

Grain size distribution and particle shape effects on shear strength of sand – gravel mixtures

Effets dus à la granulométrie et à la forme des particules sur la résistance au cisaillement du sable – mélanges de gravier

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ABSTRACT

This paper focuses on the grain size distribution and shape effects on shear strength of three sand - gravel mixtures from Switzerland. A total of 28 direct shear tests in a large direct shear box apparatus have been performed to investigate the strength and dilatancy of sand - gravel mixtures. A relation between the grain size characteristics, the shape and the shear resistance has been found. For each mixture, the void ratio extent ($e_{\max} - e_{\min}$), and angle of repose, ϕ_{cv} are determined following the standard ASTM procedure. The experimental results are analysed in terms of the frictional and dilatant contributions to the strength of mixtures as a function of the grain size distribution, shape effects and their relative density. The particle shape effects are evaluated using the criteria used by Cho et al. (2006) for natural and crushed sands. The results show the dependency of the dilation gradient on the grain size distribution and the shape of the particle. The minimum and maximum void ratios are also dependent on the grading and the shape of the particles. The shear tests are interpreted in terms of friction angle as function of the dilation angle both evaluated from the tests at peak value and at the end of the shear phase. A useful empirical equation has been developed to evaluate the friction angle at constant volume and the gradient of dilation.

RÉSUMÉ

Cet article se concentre sur les effets sur la résistance au cisaillement dus à la granulométrie et à la forme des particules pour 3 mélanges de sable et de gravier en provenance de Suisse. Un total de 28 tests de cisaillement direct a été effectué dans une grande machine de cisaillement, afin d'obtenir des informations sur la résistance et la dilatation de mélanges sable – gravier. Une relation entre les caractéristiques granulométriques, la forme et la résistance au cisaillement a été découverte. Le indice des vides ($e_{\max} - e_{\min}$) ainsi l'angle de frottement résiduel ϕ_{cv} sont déterminés grâce à des procédures ASTM standard. Les résultats des expériences sont analysés en termes de contributions de dilatation et de friction à la résistance du mélange en fonction de la granulométrie, des effets dus à la forme et à la densité relative. Les effets dus à la forme sont évalués à l'aide des critères utilisés par Cho et al. 2006 pour des sables naturels et broyés. Les résultats montrent la dépendance de la dilatation du gradient de dilatation par rapport à la granulométrie et à la forme de la particule. Les indices des vides maximum et minimum dépendent eux aussi de la granulométrie et de la forme des particules. Les essais de cisaillement sont interprétés en termes d'angle de frottement en fonction de l'angle de dilatation, tous deux évalués à partir de tests à leur valeur maximale et à la fin de la phase de cisaillement. Une équation empirique utile a été évalué afin d'évaluer l'angle de frottement résiduel et le gradient de dilatation.

Keywords: coarse granular material, shear strength, grain size distribution, shape, void ratio extent.

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1 INTRODUCTION

Coarse materials exhibit dilatancy depending on their relative density, vertical stress, shape of the particles and grain size distribution (e.g. Simoni et al. 2006; Chakraborty et al. 2010). Bolton (1986) proposed a relationship to describe the dependency as a function of confining stress and relative density for sand-size quartzitic soils. The effects of grain size distribution and shape on shear strength behaviour are analysed in this paper. The tests are performed at low vertical stress representing the vertical stress acting on steep slope along the slip surface of shallow landslide.

In past studies (Simoni & Houlsby 2006), the addition of gravel to the mixtures, even at low fractions (less than 0.1 by volume), was found to cause an increase in peak friction angle (φ'_{peak}), which results both from higher dilatancy at failure (ψ_{max}) and higher constant volume friction angle (φ'_{cv}). Use of the minimum voids ratio (e_{min}) of the materials allowed the data for the two families of mixtures to be normalized and interpreted in terms of φ'_{cv} and the ratio $(\varphi'_{peak} - \varphi'_{cv}) / \psi_{max}$.

Experimental results are analysed in terms of the frictional and dilatant contributions to the strength of mixtures, as a function of their grain size distribution, expressed as α and d_{max} and their shape. The effect of grain size distribution and shape of sand – gravel mixtures on shear strength are explained on a physical basis, and empirical equations on the basis of α , d_{max} and shape are developed.

