Experimental investigation of the evolution of grading of an artificial material with crushable grains under different loading conditions

Casini, Francesca
Institute for Geotechnical Engineering, ETH Zurich, Switzerland, francesca.casini@igt.baug.ethz.ch

Viggiani, Giulia M.B.
Dipartimento di Ingegneria Civile, Università di Roma Tor Vergata, Italy, viggiani@uniroma2.it

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ABSTRACT: The basic constitutive properties of granular materials depend on their grading. Crushing or breakage of particles under compression or shear modify the grain size distribution, with a tendency for the percentage of fine material to increase. It follows that the frictional properties of the material and the critical states are modified as a consequence of the changes in grain size distribution and the available range of packing densities. This paper shows the results of an experimental investigation of the evolution of the grading of an artificial granular material, consisting of crushed expanded clay pellets, most commonly known under the brand name LECA, under different loading conditions. The changes of grading of the material after isotropic, one-dimensional and constant mean effective stress triaxial compression were described using a single parameter based on the ratio of the areas under the current and an ultimate cumulative particle size distribution which were both assumed to be consistent with self similar grading with varying fractal dimension.

1. INTRODUCTION

Loading of geotechnical structures such as embankments, foundations, and pavements may result in particle breakage of the constituent granular materials when the stresses imposed on the particles exceed their strength. Particle breakage depends on a number of ‘macro-scale’ parameters, such as stress level, grading, and voids ratio, as well as on the characteristics of the constituent particles, such as size, shape, strength, and mineral composition (Hardin, 1985; Coop, 1990; Lade *et al*., 1996; McDowell and Bolton, 1998; Nakata *et al*., 1999; Coop *et al*., 2004).

Different measures have been suggested to quantify the amount of breakage undergone by a sample of granular material. Hardin (1985) introduced the relative breakage, B_r, based on the relative position of the current cumulative distribution from the initial cumulative distribution and a cut-off value of ‘silt’ particle size (of 0.074 mm). The use of the latter implied that, in the fragmentation process, all particles will eventually become finer than the (arbitrary) cut-off value. This conflicts with the growing understanding that the grain size distribution of aggregates of any initial grading, under extremely large confining pressure and extensive shear strains, tends to a self-similar (fractal) distribution (Turcotte, 1986; McDowell and Bolton, 1998).

Several studies (*e.g.*: Sammis *et al*., 1987; Tsoungui *et al*.; 1999) have confirmed that the predominant effect of particle crushing is to increase the proportion of fine material without particularly changing the size of the largest particles. This has been explained with the tendency of larger particles to get cushioned by surrounding smaller particles, which gives them higher coordination numbers and makes them more resistant to crushing. Smaller particles, with smaller coordination numbers, are more likely to be crushed in the fragmentation process. In other words, the cushioning effect due to the large coordination number for larger particles outweighs the effect of reducing strength with increasing particle size.

The exact form of the ultimate distribution is still an open question, which would not necessarily be fractal, although it has been suggested that a fractal distribution with a fractal dimension a of around 2.5–2.6 (Sammis *et al.*, 1987; Palmer and Sanderson, 1991; McDowell *et al*., 1996) may be used in most practical cases. Fractal characteristics may not manifest over an infinite range of sizes, because particles smaller than a so-called "comminution" limit do not fracture (Kendall, 1978; McDowell, 2005). Lade *et al*. (1996) found that if uniform sand is crushed, the resulting grain size distribution approaches that of a well graded soil for a large compressive loads. However, before reaching a well graded particle distribution, the granular assembly will experience gradual changes in particle size depending in the level of progressive load being applied.

Einav (2007) has suggested to adjust the original definition of the relative breakage by Hardin (1985) to weigh from zero to one the relative proximity of the current grain size cumulative distribution, from an initial cumulative distribution and an ultimate cumulative distribution. In his work, the Author also assumed that the breakage of the different fractions is fractionally independent, *i.e.* is the same for all the particle fractions, and tested, with some success, the effectiveness of this hypothesis against the experimental results obtained by Coop *et al*. (2004) on a initially uniformly graded Dog's Bay sand subjected to progressively larger shear strains in a ring shear apparatus, after one dimensional compression.

Wood and Maeda (2008) suggested to adopt a single initial cumulative distribution as a vertical line through the maximum grain diameter. They also noted that the implication of a fractal final grain size distribution is that the grading continues indefinitely to finer and finer particles, while, as noted above, there may be some comminution limit below which particles do not break, *i.e.* that there could exist a limiting grading to which a soil might tend to as its particles crush and that this limiting grading does not have to be fractal. The Authors also defined a grading state index, I_G, comparing areas under the current particle size distribution and under the limiting (possibly but not necessarily fractal) distribution. The grading index is zero for a single size material and 1 at the (possibly) fractal limit. Any given soil sample would have an initial intermediate value of I_G.

Nakata *et al.* 1999, used experimental results to relate the individual particle crushing to the particle strength variability. They also studied the influence of various soil parameters such as the uniformity coefficient, the initial grain size distribution, and the void ratio on the compression characteristics of the soil, showing that the amount of grain crushing under isotropic loading conditions is much lower than under shearing. Luzzani and Coop (2002) studied the relationship between volume change and particle breakage during shearing of sand in ring-shear and direct shear apparatus. They also found that the breakage caused by shearing is much larger than the one caused by compression.

