Comparison of Localization Algorithms for Mode – S Multilateration (MLAT) Systems in Airport Surface Surveillance

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Abstract

A comparison among algorithms for passive localization in Multilateration (MLAT) systems for airport surface surveillance, based on Mode S signals (replies and 1090 MHz ES) is presented. A general framework for the comparison of these algorithms is proposed and described. Finally, an analysis with real data is performed and some guidelines and conclusions are provided.

1. Introduction

In Mode S Multilateration systems, ground receiving stations are placed in some strategic locations around the area to be covered. The system uses the Mode A/C and Mode S signals, i.e. the spontaneous Mode S squitter, the asynchronous transponder replies as well as the responses to interrogations elicited by the system itself. Then, the received signals are sent to the Central Processing Subsystem (CPS) where the transponder position is calculated [1-2]. This calculation is based on the Time Difference of Arrival (TDOA) principle, see Fig. 1.

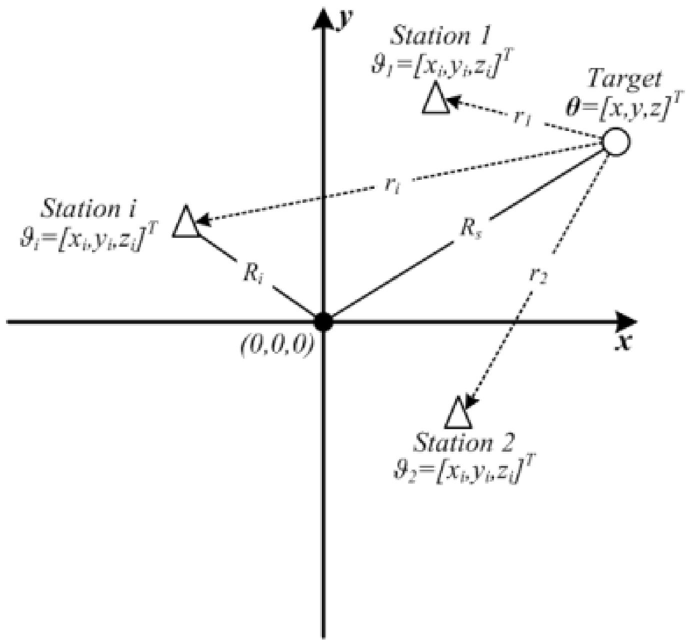

Fig. 1. MLAT system TDOA principle.

The TDOA principle consists in relating the unknown target position $\boldsymbol{\theta} \equiv \{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}$ with a set of known parameters. These known parameters are the TDOA measurements, which can be seen as the TOA difference between the ith receiving station and a reference one (the station number

1), and the stations position. The resulting function geometrically represents a hyperbola (or hyperboloid) and it can be expressed as follows:

$$\widehat{TDOA}_{i,1}(\boldsymbol{\theta}) = \frac{1}{c}(\|\boldsymbol{\theta} - \boldsymbol{\vartheta}_i\| - \|\boldsymbol{\theta} - \boldsymbol{\vartheta}_1\|) + n_{i,1}, \quad i = 2, \dots, N_s \tag{1}$$

where the superscript $\widehat{}$ denotes that is a measured (estimated) quantity, $n_{i,1}$ is a TDOA random noise term, which generally is assumed to be zero mean Gaussian distributed, c is the speed of light, and N_s is the total number of receiving stations. The TDOA measurements in (1) are proportional to range difference measurements:

$$\widehat{m}_{i,1}(\boldsymbol{\theta}) = c\widehat{TDOA}_{i,1} = (r_i - r_1) + n_{i,1} \tag{2}$$

where $r_i = \|\boldsymbol{\theta} - \boldsymbol{\vartheta}_i\|$. Finally, in order to obtain a numerical data for $\boldsymbol{\theta}$, a localization algorithm manipulates the set of measurements in the form of (2) to construct and to solve a system of equations, which is basically an inverse problem.