2 MATERIALS

Three soils used in this investigation were reconstituted to the specific grading from the following materials:

- Soil 1: sand-gravel mixture from fluvial deposits taken in the vicinity of Weiach (canton Zürich). The grain shape is mainly spheroidal and rounded (Fig. 1a);
- Soil 2: sand-gravel mixture taken in the vicinity of the Gruben glacier (canton Valais). The

shape of the grains is elongated but the edges are less sharp than those of Soil 2 (Fig. 1b).

- Soil 3: sand-gravel mixture taken at an altitude of about 2500m on a 40° steep slope from Schafberg in the vicinity of Pontresina (canton Graubünden). The shape of the grains is elongated with sharp edges (Fig. 1c);
- Four grain size distributions (GSD) have been chosen for each soil, as reported in Figure 2. The grain size distributions are characterized by equation 1:

$$P (\%) = \left[\frac{(d - d_{min})}{(d_{max} - d_{min})} \right]^\alpha \quad (1)$$

where $d_{min} = 0.06$ mm is the minimum diameter, $d_{max} = 16-8$ mm is the maximum diameter and $\alpha = 0.5-0.8$ is the exponential parameter

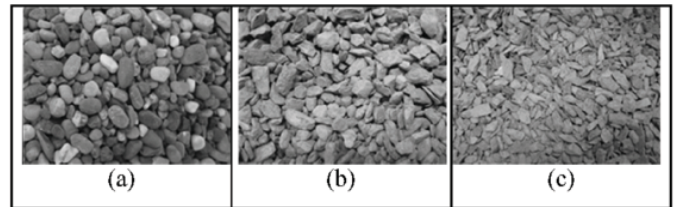


Figure 1. Sand – gravel mixtures: (a) Weiach; (b) Gruben; (c) Pontresina.

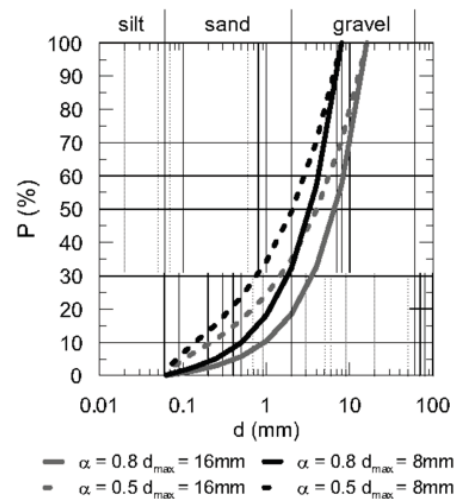


Figure 2. Grain size distribution tested.

3 SHEAR BOX APPARATUS

The shear tests were carried out in a large shear box, which was constructed at ETH-IGT. The box was originally designed in order to reproduce the large shear box for in situ testing, de-

signed by Teyssere (2005). It consists of a square aluminum box hosting a 250x250x110 mm specimen. A high entry value porous stone is set into the lower platen, whereas the box is closed by a 15 mm thick steel plate and a standard porous stone. Shearing is applied by moving the lower part generally at constant rate.

Standard instrumentation of the box consists of: 1 horizontal LVDT (Linear Variable Differential Transformer, to measure displacement with sub-millimetric precision); 3 vertical LVDTs; 2 horizontal “force measurement” cells; 1 vertical load cell; and a regulator to apply both horizontal and vertical forces

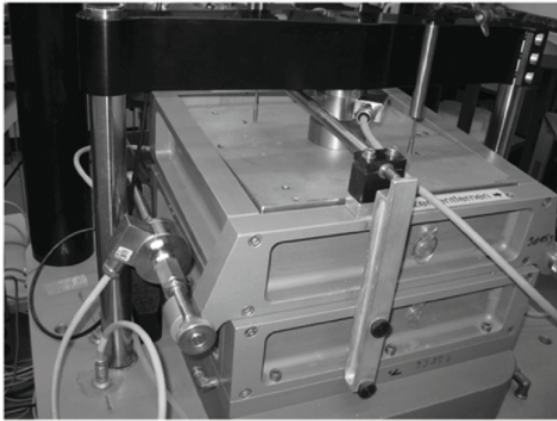


Figure 3. Large (250x250x150 mm) shear box apparatus used

The shear tests were performed at three vertical stresses $\sigma_v=25-50-100$ kPa, respectively as summarized in Table 1.