The long term objective of the work described in this paper is a deeper understanding of the mechanical behaviour of soils with crushable grains, weak rocks, or cemented aggregates whose bonds suffer progressive degradation due to applied loads. From the point of view of constitutive modelling, the question to be addressed is how microscopic degradation phenomena, such as grain crushing, affect the macroscopic properties of a granular aggregate. There are many angles of attack to this question. For plasticity based models this amounts to asking how the evolution of the micro-structure affects yield surfaces, flow rules and hardening laws (De Simone and Tamagnini 2005). To this purpose the limits of applicability of existing plasticity-based models may be probed experimentally, to single out those macroscopic parameters which are sensitive to changes of microstructure, thus identify the new ingredients that are needed to capture the macroscopic fingerprints of the microscopic processes (Cecconi *et al*. 2002). Another possibility, explored, *e.g.* by McDowell and Bolton (1998), is to try and deduce macroscopic constitutive equations from the underlying microscopic process. More recently, the idea of a fractal evolution of particle size has been adopted by introducing a breakage parameter into energy dissipation assumptions of a crushable soil treated as a continuum (Einav, 2007), and through links between particle crushing and critical states in soils (Muir Wood and Maeda, 2006; Russell *et al.* 2009).

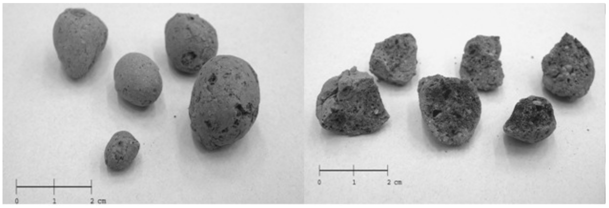
Whatever the point of view adopted in constitutive modelling, the evolution of the grading of the material under different loading conditions must be examined experimentally to test the constitutive assumptions of the models.

2. EXPERIMENTAL WORK

2.1 Tested material

The degradation processes associated with grain crushing affect the natural behaviour of many natural geotechnical materials such as pyroclastic weak rocks, carbonate sands, calcarenites and residual soils. However, systematic experimental investigation of grain crushing for natural materials is often difficult due to the relatively high stress required to crush the grains and the variability and heterogeneity of natural deposits, which makes it difficult to obtain repeatable results. For these reasons the experimental work was carried out on an artificial granular material consisting of crushed expanded clay pellets, whose grains break at relatively low stress. The material is commercially available under the acronym LECA (Light Expanded Clay Aggregate) and is obtained through an industrial process.

The clay is extracted from relatively shallow mines and then homogenized, moistened, and broken up with grinding equipment and rolling mills. The main phase of the production cycle takes place in a long rotary kiln. The clay enters the kiln from one end and moves along it, gradually increasing its temperature. At the other end of the kiln the temperature reaches approximately 1200 °C, at which point the clay is in a molten state and the expansion process commences providing a cellular vitreous interior to each pellet. Rolling of the pellets within the kiln gives them a round shape and creates a hard outer shell (see Fig. 1). The expanded clay pellets are then screened into their various fractions and made commercially available both as intact (so-called "granular") or crushed, in different grain sizes.



(a) (b)

Figure 1. LECA pellets: (a) whole, (b) broken, showing harder external shell and porous interior

The cumulative grain size distributions by weight of samples of the material obtained from the producer in the finer grain sizes (0-2 mm and 2-4 mm for crushed material, and 0-4 and 4-6, for the granular material) were determined by sieving and compared with the nominal ones. The grading curves obtained for the granular material are within the nominal range, while those obtained for the crushed material show a significant increase of the percentage of material in the range of grain sizes between 0.1 mm and 0.2 mm and a reduction in the range between 0.2 mm and 0.5 mm. This is likely to be due to further crushing during carriage and handling, favoured by the relatively high angularity of the grains ($A_{2D} \cong 500$, Miura *et al.*, 1997). Because of its high susceptibility to crushing, the experimental programme described in this work was carried out on crushed material in the 0-2 mm range of grain sizes.

The main physical characteristic of the material is the very low apparent unit weight of the particles; this is due to the existence of a double order of porosity: "inter-granular", *i.e.* voids existing between particles, and "intra-granular", *i.e.* closed voids existing within individual particles (see Fig. 2).

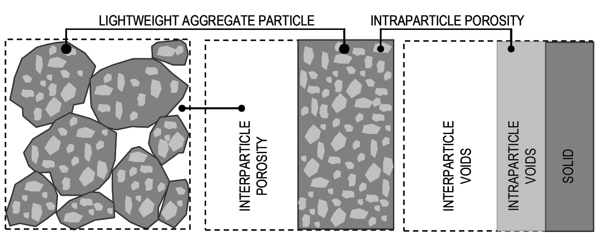


Figure 2. Inter-granular and intra-granular porosity.

