Isospin Mixing of Narrow Pentaquark States

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Abstract

Interpreting the recently discovered narrow exotic baryons as pentaquark states, we discuss, along an old argument of ours, the isospin mixing occurring within the two doublets of $Q = -1$ and $Q = 0$ states lying inside the $S = -2$ (Ξ-cascade) sector. We argue that, at least within the Jaffe-Wilczek assignment, presently available data already indicate that mixing should occur at an observable level in both charge sectors, with mixing angles that can be predicted in terms of ratios of observable mass splittings.

1 Introduction

Thirty five years after their existence had been predicted from duality arguments [1], claims that exotic hadrons have been discovered in various channels as narrow baryonic states, have been made by a number of experiments [2]. Although care must be exerted in their interpretation [3, 4], evidence in favour of the existence of several such states has been steadily mounting in recent months and will be accepted hereafter.

The old, pre-QCD argument of [1] was reconsidered, some ten years later, on the basis of a simple topology-based expansion of QCD [5, 6]. It was argued that, from that more modern viewpoint, duality in $B\bar{B}$ scattering has to be interpreted as a relation between states in scattering and annihilation channels. More precisely, QCD baryons resemble Y-shaped strings with quarks at the three ends and a junction, keeping track of baryon number, sitting in the middle. To leading order in the expansion, annihilation channels in one-, two-, or three-$q\bar{q}$ resonances are dual to channels where three new families of mesons (collectively called "baryonia") are exchanged. They correspond, respectively, to bosonic states with four, two, and no quarks, and two string junctions. Baryonia decay preferentially (i.e. through string breaking) in $B\bar{B}$ channels (thus conserving the number of junctions), hence were argued to be narrow if near or below the $B\bar{B}$ threshold.

Generalization of standard duality arguments allowed to make predictions for baryonium intercepts and slopes and led to an elegant explanation for the seemingly large breaking of exchange degeneracy of baryon trajectories. In the scheme there is naturally (or even necessarily [5, 6]) room for baryonic pentaquark states of the type found in the experiments mentioned above. Having a dynamically favoured decay into a $B\bar{B}B$ final state, sufficiently light pentaquark states are also naturally narrow. The reason why preliminary evidence for narrow baryonia [6] was never confirmed is not completely clear to us. If accepted as an experimental fact, the non existence of such states would mean that baryonia, that are sufficiently heavy for their decay into mesons to be suppressed, are already significantly above the $B\bar{B}$ threshold. The situation would be different for the pentaquarks because of the higher threshold for decay in their preferred (three baryon) channels.

Last but not least, mixings among nearby tetraquark baryonium states, resulting in large violations of isospin symmetry, were shown to be possible for sufficiently narrow states [7]. This phenomenon was argued to be quite general: indeed, at the very end of [7], it was suggested that appreciable isospin mixing should occur in other exotic multiquark systems, such as pentaquarks.

Multiquark states of the pentaquark type were also predicted in the chiral soliton model [8], together with many other exotic baryonic states with increasingly large spin and isospin quantum numbers.

Naturally the discovery of genuinely exotic resonances has prompted a good deal of new interesting investigations on possible dynamical mechanisms that would provide a theoret-

ical basis for the existence and the narrowness of families of pentaquark states in QCD. Models of different kinds have been proposed complementing or in alternative to the old ideas of refs. [5] and [8]. Among them we should quote the diquark model of ref. [9] to which we will come back in the next section and the works of refs. [10] and [11]. In [10] the narrowness of the Θ^+ states is explained as an interference effect between two almost degenerate states, while in [11] it is suggested that the flavour structure of the Θ^+ wave-function is such that after the meson is formed the residual three-quark piece has little overlap with the octet baryon wave-function (actually orthogonality becomes exact in the flavour $SU(3)$ symmetry limit).

In this note we extend the idea of mixing among narrow states suggested in ref. [7] for tetraquarks to the case of the so-called Ξ-cascade, taking for granted the classification of these states suggested in ref. [9]. It should be stressed, however, that the arguments we shall develop are rather general and apply even if the detailed pattern of degeneracies of the Ξ states is different from the one advocated in [9]. The key condition which is required for our analysis to work is the existence of almost degenerate states that can mix with a mass matrix, $\mathcal{M}$, in which electromagnetic and $(m_d - m_u)$ effects are comparable to (if not larger than) those due violations of the so-called OZI (Zweig) rule [12]. Details of the predictions, however, will depend on the classification scheme one adopts and can be used, in principle, to distinguish and confront different dynamical models.