The position accuracy of MLAT systems depends on three elements, namely, the measurements accuracy, the spatial distribution of the stations (also called system geometry) that is quantified by the Dilution of Precision (DOP) parameter, and the localization algorithm used to convert the TOA/TDOA measurements space (i.e. the set of measurements in (1)) into the position one, commonly referenced to the Cartesian space (*x,y,z*). It is assumed that the MLAT position errors can be modelled as some statistical distribution. Generally, an unbiased Gaussian distribution with a given variance is assumed, where the measurements accuracy and the DOP set the minimum variance values of such a distribution [3]. Whether the operational system (regardless of the practical implementation) reaches or not this best case depends on the efficiency (statistically and numerically) of the localization algorithm.

The paper is organized as follows: the general framework, the data model and numerical algorithms descriptions are provided in Section 2, whilst the corresponding simulations and results, and the numerical analysis are given in Section 3. Finally, we provide some conclusions about the analysis performed in this paper.

2. The General Framework for Localization Algorithms

The localization problem in MLAT systems consists of estimating the target position in a given geographical reference system, based upon a set of physical measurements of signals emitted by aircraft or vehicles, and received by a set of ground receiving stations. In general terms, the target position is that numeric parameter that satisfies the set of equations (2), relating it with each measurement.

The structure of a localization algorithm is shown in Fig. 2. Such as algorithm starts by establishing a characteristic equation that can relate the unknown target position $\boldsymbol{\theta}$ with the measurements $\widehat{\boldsymbol{m}}$ and the position $\boldsymbol{\vartheta}_i$ of a finite number of stations, and optionally another measurement of the target position, which is denoted in Fig. 2 as $g(\boldsymbol{\theta})$. This inverse problem can be generally composed by a coefficient matrix $\boldsymbol{G}$, an unknown vector $\boldsymbol{\theta}$, and a known measurement vector $\widehat{\boldsymbol{m}}$. Likewise, different pairs of coefficient matrix and measurement vector will result in different localization problems. Finally, the inverse problem can be solved by a suited numerical algorithm, and thus the solution for the target position $\widehat{\boldsymbol{\theta}}$ is obtained.

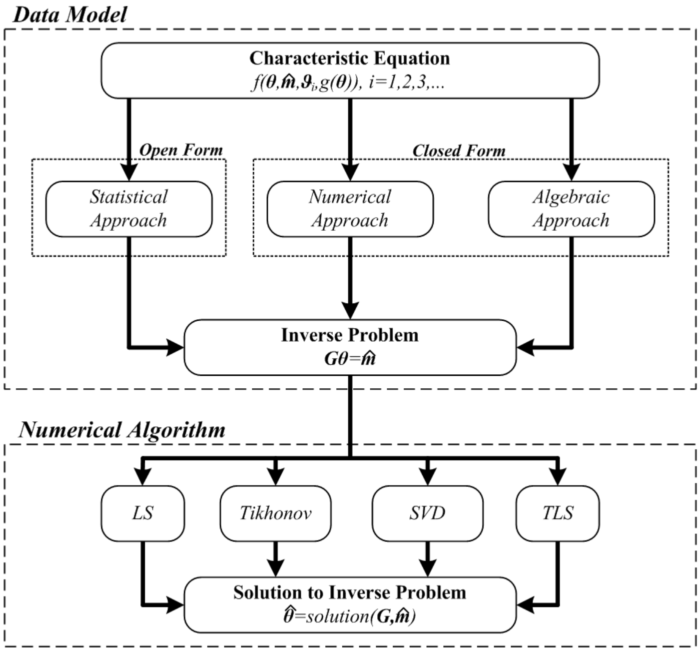

Fig. 2. General framework for localization algorithms.

2.1. The Data Model

The characteristic equation is used (Fig. 2) in different localization algorithms (i.e., the data models used by the different localization algorithms) that can be classified as based on statistical models, numerical models, and algebraic models. In the following the main characteristics of each of these are described.