Figure 3 shows the "apparent" unit weight of the particles, γ_s, as a function of their size, d, as determined in the laboratory on different fractions of crushed material. For comparison, the same figure shows also the variation of the apparent unit weight of particles with grain size for different pyroclastic materials of the Colli Albani (Cecconi *et al.*, 2009). In this case, the dependency of apparent unit weight on particle size is much less pronounced showing the smaller significance of intra-granular porosity for the natural materials. For LECA, the value of γ_s increases significantly with decreasing grain size and tends to the unit weight of the constituent clay, $\gamma_a \cong 26.5\ \mathrm{kN/m^3}$. The experimental values of γ_s were fitted with the following equation:

$$\gamma_s(\mathrm{kN/m^3}) = a \cdot d(\mathrm{mm})^{-b} \tag{1}$$

with the values of a and b reported in Figure 3, down to $d = 0.063$ mm, and were taken to be constant and equal to $\gamma_a = 26.5$ for smaller diameters.

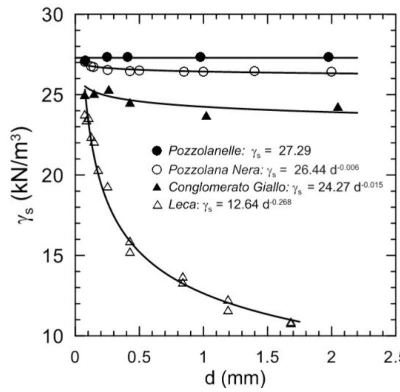


Figure 3. Apparent unit weight of particles as a function of grain size, for different materials (after Cecconi *et al.*, 2009) .

2.2 *Dependence of physical properties on initial grading*

The material was reconstituted at different grain size distributions by weight with the same value of $D_{50} = 0.5$ mm and different values of the coefficient of uniformity, $U\ (= D_{60}/D_{10}) = 3.5$, 7, 14, and 28, or with the same $U = 7$, and different values of $D_{50} = 0.25$, 0.5, and 1 mm. For each grading, the maximum and minimum voids ratio were determined using non standard procedures meant to avoid further crushing; in particular the minimum voids ratio was obtained vibrating the samples at very low energy. The experimental values of $(e_{max} - e_{min})$ obtained for the granular material at different values of U and D_{50}, of the order of $0.4 \div 0.5$, are within the range of values determined by Miura *et al*. (1997) for different granular materials, while the values obtained for the crushed material, of the order of $0.9 \div 1.0$, are much larger than those obtained for other granular materials with permanent grains, including natural sands, light weight aggregates and glass ballottini. These very high values of voids ratio at which it is possible to reconstitute the material are probably due to the very rough surface of the particles of crushed material, which favours a very "open" structure.

Both for granular and crushed LECA, consistently with the available data for granular materials with permanent grains (Miura *et al*. 1997) the values of $(e_{max} - e_{min})$ is independent of U; for the crushed material, contrary to what reported in the literature, $(e_{max} - e_{min})$ does nor decrease with D_{50}. The values of the constant volume friction angle of the material ($\varphi'_{cv} = 35° \div 39°$), determined forming a cone of sand by pluviation and measuring the inclination of its slopes, show a slight increase of the friction angle with D_{50} and U, and are in good agreement with those measured on other granular materials with permanent grains (Miura *et al.*, 1998).

2.3 Experimental programme

The experimental programme includes isotropic, one dimensional and triaxial constant mean stress compression test, carried out at increasing confining pressures, on samples of each of the reconstituted grain size distributions (see Fig. 4).

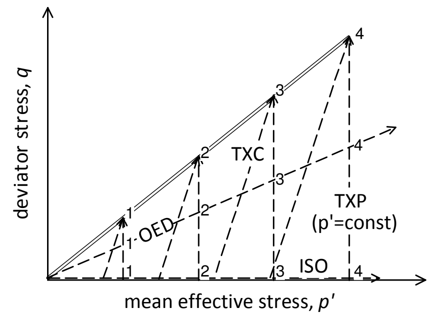


Figure 4. Experimental programme.

In this paper, a limited set of results obtained on samples having all the same an initial grading with $D_{50} = 0.5$ mm and $U = 7$ will be discussed.

3. EXPERIMENTAL RESULTS

For a material such as LECA, in which the intra-granular porosity causes the apparent unit weight of particles γ_s to depend on their size, it is necessary to distinguish between grain size distribution by weight and grain size distribution by volume.

The grading of a sample of granular material is generally determined experimentally using a discrete series of sieves with dimension Δ_i ($i = 1, 2, \ldots, n$) to measure the weights of the particles that are finer than the sieve size Δ_i and larger than the sieve size Δ_{i-1}:

$$W_i = W(\Delta_{i-1} < d < \Delta_i) \tag{2}$$

The cumulative grain size distribution by weight is then computed as:

$$F(d_j) = \frac{W(d < \Delta_1) + \sum_{2}^{j \le n} W_i}{W_\mathrm{T}} \tag{3}$$

in which $W(d < \Delta_1)$ is the material passing through all the sieves in the series and:

$$W_\mathrm{T} = W(d < \Delta_1) + \sum_{2}^{n} W_i \tag{4}$$

is the total weight of the sample. The volumes of particles that are finer than the sieve size Δ_i and larger than the sieve size Δ_{i-1}, can be computed as $V_i = W_i/\gamma_{si}$, in which γ_{si} is the average apparent unit weight of the particles in the size range $\Delta_{i-1} < d < \Delta_i$. Using eq.(1) it is easy to show that:

$$\gamma_{si} = \frac{1}{\Delta_i - \Delta_{i-1}} \int_{\Delta_{i-1}}^{\Delta_i} \gamma_s(d)\,\mathrm{d}d = \frac{a(\Delta_i^{1-b} - \Delta_i^{1-b})}{(\Delta_i - \Delta_{i-1})(1-b)} \tag{5}$$

