Orthogonal Waveforms for Multistatic and Multifunction Radar

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Abstract — **Modern radar include more and more multiple functions and multiple channels as in the case of MPAR (Multifunction Phased Array Radar) and MIMO (Multiple Input Multiple Output) radar. In this context sophisticated waveforms and related processing (pulse compression, extraction of information) are needed. Moreover, to avoid changes in target RCS and undesirable Doppler effects, the encoded waveforms (one for each channel) have to be transmitted simultaneously and at the same carrier frequency. The orthogonal property is required for the transmitted signals to separate them in reception. A good candidates to design signals that satisfy the orthogonal requirements are the Costas codes (CS) and the Phase Noise (PhaNo) signals. In this paper we present a comparison of the main characteristics of the CS and PhaNo considering as a reference the pair of "up" and "down" chirp (Linear-FM and Non-LFM).**

Keywords – Multistatic radar; multifunction radar, MPAR; MIMO, orthogonal waveforms.

I. Introduction

The concept of multifunction radar has been developed in the last years with the aim to implement multiple functions in a single apparatus with improvements of the cost/effectiveness ratio. For example a MPAR (Multifunction Phased Array Radar) offers the potential for significant performance improvements and reduced lifecycle costs for the system aimed to aircraft and weather surveillance by performing these different functions with a single radar unit [1], [2]. A reasonable cost of the basic building block of the MPAR, i.e. for the Transmit-Receive Module (TRM), can only be obtained by lowering the peak power to the order of one watt by pulse-compression [3]. Therefore, MPAR systems call for "long" transmitted signals, i.e. with high duty cycle, and for low range sidelobes (low Peak-Side-Lobe-Ratio, PSLR, for point targets and low Integrated-SideLobe-Ratio, ISLR, for distributed targets such as rain) after pulse compression even in the presence of a significant Doppler shift. A possible solution to reach low PSLR and ISLR is based on the use of a pair of orthogonal codes "up" and "down" chirp nested into a pair of complementary codes as described in [4], [5].

The multifunction radar concept can include other functions such as the addition of communication capacity [6], and the simultaneous measurement of the elements of the scattering matrix [7]. These functions calls for the simultaneous transmission of orthogonal signals to separate them in reception.

Recently, a new field of radar research called MIMO (Multiple Input Multiple Output) has been developed [8]. It can be thought as a generalization of the multistatic radar concept. This kind of radar has multiple antennas to simultaneously transmit arbitrary waveforms and different multiple antennas to receive back-scattered signals. The transmit and receive antennas may be in the form of an array and the transmit and receive arrays can be widely separated or co-located like in phased array systems [9], [10]. In most of the current literature it is assumed that the waveforms coming from each transmit antenna are orthogonal. Although this is not a strict requirement for MIMO radar, orthogonality can facilitate the process of separation in reception.

Orthogonality may be imposed in time domain, in frequency domain or in signal space. Time division or frequency division multiplexing are simple approaches but they can suffer from potential performance degradation because the loss of coherence of the target response [11]. In fact the scattering response of the target or of the background is commonly time-varying or frequency selective, limiting the ability to coherently combine the information from the antenna pairs. As a consequence, obtaining the orthogonality in signal space domain is the best choice.

The paper is organized as follows. Chapter II describes: (a) the orthogonal LFM and NLFM "up" and "down" chirp codes (they will be taken as reference for the following comparison), (b) the Costas codes and (c) the phase noise signals. In Chapter III the properties of the signals will be analyzed in the time domain (auto and cross-correlation) and in the frequency domain (density spectrum, cross-density spectrum and bandwidth). Chapter IV contains some final considerations and future perspectives.

II. Orthogonal Codes in Signal Space

In addition to the well-known *up* and *down* chirp signals, see for instance [12], more recent researches about the othogonal signals proposed the use of *normal* or *interleaved* OFDM techniques [13]. The main limitation to this approach is due to the non-constant envelope of the signals, i.e. the transmitter does not work to the maximum power. In this paper we only consider the orthogonal signals with constant envelope, i.e. with phase-only modulation.

