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# Switching current distributions in Josephson junctions at very low temperatures

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**Abstract** – Swept bias experiments carried out on Josephson junctions yield the distributions of the probabilities of early switching from the zero-voltage state. Kramers' theory of the thermally activated escape from a one-dimensional potential is well known to fall short of explaining such experiments when the junctions are at millikelvin temperatures. We propose a simple revision of the theory which is shown to give extremely good agreement with experimental data.



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When a Josephson junction is biased with a steady current, the phase variable behaves like the position coordinate that describes a “particle” sitting at the bottom of a well in a washboard potential [1]. An external dc bias current has the effect of tilting the washboard and causing the wells to become shallower and disappear when the bias is equal to the junction critical current. At bias currents smaller than the critical value, the “particle” remains in a well unless it is able to escape from it via some mechanism. The first escape mechanism to be recognized was classical thermal activation (TA) [2] in which the “particle” jumps over the barrier and then bounces down the washboard. In Josephson terms this effect results in a non-zero dc voltage across the junction.

The analogy of the Josephson potential with the general class of one-dimensional (single-coordinate) potentials makes the investigation of its features particularly interesting. One of the most significant developments in recent years began with the experiments of Voss and Webb [3]. A Josephson junction was repeatedly subjected to a smoothly increasing bias. The distribution of values of the current at which the junction switched to a finite-voltage state, also termed the switching current

distribution (SCD), was recorded. The beauty of the original swept bias experiment was its conceptual simplicity. In a previous paper [4] we noted that the escape peaks reported in [3] did not freeze at a given temperature but rather continued to advance towards higher bias currents as the temperature was lowered. We raised the possibility that this behavior could be evidence of classical escape dynamics.

In the present paper, using more recent experimental data by Yu *et al.* [5], we assess in greater detail the predictions of TA theory. This source of data was chosen because that experiment was fully characterised and, most particularly, because in fig. 2 of [5] data pairs are shown for the observed peak positions and widths at 19 different temperatures ranging from just below 0.8K down to 25 mK. Observations very similar to those in [5] have been reported many times in the literature, *e.g.* [6–9], which leads us to be confident that our conclusions are *generally* applicable to Josephson junctions at low temperatures. We draw the following two conclusions from the work reported here: first, that a small elevation of the sample temperature above the dilution refrigerator temperature is sufficient to explain experimental observations using a classical thermal activation model, and second, that published data do not exhibit saturation at

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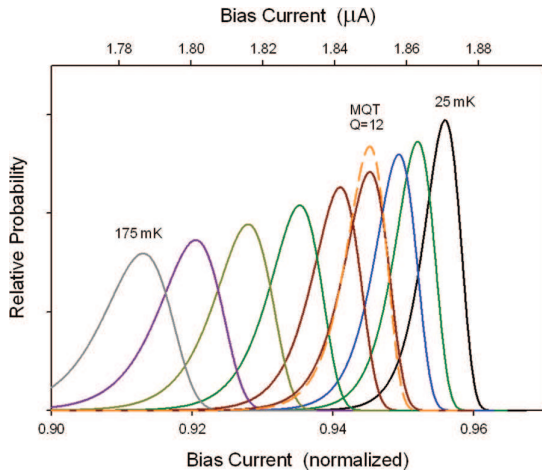


Fig. 1: (Colour on-line) Escape probability distributions (simulation) at various temperatures. The MQT distribution (dashed line) was simulated with  $Q = 12$  and coincides with the thermal activation peak at  $T = 65$  mK.

temperatures below a presumptive crossover as required by Macroscopic Quantum Tunneling (MQT) theory.

A full description of the simulation model for a swept bias experiment was presented in [4]. The key input parameters for the computer program are: the Josephson critical current (for Yu *et al.*  $1.957 \mu\text{A}$ ), the plasma frequency (for Yu *et al.*  $15.59$  GHz), the bias ramp time (for Yu *et al.*  $4.89$  ms), and the temperature.

A central role is played by the escape rate  $\Gamma(t)$  which for TA theory applied to a Josephson junction is

$$\Gamma(t) = f \exp\left(-\frac{\Delta U}{k_B T_J}\right), \quad (1)$$

where  $f$  is the plasma frequency for the well,  $\Delta U$  is the height of the potential barrier, and  $T_J$  is the junction temperature.

Using the system values given in [5], the swept bias simulation was run at various junction temperatures. The results are shown in fig. 1. These simulation peaks closely match the experimental data represented in fig. 1 of [5].