The cumulative grain size distribution by volume can then be computed as:

$$F_V(d_j) = \frac{W(d < \Delta_1)/\gamma_{s1} + \sum_{2}^{j \le n} V_i}{V_\mathrm{T}} \tag{6}$$

in which γ_{s1} is the average unit weight of the material with dimensions smaller than Δ_1 and:

$$V_\mathrm{T} = W(d < \Delta_1)/\gamma_{s1} + \sum_{2}^{n} V_i \tag{7}$$

is the total (apparent) volume of the solids in the sample. In the following we shall refer to the grain size distribution by weight, unless explicitly stated.

Figure 5 shows the cumulative grain size distributions obtained at the end of isotropic, one-dimensional, triaxial and constant mean stress compression for final values of the mean effective stress equal to 770 kPa and 1450 kPa, respectively. In all cases, the cumulative grain size distribution at the end of the test is rotated upwards and translated leftwards, with an increase of the fine fraction at an almost constant value of the maximum particle size, d_M. It has to be noted, however, that the maximum particle size d_M is likely to be different from Δ_n (maximum dimension of the sieve series) and unknown, even if, of course, $d_\mathrm{M} \le \Delta_\mathrm{n}$. Small changes of d_M with load and stress path are difficult to detect in the laboratory because the spacing of two following sieves around d_M is finite and not fine enough.

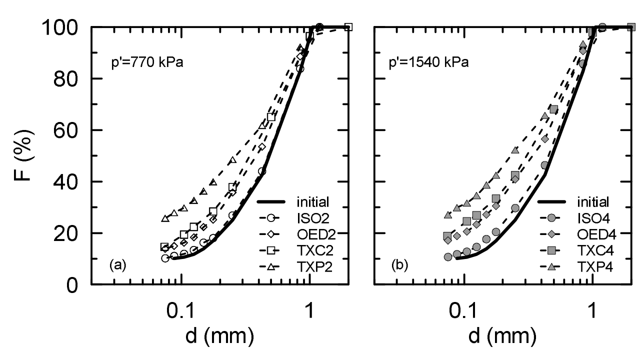


Figure 5. Cumulative grain size distributions for different tests carried out at same values of mean effective stress

Figure 6 shows the cumulative grain size distributions obtained at the end of all isotropic, one-dimensional, triaxial, and constant mean stress compression tests. It is evident that isotropic compression (Fig. 6(a)), even at relatively large mean effective stress (ISO4,

p' = 1450 kPa), is not causing major crushing with only very limited changes of the cumulative grain size distribution. One-dimensional compression and conventional triaxial compression cause a similar amount of noticeable crushing; very marked changes of grain size distribution due to grain crushing are obtained after shearing the material at constant mean effective stress.

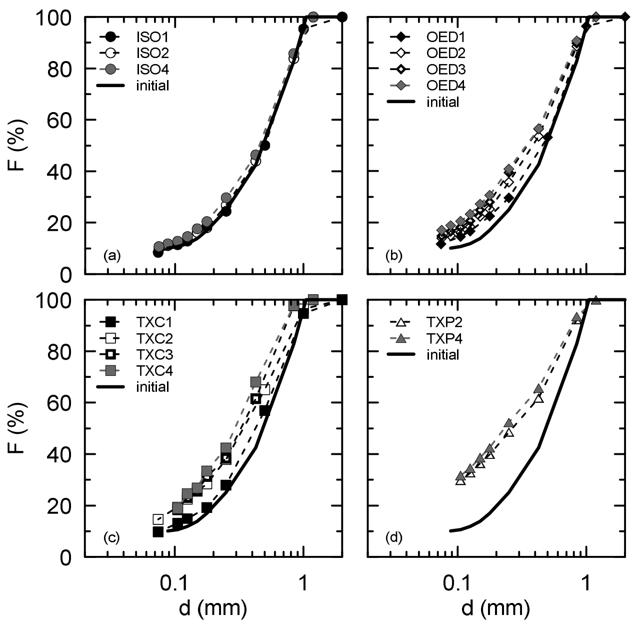


Figure 6. Cumulative grain size distributions at the end of all (a) isotropic, (b) one dimensional, (c) conventional triaxial, and (d) constant p' triaxial compression tests