Table 1. Experimental program

| Soil | $d_{max} = 16\text{mm}$ | | $d_{max} = 8\text{mm}$ | |
|------|-------------------------|----------------|------------------------|----------------|
| | $\alpha = 0.5$ | $\alpha = 0.8$ | $\alpha = 0.5$ | $\alpha = 0.8$ |
| 1 | 25 kPa | 25 kPa | 25 kPa | 25 kPa |
| | 50 kPa | 50 kPa | 50 kPa | 50 kPa |
| | 100 kPa | 100 kPa | 100 kPa | 100 kPa |
| 2 | 25 kPa | 25 kPa | 25 kPa | 25 kPa |
| | 50 kPa | 50 kPa | 50 kPa | 50 kPa |
| | 100 kPa | 100 kPa | 100 kPa | 100 kPa |
| 3 | 25 kPa | - | 25 kPa | - |
| | 50 kPa | - | 50 kPa | - |

4 LABORATORY RESULTS

4.1 Maximum minimum void ratio, angle of repose

The specific gravity G_s has been evaluated for the three soils and is reported in Table 2.

Table 2. Specific gravity

| Material | G_s |
|----------------|-------|
| 1 - Weiach | 2.65 |
| 2 - Grüben | 2.76 |
| 3 - Pontresina | 2.62 |

The maximum void ratio e_{max} and e_{min} and the angle of repose ϕ_{cv} for the material were determined following the standard ASTM procedure (D4253; D4254, C1444). The results for e_{max} , e_{min} and void ratio extent are reported in Figure 3.

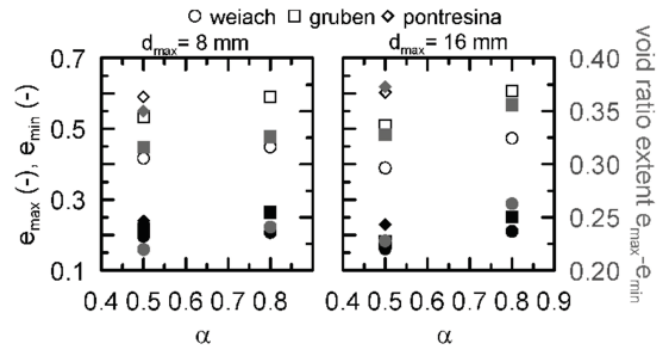


Figure 4. Maximum and minimum void ratios, void ratio extent versus α : (a) $d_{max}=8\text{mm}$; (b) $d_{max}=16\text{mm}$

The values measured increase with α and with the angularity of the particles. The value of ϕ_{cv} are reported in Table 3.

Table 3. Angle of repose

| ϕ_{cv} [°] | $d_{max} = 8$ | | $d_{max} = 16$ | |
|-----------------|----------------|----------------|----------------|----------------|
| | $\alpha = 0.5$ | $\alpha = 0.8$ | $\alpha = 0.5$ | $\alpha = 0.8$ |
| Weiach | 35 | 33 | 37 | 35 |
| Gruben | 38 | 36 | 38 | 35 |
| Pontresina | 38 | - | 37 | - |

4.2 Particle shape

The shape of the particles can be identified through the sphericity and the roundness of the particles, by visual comparison with charts. The sphericity and the roundness are determined in this work by observing individual grains through

a stereomicroscope and comparing the observation against a two-dimensional (2D) chart (Figure 5) as suggested by Cho et al. (2006).

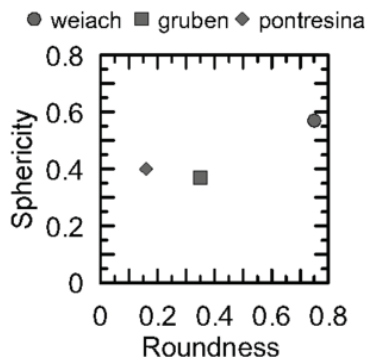


Figure 5. Sphericity and Roundness of the particles obtained comparing stereomicroscope and 2D chart (after Cho et al 2006).

4.3 Shear tests

The shear results for the three soil with different gradings are reported in Figures 6-8 in terms of shear stress τ - horizontal displacement h and vertical displacement v - horizontal displacement h .

