

# **$K$ and $D$ oscillations in the Standard Model and its extensions from $N_f = 2 + 1 + 1$ Twisted Mass LQCD**

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We present the first  $N_f = 2 + 1 + 1$  results for the matrix elements of the operators describing neutral  $K$  and  $D$  mixing in the Standard Model and its extensions. The combination of maximally twisted sea quarks and Osterwalder-Seiler valence quarks ensures  $\mathcal{O}(a)$ -improvement and continuum like renormalization pattern. We have used the  $N_f = 2 + 1 + 1$  dynamical quark gauge configurations generated by ETMC. Simulations include three lattice spacings in the interval  $[0.06 : 0.09]$  fm and pseudoscalar meson masses in the range  $[230 : 500]$  MeV. Our results are extrapolated to the continuum limit and to the physical quark masses. The calculation of the renormalization constants has been performed non-perturbatively in the RI-MOM scheme.

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## 1. Introduction

We provide the first  $N_f = 2 + 1 + 1$  accurate lattice determination of the  $\Delta S = 2$  and  $\Delta C = 2$  bag parameters relevant for physics in the SM and beyond. In [1, 2] recent  $N_f = 2 + 1$  results are presented. Other results reported in this conference can be found in [3].

$\Delta F = 2$  processes provide some of the most stringent constraints on New Physics generalizations of the Standard Model. The most general  $\Delta F = 2$  effective Hamiltonian of dimension-six operators contributing to  $P^0 - \bar{P}^0$  meson mixing is

$$H_{\text{eff}}^{\Delta F=2} = \sum_{i=1}^5 C_i(\mu) Q_i, \quad (1.1)$$

where  $C_i$  are the Wilson coefficients which encode the short distance contributions and  $\mu$  is the renormalization scale. The operators  $Q_i$  involving light ( $\ell$ ) and strange or charm ( $h$ ) quarks read, in the so-called SUSY basis,

$$\begin{aligned} Q_1 &= [\bar{h}^a \gamma_\mu (1 - \gamma_5) \ell^a] [\bar{h}^b \gamma_\mu (1 - \gamma_5) \ell^b] \\ Q_2 &= [\bar{h}^a (1 - \gamma_5) \ell^a] [\bar{h}^b (1 - \gamma_5) \ell^b] & Q_3 &= [\bar{h}^a (1 - \gamma_5) \ell^b] [\bar{h}^b (1 - \gamma_5) \ell^a] \\ Q_4 &= [\bar{h}^a (1 - \gamma_5) \ell^a] [\bar{h}^b (1 + \gamma_5) \ell^b] & Q_5 &= [\bar{h}^a (1 - \gamma_5) \ell^b] [\bar{h}^b (1 + \gamma_5) \ell^a]. \end{aligned} \quad (1.2)$$

The long-distance contributions are described by the matrix elements of the renormalized four-fermion operators. The renormalized bag parameters,  $B_i$  ( $i = 1, \dots, 5$ ), provide the value of four-fermion matrix elements in units of the deviation from their vacuum insertion approximation. They are defined as

$$\begin{aligned} \langle \bar{P}^0 | Q_1(\mu) | P^0 \rangle &= C_1 B_1(\mu) m_P^2 f_P^2, \\ \langle \bar{P}^0 | Q_i(\mu) | P^0 \rangle &= C_i B_i(\mu) m_P^2 f_P^2 \frac{m_P^2}{(m_h(\mu) + m_\ell(\mu))^2}, \end{aligned} \quad (1.3)$$

where  $C_i = 8/3, -5/3, 1/3, 2, 2/3, i = 1, \dots, 5$ .  $|P^0\rangle$  is the pseudoscalar,  $K$  or  $D$ , state,  $m_P$  and  $f_P$  are the pseudoscalar mass and decay constant and  $m_h$  and  $m_\ell$  are the renormalized quark masses.

