Waveforms Design for Modern Radar: the Chirp signal Fifty + years later

Gaspare Galati, Gabriele Pavan, Francesco De Palo

Tor Vergata University – DIE and Centro "Vito Volterra"
Via del Politecnico, 1 – 00133 Rome, ITALY
gaspare.galati@uniroma2.it , gabriele.pavan@uniroma2.it , francesco.de.palo@uniroma2.it

Abstract — **The optimal design of chirp radar waveforms has to satisfy conflicting requirements. Optimal solutions for different values of the compression ratio are considered, evaluated and compared with each other. A proposed solution has the advantage of guaranteeing low range sidelobes (less than -50 dB below the peak for compression ratios greater than 256) while maintaining the transmitter to operate in saturation. The "chirp" waveforms, with either constant or non-constant amplitude, are also candidates for the build-up - by combination - of "semideterministic" signals for the use in noise and MIMO radar.**

Keywords—Radar Waveforms,Chirp, Pulse Compression, Non Linear FM; low PSLR, Waveforms.

I. Introduction

For any new radar the waveforms design is a key element, in which conflicting requirements have to be satisfied, i.e. (to mention the main ones) range resolution and accuracy, dynamic range, MIMO or polarimetric channels orthogonality, Low Probability of Intercept (LPI), low interference to similar apparatus, and best exploitation of the transmitter power thanks to a low (or unitary) *crest factor*. The Frequency-Modulated (FM), or *"chirp"* signal, Linear (LFM) and then Non Linear (NLFM), is one of the oldest and still more used waveforms for pulse compression radar. The requirement of reducing more and more the sidelobes level of the autocorrelation function has been satisfied either by tailoring the NLFM law or by a sidelobe-suppression filter in reception, the latter to expense of losses in the signal to noise ratio (SNR).

The design of chirp radar in the Western world dates back to 1960 [1] and, in the Eastern word, to the works (1950's) by Yakov Shirman, who received the IEEE Pioneer Award in 2009 "*For the independent discovery of matched filtering, adaptive filtering, and high-resolution pulse compression for an entire generation of Russian and Ukranian radars*" [2].

The *"chirp"* theory is well established and is related to the Stationary Phase principle [3]: for large enough values of the number of independent samples of the waveform, or BT product (also called Compression Ratio) between Time duration T, and Bandwidth B, the group delay of a LFM signal is proportional to the instantaneous frequency, and the spectrum of the LFM signal after matched filtering is rectangular, with constant amplitude and linear phase in the bandwidth B (and ideally, zero amplitude outside it). It is well known that the resulting autocorrelation has a duration $1/B$ and an unacceptable Peak Side-Lobes Ratio level (PSLR) about 13.2 dB below the main peak. The sidelobes can be lowered by a NLFM law, which can be easily designed by means of the well-known spectral windows [3]-[8]. However this method, relying on the Stationary Phase principle, in terms of PSLR values when BT is "large" (in practice, greater than a few thousands), and is less and less effective when a low BT is required, as it happens in various applications like Air Traffic Control radar and the new Coherent Marine (or Navigation) radar.

A new design method to cope with this problem has been presented in [9] and [10], leading to a kind of hybrid NLFM (HNLFM), whose amplitude, however, is not constant during the duration T of the signal, thus creating implementation problems with the widely-used saturated (C-class) power amplifiers. This family of waveforms is enhanced and analyzed in the following.

The paper is organized as follows: section II describes deterministic waveforms design and compares classical pulse compression with the innovative ones; section III shows a different approach to pulse compression based on random waveforms (noisy signals); finally section IV reports conclusions and future perspectives.

II. Deterministic Waveforms Design

The complex envelope of a general signal, modulated both in Amplitude and in Phase (AM-PM), is given by:

$$s(t) = a(t)e^{j\phi(t)} \tag{1}$$

where $a(t)$ and $\phi(t)$ denote respectively the AM and PM modulation. Fig. 1 summarizes four cases of generation and processing of Non-Linear-FM waveforms that will be examined to implement pulse compression: case (a) *Millett* waveform [8], i.e. a *"cosine squared on a pedestal"* weighting (this is historically the oldest NLFM, considered here as reference); case (b) Hybrid Non-Linear FM [9], [10] with amplitude $a(t)$ shown in Fig. 2 and phase $\phi(t)$ derived from eq. (4); case (c) a Hybrid-NLFM with the same $\phi(t)$ of case (b) and a pseudo trapezoidal amplitude for $a(t)$ as shown in Fig. 3; finally case (d) a Hybrid-NLFM with the same $\phi(t)$ of case (b) with constant (unit) amplitude. The use of those waveforms is aimed to solve a relevant problem of pulse compression radar, i.e. to have a sufficiently low Peak-to-Side-Lobes Ratio, PSLR (an indicator of the level of range sidelobes). A desirable condition to resolve targets in space domain is to have a PSLR of the same order, or better, than the *two-way* antenna sidelobes level. For cases (b), (c) and (d) the reception filter is matched to the optimized signal of case (b).

