i,j} [\tau_{\mathcal{A}_t}(c_i^* \circ_s (a_i^* \circ_t a_j) \circ_s c_j) - \tau_{\mathcal{A}_t}(c_i^* \circ_s (a_i^* \circ_s a_j) \circ_s c_j)],$$

is positive with $f(t) = 0$ and hence $f'(t) \geq 0$. Consequently, if we use the formal notation $a \circ_t' b = \frac{\mathrm{d}}{\mathrm{d}s}[a \circ_s b]|_{s=t}$, which corresponds to $\eta(a,b,c) = \tau_{\mathcal{A}_t}((a \circ_t' b) \circ_t c)$, then $f'(t) \geq 0$, gives

$$\tau_{\mathcal{A}_t}(c_i^* \circ_s (a_i^* \circ_t' a_j) \circ_s c_j).$$

Remark. *One may use the positivity of η_t to deduce (as in [15]) that the product $a \circ_t' b$ defines a bimodule structure on $\mathcal{A}_t \otimes \mathcal{A}_t$. The $\mathcal{A}_t$ valued scalar product is defined by the formula*

$$< a \otimes b, a \otimes b >= b^*(a^* \circ_t' a)b.$$

We note that in the case when the algebras $\mathcal{A}_s$ are all equal and if $\Psi_{s,t}$ has the property that $X_t = \frac{\mathrm{d}}{\mathrm{d}s}\Psi_{s,t}|_{s=t}$ exists, in the strong operator topology, pointwise, then

$$\eta_t(a, b, c) = \tau\{[X(ab) - aX(b) - X(a)b]c\},$$

$$\theta_t(a, b, c) = \tau\{[(\mathrm{Im}\ X(ab) - a\ \mathrm{Im}\ X(b) - \mathrm{Im}\ X(a)b]c\},$$

for a, b, c in a dense subalgebra. Finally, one may use the Bhat-Parthasarathy dilation theorem [3] to our context:

Remark. *In the setting of the definition of a formal deformation quantization, there exists embedings $j_s : \mathcal{A}_s \to \tilde{\mathcal{A}}_s \subseteq B(\tilde{H})$ in the algebra of all bounded operators on a Hilbert space $\tilde{H}$ for all $s > s_0$. We identify the algebras $\tilde{\mathcal{A}}_s$ with the original algebras. Also there exists a decreasing (with s) net of projections p_s in $B(\tilde{H})$ such that the following formula, (for $s_0 < t \le s$),*

$$p_s a p_s b p_s = p_s(a *_s b)p_s, a, b \in \mathcal{A}_t,$$

defines, uniquely, a deformation product on (a weakly dense subalgebra of) $\mathcal{A}_t$.

We specialize now this construction for the Berezin's quantization of the upper half plane $\mathbb{H}$ (equivariant with respect a discrete lattice in $PSL(2, \mathbb{R})$) . Let Γ be a discrete subgroup of $PSL(2, \mathbb{R})$ of finite covolume.

Recall that the setting for this quantization is as follows: let H_r be the Hilbert space of analytic functions on the upper halfplane $\mathbb{H}$, square integrable with respect to the measure $\nu_r = (\mathrm{Im}\ z)^{r-2}\mathrm{d}z\mathrm{d}\overline{z} = (\mathrm{Im}\ z)^r\nu_0$, ($\nu_0$ is the $PSL(2, \mathbb{R})$ invariant measure). On the Hilbert spaces H_r there exists projective, unitary representations π_r of $PSL(2, \mathbb{R})$, (see [14]), defined by the Móebius transforms. Let, for $z \in \mathbb{H}$, e_z^r, be the point evaluation vectors in H_r; these vectors are subject to the relation $f(z) =< f, e_z^r >$ for f in H_r.

The Berezin's (contravariant symbol) is defined for A in $B(H_r)$ by the formula

$$\hat{A}(\overline{z}, \zeta) = \frac{< Ae_z^r, e_\zeta^r >}{< e_z^r, e_\zeta^r >}, z, \zeta \in \mathbb{H}.$$

Note that $\hat{A}$ is antianalytic in the first variable and analytic in the second. If A commutes with $\pi_t(\Gamma)$ then the symbol $\hat{A}$ is Γ-invariant: that is $\hat{A}(\overline{z}, \zeta) = \hat{A}(\overline{\gamma z}, \gamma\zeta)$, for all γ in Γ.