With inputs taken from experiments, and reasonable guesses about other, presently unknown, parameters entering the mass matrix, we find that, in the $Q = -1$ as well as in the $Q = 0$ charge sector, the eigenstates of $\mathcal{M}$ should be appreciably different from pure isospin states, while an almost maximal violation of isospin symmetry ($SU(2)$ "ideal" mixing) is all but excluded. Such a theoretical picture has obvious experimental consequences for the production mechanisms and the decay of these narrow states.

2 A classification of Ξ pentaquark states

The recently discovered Ξ states can be classified, within the scheme proposed in ref. [9], as belonging to an ideally mixed $\mathbf{8} + \overline{\mathbf{10}}$ of flavour $SU(3)$. In this scheme the Ξ states consist of five valence quarks arranged as $(qs)(qs)\bar{q}$ where q stands for u or d, and the two-diquark system belongs to an isospin triplet (i.e. flavour-symmetric) state. As a result one has a total of six states with charge Q going from -2 to $+1$. Out of these the sectors with $Q = -2$ and $Q = +1$ are pure $I = 3/2$ states, while those with $Q = -1$ and $Q = 0$ appear in pairs with both $I = 1/2$ and $I = 3/2$ components. The states with a given quark content are those with a definite I_3 of the two diquark system and of the antiquark. We shall use the following notation:

$$\begin{aligned}
\Xi_d^- &= |-1\rangle\,|+1/2\rangle \sim (ds)(ds)\bar{d}\,, & \Xi_u^- &= |0\rangle\,|-1/2\rangle \sim (ds)(us)\bar{u}\,, \\
\Xi_u^0 &= |+1\rangle\,|-1/2\rangle \sim (us)(us)\bar{u}\,, & \Xi_d^0 &= |0\rangle\,|+1/2\rangle \sim (us)(ds)\bar{d}\,.
\end{aligned} \tag{1}$$

The pure isospin states are related to those of (1) by standard Clebsh-Gordan (CG) coefficients. Using the conventions of the PDG one finds:

$$\Xi_d^- = \frac{1}{\sqrt{3}}\left[\Xi_{3/2}^- - \sqrt{2}\Xi_{1/2}^-\right], \quad \Xi_u^- = \frac{1}{\sqrt{3}}\left[\Xi_{1/2}^- + \sqrt{2}\Xi_{3/2}^-\right],$$

$$\Xi_u^0 = \frac{1}{\sqrt{3}}\left[\Xi_{3/2}^0 + \sqrt{2}\Xi_{1/2}^0\right], \quad \Xi_d^0 = \frac{1}{\sqrt{3}}\left[\Xi_{1/2}^0 - \sqrt{2}\Xi_{3/2}^0\right]. \tag{2}$$

These equations can be immediately inverted to give

$$\Xi_{3/2}^- = \frac{1}{\sqrt{3}}\left[\Xi_d^- + \sqrt{2}\Xi_u^-\right], \quad \Xi_{1/2}^- = \frac{1}{\sqrt{3}}\left[\Xi_u^- - \sqrt{2}\Xi_d^-\right],$$

$$\Xi_{3/2}^0 = \frac{1}{\sqrt{3}}\left[\Xi_u^0 - \sqrt{2}\Xi_d^0\right], \quad \Xi_{1/2}^0 = \frac{1}{\sqrt{3}}\left[\Xi_d^0 + \sqrt{2}\Xi_u^0\right]. \tag{3}$$

3 The mass matrices

We denote the 2×2 mass matrices in the two charge sectors by $\mathcal{M}_q^-$, $\mathcal{M}_I^-$, $\mathcal{M}_q^0$, $\mathcal{M}_I^0$ depending on the basis used, with the convention that the first row and column will denote $I = 1/2$ in the isospin basis and Ξ_d^- or Ξ_u^0 in the quark basis. If $q\bar{q}$ annihilation diagrams (i.e. violations of the OZI rule) as well as quark masses and electromagnetic (EM) interactions are neglected, all six states are exactly degenerate. Effects of OZI violations, quark masses and EM interactions will be added to first order. We will therefore discuss, in turn, OZI violations neglecting quark mass and EM contributions, and then the isospin violations due to the latter two effects in the OZI limit. This approximation will be justified, a posteriori, if both kinds of corrections are small and comparable.