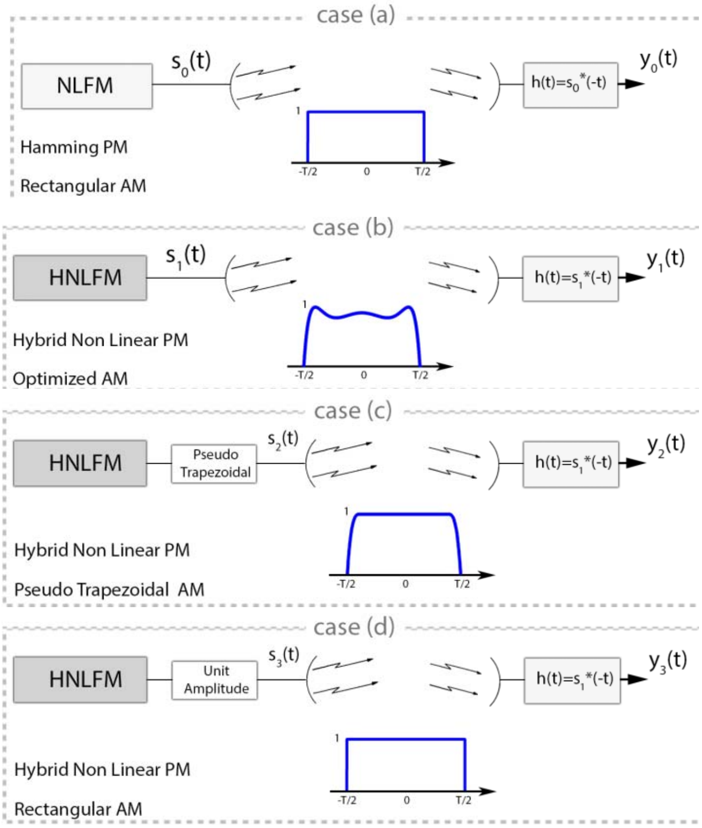

Figure 1. Scheme of the considered waveforms and their processing.

The stationary phase principle establishes that the amplitude spectrum $|S(\omega_t)|^2$ of the signal $s(t)$ at the instantaneous frequency ω_t can be approximated as:

$$|S(\omega_t)|^2 \cong 2\pi \frac{a^2(t)}{|\phi''(t)|} \tag{2}$$

i.e. the energy spectral density at the frequency $\omega_t = \phi'(t)$ (in radians) is *larger* when the rate of change of ω_t is *smaller*.

From equation (2) the amplitude modulation function $a(t)$ can be evaluated as:

$$a(t) \cong \sqrt{\frac{1}{2\pi}\left|S\big(\phi'(t)\big)\right|^2 |\phi''(t)|} \tag{3}$$

However the stationary phase approximation depends on the compression ratio BT, as shown in Fig. 4 where the PSLR of the compressed pulse is plotted versus BT. Considering the four cases as previously defined, in case (a), i.e. the Millett waveform [8], the output of the Matched Filter (MF) reaches the theoretical PSLR of -42 dB only when BT > 1000. In case (b) we choose the frequency modulation function (in radians) as a weighted sum between the Non Linear tangent FM term and the LFM one [9], [10], hence the name Hybrid-NLFM:

$$\phi'(t) = \pi B\left\{\alpha \frac{1}{tg(\gamma)} tg\left(\frac{2\gamma B}{T}\right) + (1-\alpha)\frac{2t}{T}\right\} \tag{4}$$

where $\alpha \in (0,1)$ is the relative weight, B is the sweep frequency, γ is the Non-Linear-tangent FM rate, being $t \in \left[-\frac{T}{2}, +\frac{T}{2}\right]$ and T denotes the pulse-width.