## 2. Lattice setup

Simulations have been performed at three values of the lattice spacing using the  $N_f = 2 + 1 + 1$  dynamical quark configurations produced by ETMC [4]. In the gauge sector, the Iwasaki action has been used while the dynamical sea quarks have been regularized employing the Twisted Mass action [5] at maximal twist which ensures  $\mathcal{O}(a)$  improvement [6, 7]. The fermionic action for the light doublet reads

$$S_\ell = \sum_x \bar{\psi}_\ell(x) \left\{ \frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla_\mu^*) - i\gamma_5 \tau^3 \left[ M_{\text{cr}} - \frac{a}{2} \sum_\mu \nabla_\mu^* \nabla_\mu \right] + \mu_{\text{sea}} \right\} \psi_\ell(x) \quad (2.1)$$

where we follow the notation of [8]. In the heavy sector the sea quark action is

$$S_h = \sum_x \bar{\psi}_h(x) \left\{ \frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla_\mu^*) - i\gamma_5 \tau^1 \left[ m_0 - \frac{a}{2} \sum_\mu \nabla_\mu^* \nabla_\mu \right] + \mu_\sigma + \mu_\delta \tau_3 \right\} \psi_h(x). \quad (2.2)$$

Continuum-like renormalization pattern for the four-fermion operators and  $\mathcal{O}(a)$ -improvement are achieved using a mixed action setup. We introduce Osterwalder-Seiler [9] valence quarks allowing for a replica of the heavy ( $h, h'$ ) and the light ( $\ell, \ell'$ ) quarks [10]. The valence quark action reads

$$S^{\text{OS}} = \sum_x \sum_{f=\ell, \ell', h, h'} \bar{q}_f \left\{ \frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla_\mu^*) - i\gamma_5 r_f \left[ M_{\text{cr}} - \frac{a}{2} \sum_\mu \nabla_\mu^* \nabla_\mu \right] + \mu_f \right\} q_f(x), \quad (2.3)$$

where the Wilson parameters are conveniently chosen such that  $r_h = r_\ell = r_{h'} = -r_{\ell'}$  [10].

In table 1 we give the details of the simulation and the values of the sea and the valence quark masses at each lattice spacing. The smallest sea quark mass corresponds to a pion of about 230 MeV and is attained at  $\beta = 2.10$ . We simulate three heavy valence quark masses  $\mu_{s''}$  around the physical strange one and three  $\mu_{c''}$  around the physical charm mass to allow for a smooth interpolation to the physical strange and charm quark masses. For the extrapolation/interpolation to the physical quark masses we use the preliminary ETMC values [11].

For the inversions in the valence sector we used the stochastic method with propagator sources located at random timeslices [12, 13]. Gaussian smeared quark fields [14] are implemented in the case of  $D$  mesons to improve the determination of the ground state contribution with respect to the case of simple local interpolating fields. The value of the smearing parameters are  $k_G = 4$  and  $N_G = 30$ . In addition, we apply APE-smearing to the gauge links in the interpolating fields [15] with parameters  $\alpha_{APE} = 0.5$  and  $N_{APE} = 20$ .

$\beta$	$L^3 \times T$	$a\mu_\ell = a\mu_{\text{sea}}$	$a\mu_{s''}$	$a\mu_{c''}$
1.90 ( $a^{-1} \sim 2.19$ GeV) $\mu_\sigma = 0.15$ $\mu_\delta = 0.19$	$24^3 \times 48$	0.0040	0.0145 0.0185 0.0225	0.21256 0.25 0.29404
		0.0060		
		0.0080		
		0.0100		
	$32^3 \times 64$	0.0030	0.0145 0.0185 0.0225	0.21256 0.25 0.29404
		0.0040		
		0.0050		
1.95 ( $a^{-1} \sim 2.50$ GeV) $\mu_\sigma = 0.135$ $\mu_\delta = 0.17$	$24^3 \times 48$	0.0085	0.0141 0.0180 0.0219	0.18705 0.22 0.25875
		0.0025	0.0141 0.0180 0.0219	0.18705 0.22 0.25875
		0.0035		
		0.0055		
	$32^3 \times 64$	0.0025	0.0141 0.0180 0.0219	0.18705 0.22 0.25875
		0.0035		
		0.0055		
		0.0075		
2.10 ( $a^{-1} \sim 3.23$ GeV) $\mu_\sigma = 0.12$ $\mu_\delta = 0.1385$	$48^3 \times 96$	0.0015	0.0118 0.0151 0.0184	0.14454 0.17 0.19995
		0.0020		
		0.0030		