3.1 OZI violating contribution

The OZI-violating diagrams are shown in Fig. 1. Clearly they contribute only to $I = 1/2$ states. Their contribution to the mass matrix is thus particularly simple in the isospin basis in which, in both charge sectors, it reads:

$$\delta\mathcal{M}_I = \begin{pmatrix} \delta & 0 \\ 0 & 0 \end{pmatrix}, \tag{4}$$

where, by definition, δ is the mass splitting between $I = 1/2$ and $I = 3/2$ states induced by OZI violations. We will comment later on the magnitude of δ. In the quark basis (4) gives

$$\delta\mathcal{M}_q^- = \begin{pmatrix} \frac{2}{3}\delta & -\frac{\sqrt{2}}{3}\delta \\ -\frac{\sqrt{2}}{3}\delta & \frac{1}{3}\delta \end{pmatrix}, \quad \delta\mathcal{M}_q^0 = \begin{pmatrix} \frac{2}{3}\delta & \frac{\sqrt{2}}{3}\delta \\ \frac{\sqrt{2}}{3}\delta & \frac{1}{3}\delta \end{pmatrix}. \tag{5}$$

OZI violations tend to align eigenstates along pure $SU(2)$ or $SU(3)$ representations. In the case of $SU(3)$, the explicit breaking due to the strange-quark mass supposedly induces
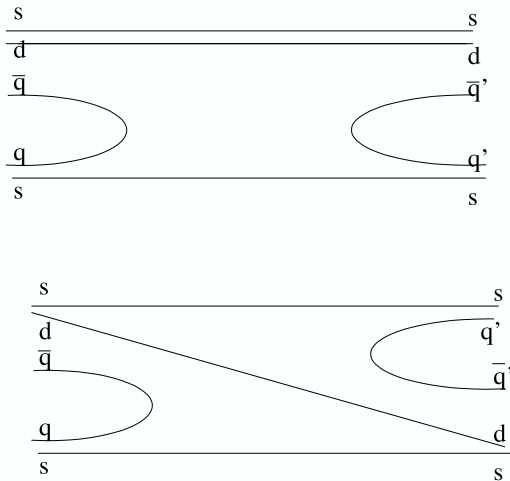
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Figure 1: Diagrams that violate the OZI rule and mix different flavours in the $Q = -1$ sector.

strong $SU(3)$ mixing so that the true eigenstates are expected to be close to those with definite strange-quark content. This is the case for the $\mathbf{8} + \overline{\mathbf{10}}$ classification of pentaquarks in the proposal of ref. [9]. Here we are addressing, instead, the question of the relative magnitude of δ and of the explicit $SU(2)$ violating contributions that we are now going to discuss.

3.2 Isospin violations from quark masses and EM interactions

Quark masses contribute to the QCD Hamiltonian with an iso-singlet piece proportional to $m = (m_u + m_d)/2$ and an iso-triplet term proportional to $\delta m \equiv (m_d - m_u)$. The former provides a common mass-term for all six states: being interested only in mass differences we will neglect it. The latter piece reads:

$$\delta H_m = -\frac{1}{2}\delta m \ \bar{q}\tau_3 q \,, \tag{6}$$

and transforms as an $I = 1, I_3 = 0$ operator. Its contributions to the mass matrices can be expressed in terms of three independent reduced matrix elements. Two are the diagonal matrix elements corresponding to the $I = 1/2$ and $I = 3/2$ states and the third is the off-diagonal (transition) matrix element. Using standard CG coefficients, this gives:

$$\delta\mathcal{M}_I^- = \begin{pmatrix} \Delta_{1/2}^{(m)} & \Delta_{off}^{(m)} \\ \Delta_{off}^{(m)} & \Delta_{3/2}^{(m)} \end{pmatrix}, \quad \delta\mathcal{M}_I^0 = \begin{pmatrix} -\Delta_{1/2}^{(m)} & \Delta_{off}^{(m)} \\ \Delta_{off}^{(m)} & -\Delta_{3/2}^{(m)} \end{pmatrix}. \tag{7}$$