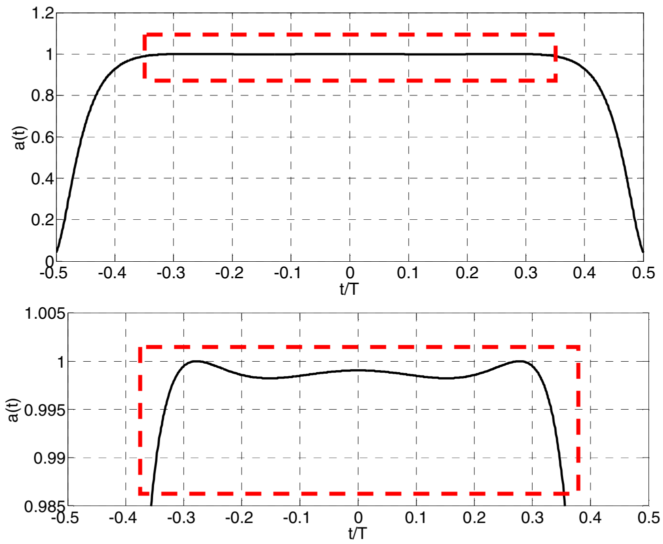

Figure 2. Normalized amplitude weighting for HNLFM case (b).

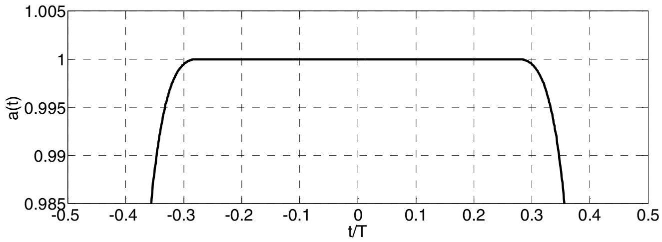

Figure 3. Normalized amplitude weighting for case (c), pseudo trapezoidal Amplitude Modulation.

If $s(t)$ is a signal with a Gaussian spectrum: $|S(\omega_t)|^2 = exp\left(-\beta\frac{\omega_t^2}{B^2}\right), \beta = 1$, using the optimized values of α and γ as evaluated in [9], [10], the loss results as low as 0.58 dB only. Fig. 4 shows the PSLR of the matched filter output. Analyzing case (b) with BT decreasing the approximation due to the principle of the stationary phase becomes worse causing an increase in the PSLR. However with BT = 64 the PSLR (-51 dB) is still compatible with many applications and for BT = 128 the PSL is -75 dB, suited to most applications. These excellent results shown in Fig. 4 case (b) are possible *if and only if* the amplitude weighting is strictly the one shown in Fig. 2. In practice it is hard to implement the requirement on $a(t)$, as ripples of the order of 1 over 1000 are not acceptable, see zoom of Fig. 2. It is preferable that the amplitude of the transmitted signal should be constant (with the amplifier working in saturation). However, as shown in Fig. 4 – case (d) – this choice leads to increase the PSLR of 25 ÷ 30 dB. An improvement can be obtained using a pseudo trapezoidal amplitude as reported in Fig. 3, where the ripples shown in Fig. 2 are removed. The corresponding PSLR result only 5 ÷ 10 dB greater than the optimized signal when $BT \geq 128$. The mismatching loss is negligible. Fig. 5 and 6 show, for cases (b), (c) and (d) the compressed pulse when BT is equal to 128 and 1024 respectively, T = 100 µs.

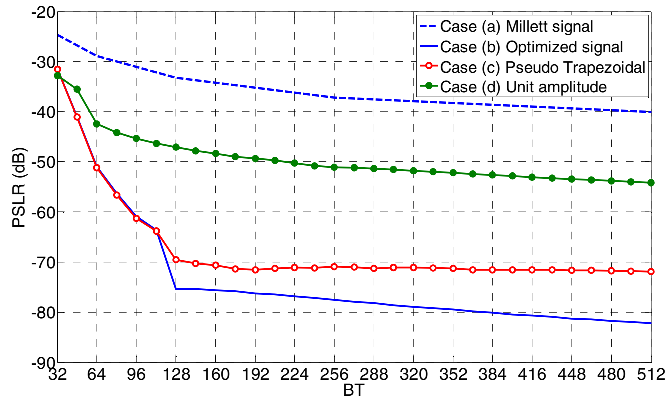

Figure 4. Effect of the compression ratio BT on the PSLR.