Table 1: Simulation details

The computation of the renormalization constants (RCs) for the relevant two- and four-fermion operators has been performed adopting the RI'-MOM scheme [16]. These RCs are computed by extrapolating to the chiral limit the RCs estimators measured at several quark mass values. For the computation of the RCs, ETMC has generated dedicated runs with  $N_f = 4$  degenerate sea quarks. In these  $N_f = 4$  simulations working at maximal twist would imply a considerable fine tuning effort to get  $am_{PCAC} \simeq 0$ . Instead, working out of maximal twist the stability of the simulations increases.  $\mathcal{O}(a)$  improvement of the RC estimators is achieved by averaging simulations with an equal value of the polar mass  $M^{sea}$  but opposite value of  $m_{PCAC}^{sea}$  and  $\theta^{sea}$ , where  $\tan \theta^{sea} = Z_A m_{PCAC}^{sea} / \mu^{sea}$ . For details see [17].

Thanks to the OS-tm mixed action setup, the renormalized values of the bag parameters are given by the formulae [10, 18, 19]

$$B_1 = \frac{Z_{11}}{Z_A Z_V} B_1^{(b)}, \quad B_i = \frac{Z_{ij}}{Z_P Z_S} B_j^{(b)} \quad i, j = 2, \dots, 5. \quad (2.4)$$

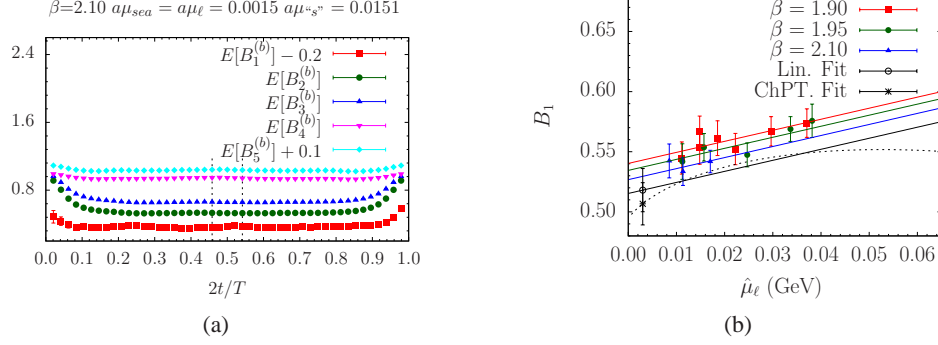


Figure 1: (a)  $B_i$  plateaux vs  $2t/T$  at  $\beta = 2.10$  and  $(a\mu_\ell, a\mu_h) = (0.0015, 0.0151)$ . The vertical dotted lines delimit the plateaux region. (b) Chiral and continuum extrapolation of  $B_1$   $K^0$  parameter renormalized in  $\overline{MS}$  scheme at 3 GeV.  $\hat{\mu}_\ell$  is the quark mass renormalised in  $\overline{MS}$  at 3 GeV. The full black line is the continuum limit curve and the dashed black line is the NLO ChPT continuum limit curve.