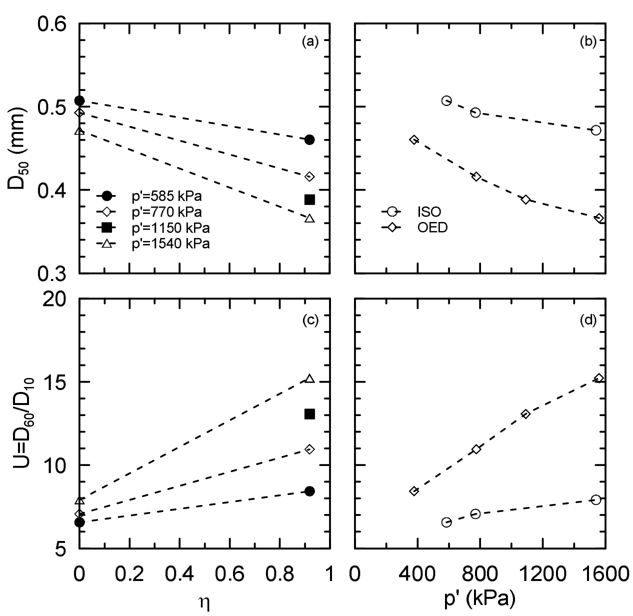


Figure 7. Observed reductions of D_{50} and increases of U: (a) and (c) as a function of η, (b) and (d) as a function of p'.

Figures 7 (a) and (b) summarise the results obtained for all tests carried out at constant η, *i.e.* isotropic compression ($\eta = 0$) and oedometer tests ($\eta \cong 0.92$).

It appears that grain crushing, expressed as a first approximation by the reduction of D_{50}, depends on both the stress ratio, η (= q/p') and on the maximum mean effective stress reached at the end of the test, p'. The same trend is also confirmed by the increase of the uniformity $U = D_{60}/D_{10}$ as shown in Figure 8 (c) and (d).

4. ANALYSIS OF RESULTS

For a fractal grain size distribution, the number of particles that have dimension Δ larger than d is:

$$N(d > \Delta) = Cd^{-\alpha} \tag{8}$$

in which C is a constant and α is the so-called fractal dimension (Turcotte, 1986), while the number of particles in a fraction is (McDowell *et al.*, 1996):

$$dN(\Delta) = \alpha C d^{-\alpha-1} d\Delta \tag{9}$$

The (continuous) cumulative grain size distribution is equal to:

$$F(d) = \frac{W(d < \Delta)}{W_{\mathrm{T}}} = \frac{\int_0^d s\gamma_s(\Delta)\Delta^3 dN(\Delta)}{\int_0^{d_{\mathrm{M}}} s\gamma_s(\Delta)\Delta^3 dN(\Delta)} \tag{10}$$

in which s is a shape factor and d_{M} is the maximum particle size. Substituting eqs. (1) and (9) into eq.(10), the following expression for the cumulative grain size distribution by weight of a fractal grain size distribution is obtained:

$$F(d) = \frac{W(d < \Delta)}{W} = \left(\frac{d}{d_{\mathrm{M}}}\right)^{3-\alpha-b} = \left(\frac{d}{d_{\mathrm{M}}}\right)^{\beta} \tag{11}$$

In which d_{M} is the largest particle size in the sample.

Using equation (1), it is easy to show that the corresponding cumulative grain size distribution by volume is:

$$F_{\mathrm{V}}(d) = \frac{V(d < \Delta)}{V_{\mathrm{T}}} = \left(\frac{d}{d_{\mathrm{M}}}\right)^{3-\alpha} = \left(\frac{d}{d_{\mathrm{M}}}\right)^{\beta_v} \tag{12}$$

in which $\beta_{\mathrm{v}} = \beta + b$. For a material in which the apparent unit weight does not depend on grain size ($b = 0$) the cumulative grain size distribution by weight and by volume are the same ($\beta_{\mathrm{v}} = \beta$).

We will assume that the ultimate grain size cumulative distribution by weight is fractal:

$$F_u(d) = \left(\frac{d}{d_M}\right)^{\beta_u} \tag{13}$$

in which the largest particle size in the sample, d_M, is taken not to change as grading evolves by crushing, while the exponent β_u is related to the ultimate fractal dimension, $\beta_u = (3 - \alpha_u - b)$. In the derivations above, it is implicitly assumed that the minimum grain size dimension is $d_m = 0$, *i.e.* that there is no comminution limit to the grain size.

We further assume that also the initial and current (evolving) cumulative grain size distributions may be written in a similar form as the ultimate grain size distribution:

$$F_0(d) = \left(\frac{d}{d_M}\right)^{\beta_0} \qquad F(d) = \left(\frac{d}{d_M}\right)^{\beta} \qquad (14)$$

Although the initial cumulative grain size distribution may not be self similar, in most practical cases, the cumulative grain size distributions would be reasonably fitted by an equation similar to (14) and the loading required to reduce the initial distribution to a self similar distribution would be small. For instance, even the very small variations of grading obtained under isotropic compression, significantly improve the best fit of the grain size distribution using a self similar equation.

The initial cumulative grain size distribution and those obtained at the end of each test, were fitted using equation (14). Both parameters defining equation (14), namely β and d_M, were obtained by regression of the experimental data; it turned out that d_M is practically constant, while β reduces due to grain crushing.

Figure 8 shows the values of exponent β obtained from the best fit of the final cumulative grain size distribution curves by weight or by volume for different types of tests at two values of mean effective stress. Also reported in Figure 8 with open symbols are the ideal values of β_v (= $\beta + b$); the differences between the actual value of β_v and the theoretical value are due to the fact that the variation of apparent unit weight with grain size is not exponential, as per equation (1), for the smaller diameter fraction. In the following, we will take the minimum observed value of β to be the exponent of the ultimate cumulative grain size distribution ($\beta_u = 0.483$).

