Strategies to Design and Deploy Mode-S Multilateration Systems

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Abstract— **In this paper, we study and develop some strategies to design and deploy Mode-S multilateration systems. These strategies are based on metaheuristic optimization techniques, like Genetic Algorithm (GA) and are intended to obtain useful parameters for an optimal system configuration that provides acceptable performance levels. Furthermore, these strategies are able to evaluate and improve previous system designs. Parameters such as the number of stations, the system geometry, the kind of measurements to be used and system accuracy are obtained taking into account requirements such as the line of sight, the probability of detection and the accuracy levels.**

Keywords- multilateration; air traffic control; optimization; metaheuristic methods.

I. INTRODUCTION

Multilateration Systems (MLAT Systems) are a powerful option for the surveillance function of air traffic control. These systems are intended to inform air traffic controllers of the location and identification of aircraft (taxiing, taking off / landing, approach or enroute) or vehicles equipped with an operational SSR transponder [1]. To perform these functions, a number of ground stations (at least three for 2D or four for 3D), with capabilities to measure some characteristics of the Mode-S signals, emitted by the transponders (e.g. Time of Arrival -TOA-, Round Trip Delay -RTD- or Angle of Arrival –AOA-), are placed in some strategic locations around the airport or the area to be covered and connected with a Central Processing Subsystem (CPS).

The accuracy of position estimation in MLAT systems basically depends on the stations positions [2-5]. To design and deploy these systems, one should consider multiple factors such as the Line of Sight (LoS) of each station, the probability of detection, the accuracy, the redundancy, etc., and they deploy all the stations, to obtain the maximum possible system coverage, respecting all the regulatory standards (e.g. those described in [1]) and the many constraints imposed by the particular site. In many cases, choosing the number of stations and their locations to meet all the requirements is not an obvious task and the system designer has to do several designs, by trial and error, before obtaining a satisfactory spatial distribution of the stations.

A first application of the metaheuristic optimization techniques, to design multilateration systems, was presented in ESAVS 2010. That work [6] proposes the use of Genetic Algorithms to obtain an optimal distribution (system geometry) of a given number of MLAT ground stations only taking into account the line of sight and the Dilution Of Precision (DOP). In [6] only Time Difference of Arrival (TDOA) measurements have been considered. However, there are other relevant parameters that should be taken into account in order to obtain a more realistic design. Another important aspect is that the DOP only reflects the errors due to the spatial distribution of the stations, regardless of other important sources of errors (e.g. errors due to propagation effects, which are site-dependent, instrumental errors due to time stamp, etc.).

This paper presents an evolution of the previous work [6] with the introduction of more relevant parameters and a more rigorous formulation to evaluate the system accuracy (the Cramér-Rao Lower Bound -CRLB- analysis described in [2]). The possible implementation of the system with other kind of measurements, like RTD or AOA, is also evaluated. Moreover, the strategies developed herein are able to evaluate, validate and improve previous systems designs.

II. GENERAL PROCEDURE DESCRIPTION

The strategies developed in this work are based on the design of a new standard MLAT system (e.g., with only Time Difference of Arrival -TDOA- measurements) or of its improved version (e.g., with the combination of TDOA/RTD or TDOA/AOA). In this work, the system design is obtained by calculating the minimum number of stations and their locations (sites coordinates), that maximize the line of sight coverage and system accuracy. These calculations are performed under some regulatory constraints [1] or by those that are intrinsic to the airport layout, e.g. there are forbidden areas (clearances) or the available sites are restricted to some specific areas. In all cases these constraints can be modified to satisfy some particularities of the design.

The procedure proposed here is also useful to analyze if any previous design is the optimum solution for a given resources or whether it could be improved by some feasible but not obvious position changes of the stations.