A. Up and down chirp (LFM and NLFM)

The complex envelope of a pair of orthogonal codes *up* and *down* chirp-like can be written as: $c_1(t) = e^{j\phi(t)} rect_T(t)$, $c_2(t) = e^{-j\phi(t)} rect_T(t)$ where $rect_T(t) = 1$ in $(-T/2, T/2)$ an zero elsewhere. For LFM: $\phi(t) = \frac{\mu \cdot t^2}{2}$ with $\mu = 2\pi B/T$, where T denotes the length of the non-compressed pulse having bandwidth $B \cong 1/\tau$, τ is the duration of the compressed signal; the product *BT* is the compression ratio into the chip time *T*. The autocorrelation assumes the well known expression similar to a $sinc(x)$ as reported in [14]. The amplitude of the cross-correlation has been evaluated as [15]:

$$\left|R_{12}(t)\right| = 2\left|\mathcal{F}\left(\sqrt{BT}\left(1-\frac{|t|}{T}\right)\right)\right| \qquad -T < t < T \qquad (1)$$

where $\mathcal{F}(z)$ is the Fresnel integral in complex form [15]. For NLFM the phase $\phi(t)$ is evaluated applying the principle of stationary phase supposing a Hamming weighting [14]. In the NLFM case evaluation of the auto and cross-correlation lead to very complicated expressions and its values are better derived by simulation.

B. Costas Signals (CS)

A Costas code can be obtained dividing the *time-frequency* plane in *M* sub-elements (*chips*) of equal duration t_b and band $\Delta f = 1/t_b$ [16]. The complex envelope of a Costas signal of length $T = Mt_b$ (*M* integer) is [13]:

$$s(t) = \frac{1}{\sqrt{Mt_b}} \sum_{m=1}^{M} exp(j2\pi f_m t) rect_{t_b}\left[t-(m-1)t_b + \frac{T}{2}\right] \qquad (2)$$

where t_b is the chip time and $f_m = a_m \Delta f$, with $m = 1,2,...,M$, is the carrier frequency of the chip *m*, being $a = [a_1, a_2, ..., a_M]$ the sequence of distinct integers between 1 and *M* making the code (*hopping sequence*); $rect_{t_b}(t)$ is equal to 1 for $0 \le t < t_b$ and *zero* elsewhere. The band is $B = M \cdot \Delta f$ and the resulting compression ratio is M^2. Considering a *discrete* LFM, the hopping sequence is deterministic and the frequency of the m^{th} chip is $f_m = m \cdot \Delta f$ for $m = 1,2,...,M$. The same approach can be applied to the NLFM when the non-linear-frequency law is known.

C. Phase Noise signals (PhaNo)

For phase noise signal (PhaNo) the complex envelope is $s(t) = exp\left[j\phi(t)\right] rect_T(t)$ where $\phi(t)$ is the random process modulating the phase of the signal. If $\phi(t)$ is a stationary Gaussian process with mean zero, standard deviation σ (in *rad*) and correlation coefficient $\rho(\tau)$, indicating with b_{rms} [(1)] the r.m.s. band of its power spectrum, the statistical autocorrelation of the modulated signal results [17]:

$$R(\tau) = exp\left\{-\sigma^2\left[1-\rho(\tau)\right]\right\} \qquad (3)$$

with an improvement in term of PSLR at the cost of increasing the r.m.s. band (B_{rms}) of the modulated signal: $B_{rms} = \sigma \cdot b_{rms}$ [16].

However this model can be used only with low values of σ. The Gaussian noise is used to modulate the signal phase, which is usually defined as a uniform distribution in the range $[-\pi, \pi]$ with a standard deviation of $\pi/\sqrt{3} \cong 1.8$.

Therefore, if σ is too large ($\sigma > \pi/\sqrt{3}\ rad$), the resultant phase does not have a Gaussian distribution in the interval $[-\pi, \pi]$ and the mathematical formulation introduced before cannot be used. As a consequence (3) is valid only with $\sigma \le 2\ rad$.

Hereafter we consider in the performance analysis $\sigma = 2\ rad$ and $\rho(\tau)$ a *sinc* function, i.e. in frequency domain a Gaussian noise with constant spectrum of band *B* (the r.m.s. results approximately $b_{rms} \cong B/\sqrt{12}$).

We can relate the phase noise signal with the Costas codes considering a *discrete* phase noise where the frequency of the chip *m*, $f_m = a_m \Delta f$, has been obtained by a realization of a discrete (integer) random variable from 1 to *M*.