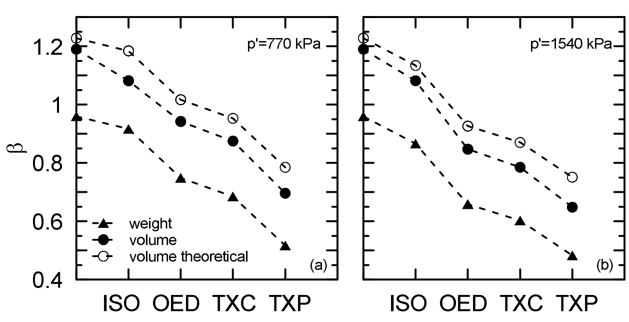


Figure 8. Values of exponent β and β_v for different tests carried out at two values of mean effective stress

Figure 9 shows the values of exponent β obtained from the best fit of the final cumulative grain size distribution curves by weight or by volume for different types of tests. Once again, isotropic compression (Fig. 9(a)), even at relatively large mean effective stress, does not causing major changes of β, one-dimensional compression and conventional triaxial compression cause a similar reduction of β, while very marked reduction of β due to grain crushing are obtained after shearing the material at constant mean effective stress.

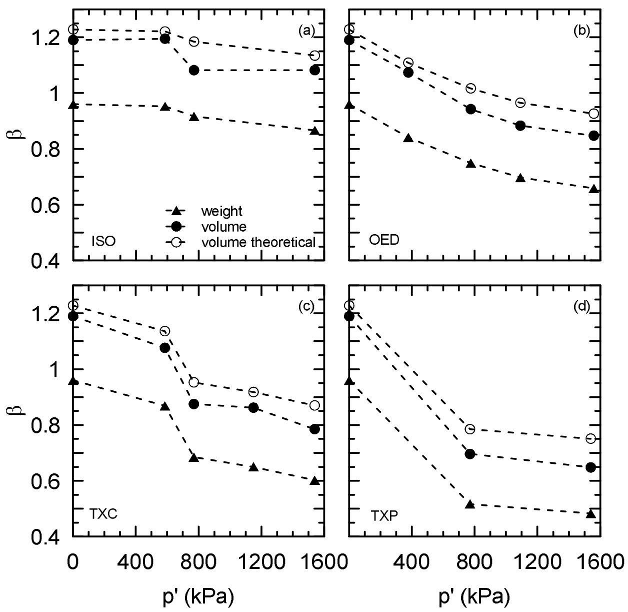


Figure 9. Values of β and β_v at the end of all (a) isotropic, (b) one dimensional, (c) conventional triaxial, and (d) constant p' triaxial compression tests

The area below the current cumulative distribution may be obtained by integration over the log$d$ scale:

$$A = \int_0^{d_M} F(d)\mathrm{d}(\log d) = \frac{1}{\ln 10}\int_0^{d_M} F(d)d^{-1}\mathrm{d}d = \frac{0.434}{\beta} \quad (15)$$

The breakage potential, B_p, and the total breakage, B_t, can then be obtained simply by subtracting the area below the initial grain size distribution from the area below the ultimate grain size distribution or below the current grain size distribution (see Fig. 10):

$$B_p = A_u - A_0 = 0.434\left(\frac{1}{\beta_u} - \frac{1}{\beta_0}\right) \qquad (16)$$

$$B_t = A - A_0 = 0.434\left(\frac{1}{\beta} - \frac{1}{\beta_0}\right) \qquad (17)$$

The definition of relative breakage follows combining equations (6) and (7):

$$B_r = \frac{A - A_0}{A_u - A_0} = \left(\frac{1}{\beta} - \frac{1}{\beta_0}\right) \bigg/ \left(\frac{1}{\beta_u} - \frac{1}{\beta_0}\right) \qquad (18)$$

Given the ultimate grain size cumulative distribution, the relative breakage is limited by $0 \le B_r \le 1$, where $B_r = 0$ denotes unbroken material and $B_r = 1$ represents complete breakage.

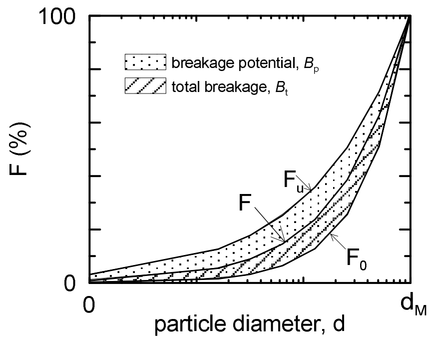


Figure 10. Definition of breakage potential, B_p, and total breakage B_t

It is of course possible to define a universal initial cumulative distribution as a vertical line through d_M as proposed, among others, by Wood & Maeda (2006). This would correspond to assuming that the initial cumulative grain size distribution is the heavy side around d_M ($\beta_0 \to \infty$; $A_0 \to 0$). In this way, B_r would still span from zero to unity as β spans from $\beta_0(\to\infty)$ to β_u, although the initial grading of the sample would correspond to a non-zero initial value of B_r. It is worth reminding that, in most practical cases, even a very uniformly graded sample would have a high but finite value of β_0 (= 50 - 60), and therefore using the universal initial grading definition, a finite, non-zero value of relative breakage.