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Unlike the previous work described in [6], where the search space (the set of available station sites) is composed by the entire airport area (i.e., a relative continuous space), in this work, due to real constraints like power supply, sites availability, etc., we have limited that search space only to a set of P sites. The latter allows obtaining more realistic designs. The complexity of this problem, for a number of N_s stations (with $N_s<P$) can be evaluated by,

$$Combinations = \frac{P!}{(P-N_s)!N_s!} \quad (1)$$

Equation (1) provides the number of possible combinations given the size of the discrete search space P and the number of stations to be deployed N_s. The procedure used in this work is based on that one proposed in [6] but, here several aspects for each step have been modified and added. The updated procedure is shown in Fig. 1. This procedure is composed of three steps, namely, Initialization, System Design Evaluation and Genetic Algorithms. In the following the updated and new aspects are described.

In the first step (Initialization) all the problem characteristics are defined. In the scenario definition the *P*-set of possible sites, to locate the stations, is selected and some areas of interest (areas to calculate the system parameters - basically LoS and theoretical accuracy-) are defined. Then, the initial stations sites (normally by a random selection) and all the variables are initialized. The variables can be classified as requirements or restrictions. The requirements are the number of stations (or a range of minimum and maximum number), the horizontal accuracy and the System Probability of Detection (SPoD) [1]. All of these are input data to the problem. On the other hand, the restrictions are the *LoS redundancy*, which is the minimum number of stations that must cover a point, in the coverage area, in order to satisfy the requirement of *SPoD* and the minimum spatial separation $\Delta_{i,j}$ between the *i*th and *j*th station. In this work, we calculate the restriction of *LoS redundancy* based on the manufacturer data about the *PoD* of each station. The *SPoD*, for a given point j, can be calculated as follows,

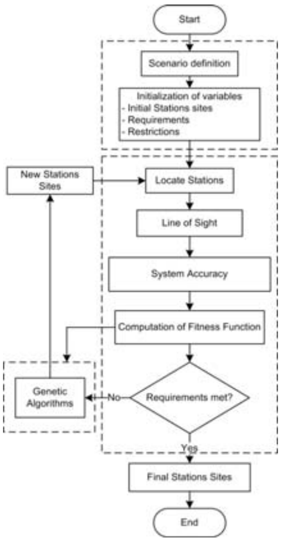

Figure 1. General design procedure.

$$SPoD_{N_{sj}} = \sum_{k=0}^{N_{sj}-4} \left(\frac{N_{sj}!}{(N_{sj}-k)!k!}\right) PoD^{N_{sj}-k}(1-PoD)^k \quad (2)$$

where *PoD* is the probability of detection of one station and it should be provided by the manufacturer and, N_{sj} is the number of stations that cover the *j*th point. In (2) it is assumed that at least four stations are needed to calculate the position. By (2) it can be estimated the minimum number of stations that make $SPoD_{N_{sj}}$ equal or greater than the corresponding requirement for the *SPoD*. This minimum value is taken as the *LoS redundancy* restriction. Moreover, this value also depends on the performance of the location algorithm used and in any case it can be modified (normally increased). However, in the remaining of this work, we assume that the *LoS redundancy* calculated by the evaluation of (2) also satisfies the location algorithms performance.

In the second step (System Design Evaluation), the quality of the partial design is evaluated. For this, the line of sight and the system accuracy are calculated and these values are introduced to a fitness function which assigns a suitable score and thus quantifies the system quality, regarding to the requirements and restrictions as defined in the first step.

The line of sight calculation is performed only in those points within the areas of interest and the system accuracy is obtained by the CRLB analysis [2] only in those points that satisfy the requirement of *LoS redundancy*. In this work, the CRLB formulation takes into account also the propagation effects, the instrumental errors, synchronization errors and the analog-to-digital converter sample period and resolution [2]. Unlike to the work presented in [6] which calculates the DOP for arrays of stations, here the CRLB for each point is calculated with all the stations with LoS for that point.