III. PERFORMANCE ANALYSIS

A. Autocorrelation (PSLR, ISLR)

In Figure 1 are shown the normalized autocorrelations of two Costas codes with M = 40 and T = 16 μs; the main lobes and the lobes around it have the same behavior similar to a *sinc* function with PSLR of -13 dB as in the LFM chirp.

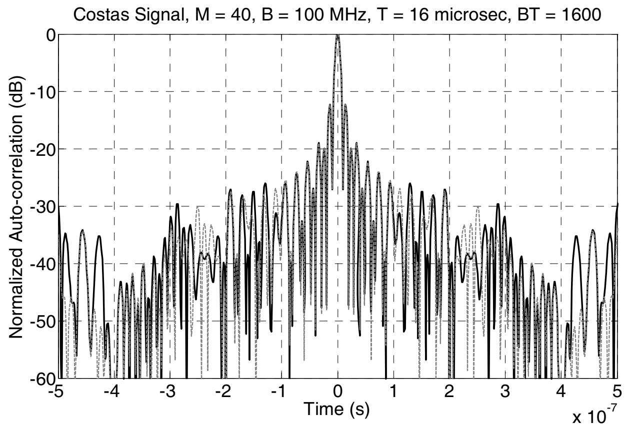

Figure 1. Normalized autocorrelation (zoom in $\pm 0.5\ \mu s$) for a pair of Costas codes with M = 40, BT = 1600.

Considering a pair of PhaNo, with $\sigma = 2\ rad$, T = 16 μs, B = 100 MHz, Figure 2 reports the normalized autocorrelations. The main lobes are the same while the side lobes are randomly distributed with a PSLR of -25 dB circa.

(1) $b_{rms} = \left[\frac{\int f^2 W(f) df}{\int W(f) df}\right]^{\frac{1}{2}}$, $W(f)$ is the spectral power density of the process.

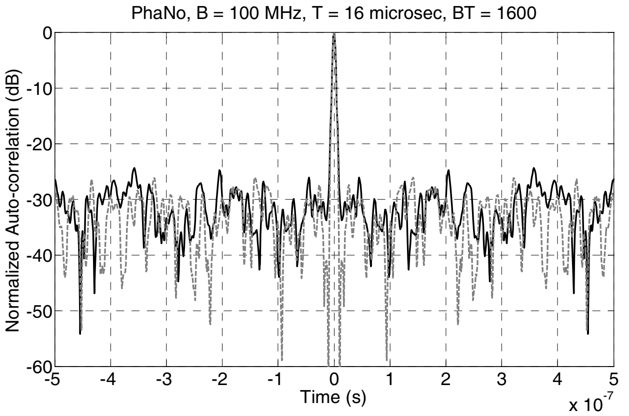

Figure 2. Normalized autocorrelation (zoom in $\pm 0.5\ \mu s$) for a pair of PhaNo, BT = 1600.

Cross-correlation

To evaluate the orthogonality between two signals with same energy, the cross-correlation, normalized to the energy, is shown in Figure 3 for a pair of Costas codes and in Figure 4 for a pair of PhaNo codes considering a compression ratio of 1600. The black and the dashed line are referred to a pair of up and down chirp, LFM and NLFM respectively. The waveforms are the same as in the previous paragraph.

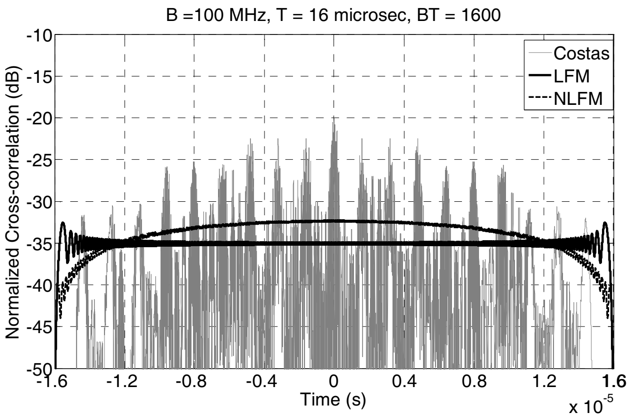

Figure 3. Normalized cross-correlation for a pair of Costas codes with M = 40 (gray), up and down chirp LFM (black line) and NLFM (dashed line).