Definition. *Let $\Psi_{s,t} : B(H_t) \to B(H_s)$ be the linear mapping that is associating to a bounded linear operator A on H_t the operator on H_s having the same symbol as the original operator A. Then $\Psi_{s,t}$ takes its values into $B(H_s)$. Consequently, the Berezin's quantization, has the properties of a formal quantization, as explained in the first definition.*

Let $\mathcal{A}_t$ be the commutant $\{\pi_t(\Gamma)\}' \subseteq B(H_t)$. As the symbols are, in this case, Γ-invariant, we obtain a deformation of $\mathbb{H}/\Gamma$ (using the restrictions of the symbols to the diagonal). Moreover $\Psi_{s,t}$ maps $\mathcal{A}_t$ into $\mathcal{A}_s$. Consequently $(\mathcal{A}_s, \Psi_{s,t})$ has also the properties of a formal deformation.

Note that if Γ has finite covolume, then (by using the generalization in ([13]) of the theorem contained in [9] (see also [2], [6])) one obtains that the Murray von Neumann dimension ([9],[10]) of H_t, as a left Hilbert module over the type II_1 factor associated to the discrete group Γ, is

$$\dim{}_\Gamma H_t = (t-1)(\text{covolume } \Gamma)^{-1}.$$

Recall that if M is a type II_1 factor and e is a selfadjoint idempotent of trace t, then M_t denotes the isomorphism class of the type II_1 factor eMe. Consequently

$$\mathcal{A}_s \cong \mathcal{L}(\Gamma)_{[(s-1)(\text{covolume } \Gamma)^{-1}]},$$

if s is an integer or Γ is $PSL(2,\mathbb{Z})$.

The following result is an easy consequence of the computations in [13]. It proves that the cyclic cocycle associated to the deformation, introduced in the first definition, is non-trivial on a suitable subalgebra:

Theorem. *Let $(\mathcal{A}_s, \Psi_{s,t})$ be the Berezin's deformation quantization, when Γ is $PSL(2,\mathbb{Z})$. Then there exists a strictly positive number C_t and an antisymmetric form χ_t on a dense subset of $L^2(\mathcal{A}_t, \tau)$ such that the following formula holds for the cocycle θ_t associated with the deformation (as in the first proposition):*

$$\theta_t(a,b,c) = C_t\tau(abc) + \chi_t(ab,c) + \chi_t(bc,a) + \chi_t(ca,b),$$

for a,b,c in a dense subspace of $\mathcal{A}_t$. In particular θ_t is non-trivial.

The following remark explains the role of the cocycle θ_t as an obstruction to the existence of a family of unital isomorphisms from the algebra $\mathcal{A}_t$ onto $\mathcal{A}_s$ that preserve a certain differentiable structure.

Remark. *Let $\beta_{s,t}$ be a unital family of endomorphisms, $\beta_{s,t} : \mathcal{A}_t \to \mathcal{A}_s$. Let δ_t be the (unbounded) quadratic form, with domain contained in $L^2(\mathcal{A}_t, \tau)$, defined by the relation*

$$\delta_t(a,b) = \frac{d}{ds}\tau_{\mathcal{A}_t}\{[\Psi_{s,t}^{-1}(\beta_{s,t}(a))]b^*\},$$

the domain of δ_t consisting of all a,b for which the above differentiation makes sense. Then the imaginary part of the quadratic form δ_t, defined by $Im\ \ \delta_t(a,b) = \delta_t(a,b) - \delta_t(b,a)$ has the property that

$$\theta_t(a,b,c) = \ Im\ \ \delta_t(ab,c) + \ Im\ \ \delta_t(bc,a) + \ Im\ \ \delta_t(ca,b),$$

for all a,b,c for which which the above forms are defined. In particular, by the result in the previous theorem, there exists no such a family $\beta_{s,t}$ such that the domain of the associated quadratic form δ_t, contains the domain of χ_t.