We now impose the constraint that comes from insisting that the OZI rule holds true, namely we require that, in the quark basis and in each charge sector, there is no off-diagonal

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(i.e. quark-flavour mixing) contribution. It is easy to check that such a constraint requires a single relation between the diagonal and off-diagonal entries in (7), namely:

$$\Delta_{off}^{(m)} = \sqrt{2}(\Delta_{3/2}^{(m)} - \Delta_{1/2}^{(m)}). \tag{8}$$

Using such a relation, the $\delta\mathcal{M}_I$ contributions to the mass matrices in the quark basis read:

$$\delta\mathcal{M}_q^- = \begin{pmatrix} 2\Delta_{1/2}^{(m)} - \Delta_{3/2}^{(m)} & 0 \\ 0 & 2\Delta_{3/2}^{(m)} - \Delta_{1/2}^{(m)} \end{pmatrix}, \tag{9}$$

$$\delta\mathcal{M}_q^0 = \begin{pmatrix} \Delta_{3/2}^{(m)} - 2\Delta_{1/2}^{(m)} & 0 \\ 0 & \Delta_{1/2}^{(m)} - 2\Delta_{3/2}^{(m)} \end{pmatrix}. \tag{10}$$

The above general result can be compared with the one obtained by simply counting the number of u and d quarks in each hadron. The resulting mass matrices are indeed of the form (9)–(10) with the additional constraint $\Delta_{1/2}^{(m)} = 5\Delta_{3/2}^{(m)}$.

Let us turn now to EM effects. Since the electromagnetic current is a mixture of $I = 0$ and $I = 1$ terms, a virtual EM contribution will involve, to order α, $I = 0, 1, 2$ terms in the effective Hamiltonian. The $I = 1$ contribution can be treated exactly as the $\delta\mathcal{M}_I^{-/0}$ contributions we have just discussed. It is enough to make the following replacements in the mass-matrices (9) and (10):

$$\Delta_{1/2}^{(m)} \to \Delta_{1/2} \equiv \Delta_{1/2}^{(m)} + \Delta_{1/2}^{(em)}, \qquad \Delta_{3/2}^{(m)} \to \Delta_{3/2} \equiv \Delta_{3/2}^{(m)} + \Delta_{3/2}^{(em)}. \tag{11}$$

By contrast, the $I = 0$ piece will in general give different contributions to the $I = 1/2$ and $I = 3/2$ diagonal matrix elements. We shall denote by Δ_0 the difference between the former and the latter. Finally, the $I = 2$ piece of the EM Hamiltonian can only contribute to the diagonal $I = 3/2$ entry, Δ_2, and to the off-diagonal ones, $\Delta_{2\,off}$. Asking as before the validity of the OZI rule allows to express the transition matrix element in terms of Δ_0 and Δ_2 through the relation $\Delta_{2\,off} = \sqrt{2}\,(\Delta_2 - \Delta_0)$.

3.3   Adding up all the contributions

We can now collect all the non-trivial contributions to the mass matrices (to be added to a common matrix $\mathcal{M}_0$ proportional to the unit matrix) and write their final form as follows:

$$\delta\mathcal{M}_I^- = \begin{pmatrix} \delta + \Delta_{1/2} + \frac{1}{2}\Delta_0 & \sqrt{2}(\Delta_{3/2} - \Delta_{1/2} + \Delta_2 - \Delta_0) \\ \sqrt{2}(\Delta_{3/2} - \Delta_{1/2} + \Delta_2 - \Delta_0) & \Delta_{3/2} + \Delta_2 - \frac{1}{2}\Delta_0 \end{pmatrix}, \tag{12}$$

$$\delta\mathcal{M}_I^0 = \begin{pmatrix} \delta - \Delta_{1/2} + \frac{1}{2}\Delta_0 & \sqrt{2}(\Delta_{3/2} - \Delta_{1/2} - \Delta_2 + \Delta_0) \\ \sqrt{2}(\Delta_{3/2} - \Delta_{1/2} - \Delta_2 + \Delta_0) & -\Delta_{3/2} + \Delta_2 - \frac{1}{2}\Delta_0 \end{pmatrix}, \tag{13}$$