2.1.1. The statistical models

This class of models assume certain statistical hypothesis about the measurements and the target position and set a probabilistic model that relates to each other. Most of statistical models are based on the Maximum Likelihood (ML) principle [4] due to the proven asymptotic consistency and efficiency of the ML estimators (MLE). These models require additional information about the measurement error distributions; normally Gaussian distributions are assumed. Furthermore, the resulting models are highly nonlinear. Then, to solve this kind of models, linear approximations and iterative numerical methods are required.

The statistical approach based models generally require two stations for setting a characteristic equation; hence, they require only a minimum of $n+1$ stations for an n-dimensional localization. Moreover, they allow of providing only one solution, and introduce only linear noise terms.

The typical problem that is solved by the localization algorithms using this kind of data models is the maximization of the likelihood function defined as follows:

$$\Lambda(\boldsymbol{\theta}) = \frac{1}{(2\pi)^{\frac{N_S-1}{2}} det\left(\boldsymbol{N}(\boldsymbol{\theta})\right)^{\frac{1}{2}}} e^{-\frac{1}{2}\left\{\left(\widehat{\boldsymbol{m}}-\boldsymbol{m}(\boldsymbol{\theta})\right)^T \boldsymbol{N}(\boldsymbol{\theta})^{-1}\left(\widehat{\boldsymbol{m}}-\boldsymbol{m}(\boldsymbol{\theta})\right)\right\}} \quad (3)$$

where $\boldsymbol{m}(\boldsymbol{\theta})$ is the (highly nonlinear) function describing the exact range difference quantity in (2), without the random noise term, $\boldsymbol{N}(\boldsymbol{\theta})$ is the covariance matrix of the measurement errors, and $\det(\;)$ denotes the determinant. Thus, the likelihood function is maximized by minimizing the following function:

$$Q(\boldsymbol{\theta}) = \left(\widehat{\boldsymbol{m}} - \boldsymbol{m}(\boldsymbol{\theta})\right)^T \boldsymbol{N}(\boldsymbol{\theta})^{-1}\left(\widehat{\boldsymbol{m}} - \boldsymbol{m}(\boldsymbol{\theta})\right) \quad (4)$$

Then this kind of models commonly use a Taylor series approximation of the function $\boldsymbol{m}(\boldsymbol{\theta})$ to obtain a linear relation that allows the construction of a linear inverse problem. This linear approximation is generally expressed as follows:

$$\boldsymbol{m}(\boldsymbol{\theta}) = \boldsymbol{m}(\boldsymbol{\theta}_0) + \boldsymbol{G}(\boldsymbol{\theta} - \boldsymbol{\theta}_0) \tag{5}$$

where $\boldsymbol{G}$ is the Jacobian differential matrix of $\boldsymbol{m}(\boldsymbol{\theta})$ (see [5-6] for details), and $\boldsymbol{\theta}_0$ is a previous estimation of the solution, which is also called starting point. The convergence of the localization algorithms initially depends on the starting point accuracy, although the numerical algorithm that it uses can allow the use of relative poor accurate starting points [6].

The most relevant localization algorithms that uses this kind of data models are those described in [7], which was statistically described and analyzed in [5], and the ones proposed in [6] and [8].

2.1.2. The numerical models

These models set a mathematical function that jointly relates the unknown target position, the measurements and a parameter derived from the target position (e.g., the target range) that of course is also unknown. The resulting models are linear in one unknown given the other one. Then, they assume certain numerical approximations between the target position and its derived parameter in order to simplify the solution. The most common assumed approximation is that of mutual independence between them. These numerical approximations are independent of the statistical distributions of the measurement errors. Most of them are based on the LS (Least Squares) principle, i.e., they set an error function whose squared norm is minimized. These models can be solved by direct optimization and do not require any previous estimation of the solution; for this reason the algorithms based on them are commonly classified as closed form algorithms. Furthermore, normally the computational cost required to solve this kind of models is less than that required for ones based on the statistical approach. On the other hand, all the algorithms based on these models introduce quadratic noise terms in the resulting inverse problem, and the solutions provided by them are biased and are not optimum in the statistical sense. Moreover, most of the localization algorithms that can be found in literature use this kind of data model.