One can also prove the following: Let X_t is be the antisymmetric unbounded operator associated with χ_t. Let 1 be the symbol of the identity operator. Then

$X_t^* 1 = (-C_t)1$. In particular if one is able to solve the differential equation $y'(s) = X_s y(s)$ for all initial values, then the associated evolution operator would induce a family of endomorphisms from $\mathcal{A}_t$ into $\mathcal{A}_s$ scaling the trace by C_t/C_s, which turns out to be $(t-1)/(s-1)$.

A possible interesting problem is the determination of an index, similar to the Power's index for semigroups of endomorphisms, for the family of endomorphisms $\beta_{s,t}$, generated by the above method, (that verifies the relation $\beta_{s,t} = \beta_{s,r} \circ \beta_{r,t}$). The formalism in ([1]) would be probably very helpful in approaching such a subject.

In the rest of this paper we analyze a larger class of completely positive maps on the algebras $\mathcal{A}_t$. These maps are similar to the Schurr multipliers on group algebras. Let F be a fundamental domain for Γ in $\mathbb{H}$. Let $d(z,w) = \frac{\operatorname{Im} z \operatorname{Im} w}{|\overline{z}-w|^2}$ for z, w in $\mathbb{H}$, $(d(z,w)$ is the hyperbolic cosine of the the hyperbolic distance between z and w in the upper half plane.

We note that the space $L^2(\mathcal{A}_t, \tau)$ is the Hilbert space $\mathcal{K}_t$ of functions (kernels) k on $\mathbb{H} \times \mathbb{H}$, antianalytic in the first variable and analytic in the second, $\Gamma-$invariant $(k(\overline{z}, \zeta) = k(\overline{\gamma z}, \gamma\zeta), \gamma \in \Gamma$. The norm of an element k in $\mathcal{K}_t$ is computed by the formula $||k||^2 = (\text{const }) \int_F \int_{\mathbb{H}} |k(z,w)|^2 (d(z,w))^t \mathrm{d}\nu_0(z) \mathrm{d}\nu_0(w)$. The constant is determined by the requirement that the norm of the identity element be equal to 1.

For complex numbers a, b, c, d let $[a, b, c, d]$ be the cross ratio $\frac{(a-b)(c-d)}{(a-c)(b-d)}$. Consider

$$e^t_{\overline{z},\zeta}(\eta_1, \eta_2) = \sum_{\gamma \in \Gamma} [\overline{\gamma z}, \gamma\zeta, \overline{\eta_1}, \eta_2]^t.$$

Then the function $e^t_{\overline{z},\zeta}$ defines an element of $\mathcal{K}_t$ for all $z, \zeta \in \mathbb{H}$ and $e^t_{\overline{z},\zeta}$ is the evaluation vector for the Hilbert space of analytic functions $\mathcal{K}_t$, that is

$$e^t_{\overline{z},\zeta} \in \mathcal{K}_t \text{ and } < k, e^t_{\overline{z},\zeta} >= k(\overline{z}, \zeta), z, \zeta \in \mathbb{H}, k \in \mathcal{K}_t.$$

The following relation (with convergence in the weak topology) gives a generators and relations presentation for the algebra $\mathcal{A}_t$:

$$(*) \qquad e^t_{\overline{z},\zeta} e^t_{\overline{z_1},\zeta_1} = \textstyle\sum_{\gamma \in \Gamma} [\overline{z}, \zeta, \overline{\gamma z_1}, \gamma\zeta_1]^t e^t_{\overline{\gamma z_1},\zeta}, \text{ for all } z, z_1, \zeta, \zeta_1 \in \mathbb{H}.$$

The following definition is analogous to the definition of the Schurr multipliers on a group algebra:

Definition. *Let Φ be a function on $\mathbb{H} \times \mathbb{H}$ that is Γ invariant ($\Phi(\gamma z, \gamma w) = \Phi(z,w)$, for all $z, w \in \mathbb{H}$ and $\gamma \in \Gamma$). We also assume that Φ is completely positive, that is for all $n \in \mathbb{N}$ and all $z_1, ..., z_n$ in $\mathbb{H}$, the matrix $[\Phi(z_i, z_j)]_{i,j}$ is positive. If $\sup_{z \in \mathbb{H}} \Phi(z,z)$ is finite then Φ defines, by pointwise multiplication, a Toeplitz operator $\mathcal{T}_\Phi$ on the Hilbert space $\mathcal{K}_t$. Moreover, if $\mathcal{T}_\Phi$ maps $\mathcal{A}_t$ into $\mathcal{A}_t$, then $\mathcal{T}_\Phi$ is a completely positive map.*