We now consider the behavior of the positions of the peaks of fig. 1 as a function of temperature. In Yu *et al.* [5] the experimental data for peak positions are shown as squares in their fig. 2. We plot their 11 lowest temperature points in fig. 2 together with our simulation results based on the classical thermal escape rate expressed by eq. (1). Note that we use a linear temperature axis in this plot.

The experimental data for peak positions fall below the TA theory results at low temperatures. This separation of theory from experiment is *smooth*, suggesting a continuous process rather than a definite transition to any different underlying escape mechanism. The TA simulation outcome (dashed curve) can be made to drop to the experimental data points (squares) by modifying the classical

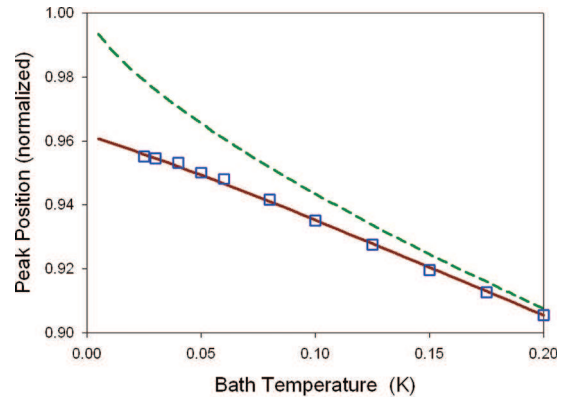


Fig. 2: (Colour on-line) Experimentally observed escape probability peak positions [5] as a function of bath temperature (blue squares). The dashed line is the classical thermal activation result, eq. (1). The solid line is the result of the modified classical theory eq. (2).

escape rate expression (1) as follows:

$$\Gamma(t) = f \exp\left(-\varepsilon \frac{\Delta U}{k_B T}\right). \quad (2)$$

It was found that

$$\varepsilon = 1.0 - \exp(-gT) \quad (3)$$

with  $g = 17.5 \text{ K}^{-1}$  yielded a revised classical result (solid curve in fig. 2) that is in outstanding agreement with the experimental data.

The exponent in eq. (2) can be regarded as containing a modified junction temperature  $T \rightarrow T/\varepsilon$ , with larger modifications occurring at low values of  $T$ . Therefore,  $T$  should be identified as the experimentally known bath temperature  $T_B$ . The key point is that  $T_B$  is provided by the cryogenic apparatus, whereas the all-important junction temperature  $T_J$  is not known. In a situation where  $T_J \neq T_B$ , using  $T_B$  in place of  $T_J$  in eq. (1) will make it *appear* that the classical theory has somehow failed. It should be noted that from this perspective, identifying, as is commonly done, the bath temperature with the junction temperature when plotting experimental data for peak position *vs.* temperature is fundamentally wrong, most especially at low bath temperatures where the junction temperature might be elevated. To illustrate this, we use the expression  $T_J = T_B/\varepsilon$  to “correct” the temperatures of experimental data points derived from fig. 2 in [5]. With these junction temperatures in place of the bath temperatures, we obtain the results shown in fig. 3. Comparing this figure with fig. 2, it is now clear that the classical TA theory gives an excellent accounting of the experiment —if the experimental bath temperatures are understood to be not the same as the temperatures of the junction.

Figure 4 covers the temperature range reported in [5] and adopts the same logarithmic temperature scale. Here

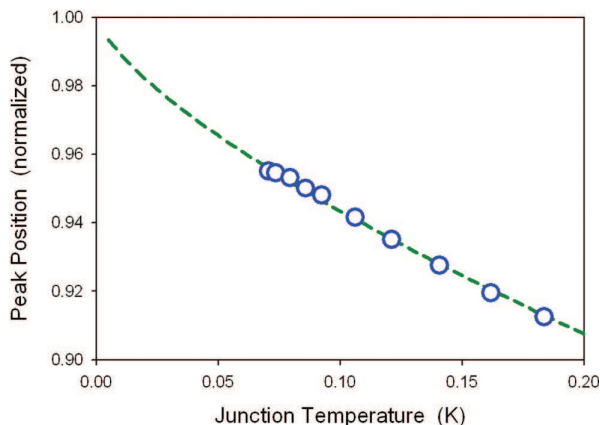


Fig. 3: (Colour on-line) Dependence of escape peak positions as a function of junction temperature from a simulation based on the escape rate, eq. (1). Experimental data from [5] (circles) have been converted from bath temperature to junction temperature.