This kind of data model requires two stations to form a characteristic equation. However, depending on the numerical assumptions, they can require $n + 1$ or $n + 2$ receiving stations for a n-dimensional localization. Moreover, some of them can provide two possible solutions for the target position as they set a quadratic data model. Thus, these ones require an additional procedure to choose one of the two possible solutions. Likewise, this data model always introduces quadratic noise terms in the resulting localization problem.

The most representative algorithms using this kind of models are the one by Smith and Abel [9], the one by Friedlander [10], the one by Schau and Robinson [11], and the one by Chan and Ho [12]. Obviously, there exist much more algorithms in the literature. However, from our point of view, the above cited ones are a suitable representation of the most of them.

2.1.3. The algebraic models

These models do not use any statistical assumptions or numerical approximations. They algebraically manipulate the hyperbolic equations until directly set an inverse problem that linearly relates the unknown target position with the known parameters (i.e., the measurements and the station positions). These models are very simple as only geometric relations are used. By contrary, they normally require more measurements and introduce quadratic and cubic noise terms in the inverse problem. As the numerical approach based models, these ones do not require any previous estimation of the solution and can be solved by direct optimization; hence, the algorithms that use these models are also classified as closed form algorithms. Moreover, the solutions provided by the algorithms based on these models are also biased and are not optimum in the statistical sense. On the other hand, most of them require the lowest computational resources.

This kind of data model can require $n+1$ or $n+2$ stations for n-dimensional localization, and, as the numerical approach based models, they can provide one or two possible solutions for the target location.

The most relevant algorithms using this data model are the one by Schmidt [13], the one by Geyer and Daskalakis [14] and the one published in the open license website Wikipedia® [15]. Particularly, the Geyer and Daskalakis [14] algorithms is a practical implementation of the Bancroft algorithm [16], which was originally proposed for the Global Positioning System (GPS) and that is based on TOA measurements rather than TDOA or range differences. For this reason, we refer this algorithm as the Bancroft algorithm.

2.2. The Numerical Algorithm

The linear inverse problem $\boldsymbol{G\theta} = \widehat{\boldsymbol{m}}$ as depicted in Fig. 1, and obtained by any data model, must be numerically solved to obtain a numerical data for the target position. Besides the particular form that can takes the matrix $\boldsymbol{G}$ and the measurement vector $\widehat{\boldsymbol{m}}$ the numerical efficiency of every particular solution strongly depends on the chosen solution for that inverse problem. As we have also commented before, the most used numerical algorithm to solve the resulting linear inverse problem is the LS, i.e., by the pseudoinverse matrix. Moreover, for the statistical approach based models that require iterative procedures because the nonlinearity of the resulting model, the Gauss-Newton method [7] is the most commonly used. However, the latter can be also seen as an iterative, recursive solution in the sense of LS.

The LS solution solves the following residual error norm minimization problem:

$$\widehat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \|\boldsymbol{G\theta} - \widehat{\boldsymbol{m}}\|^2 \tag{6}$$

The solution $\widehat{\boldsymbol{\theta}}$ provided by (6) is known to be the minimum residual norm when solved by the pseudoinverse matrix [17]. However, this solution is not always numerically stable and, hence, it does not always provide acceptable accuracies because, in many operational conditions, the coefficient matrix $\boldsymbol{G}$ has some linearly-dependent rows [3], [8].

On the other hand, also there exist other robust numerical algorithms, which have been recently implemented for the MLAT localization problem. These algorithms include the Tikhonov, SVD, and Total Least Squares (TLS) based regularization, and are intended to provide numerical stable solutions. These numerical methods solve a modified version of the residual error norm in (6).