The following proposition follows from the computations in [13]

Remark. *Let Φ be the function d^a for a positive number a. Then $\mathcal{T}_{d^a}$ is equal to $\Psi^*_{t+a,t}\Psi_{t+a,t}$ and the operators are a commuting family. Moreover, there exists a positive number $f(t)$, depending linearly on t, such that $\mathcal{T}_d$ is unitary equivalent to a resolvent $(f(t)-\Delta)^{-1}$ of the invariant laplacian Δ on the fundamental domain F.*

This remark is an easy consequence of the Berezin's transform; the completely positive maps are unitary equivalent to operators that are functions of the invariant laplacian.

Let T be any trace class operator on the Hilbert space $\mathcal{K}_t= L^2(\mathcal{A}_t,\tau)$. Then the Berezin's theory implies that the trace of T is computed by the formula

$$\mathrm{Tr}\ T = \int_F\int_{\mathbb{H}} < T e^t_{\overline{\eta_1},\eta_2}, e^t_{\overline{\eta_2},\eta_1} > d(\eta_1,\eta_2)^t \mathrm{d}\nu_0(\eta_1)\mathrm{d}\nu_0(\eta_2).$$

Let Φ be a completely positive multiplier on the algebra $\mathcal{A}_t$. Then we have the following formula for the trace of $\mathcal{T}_\Phi$ as an operator on the Hilbert space $\mathcal{K}_t$:

Proposition. *Let Let Φ be a function on $\mathbb{H}\times\mathbb{H}$ that is Γ invariant (that is $\Phi(\gamma z,\gamma w)=\Phi(z,w)$, for all $z,w\in\mathbb{H}$ and $\gamma\in\Gamma$) and assume that Φ is completely positive, that is for all $n\in\mathbb{N}$ and all $z_1,...,z_n$ in $\mathbb{H}$, the matrix $[\Phi(z_i,z_j)]_{i,j}$ is positive. Assume that $\mathcal{T}_\Phi$ is a trace class operator on the Hilbert space $\mathcal{K}_t= L^2(\mathcal{A}_t,\tau)$. Then, for any k in $\mathcal{A}_t$,*

$$Tr\ (\mathcal{T}_\Phi k) = \int_F\int_{\mathbb{H}} \tau(e^t_{\overline{\eta_1},\eta_2} e^t_{\overline{\eta_2},\eta_1} k)\Phi(\eta_1,\eta_2)(d(\eta_1,\eta_2))^t d\nu_0(\eta_1)d\nu_0(\eta_2).$$

Proof. This follows from the following (elementary) expression for the trace of $\mathcal{T}_\Phi k$:

$$\mathrm{Tr}\ \ (\mathcal{T}_\Phi k) = \int_F\int_{\mathbb{H}}\sum_\gamma k(\overline{\gamma\eta_1},\eta_1)[\overline{\gamma\eta_1},\gamma\eta_2,\overline{\eta_2},\eta_1] d(\eta_1,\eta_2)^t \mathrm{d}\nu_0(\eta_1)\mathrm{d}\nu_0(\eta_2).$$

This expression follows from the formula for the reproducing kernel of the operator $\mathcal{T}_\Phi k$.

By taking $k = e^t_{\overline{z},\zeta}$ and using the formula $(*)$ we get the formula in the statement for $e^t_{\overline{z},\zeta}$. The general result then follows by linearity.

Corollary. *Let a,b be automorphic forms of order p. Let T_a, T_b be the Toeplitz operators on the Hilbert spaces H_t into H_{t+p}, with symbols a,b. Let $k=T_aT_b^*$. Then*

$$Res_{\ \epsilon=1}\ Tr\ (\mathcal{T}_{d^\epsilon} k) = \ const\ \tau_{\mathcal{A}_t}(k)\ \ Tr_\omega(\mathcal{T}_d),$$

where $Tr_{\ \omega}$ is the Macaev trace ([5]).

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