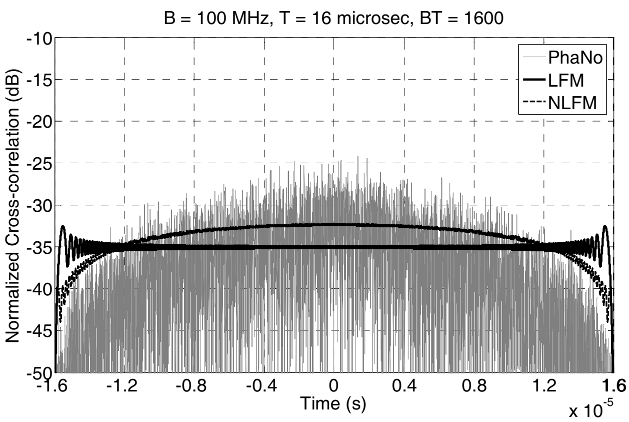

Figure 4. Normalized cross-correlation for a pair of PhaNo, up and down chirp LFM (black line) and NLFM (dashed line).

Around the time origin the cross correlation is -35 dB for LFM chirp up and down and -32 dB for NLFM, while for a pair of Costas codes the cross-correlation is about 15 dB higher than LFM, while for PhaNo it is around 10 dB higher.

When comparing pairs of different codes (chirp, Costas, Phase Noise and OFDM) it must be considered that the up and down chirp permit fairly low sidelobes for the autocorrelation function and a low (and nearly uniform) cross correlation. However, for assigned values of B and T, only one pair of chirp exist, while the other codes permit to generate a large number of orthogonal pairs with the same B and T values. By transmitting these pairs in the radar dwell time, it is possible: (a) to optimize the chosen pair, (b) to smooth, by averaging, the unwanted peaks for both the autocorrelation. In fact, supposing the transmission of *N* different PhaNo codes in the radar dwell time, the process of adding radar echoes from the many pulses allows the smoothing of the peaks of the autocorrelation function. Figure 5 shows this aspect by comparing the Normalized Auto-correlation function in the case of single pulse and $N = 64$ pulses.

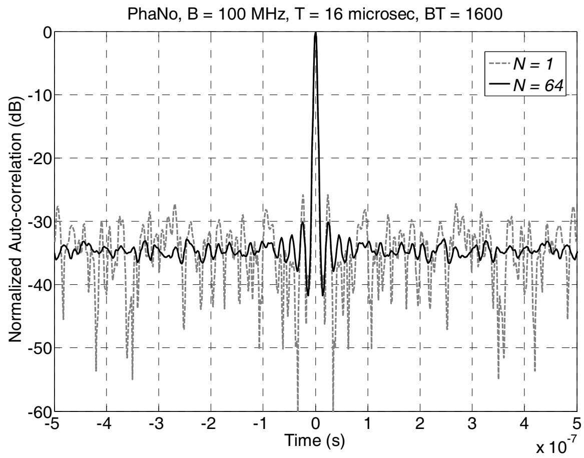

Figure 5. Normalized autocorrelation (zoom in $\pm 0.5\ \mu s$) in the case of single pulse and $N=64$ pulses averaged.

B. Density spectrum

In frequency domain if the signals are up and down chirp-like (LFM, NLFM) the absolute value of the density spectrum and of the density cross-spectrum coincide. The chirp signals up and down make more uniform in time the behavior of the cross-correlation, reducing the PSLR, but making the ISLR at the level of the one due to the cross-correlation. This effect is emphasized for LFM confirming that that the cross-correlation interference can only be reduced by spreading the spectrum. Figure 6 and Figure 7 reports for up and down chirp LFM (black line) and NLFM (dashed line) the density spectrum. For a pair of Costas codes and PhaNo the density spectrum and the density cross-spectrum are a bit different and in Figures 6 and 7 in gray the only density spectrum of the two signals is shown.

For PhaNo the theoretical r.m.s. band results: $B_{rms} \cong \dfrac{\sigma B}{\sqrt{12}}$ if the modulating noise has an uniform spectrum in $-B/2$, $+B/2$. For $B = 100\ MHz$ and $\sigma = 2$, $B_{rms} \cong 58\ MHz$.