### 3. $K^0 - \overline{K}^0$

The lattice estimators of bare  $B_i$  parameters are obtained from the plateaux of the ratios

$$E[B_1^{(b)}](x_0) = \frac{C_1(x_0)}{C_{AP}(x_0)C'_{AP}(x_0)}, \quad E[B_i^{(b)}](x_0) = \frac{C_i(x_0)}{C_{PP}(x_0)C'_{PP}(x_0)}, \quad (3.1)$$

at times  $x_0$  such that  $y_0 \ll x_0 \ll y_0 + T_{\text{sep}}$  where  $T_{\text{sep}}$  is the separation between the two pseudoscalar meson walls. For  $K^0 - \overline{K}^0$  we fix  $T_{\text{sep}} = T/2$ . The involved correlators are defined as in [18]

$$\begin{aligned} C_i(x_0) &= \sum_{\vec{x}} \langle \mathcal{P}_{y_0+T_{\text{sep}}}^{43} Q_i(\vec{x}, x_0) \mathcal{P}_{y_0}^{21} \rangle, \\ C_{PP}(x_0) &= \sum_{\vec{x}} \langle P^{12}(\vec{x}, x_0) \mathcal{P}_{y_0}^{21} \rangle, \quad C_{AP}(x_0) = \sum_{\vec{x}} \langle A^{12}(\vec{x}, x_0) \mathcal{P}_{y_0}^{21} \rangle, \\ C'_{PP}(x_0) &= \sum_{\vec{x}} \langle \mathcal{P}_{y_0+T_{\text{sep}}}^{43} P^{34}(\vec{x}, x_0) \rangle, \quad C'_{AP}(x_0) = \sum_{\vec{x}} \langle \mathcal{P}_{y_0+T_{\text{sep}}}^{43} A^{34}(\vec{x}, x_0) \rangle, \end{aligned} \quad (3.2)$$

where  $\mathcal{P}$  are the pseudoscalar meson sources

$$\mathcal{P}_{y_0}^{21} = \sum_{\vec{y}} \bar{q}_2(\vec{y}, y_0) \gamma_5 q_1(\vec{y}, y_0), \quad \mathcal{P}_{y_0}^{43} = \sum_{\vec{y}} \bar{q}_4(\vec{y}, y_0 + T_{\text{sep}}) \gamma_5 q_3(\vec{y}, y_0 + T_{\text{sep}}), \quad (3.3)$$

and

$$P^{ij} = \bar{q}_i \gamma_5 q_j, \quad A^{ij} = \bar{q}_i \gamma_0 \gamma_5 q_j. \quad (3.4)$$

In figure 1a we display the quality of the  $B_i$  plateaux at  $\beta = 2.10$  and the smallest value of the light quark mass. Chiral and continuum extrapolations are carried out in a combined way. As an example, in figure 1b we show the combined fit for the  $B_1$   $K^0$  parameter, renormalized in the  $\overline{MS}$  scheme at 3 GeV, against the renormalized light quark mass.

Alternatively, we can consider the matrix elements ratio

$$R_i = \frac{\langle \overline{K}^0 | Q_i | K^0 \rangle}{\langle \overline{K}^0 | Q_1 | K^0 \rangle}. \quad (3.5)$$

as first proposed in [21]. Bare  $R_i$  parameters are obtained from the asymptotic time behaviour of

$$E[R_i^{(b)}](x_0) = \frac{C_i(x_0)}{C_1(x_0)}. \quad (3.6)$$

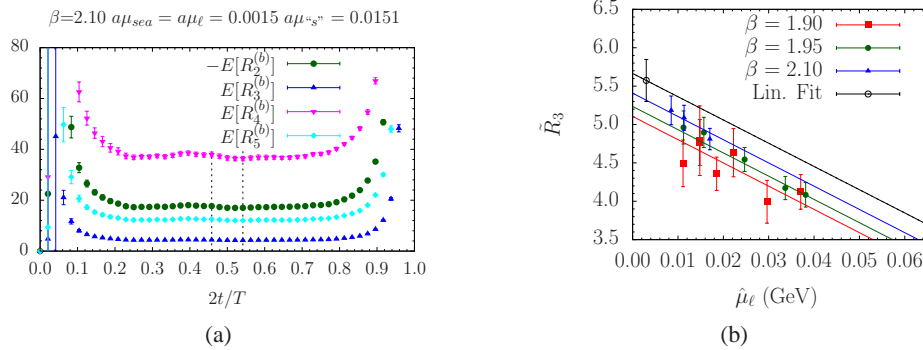


Figure 2: (a)  $R_i$  plateaux vs  $2t/T$  at  $\beta = 2.10$  lattice and  $(a\mu_\ell, a\mu_h) = (0.0015, 0.0151)$ . (b) Chiral and continuum extrapolation of  $R_3 K^0$  parameter renormalized in  $\overline{MS}$  scheme at 3 GeV.