The quality of system design is evaluated and quantified by a fitness function (cost function) that takes into account the set of design requirements, i.e. the technical and economic aspects. The technical aspects are related with satisfying the requirements and restrictions and the economic aspects are related with the number of stations used. This last aspect is useful to those simulations which seek to optimize the number of stations. The fitness function is particular to each problem but, in a general sense the function proposed in this work takes the following form,

$$f = 1 - \sum_{i=1}^{cond} \delta w_i c_i\,; \quad \delta = \begin{cases} 1, & c_i\text{: }requirement \\ -1, & c_i\text{: }restriction \end{cases} \quad (3)$$

where *cond* is the total number of requirements and restrictions, c_i is the cost of the *i*th requirement or restriction and w_i is a weight factor that controls the importance of c_i on the design. The corresponding values of w_i and the functions to obtain c_i, for each application, are shown in the next section.

Finally, in the third step (Genetic Algorithm -GA-), a genetic algorithm is used to iterate and to modify the partial solution which will be evaluated by the iterative procedure

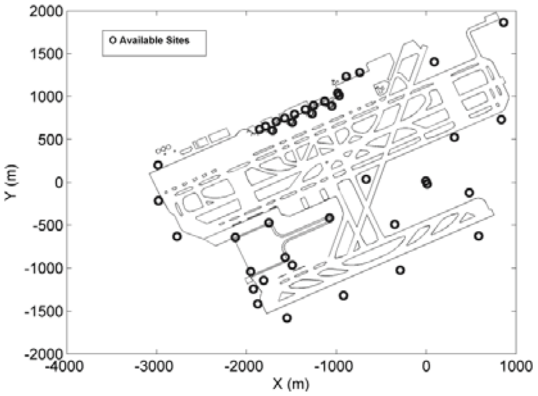

Figure 2. Barcelona airport layout.

described in Fig. 1. The genetic algorithm used in this work is basically the same used in [6]. Therefore, it is not the aim of this work to describe that algorithm. The only difference is that, due to the discretization of the search space to P possible options, here, an individual consists of an N_s-array of integer numbers, where the value of the ith array position represents the index of the selected site for the ith station. Instead, in [6] each individual is composed by the set of (x,y) coordinates of the stations. Moreover, it is worth to say that the information contained in a specific individual position can change and depends on the parameters to be optimized in the design. This particularity is commented in the next section.

III. SIMULATION AND RESULTS

To validate the strategies proposed in this work three different simulations have been carried out over the layout of Barcelona (Spain) Airport. The common objective for all the simulations is to obtain a MLAT system which cover the three runways, the taxiways and the apron centrelines, given a set of requirements and restrictions. The first simulation consists in the design of a MLAT system with a fixed number of Time Difference of Arrival (TDOA) stations. The second one consists in the design of a MLAT system with a variable number of stations. In this simulation, the objective is to find a design that satisfies all the requirements and restrictions by using the possible minimum number of TDOA stations. The last simulation consists in the design of a MLAT system with a fixed number of TDOA and AOA stations. Fig. 2 shows the Barcelona airport layout and the P-set of available sites for the simulations. For these simulations P=41.

For all the simulations, the antenna station height (mast length) has been assumed to be equal to 2 m and the calculations for LoS and CRLB are performed for a spatial grid of 5m × 5m. This spatial grid is also in concordance with the Digital Terrain Model (DTM) used to calculate the LoS. The Genetic Algorithms (GA) parameters for all the simulations are those described in [6].

A. MLAT System with a Fixed Number of TDOA Stations

The first scenario shows the first and the standard strategy proposed herein. It consists in the design of a MLAT system for a given set of requirements and restrictions. The requirements for this particular simulation are based on those described in [1], which are basically: Horizontal accuracy must be within 3.75 m and the System Probability of Detection must be better than 99.9%. The number of stations to use in this design is twelve and they measure only the TDOA parameter. The restriction of *LoS redundancy*, using a station probability of detection of *PoD*=97%, provided by a quick evaluation of (2) is 7 and the minimum spatial separation is Δ_{min}= 400 m.