The Tikhonov regularization solves a linear combination of the residual error norm in (6) and an auxiliary norm called smoothed norm, as follows:

$$\widehat{\boldsymbol{\theta}}_{Tikhonov} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \{\|\boldsymbol{G\theta} - \widehat{\boldsymbol{m}}\|^2 + \lambda^2\|\boldsymbol{L\theta}\|^2\} \tag{7}$$

where λ and $\boldsymbol{L}$ are called regularization parameter and matrix of Tikhonov, respectively, and the term $\|\boldsymbol{L\theta}\|$ is the smoothed norm. The regularization parameter and matrix must be previously estimated. The authors in [6] provide and simple, but efficient procedure for that estimation in MLAT localization.

The SVD based regularization, specifically the Truncated SVD (T-SVD), solves an equivalent minimization problem to (6) but by using a modified (truncated) version of matrix $\boldsymbol{G}$, as follows:

$$\widehat{\boldsymbol{\theta}}_{T-SVD} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \|\boldsymbol{G}_k\boldsymbol{\theta} - \widehat{\boldsymbol{m}}\|^2 \tag{8}$$

where $\boldsymbol{G}_k$ is the modified version of $\boldsymbol{G}$ and k is known as the truncation parameter. This parameter must also be previously estimated. A procedure to obtain the truncation parameter and the corresponding modified matrix $\boldsymbol{G}_k$ are described in [8] for MLAT localization.

The TLS based regularization, specifically the Truncated TLS (T-TLS), assumes that also the coefficient matrix $\boldsymbol{G}$ is perturbed by some errors and, under this assumption, solves the following LS problem:

$$\widehat{\boldsymbol{\theta}}_{T-TLS} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \left\| [\boldsymbol{G}, \widehat{\boldsymbol{m}}]_{k_{T-TLS}} - \left[\widetilde{\boldsymbol{G}}, \widehat{\boldsymbol{m}}'\right] \right\|_F \quad \text{subject to} \quad \widehat{\boldsymbol{m}}' = \widetilde{\boldsymbol{G}}\boldsymbol{\theta} \tag{9}$$

where $\widetilde{\boldsymbol{G}}$ is the perturbed coefficient matrix, $\widehat{\boldsymbol{m}}'$ is the equivalent perturbed version of the measurement vector, $\| \quad \|_F$ denotes de Frobenius norm of a matrix [17], k_{T-TLS} is the corresponding T-TLS truncation parameter, and the matrix $[\boldsymbol{G}, \widehat{\boldsymbol{m}}]_{k_{T-TLS}}$ is a modified version of matrix $[\boldsymbol{G}, \widehat{\boldsymbol{m}}]$. Some particular application of T-TLS method, for some numerical approach based model algorithms, can be found in [18-19].

3. Simulation and Results

To perform the comparison of localization algorithms, the company ERA A.S. has provided us a data base of TOA measurements of one of its operational system, the MLAT system installed at Tallinn airport (Tallinn, Estonia). This system is intended for surface surveillance and is composed of fourteen receiving stations. The record of TOA measurements was taken through the entire airport surface following the requested procedures by the European regulatory bodies [1]. The data base contains more than 4000 records (with an average period of 1 s), where each record contains set of TOA measurements. Moreover, also the company above mentioned has provided as of highly accurate position reference data, which was simultaneously recorded with a GPS receiver with differential correction capabilities (DGPS). This data is used to calculate the 2D error of every analyzed localization algorithm. The system layout and the reference position data are depicted in Fig. 3.

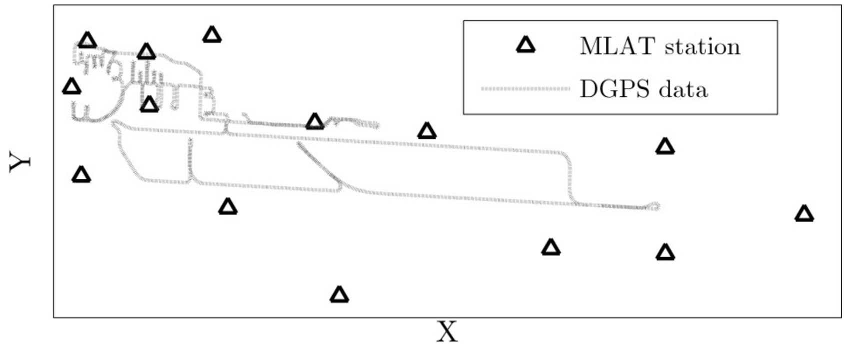

Fig. 3. Tallinn MLAT system.