$$\delta\mathcal{M}_q^- = \begin{pmatrix} \frac{2}{3}\delta + 2\Delta_{1/2} - \Delta_{3/2} - \Delta_2 + \frac{3}{2}\Delta_0 & -\frac{\sqrt{2}}{3}\delta \\ -\frac{\sqrt{2}}{3}\delta & \frac{1}{3}\delta + 2\Delta_{3/2} - \Delta_{1/2} + 2\Delta_2 - \frac{3}{2}\Delta_0 \end{pmatrix}, \tag{14}$$

$$\delta\mathcal{M}_q^0 = \begin{pmatrix} \frac{2}{3}\delta - 2\Delta_{1/2} + \Delta_{3/2} - \Delta_2 + \frac{3}{2}\Delta_0 & \frac{\sqrt{2}}{3}\delta \\ \frac{\sqrt{2}}{3}\delta & \frac{1}{3}\delta - 2\Delta_{3/2} + \Delta_{1/2} + 2\Delta_2 - \frac{3}{2}\Delta_0 \end{pmatrix}. \tag{15}$$

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One can check that simple toy models of Coulomb energy effects (like the one where quarks are taken to be equidistant [13]) give $\Delta_2 = \Delta_0$. However this is not a necessity. For instance, in a model where the average distance of quark pairs is different than the one between the quarks and the antiquark, it is possible to generate a non vanishing $\Delta_2 - \Delta_0$.

Note that these mass matrices provide immediately, in each charge sector, an average mass $\bar{M}^-$, $\bar{M}^0$:

$$\bar{M}^- = M_0 + \frac{1}{2}(\delta + \Delta_{1/2} + \Delta_{3/2} + \Delta_2)\,, \quad \bar{M}^0 = M_0 + \frac{1}{2}(\delta - \Delta_{1/2} - \Delta_{3/2} + \Delta_2)\,, \quad (16)$$

giving a gap $\Delta_{1/2} + \Delta_{3/2}$ between the average mass in the two doublets. On the other hand, splitting and mixing within each sector only depends on δ and on the combinations

$$\delta^- \equiv \Delta_{1/2} - \Delta_{3/2} - \Delta_2 + \Delta_0\,, \qquad \delta^0 \equiv -\Delta_{1/2} + \Delta_{3/2} - \Delta_2 + \Delta_0 \quad (17)$$

for the $Q = -1$ and $Q = 0$ sectors, respectively.

We also give, within our first-order approximation, the mass of the two pure $I = 3/2$ states. They receive contributions that are completely fixed by group theory in terms of Δ_0, $\Delta_{3/2}$ and Δ_2. One finds:

$$M^{--} = M_0 + 3\Delta_{3/2} - \Delta_2 - \frac{1}{2}\Delta_0\,, \quad M^+ = M_0 - 3\Delta_{3/2} - \Delta_2 - \frac{1}{2}\Delta_0\,, \quad (18)$$

As a check note that, for $\Delta_{1/2} = 5\Delta_{3/2}$ one gets identical quark-mass contributions to the mass of states having the same number of $u + \bar{u}$ and $d + \bar{d}$ quarks (such as Ξ^{--} and Ξ_d^0), in agreement with the naive quark-counting approximation.

One should also note the following relations:

$$(\bar{M}^- + \bar{M}^0) - (M^{--} + M^+) = \delta + 3\Delta_2 + \Delta_0\,, \quad (19)$$

$$M^{--} - M^+ = 6\Delta_{3/2}\,, \quad (20)$$

$$3(\bar{M}^- - \bar{M}^0) - (M^{--} - M^+) = 3(\Delta_{3/2} - \Delta_{1/2})\,. \quad (21)$$

Summarizing, we have been able to express all mass matrix elements of the six Ξ states in terms of a common mass M_0 and of *five* parameters $\delta, \Delta_0, \Delta_{1/2}, \Delta_{3/2}, \Delta_2$. Since, in principle, one can measure six masses and two mixing angles, our scheme makes *two* testable predictions.