4.3.1 Soil 1: Weiach

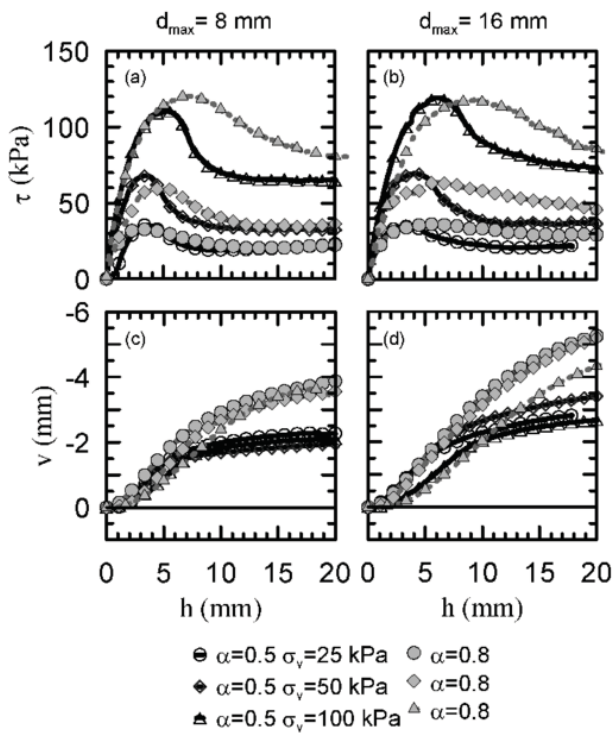


Figure 6. Shear test results for the Weiach soils: (a) τ - h $d_{max} = 8$ mm; (b) τ - h $d_{max} = 16$ mm; (c) v - h $d_{max} = 8$ mm; (d) v - h $d_{max} = 16$ mm.

The different gradings of this well rounded mixture change the form of the curves in the τ - h plane for the higher vertical stress, while the sample under smaller stresses behave in quite a similar way (Fig. 6a). This is consistent for both value of d_{max} (Fig. 6b).

In the v - h plane, the more well graded curves with $\alpha=0.5$ shows less dilatant behavior than those with $\alpha = 0.8$ for the same vertical stress (Fig. 6c and 6d). It is possible to distinguish two families of curves: those with a $v_{fin} \sim -2$ mm and the others with $v_{fin} \sim -3.75$ mm for $d_{max} = 8$ mm. Increasing d_{max} also increases the value of v_{fin} (Fig. 6d).

4.3.2 Soil 2: Gruben

As for Weiach, the different gradings change the form of the curve for $\sigma_v = 100$ kPa in the τ - h plane, while sample tested under $\sigma_v = 25$ -50 kPa behave quite similar (Fig. 7a). The behavior is also similar, irrespective of d_{max} (Fig. 7b).

The dilatant behaviour is less pronounced for the more well graded soil ($\alpha=0.5$), with the same $v_{fin} \sim -2.75$ mm for both value of d_{max} (Fig. 7c),

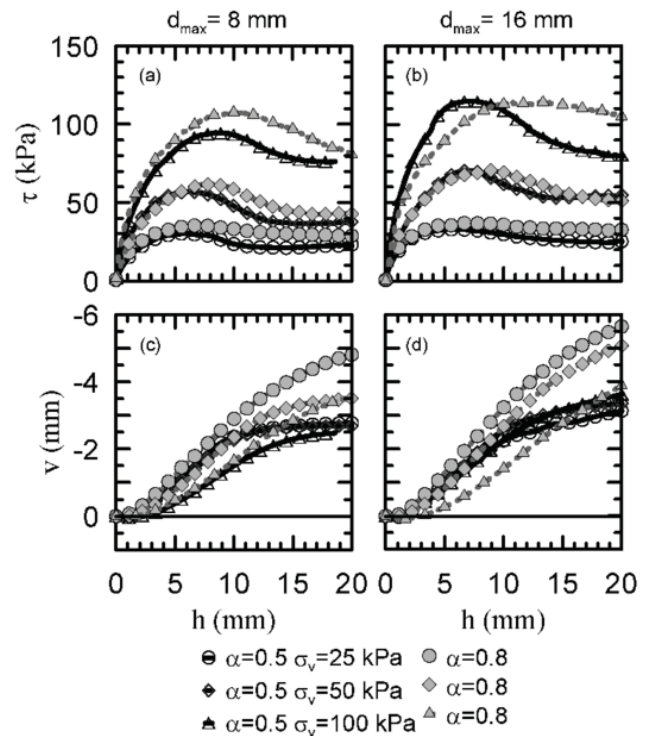


Figure 7. Shear tests results for Gruben soils: (a) τ - h $d_{max} = 8$ mm; (b) τ - h $d_{max} = 16$ mm; (c) v - h $d_{max} = 8$ mm; (d) v - h $d_{max} = 16$ mm.