For this scenario, an individual is an array of 12×1 size, where the ith position represents the index of the possible position for the ith station and it can be written as $\boldsymbol{x} = [p_l, \dots, p_m]^T$, where p_l and p_m are elements of the search space, i.e., the P-set of available sites shown in Fig. 2. The fitness function for this scenario takes the following form,

$$f(\boldsymbol{x}_t) = 1 - \left(w_1 f_{TC}(\boldsymbol{x}_t) - w_2 f_{RoS}(\boldsymbol{x}_t)\right) \quad (4)$$

where f_{TC} is a function which quantifies the requirement of total coverage for a partial solution $\boldsymbol{x}_t$ at time t, i.e., the percentage of points that are covered for more than *LoS redundancy* stations within a horizontal accuracy better than the corresponding value stipulated in the requirements and, f_{RoS} is a function which quantifies the restriction of minimum spatial separation between two stations for a partial solution $\boldsymbol{x}_t$ at time t. These two functions can be calculated as follows,

$$f_{TC}(\boldsymbol{x}_t) = \frac{Points\ with\ total\ coverage}{Total\ points\ evaluated} \quad (5)$$

and

$$f_{RoS}(\boldsymbol{x}_t) = \frac{Total\ of\ \Delta_{i,j}\ with\ \Delta_{i,j}<\Delta_{min}}{Total\ of\ \Delta_{i,j}} \quad (6)$$

Finally, the value of the weight factors depends on the importance given to each requirement or restriction on the design; they can be chosen by the designer. Here, we have used w_1=0.95 and w_2=0.05. The only condition that they must satisfy is that the sum of these must be equal to 1. The function in (6) penalizes those solutions with stations close to each other a distance smaller than Δ_{min}. However, there exists the possibility to obtain solutions with two (or more than two) stations in the same site. These particular situations are penalized directly in (4) instead in (6). In this way the final expression for the fitness function takes the following form,

$$f(\boldsymbol{x}_t) = \begin{cases} 1 - F_{R_1}(\boldsymbol{x}_t), & if\ all\ \Delta_{i,j} > 0 \\ 1, & if\ at\ least\ one\ \Delta_{i,j} = 0 \end{cases} \quad (7)$$

where $F_{R_1}(\boldsymbol{x}_t) = w_1 f_{TC}(\boldsymbol{x}_t) - w_2 f_{RoS}(\boldsymbol{x}_t)$.

Fig. 3 shows the horizontal accuracy for this scenario and how the interested airport areas are covered with the assumed requirements. From the theory [2], [4-5] it is well known that a correct system geometry, to obtain high accuracy levels, is to set the stations in a polygon enclosing the interest area. In Fig. 3 it can be observed that the proposed procedure provides a solution that is in the line of this theoretical aspect. Finally, Fig. 4 shows the procedure convergence. In this scenario, the

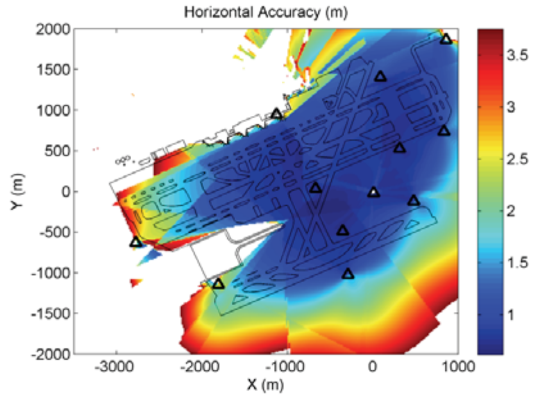

Figure 3. Horizontal accuracy for the design with a fixed number of TDOA stations.