4 Eigenstates and their phenomenology

In order to determine the mass eigenstates and eigenvalues (hence the mixing angles) in the two charge sectors it is useful to rewrite the mass-matrices in the isospin basis in the form:

$$\mathcal{M}_I^- = \bar{M}^- \times \mathbb{1} + \begin{pmatrix} (\delta + \delta^-)/2 & -\sqrt{2}\,\delta^- \\ -\sqrt{2}\,\delta^- & -(\delta + \delta^-)/2 \end{pmatrix}\,, \quad (22)$$

$$\mathcal{M}_I^0 \;=\; \bar{M}^0 \times \mathbb{1} + \begin{pmatrix} (\delta + \delta^0)/2 & \sqrt{2}\,\delta^0 \\ \sqrt{2}\,\delta^0 & -(\delta + \delta^0)/2 \end{pmatrix} . \tag{23}$$

Diagonalization is of course trivial. The mass eigenvalues are given by:

$$M_{1,2}^- \;=\; \bar{M}^- \pm \frac{1}{2}\sqrt{(\delta + \delta^-)^2 + 8(\delta^-)^2}\,, \tag{24}$$

$$M_{1,2}^0 \;=\; \bar{M}^0 \pm \frac{1}{2}\sqrt{(\delta + \delta^0)^2 + 8(\delta^0)^2}\,, \tag{25}$$

while, defining the mixing angle as $\Xi_1 = \cos\theta\,\Xi_{1/2} + \sin\theta\,\Xi_{3/2}$, we find:

$$\tan\theta^- \;=\; \frac{1}{2\sqrt{2}\epsilon^-}\left[(1 + \epsilon^-) - \sqrt{1 + 2\epsilon^- + 9(\epsilon^-)^2}\right], \tag{26}$$

$$\tan\theta^0 \;=\; -\frac{1}{2\sqrt{2}\epsilon^0}\left[(1 + \epsilon^0) - \sqrt{1 + 2\epsilon^0 + 9(\epsilon^0)^2}\right], \tag{27}$$

where we have set:

$$\epsilon^- = \frac{\delta^-}{\delta}\,, \quad \epsilon^0 = \frac{\delta^0}{\delta}\,. \tag{28}$$

It is easy to check that the mixing angles approach the expected values in the two limiting cases: zero mixing when $\epsilon = 0$ and maximal (or $SU(2)$ "ideal") mixing (meaning here $\tan\theta^- = -1/\sqrt{2}$ and $\tan\theta^0 = -\sqrt{2}$) for $\epsilon^- \to \infty$ and $\epsilon^0 \to -\infty$, respectively. The mixing angles depend only on the two quantities ϵ^- and ϵ^0. The latter can be determined, for instance, from the two independent ratios that one can form from the differences $M_2^- - M_1^-$, $M_2^0 - M_1^0$ and the combination (21).

Although we are still far, experimentally, from being able to estimate the mixing angles this way, we claim that we can nevertheless already exclude very small mixing angles (pure isospin states). The argument goes as follows. Assume mixing to be small in both sectors, i.e. $\epsilon \ll 1$. In this case the observed mass splitting between the Ξ^{--} and the Ξ^- (which, being observed to decay in $\Xi^{0*}(1530)\pi^-$, is identified [9] with the $\Xi_{1/2}^-$ state of eq. (3)) implies $\delta = -7 \pm 3$ MeV. This, however, is also the expected order of magnitude of the isospin breaking parameters δ^- and δ^0. Indeed, since $3\delta^-$ is the isospin-breaking contribution to the difference between the two diagonal matrix elements of the matrices (14) and (15), a naive quark-counting argument, together with a simple model of Coulomb interaction energy with equidistant quarks and antiquark [13, 7], leads to the estimate

$$\delta^- = -\delta^0 = \frac{2}{3}\delta m + \frac{1}{9}\alpha\left\langle\frac{1}{r}\right\rangle . \tag{29}$$

A rough estimate of the r.h.s. of eq. (29), using [13] $\delta m \sim 4.5$ MeV [1] and $\left\langle\frac{1}{r}\right\rangle \sim 240$ MeV, gives

$$\delta^- = -\delta^0 \sim 3.2 \text{ MeV} \sim -0.46\,\delta, \tag{30}$$

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[1] Lattice estimates give $\delta m(2\text{Gev})_{\overline{MS}} = 2.3 \pm .2\,(\text{stat}) \pm .5\,(\text{syst})$ Mev [14], where most of the large systematic error comes from unquenching. We remark that the number in eq. (29) is the down-up mass difference at a substantially smaller scale, of order Λ_{QCD}.

with a large error. In any case (30) is in contradiction with the assumption of very small $|\epsilon|$'s. A slightly more sophisticated model, where the average distance of quarks inside a diquark is smaller than the average diquark-diquark or diquark-antiquark distance, relates $\delta^- = -\delta^0$ to $m(n) - m(p) + m(\Xi^-) - m(\Xi^0) \sim 8 \, \text{MeV}$, giving $\delta^- = -\delta^0 \sim -0.4 \, \delta$.