$B_1$	$B_2$	$B_3$	$B_4$	$B_5$
0.51(02)	0.46(02)	0.81(05)	0.76(03)	0.47(04)
$R_2$		$R_3$	$R_4$	$R_5$
-15.1(6)		5.4(3)	30.1(1.5)	6.0(3)

Table 2: Continuum limit results for  $B_i$  and  $R_i$  parameters of the  $K^0 - \overline{K}^0$  system renormalized in the  $\overline{MS}$  scheme of [22] at 3 GeV.

As in [18, 20] we choose to evaluate the rescaled renormalized quantity defined as

$$\tilde{R}_i = \left( \frac{f_K}{m_K} \right)_{\text{exp}}^2 \left[ \frac{M^{12} M^{34} Z_{ij}}{F^{12} F^{34} Z_{11}} R_j^{(b)} \right]_{\text{latt}}, \quad (3.7)$$

in order to compensate for the chiral vanishing of the  $\langle \overline{K}^0 | Q_1 | K^0 \rangle$  matrix elements and reduce the lattice artefacts due to the different lattice discretizations of the kaon mesons. In Equation (3.7) we have normalized with the ratio of the experimental quantities  $f_K^{\text{exp}} = 156.1$  MeV and  $m_K^{\text{exp}} = 494.4$  MeV. Notice that in the continuum limit the quantity  $\tilde{R}_i$  of Equation 3.7 provides the right estimate for the ratio of the renormalized matrix elements.

Figure 2a is an example of the plateaux quality of the four-fermion operator ratios and figure 2b shows the combined chiral and continuum fit for the ratio  $\tilde{R}_3$  against the renormalized light quark mass. In table 2 we gather our final continuum results for  $B_i$  and  $R_i$  in the  $\overline{MS}$  scheme of [22] at 3 GeV with their total error. The systematic error, resulting from discretization effects, the chiral fit and the renormalization procedure, is added in quadrature to the statistical uncertainty.

#### 4. $D^0 - \overline{D}^0$

$B_i$  and  $R_i$  parameters for the  $D^0 - \overline{D}^0$  oscillations can be determined following a similar strategy. However, due to the experimental uncertainty on  $f_D$  we modify Equation (3.7). The renormalized  $R_i$  parameters for  $D^0 - \overline{D}^0$  are defined as

$$\tilde{R}_i = \left( \frac{1}{m_D^2} \right)_{\text{exp}} \left[ M^{12} M^{34} \frac{Z_{ij}}{Z_{11}} R_j^{(b)} \right]_{\text{latt}}. \quad (4.1)$$

Previous exploratory studies with the ETMC  $N_f = 2$  data show that using Gaussian smeared sources and choosing a time separation between meson sources smaller than  $T/2$  is crucial for