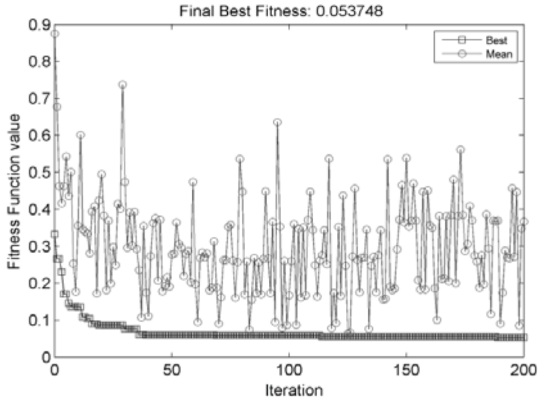

Figure 4. GA convergence for the design with a fixed number of TDOA stations.

number of possible combinations, provided by (1), is 7.8987×10^9 and a relative good solution is obtained within 50 iterations, which means only 500 problem evaluations. However, it is advisable to expend more iterations (up to 200) because the random component of the GA allows to the procedure the exploration of new values in the search space. In any case the total number of problem evaluations is much smaller than that value provided by (1).

B. MLAT System with a Variable Number of Stations

The second scenario consists in the design of a MLAT system with a variable number of TDOA stations. In this kind of scenario, the objective is not only to calculate the stations sites but it is also to calculate a relative minimum number of stations that satisfy all of the assumed requirements and restrictions. All requirements and restrictions for this problem are those described for the first problem. Moreover, for this problem it is necessary to stipulate a range for the number of stations. For this work, we have used a range of $R_{N_s} = [7,15]$.

For this scenario, an individual is an array of variable length, where the first position sets the length of this. It can be written as $\boldsymbol{x} = [N_s^t, p_l, \dots, p_m]^T$, where N_s^t is the number of stations calculated at time *t*. The fitness function for this scenario takes the following form,

$$f(\boldsymbol{x}_t) = \begin{cases} 1 - F_{R_2}(\boldsymbol{x}_t), & if\ all\ \Delta_{i,j} > 0 \\ 1, & if\ at\ least\ one\ \Delta_{i,j} = 0 \end{cases} \qquad (8)$$

where $F_{R_2}(\boldsymbol{x}_t) = w_1 f_{TC}(\boldsymbol{x}_t) - w_2 f_{RoS}(\boldsymbol{x}_t) - w_3 f_{RoNS}(\boldsymbol{x}_t)$ and f_{RoNS} is a function that quantifies the importance given to the requirement of number of stations. This function is expressed as follows,

$$f_{RoNS}(\boldsymbol{x}_t) = \frac{x_t(1) - \min(R_{N_S})}{\max(R_{N_S}) - \min(R_{N_S})} \qquad (9)$$

Finally, the weight factors values used for this problem are w_1=0.85, w_2=0.05 and w_3=0.1.

Fig. 5 shows the results for the horizontal accuracy. Also in this scenario, all the areas of interest are covered satisfying all requirements and restrictions. The important aspect in this scenario is that the minimum number of stations calculated is 11, it is, one less station than in the first scenario. This kind of simulation is useful to know an approximate minimum number of stations that meets the requirements and restrictions. However, due to the random component of the GA it is advisable to run the procedure, for this scenario, once or twice more, just to validate the calculated minimum number. Finally, Fig. 6 shows the procedure convergence for this scenario, for this scenario a good solution is found after 150 iterations. It can be understood because the complexity of this problem (number of possible combinations) is much greater than that of the first scenario.

C. MLAT System with a Fixed Number of TDOA/AOA stations

This scenario consists in the design of an improved MLAT system with a fixed number of TDOA/AOA stations. Normally, the AOA measurement capabilities are added to improve the horizontal accuracy in surface movement applications [2]. For this scenario the requirements and restrictions are those described for the first problem and the AOA measurements capabilities are added only to the station number 1 (the AOA measurements error is assumed to be 10^{-3} rad).