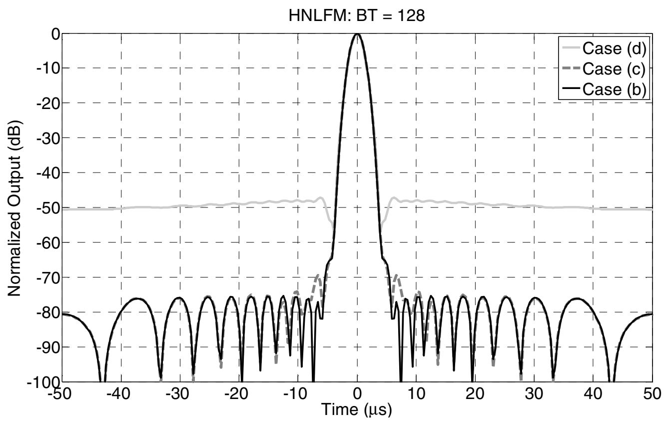

Figure 5. Compressed pulse: BT = 128, T = 100 µs, cases (b), (c) and (d).

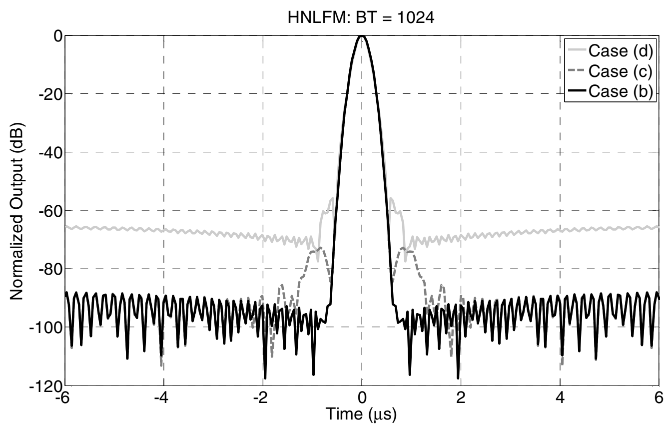

Figure 6. Compressed pulse: BT = 1024, T = 100 µs, cases (b), (c) and (d).

III. RANDOM WAVEFORM DESIGN

As seen above, the Hybrid-NLFM provides good PSLR. Changing the phase of 180° it can be obtained another signal with the same PSLR characteristics and a good orthogonality. However only two orthogonal deterministic waveforms are available in this way.

To design noisy waveforms that satisfie good PSLR and good cross-correlation, with a large number of generated sequences, noisy waveforms, named H-APCN (Hybrid-Advanced-Pulse-Compression-Noise) will be described. We will show that a trade-off between good PSLR and low cross-correlation is necessary.

The APCN technique has been introduced in [11], with a signal of form:

$$s_{APCN}(t) = a(t)e^{j\varphi(t)} \cdot e^{jk\cdot\varphi_{noise}(t)} \tag{5}$$

In our applications we have considered the deterministic terms $a(t)$ and $\varphi(t)$ equal to the one employed above for HNLFM case (c) with $\varphi_{noise}(t)$ *uniform-distributed* in $[0, 2\pi]$; the random phase parameter k (varying from 0 to 1) defines the interval for the random phase $[0, 2k\pi]$. The noise phase signal $e^{jk\cdot\varphi_{noise}(t)}$ is band limited with a Gaussian law having the same bandwidth of the deterministic signal $a(t)e^{j\varphi(t)}$. Increasing the noisy contribution, i.e. k, the band of the spectrum tends to the noisy one, as shown in Fig. 7 where the spectrum of the signal is shown with k varying.

Each generated signal is characterized by the autocorrelation and the cross-correlation with another realization having the same k.

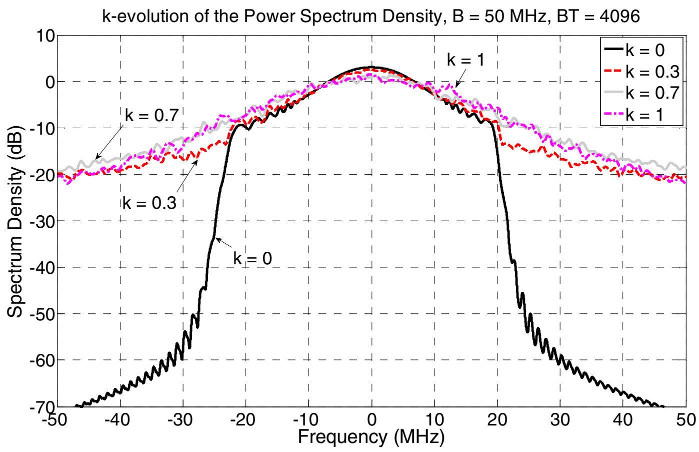

Figure 7. Spectrum versus k-phase parameter.