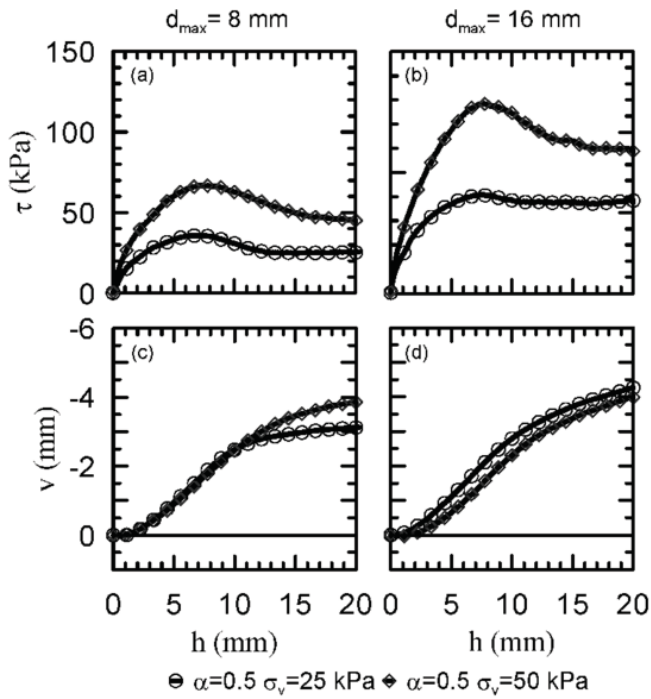


Figure 8. Shear tests results for Pontresina soils: (a) $\tau - h$ $d_{max} = 8$ mm; (b) $\tau - h$ $d_{max} = 16$ mm; (c) $v - h$ $d_{max} = 8$ mm; (d) $v - h$ $d_{max} = 16$ mm.

while those with $\alpha = 0.8$ show a more dispersed final value.

The v_{fin} also increases for the different gradings with $d_{max} = 16$ mm (Fig. 7d).

4.3.3 Soil 3: Pontresina

Increasing of d_{max} has a much greater influence for the Pontresina soil in the $\tau - h$ plane for both the peak and the final values (Figs. 8a & b). The increase in d_{max} also increases the final value of v_{fin} .

5 ANALYSES OF THE RESULTS

The direct shear tests can be interpreted in different way (e.g. Bolton 1986, Simoni & Houlsby 2006). The measurement of vertical v and horizontal h displacements are used to calculate the rate of dilation ($\delta v / \delta h$). Assuming that the horizontal plane in the shear box is a zero extension line, the angle of dilation can be deduced from the Mohr's circle of strain increments as $\tan \Psi = -\delta v / \delta h$.

The dependency of the friction angle $\phi' = \tan^{-1}(\tau / \sigma'_v)$ on the angle of dilatancy $\Psi = -\tan^{-1}(\delta v / \delta h)$ has been analysed (Figure 9).