For this scenario, an individual is represented as in the first scenario, i.e., as an array of 12×1 size $\boldsymbol{x} = [p_l, \dots, p_m]^T$. The difference lies in that, for this scenario, the pertaining LoS coverage of the station number 1 is relatively more important than those of the remaining stations. This particular aspect is introduced in the fitness function as follows,

$$f(\boldsymbol{x}_t) = \begin{cases} 1 - F_{R_3}(\boldsymbol{x}_t), & if\ all\ \Delta_{i,j} > 0 \\ 1, & if\ at\ least\ one\ \Delta_{i,j} = 0 \end{cases} \qquad (10)$$

where $F_{R_3}(\boldsymbol{x}_t) = w_1 f_{TC}(\boldsymbol{x}_t) + w_2 f_{LoS}(\boldsymbol{x}_t(1)) - w_3 f_{RoS}(\boldsymbol{x}_t)$ and f_{LoS} is a function that quantifies the relative LoS coverage of the station number 1 and it can be calculated as follows,

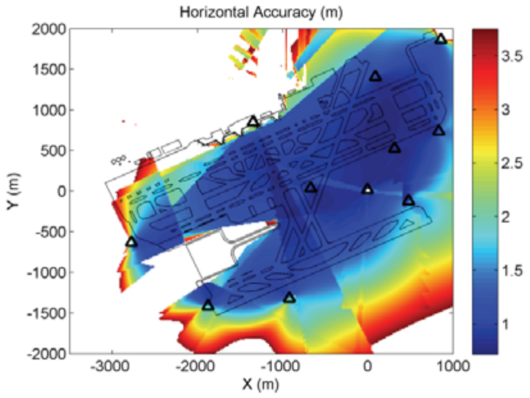

Figure 5. Horizontal accuracy for the design with a variable number of TDOA stations.

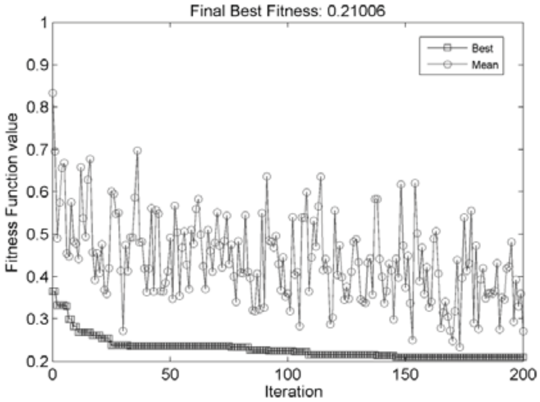

Figure 6. GA convergence for the design with a variable number of TDOA stations.

$$f_{LoS} = \frac{Number\ of\ points\ covered\ by\ x_t(1)}{Total\ points\ evaluated} \tag{11}$$

Finally, the weight factors values used for this problem are w_1=0.9, w_2=0.05 and w_3=0.05.

Fig. 7 shows the horizontal accuracy for this scenario. The complexity of this problem is basically of the same order than that of the first one but, here the CRLB calculation has been carried out by taking into account the accuracy improvement provided by the TDOA/AOA station [2]. The final site for this station is shown in Fig. 7 as the diamond. Also for this kind of scenario it is advisable to run the procedure once or twice more. Similarly to the first problem, here a good solution is found after 50 iterations (see Fig. 8).

IV. CONCLUSION

In this work, a set of practical and useful strategies to design and deploy Mode-S Multilateration systems has been presented. These strategies are based on the use of genetic algorithms along with the well-known CRLB analysis. A general procedure to use these strategies is also proposed and it is useful to design new MLAT systems but also to validate whether a previous system design could be the optimum solution regarding to a set of available resources.