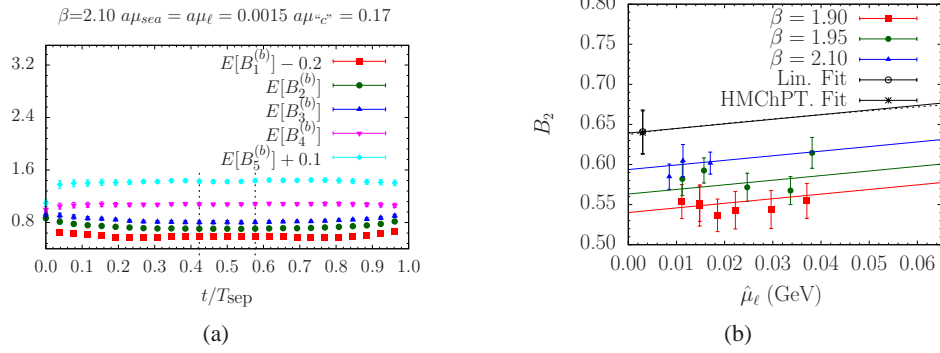


Figure 3: (a)  $B_i$  plateaux vs  $t/T_{\text{sep}}$  at  $\beta = 2.10$  and  $(a\mu_\ell, a\mu_h) = (0.0015, 0.17)$ . The vertical dotted lines delimit the plateaux region. (b) Chiral and continuum extrapolation of  $B_2$   $D^0$  parameter renormalized in  $\overline{MS}$  scheme at 3 GeV. The full black line is the continuum limit curve while the dashed black line represents the continuum limit curve in the case of a NLO HMChPT ansatz.

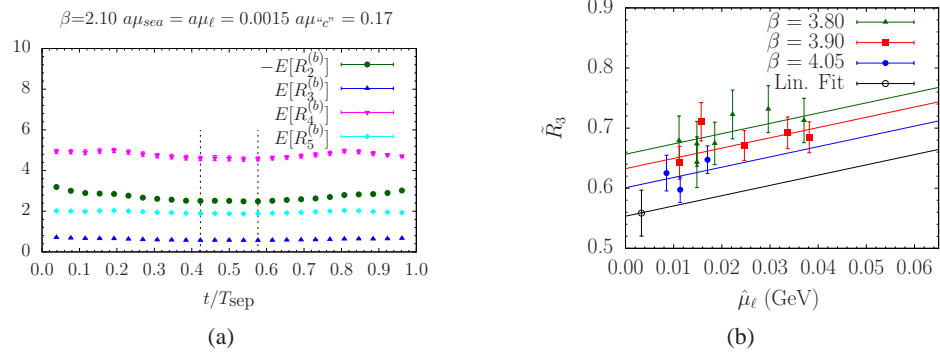


Figure 4: (a)  $R_i$  plateaux vs  $t/T_{\text{sep}}$  at  $\beta = 2.10$  lattice and  $(a\mu_\ell, a\mu_h) = (0.0015, 0.17)$ . (b) Chiral and continuum extrapolation of  $R_3$   $D^0$  parameter renormalized in  $\overline{MS}$  scheme at 3 GeV.

$B_1$	$B_2$	$B_3$	$B_4$	$B_5$
0.76(04)	0.64(02)	1.02(07)	0.92(03)	0.95(05)
	$R_2$	$R_3$	$R_4$	$R_5$
	-1.67(09)	0.53(05)	3.00(15)	1.02(07)

Table 3: Continuum limit results for  $B_i$  and  $R_i$  parameters of the  $D^0 - \overline{D}^0$  system renormalized in the  $\overline{MS}$  scheme of [22] at 3 GeV.

quark masses around the physical charm and above [23, 24]. In particular, we set  $T_{\text{sep}} = 18$  at  $\beta = 1.9$ ,  $T_{\text{sep}} = 20$  at  $\beta = 1.95$  and  $T_{\text{sep}} = 26$  at  $\beta = 2.10$ .

For illustration, in figure 3a and 4a we display the plateaux quality for  $B_i$  and  $R_i$  respectively at  $\beta = 2.10$  and  $(a\mu_\ell, a\mu_h) = (0.0015, 0.17)$ . Figures 3b and 4b show examples of the chiral and continuum extrapolation for  $B_2$  and  $R_3$ . Finally, in table 3 we collect our final results for  $B_i$  and  $R_i$  in the  $\overline{MS}$  scheme of [22] at 3 GeV.

Using as inputs  $R_i$ , the value of  $B_1$  and the renormalized quark masses one can compute indirectly the  $B_i$  ( $i = 2, 3, 4, 5$ ) parameters. The indirect evaluation leads to results compatible within errors with the results shown in table 2 and table 3 but with larger uncertainties.

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