Figure 11 shows the relative breakage obtained at the end of the different tests as a function of the plastic work, L_p. In Figure 11(a) B_r was computed using the initial grading as the reference cumulative grain size distribution, while in Figure 11(b) the reference cumulative grain size distribution is the heavy side around d_M, and therefore, the initial grain size distribution is characterised by a non zero value of relative breakage.

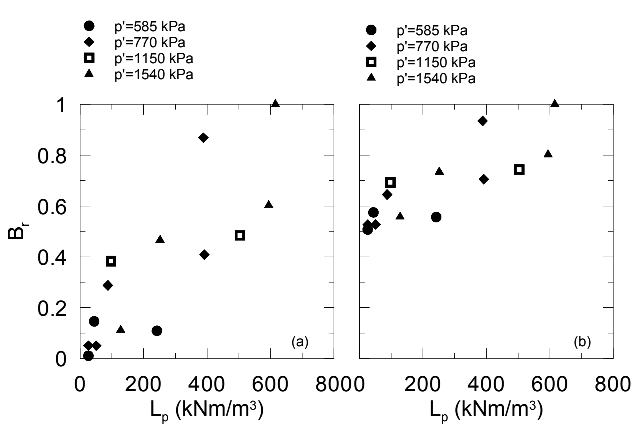


Figure 11. Relative breakage as a function of plastic work: (a) initial β_0 (= 0.96); (b) $\beta_0 \to \infty$

The plastic work at the end of the test was computed as:

$$L_p = \int p' \cdot \delta\varepsilon_v^p + \int q \cdot \delta\varepsilon_s^p \qquad (19)$$

in which the plastic strains were obtained by subtracting the elastic strains from the total strains:

$$\delta\varepsilon_v^p = \delta\varepsilon_v - \delta\varepsilon_v^e = \delta\varepsilon_v - \frac{C_s}{2.303}\frac{\delta p'}{p'}\frac{1}{(1+e)} = \frac{\delta p'}{K} \qquad (20)$$

$$\delta\varepsilon_s^p = \delta\varepsilon_s - \delta\varepsilon_s^e = \delta\varepsilon_s - \frac{2(1+\nu)}{9(1-2\nu)}\frac{\delta q}{K} \qquad (21)$$

Finally, it is worth noting that the grain size distribution corresponding to a fractal cumulative distribution is:

$$p(d) = \frac{\beta}{d_M}\left(\frac{d}{d_M}\right)^{\beta-1} \qquad (22)$$

Therefore, different from what assumed by Einav (2007), the implication of the assumed grain size evolution upon loading is that the fractional breakage $B_r(d)$ is not constant with grain size:

$$B_r(d) = \frac{p(d) - p_0(d)}{p_u(d) - p_0(d)} = \frac{\dfrac{\beta}{\beta_0}\left(\dfrac{d}{d_M}\right)^{\beta-\beta_0} - 1}{\dfrac{\beta_u}{\beta_0}\left(\dfrac{d}{d_M}\right)^{\beta_u-\beta_0} - 1} \qquad (23)$$

The experimental values of fractional breakage $B_r(d)$ computed using the heavy side around d_M as the reference cumulative grain size distribution are plotted as a function of d in Figure 12. Despite the scatter of experimental data, it is evident that fractional breakage is not constant with grain size and it follows the trend suggested by eq. (23).

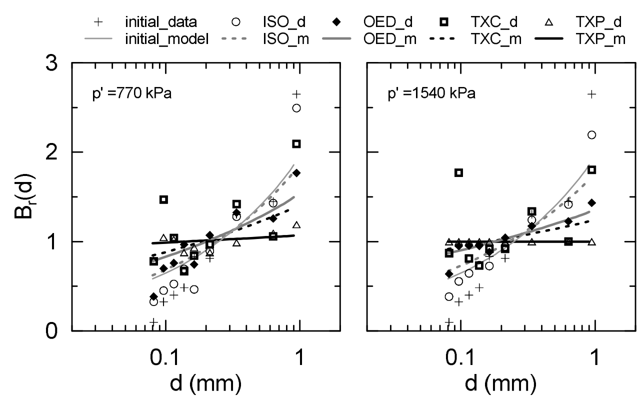


Figure 12. Fractional relative breakage as a function of grain size: (a) p' = 770 kPa, (b) p' = 1540 kPa

5. CONCLUSIONS

This paper has presented the preliminary results of an ongoing experimental investigation of the mechanical behaviour of granular soils with crushable grains. The final goal of the work is to gain a deeper understanding of the mechanical behaviour of soils with crushable grains, but also of weak rocks, or cemented aggregates whose bonds suffer progressive degradation due to applied loads in order to develop suitable constitutive equations that incorporate the (evolving) grain size distribution. In particular, this paper examines the evolution of grain size distribution due to loading along different stress paths.