Fig. 8 shows the PSLR and the cross-correlation average level (averaging 10 realizations) versus k, with a confidence interval of 2 standard deviations, considering two values for the compression ratio: 128 and 4096. The PSLR in Fig. 8 starts from the level of -73 ÷ -69 dB that it occurs for HNLFM signal case (c) when the noise is not present ($k = 0$). The crosscorrelation is very high because the two signals involved are the same. As the k increases the PSLR level worsens and the cross-correlation level improves. Both auto and cross-correlation, when $k \to 1$, tend to the levels of a pure noisy signals [14].

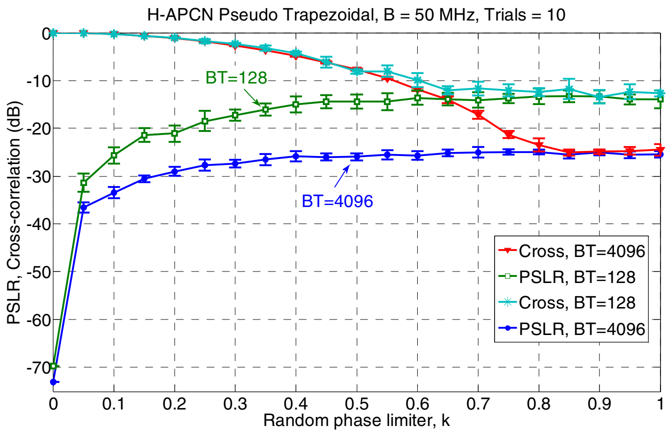

Figure 8. PSLR and Cross-correlation of H-APCN versus the random phase parameter k.

Fig. 9 shows the autocorrelation versus both random phase parameter k and range resolution (in meters) for a BT = 4096 with a band of 50 MHz. It is quite clear that as k increases towards one, the main lobe width decreases causing the rise of the sidelobes.

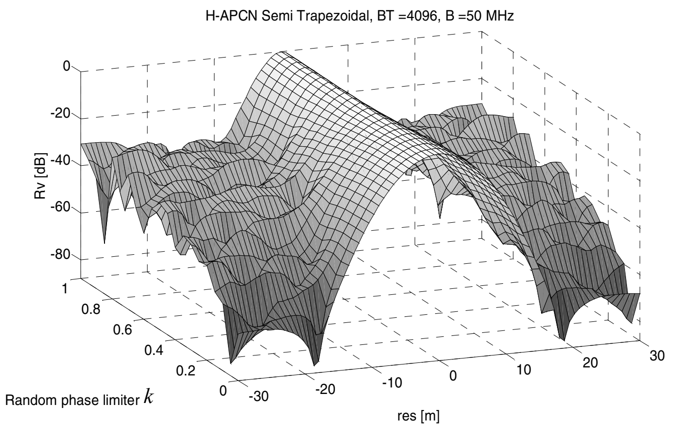

Figure 9. Autocorrealtion of H-APCN versus the random phase parameter and the range resolution.

IV. Conclusions and Future Perspectives

Modern radars are more and more demanding with respect to the quality of the transmitted waveforms, both in military applications, in which topics are the Low Probability of Intercept and the resistance to jamming (ECCM capability), and in civilian applications, where a main item is the resolution (in clear and in clutter) of closely-spaced targets with a high dynamic range. The emerging low-cost technologies make of increasing interest coherent solutions with a solid-state transmitter also for marine radars, which traditionally use the magnetron transmitter [12].

For deterministic pulse compression, the proposed solution, based on a Hybrid-NLFM, has the advantage of guaranteeing low PSLR, less than -50 dB for compression ratios greater than 256, while maintaining the transmitter to operate in saturation. To improve the PSLR performance an amplitude modulation is needed, reaching low PSLR, less than -70 dB for compression ratios greater than 128 tolerating a SNR loss of only 0.5 dB.

Moreover, the novel Noise Radar Technology and MIMO radar call for sophisticated waveforms that may be synthesized by combining in a suited way a deterministic, low-sidelobes signal (such as the ones described in this paper) with a noisy component, [13] - [16].

The main limit of this technique is due to the large compression ratio (of the order of thousands or hundred thousands) needed to reach acceptable low PSLR values. Further research is ongoing on both sides.

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