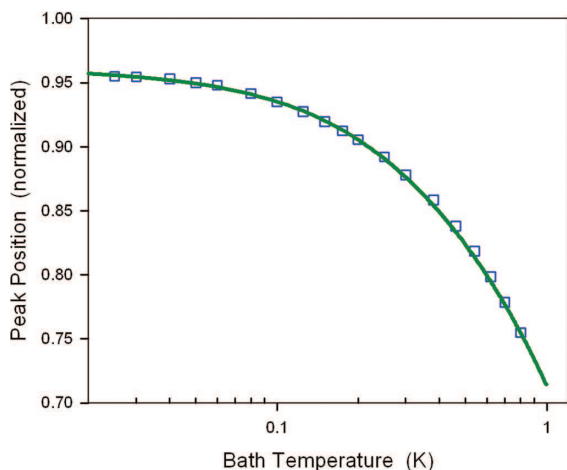


Fig. 4: (Colour on-line) Peak position *vs.* temperature. Squares are experimental data points from [5]. The solid line was produced from the modified classical escape rate, eq. (2).

the success of the modified exponent in matching theory and experiment over the full temperature range is quite apparent.

The same question regarding the apparent discrepancy between experiment and TA theory was answered differently in Voss and Webb [3]. They invoked the hypothesis that below a crossover temperature the junction phase would behave as a macroscopic quantum variable and the junction could escape to a running state by tunneling out of the well. In the MQT regime the escape rate is expected to be [10]

$$\Gamma(t) = a_q f \exp \left[ -7.2 \frac{\Delta U}{hf} \left[ 1 + \frac{0.87}{Q} \right] \right], \quad (4)$$

where

$$a_q = \left[ 120\pi \left( \frac{7.2\Delta U}{hf} \right) \right]^{\frac{1}{2}}. \quad (5)$$

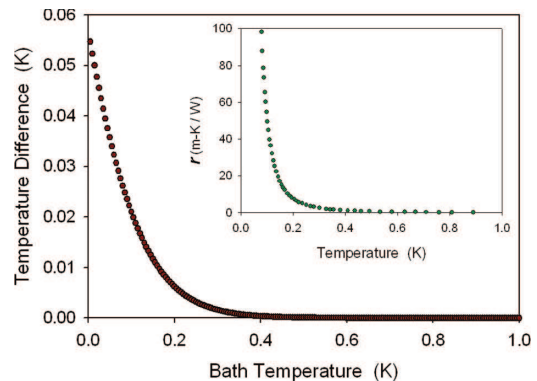


Fig. 5: (Colour on-line) Difference between junction temperature and bath temperature *vs.* bath temperature, as predicted by the empirical expression for  $\varepsilon$ . Inset: data for the thermal resistivity ( $r$ ) of niobium.

We performed a bias sweep simulation using the MQT escape rate given in the above eq. (4) setting  $Q = 12$  (see footnote <sup>1</sup>). The resulting SCD peak is included in fig. 1 where it can be seen to coincide with the SCD peak for TA at  $T = 65$  mK —essentially the crossover temperature. Note that for TA the escape rate is temperature dependent, whereas the MQT escape rate, eq. (4), does not include temperature. A direct consequence of this is that following a transition into the macroscopic quantum regime, if that occurs, the resulting escape process should become temperature independent. However, such a change is not what the experimental data exhibit—the experimental points actually follow a very smooth path with no real sign of a transition.

As noted already,  $T_B/\varepsilon$  represents the junction temperature  $T_J$ , and for  $\varepsilon < 1$ ,  $T_J > T_B$ . Thus,

$$T_J - T_B = T_B [\varepsilon^{-1} - 1] \quad (6)$$

indicates the amount by which the junction temperature is elevated above the bath temperature. This is plotted in fig. 5. Below  $T_B \approx 400$  mK the junction temperature begins to rise and, for the experimental conditions in [5], would reach a maximum of about 55 mK above the bath.

Seemingly elevated sample temperatures have been noted before [4,11,12]. It is well known that a sample subjected to input power will experience an elevation of temperature proportional to the product of that power and the thermal resistivity of the link to cooler surroundings. When the thermal path out of a junction becomes more resistive, the junction temperature will rise. This point was made by Kumano *et al.* [13] who noted that for samples of organic molecular crystals characterized by extremely low thermal conductivity, a “temperature *difference* (will exist) between the sample and the thermometer” for measurements taken in a dilution refrigerator.

<sup>1</sup>This value of  $Q$  was found to give a very good match of the MQT peak with the TA peak at 65 mK. From the junction parameters given in table 1 of [5] and  $Q = \omega_P RC$ , we find  $Q \approx 18$ , a value not significantly different from our “best-fit” estimate.