We have simulated the localization algorithms by Schmidt [13], Foy [7], Smith and Abel [9], Friedlander [10], Schau and Robinson [11], Chan and Ho [12], the application of Bancroft by Geyer and Daskalakis [14], and the Wikipedia® [15]. All of these algorithms are solved in the sense of LS. Moreover, we also have simulated the particular applications of Tikhonov [6] and T-SVD [8] regularization methods. Particularly, for the localization algorithms using a statistical approach based model, which require for a starting point, we use the one as provided by the Schau and Robinson algorithm[11]. Moreover, for those algorithms that require choosing one of the two possible solutions (i.e., the Schau and Robinson algorithm, the Chan and Ho[12], and the Bancroft) we have used the same selection procedure.

To evaluate accuracy of the above mentioned algorithms, we have calculated the target position through the entire path depicted in Fig. 3 and obtained, for each of these, the corresponding 2D (x, y) error versus the DGPS reference position data. Then, the standard deviation and mean of each error distribution are obtained. These values are shown in Table 1.

Table 1. Standard deviation and mean for the error distribution of every localization algorithm. Values in meters (N: numerical, S: statistical, A:algebric model).

Model	Algorithm	σ_{2D} (m)	$Mean_{2D}$ (m)
A	Schmidt	385.93	39.74
S	Foy	1.20E+18	4.82E+16
N	Smith and Abel	614.66	67.16
N	Friedlander	385.15	36.67
N	Schau and Robinson	616.26	108.62
N	Chan and Ho	737.49	876.57
A	Bancroft	456.06	97.61
A	Wikipedia	384.66	35.38
S	Tikhonov	61.79	12.57
S	T-SVD	65.03	12.75

As we can see from Table 1, all the localization algorithms solved in the sense of LS provide very large values of standard deviation and mean, significantly much greater than the requested one for surface surveillance [1]. In this sense, the Schmidt, Friedlander, and Wikipedia algorithms provide the smallest values of standard deviation and mean (385 m and 37 m respectively). Then, the Smith and Abel, Schau and Robinson, Bancroft, algorithms provide greater values. The Foy algorithm is the one with the greatest errors. It is mainly due to the impossibility of this algorithm to converge through the entire surface path. Regarding to the Chan and Ho algorithm, which provides the largest value of mean (after the Foy algorithm), the reason is different from that of Foy algorithm. It is due to its low capability of jointly obtaining the target range and the target position. In this case, when this algorithm applies the quadratic correction [12], as the target range is highly inaccurate (in some cases negative), this correction also leads to a highly inaccurate positions. We have also found that if only the first solution of this algorithm (i.e., before the quadratic correction), for target position, is taken as the final one, it presents an equivalent performance as Smith and Abel algorithm.

Finally, the localization algorithms solved in the sense of Tikhonov and T-SVD provide the best values of standard deviation and mean.

4. Conclusions

We have proposed a general framework to understand, and compare the localization algorithms. We have tested all the described localization algorithms for a real data scenario and we have found the most statistically optimal solutions are provided by the algorithms using a statistical approach based model, as long as the statistical hypotheses are met and the algorithm convergence is reached.

However, the convergence of these algorithms when solved in the sense of LS is unstable and, hence, not always guaranteed. In this sense, we have shown that the corresponding convergence of these algorithms can be guaranteed when using regularization techniques like Tikhonov or T-SVD.

In order to obtain the most statistical and numerically efficient localization strategy, it is always advisable to use the combination of a statistical approach based model algorithm (that is an open form algorithm) along with either a numerical or an algebraic approach based model algorithm (that are closed form algorithms).

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