Three kinds of scenarios have been presented. The first one is able to design new MLAT systems with a fixed number of TDOA stations but also to validate whether a final design (clearly before the implementation) can be improved by feasible but not obvious sites changes. The second one provides a strategy to obtain a minimum number of stations which satisfy all the stipulated requirements and restrictions. The third scenario is proposed to design improved MLAT systems, i.e., by using other type of measurements like AOA or RTD. For this third scenario, an example with a MLAT system using TDOA/AOA stations has been presented but, the use with other measurements combinations is straightforward. Finally, it is worth to say that also these strategies can be used together in order to obtain more reliable results, e.g., firstly the second scenario can be used to obtain a possible minimum number of stations that meets all the requirements and restrictions and then, by means of the first scenario, obtain the optimum sites or just to validate that set obtained with the second scenario.

The use of new requirements or restrictions is also possible

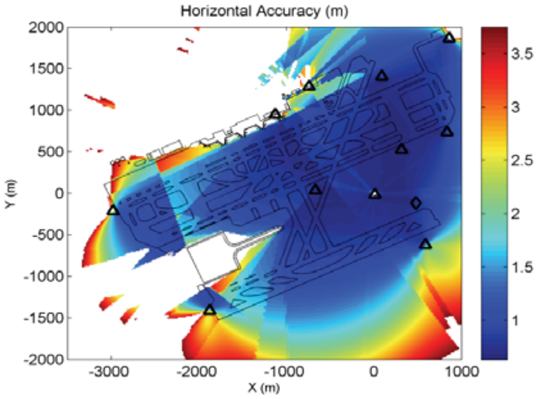

Figure 7. Horiztonal accuracy for the design with a fixed number of TDOA/AOA stations.

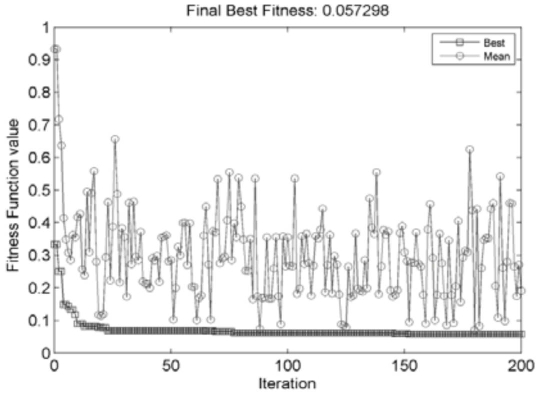

Figure 8. GA convergence for the design with a fixed number of TDOA/AOA stations.

only by modifying the corresponding cost function and their weight factors.

REFERENCES

[1] "Minimum Operational Performance Specification for Mode S Multilateration Systems for Use in Advanced Surface Movement Guidance and Control Systems (A-SMGCS)," EUROCAE, 2003.

[2] G. Galati, M. Leonardi, and M. Tosti, "Multilateration (Local and Wide area) as a distributed sensor system: Lower bounds of accuracy," in *European Radar Conference, EuRAD*, Amsterdam, 30-31 Oct. 2008, pp. 196-199.

[3] H. B. Lee, "Accuracy Limitations of Hyperbolic Multilateration System," *IEEE Transactions on Aerospace and Electronic System,* vol. AES-11, January 1975.

[4] N. Levanon, "Lowest GDOP in 2-D scenarios," *IEE Proc. Radar, Sonar Navig.,* vol. 147, pp. 149-155, June 2000.

[5] D. J. Torrieri, "Statistical Theory of Passive Location Systems," *IEEE Transactions on Aerospace and Electronic System,* vol. AES-20, pp. 183-198, March 1984.

[6] I. A. Mantilla-G, R. F. Ruiz, J. V. Balbastre-T, and E. d. l. Reyes, "Application of Metaheuristic Optimization Techniques to Multilateration System Deployment," in *Enhanced Solutions for Aircraft and Vehicle Surveillance Applications, ESAVS 2010, German Institute of Navigation (DGON)*, Berlin, Germany, 16-17 March 2010, p. Session 2B/3.