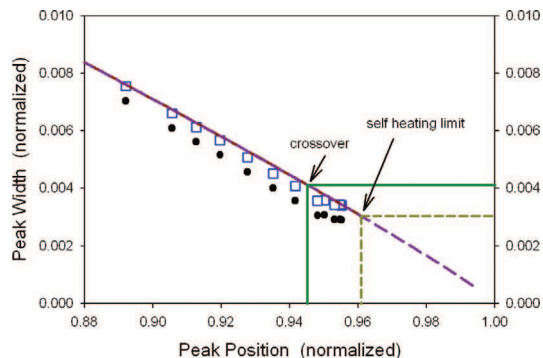


Fig. 6: (Colour on-line) Relationship between the position of the peak in the escape probability distribution and the width of that peak (numerical simulation using system parameters from [5]). Dots correspond to experimental data values from fig. 2 in [5]. Squares are experimental data points shifted upwards by 1 nA. Solid lines point to the location of the crossover peak; dashed lines point to the escape peak in the limit of zero bath temperature (see fig. 2) with position 0.961 and width 0.003.

Similar comments concerning the likelihood of elevated sample temperatures were expressed to us by Bradley [14]. The inset to fig. 5 shows thermal resistivity data for niobium [15]. There is a marked increase in thermal resistivity below a few hundred millikelvins, just where our parameter  $\varepsilon$  implies that the junction temperature will begin to grow significantly above  $T_B$ . Thus, the shape of the temperature elevation curve in fig. 5 strikingly echoes the underlying thermal property of the sample material.

Finally, we comment on the relationship of SCD peaks and their widths. Voss and Webb [3] focused on the temperature dependence of the peak width and emphasized that the width should become independent of temperature at low temperature because of the expected transition from TA to MQT. Their plot of the peak width employed a logarithmic temperature scale which, of course, visually stretches the temperature axis leading to a perceptual impression of flattening. In other words, what can seem to be evidence of width saturation might not really be the case. This initial focus on peak width as opposed to peak position has persisted in the literature. In truth there is no more information to be found in the peak widths than in the peak positions. As can be clearly seen in fig. 1 there is a clear linkage between peak positions and peak widths—the higher the peak position, the narrower the peak (a similar observation was made in [16]). This is shown explicitly in fig. 6.

Peak positions are typically much larger numbers ( $\mu\text{A}$ ) than widths (nA) and, in addition, in the crucial domain of very low temperatures the peaks become especially narrow and thus subject to increased measurement error. So the positions are more accurately known. If the width freezes, then the position must also freeze. Therefore, the observed temperature dependence of the peak positions is preferable in assessing evidence or lack of it for a crossover from TA to MQT.

Over the years, MQT has gone from being a possibility to an apparently confirmed theory, and experiments on Josephson junctions at millikelvin temperatures are routinely discussed in the language of quantum mechanics. The wells in the washboard potential are expected to possess quantized levels, and microwaves are presumed to stimulate level transitions [4,17]; the classical thermal model is no longer considered in the interpretation of experimental data. But we have shown here that published data at millikelvin temperatures do agree with classical thermal activation theory while the same experimental data are at odds with the MQT hypothesis. As noted already, experimentally observed SCD peaks persist beyond the crossover point shown in fig. 6.

Our analysis indicates that classical escape models convincingly explain observed switching current distributions in Josephson junctions at millikelvin temperatures. We believe that this close analysis of the experimental data reported in [5] would apply equally to other published results for swept bias experiments in the absence of microwaves. Voss and Webb [3] asserted that evidence for MQT should appear in SCD data from swept bias experiments. The point is not that such evidence might be present, but that it *must* be present. However, the expected unambiguous transition to macroscopic quantum behavior is not, actually, evidenced in experimental data.

It is worth recalling that the original analysis by Affleck [18] relied on the assumption that macroscopic quantum tunneling effects in a one-dimensional potential can be observed only if the system does not respond to thermal effects. However, even if the Josephson excitation energies, namely the height of the washboard potential, are above the thermal  $k_B T$  level, the current and phase variables fluctuate according to classical distributions. Further, from a previously published analysis [4] it is known that the observations of inferred quantized energy levels inside the wells of the washboard potential at very low temperatures can be interpreted alternatively as nonlinear resonances. Thus, neither conditions outlined by Affleck [18] for a reliable observation of quantum effects (existence of quantized energy levels and non-thermal statistical distributions) are convincingly present in systems of Josephson junctions. But, as we have shown here, there is a possibility that the observations are dominated by enhanced thermal fluctuations caused by overheating. We believe that an important issue for future experiments will be the determination of an effective temperature of the junctions because an estimate of this parameter could lead to a possible distinction between classical and macroscopic quantum tunnelling regimes.

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