Obviously, once mixing is non-negligible, the above estimates of δ^- and δ^0 have to be reconsidered. Within a definite model for isospin breaking effects, a determination of δ is possible from the measured splitting between the Ξ^{--} and the observed Ξ^- (now interpreted as one of the two negative-charge mass eigenstates). This eventually leads to an evaluation of ϵ^- and the mixing angle θ^- using (26). We find that, in some cases, δ comes out so small that almost ideal mixing is expected. However, more precise data on the mass splittings and a better theory of isospin breaking terms is needed before one can make definite predictions. In order to show how sensitive the mass splittings are to mixing we show, in Fig. 2, the structure of the six mass eigenstates in the two extreme situations of very small and very large (i.e. ideal) mixing. Eqs. (26) and (27) imply that for $|\epsilon| > 0.3$, mixing angles are
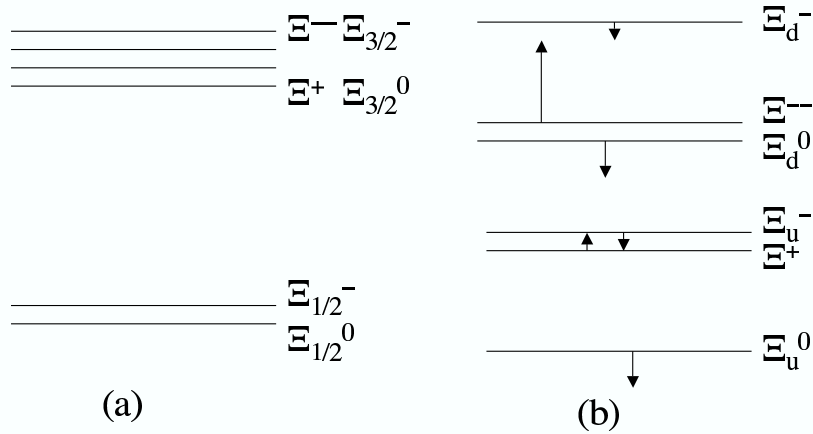

Figure 2: Schematic structure of the six energy levels in the two extreme cases: (a) very small isospin mixing, i.e. $\epsilon^0, \epsilon^- \ll 1$; (b) almost ideal mixing. In (b) the line levels take into account quark and diquark mass differences, while the arrows show direction and relative magnitude of Coulomb effects for each state in the diquark picture. The relative normalization of the two isospin-breaking effects is not fixed

sufficiently large for their effects to be easily detectable. Indeed, for not too small values of θ^- (θ^0), we should see two quasi-degenerate peaks in the $Q = -1$ ($Q = 0$) sector with large violation of isospin present in their decays. For instance, in the $Q = -1$ sector only

one of the two peaks (the one which is prevalently the $\Xi_u^- = (ds)(us)\bar{u}$ quark state) should be easily visible in the $\Xi^{*0}\pi^-$ channel, while also the second peak (corresponding to a state close to the $\Xi_d^- = (ds)(ds)\bar{d}$ quark state) should appear in isospin-related channels with a π^0 in the final state.

We conclude by mentioning that, in the pentaquark picture assumed here, isospin mixing is also be expected to occur in the $S = -1$ sector. In fact, since the states that can mix must differ by the replacement of a $u\bar{u}$ with a $d\bar{d}$ pair, the only other interesting doublet consists of the two neutral Σ/Λ-like $(ud)(qs)\bar{q}$ states. These are indeed the two lightest states with $Q = 0$, $S = -1$ once $SU(3)$ mixing has lifted up those with three strange quarks. Similar phenomena should also occur in pentaquarks with one heavy quark (but not in those with a heavy antiquark). If the values of the ϵ-like parameters we have introduced, will turn out to be in the appropriate range in some of the channels, rather spectacular phenomena will be able to put many of the current theoretical models to an interesting test.

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