From these figures it results that in order to exploit the same available frequency interval as the chirp, the Costas (in very small extent) and the PhaNo (in a much greater extent) must be filtered before power amplification.

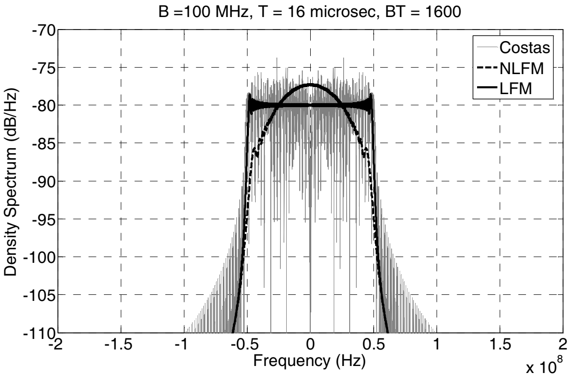

Figure 6. Density spectrum for a pair of Costas codes with M = 40 (gray), up and down chirp LFM (black line) and NLFM (dashed line).

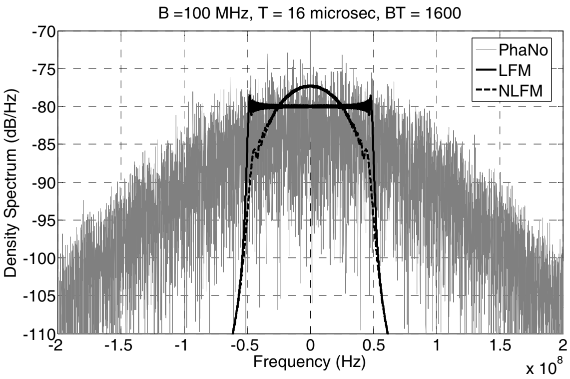

Figure 7. Density spectrum for a pair of PhaNo codes with BT=1600, up and down chirp LFM (black line) and NLFM (dashed line).

As regard the PhaNo, Figure 8 shows the effect on the autocorrelation after applying a Hamming Window from -50 to + 50 MHz. After the windowing the PhaNo presents the same frequency interval of the chirp, maintaining about the same level of the side lobes (PSLR of circa -28 dB), but with a consequent widening of the main lobe.

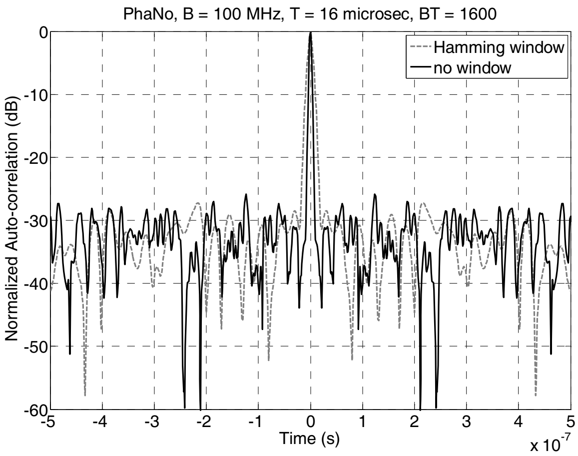

Figure 8. Normalized autocorrelation (zoom in $\pm 0.5\ \mu s$) after and before applying the Hamming Window.

IV. CONCLUSIONS

In this paper, we underlined the importance of transmitting orthogonal waveforms for MPAR and MIMO radar. In detail we analyzed the Costas codes and the Phase Noise signals, considering as a reference the up and down chirp.

By comparing the main characteristics of these signals we have taken into account that, although the up and down chirp permit lower sidelobes for the autocorrelation function and a lower cross correlation, for assigned values of B and T, only one pair of chirp exist. On the other hand the CS and the PhaNo, despite a little loss of orthogonality due to an higher level of the cross-correlation, allow the generation of a large number of orthogonal signals. Therefore they are good candidates for MIMO radar. Moreover PhaNo signal has a further advantage about the detectability, placing limitation on the detection, the identification and the eventual regeneration of the signal, aspect of great importance in many military applications which require low detectability of the active system.

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