The experimental work, currently under way, is being carried out on an artificial material composed of crushed expanded clay pellets, that break at relatively low stress, reconstituted to obtain repeatable grain size distributions with different values of D_{50} and $U(= D_{60}/D_{10})$. This paper shows preliminary results on the evolution of the grain size distribution of samples whose initial grading was characterised by $D_{50} = 0.5$ mm and $U = 7$ after testing in isotropic, one-dimensional and constant mean effective stress triaxial compression at increasing stress levels.

Because of the existence of a double order of porosity, i.e. "inter-granular" and "intra-granular" porosity it is necessary to distinguish between the cumulative grain size distribution by weight and the cumulative grain size distribution by volume.

Simple measures of grain crushing, such as the reduction of D_{50} or the increase of the uniformity coefficient U, demonstrate that, for tests carried out at constant η, crushing depends both on the stress ratio and on the value of mean effective stress reached in the test.

In the analysis of the results, the ultimate, current, and initial cumulative grain size distribution were all taken to be self similar, with a constant value of the maximum grain dimension, d_M, and an evolving exponent, β. The description of breakage through only one scalar parameter β, conceptually similar to Einav's (2007) assumption of fractional independency, is equivalent to the use of a single internal variable in elastic-plastic models for granular media with crushable grains.

The values of β, describing the evolution of grain size distribution with loading, have been shown to be related to the plastic work in the tests. Of course, in the loading process, granular materials may undergo several dissipative mechanisms, which include energy dissipation from the breakage of the particles (here described via B_r and hence by exponent β), but also from granular rearrangement with relative frictional sliding and spin.

REFERENCES

Cecconi M., De Simone A., Tamagnini C., Viggiani G.M.B. (2002). "A constitutive model for granular materials with grain crushing and its application to a pyroclastic soil". Int. J. Numer. Anal. Meth. Geomech., **26**:1531–1560

Cecconi M., Scarapazzi, M. Viggiani G.M.B. (2009). "On the geology and the geotechnical properties of pyroclastic flow deposits of the Colli Albani". Bull. Eng. Geol. Environ. **69**:185–206

Coop, M.R. (1990). "The mechanics of un-cemented carbonate sands". Geotéchnique, **40** (4): 607-626

Coop, M.R., Sorensen, K. K., Bodas Freitas, T., Georgoutsos, G. (2004). "Particle breakage during shearing of a carbonate sand". Géotechnique **54**(3), 157–163

De Simone A., Tamagnini C. (2005). "Stress–dilatancy based modelling of granular materials and extensions to soils with crushable grains". Int. J. Numer. Anal. Meth. Geomech., **29**: 73–101

Einav I. (2007). "Breakage mechanics—Part I: Theory". J. of the Mech. and Phys. of Solids, **55** (2007) 1274–1297

Hardin, B.O. (1985). “Crushing of soil particles”, J. Geotechnical Engng., ASCE, 111, 1177-1192

Kendall K. (1978). "Complexities of compression failure". Proc. R. Soc. Lond. A **361**: 245-263

Lade, P.V., Yamamuro, J.A., Bopp, P.A. (1996). "Significance of particle crushing in granular materials" J. Geotech. Eng., ASCE, **122**(4), 309–316.

Luzzani, L., Coop, M.R. (2002). “On the relationship between particle breakage and the critical state of sands,” Soils Found., 42(2), 71–82.

McDowell, G.R., Bolton, M.D. (1998). "On the micromechanics of crushable aggregates". Géotechnique **48**(5), 667-679

McDowell, G. R. (2005). “A physical justification for log(e)–log(σ'_v) based on fractal crushing and particle kinematics,” Géotechnique, **55**(9), 697–698

McDowell, G. R., Bolton, M. D., Robertson D. (1986). "The fractal crushing of granular materials" J. of the Mech. and Phys. of Solids, **44**(12), 2079-2101,constitutive behaviour; B. granular material; C. probability and statistics

McDowell, G.R., Bolton, M.D., Robertson D. (1996). "The fractal crushing of granular materials," J. of the Mech. and Phys. of Solids, **44**(12), 2079-2101.

Miura K., Maeda K., Furukawa M., Toki S. (1997). "Physical characteristics of sands with different primary properties". Soils and Foundations **37**(3);53-64.

Miura K., Maeda K., Furukawa M., Toki S. (1998). "Mechanical characteristics of sands with different primary properties". Soils and Foundations **38**(4);159-172.

Nakata, Y., Hyde, A. F. L., Hyodo, M., Murata, H. (1999). " A probabilistic approach to sand particle crushing in the triaxial test". Géotechnique **49**(5), 567-583

Palmer A.C., Sanderson T.J.O. (1991). "Fractal crushing of ice and brittle solids". Proc. R. Soc. Lond. A **433**: 469-477

Russel A.R., Muir-Wood D., Kikumoto M. (2009). "Crushing of particles in idealised granular assemblies". J. o of the Mech. and Phys. of Solids, **57**, 1293–1313

Sammis C. King G., Biegel R. (1987). "The Kinematics of Gouge Deformation" Pure and Applied Geophysics, **125**(5): 777-812.

Turcotte D.L. (1986). " Fractals and fragmentation". J. Geophys. Res., **91**(B2), 1921–1926.

Wood, D.M., Maeda, K. (2008). "Changing grading of soil: effect on critical states". Acta Geotechnica **3**, 3-14.