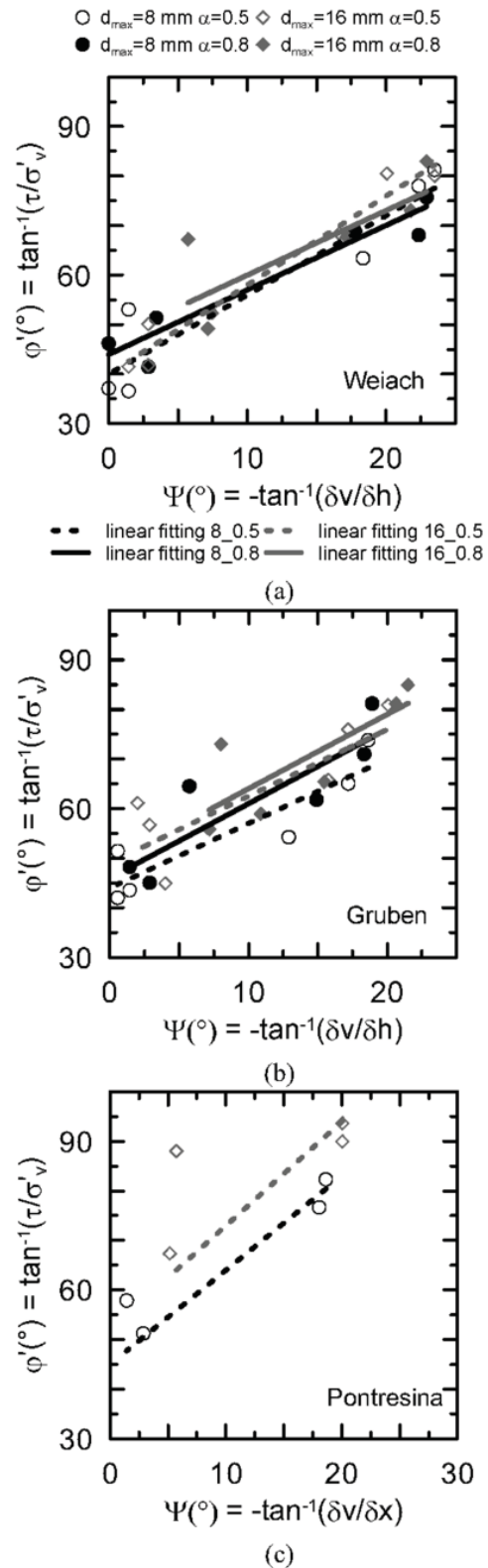


Figure 9. Friction angle versus dilation angle: (a) Weiach; (b) Gruben; (c) Pontresina.

The point representative of the peak and ultimate value for each set of data has been fitted with the following equation:

$$\varphi' = b \cdot \Psi + \varphi'_{cv} \quad (2)$$

where b is related to the dilation and φ'_{cv} the friction angle at constant volume.

The parameters obtained are summarized in Table 4 and the best fit line gives φ'_{cv} as the shearing resistance of a sample which would exhibit zero dilatancy at failure. This approach also facilitates the comparison with Bolton's empirical equation, which predict a constant $b = 0.8$.

Table 4. Parameters for equation (2)

| Soil | d_{max} (mm) | α (-) | φ'_{cv} (°) | b (-) |
|------------|-------------------|-----------------|------------------------|------------|
| Weiach | 8 | 0.5 | 40 | 1.6 |
| Weiach | 8 | 0.8 | 44 | 1.3 |
| Weiach | 16 | 0.5 | 40 | 1.8 |
| Weiach | 16 | 0.8 | 47 | 1.3 |
| Gruben | 8 | 0.5 | 44 | 1.3 |
| Gruben | 8 | 0.8 | 46 | 1.5 |
| Gruben | 16 | 0.5 | 49 | 1.3 |
| Gruben | 16 | 0.8 | 49 | 1.5 |
| Pontresina | 8 | 0.5 | 45 | 1.9 |
| Pontresina | 16 | 0.5 | 52 | 2.1 |

Increasing the gravel fraction for soil 1 increases φ'_{cv} and decreases the gradient of dilation. The same arising d_{max} . The rate of dilation b rises with the gravel fraction for soils 2 and 3, which also exhibit a strong dependency of the φ'_{cv} on d_{max} . More data would be required for more general conclusion.

It is interesting to note that the shape of particles play an important role on the value of the gradient of dilation b . It seems that the gradient of dilation b increases with gravel fraction for the less round and spherical particles (Gruben, Pontresina) whereas it decreases for the more rounded Weiach soil as α rises. This behaviour would be likely to change under higher normal stresses where abrasion is likely to occur on the

angular soils. The friction angle at constant volume increases mostly with increasing gravel fraction.

The shape of the particles in terms of roundness and sphericity has been evaluated comparing the microscope observation with a 2D chart.

CONCLUSION

An experimental investigation has been performed on reconstituted sand-gravel mixtures from three different places in Switzerland. The vertical stresses were chosen to represent conditions for shear surfaces shallow landslides, characterised by low stresses.

The results shows a dependency of the peak friction angle on dilatancy that change with grain size distribution and shape of the particles. The gradient of dilation decreases with increasing gravel fraction for the more round and spherical particles, while the opposite is observed for more angular and elongated particles.

The addition of coarse particles causes several effects on the behaviour of the resulting material. The void ratio extent increases with addition of gravel (increasing α) and with roundness of the particle. Further investigation would be required to complete the study of the effects of particle shape and grain size distribution.

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