

Experimental violation of Bell's inequality by local classical variables

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I DESCRIPTION OF THE EXPERIMENT

We describe a simple experiment in which two experimenters, by performing independent, local, binary choices on a common source of randomness and computing the empirical correlations among their results, arrive to a violation of Bell's inequalities.

Let D denote the unit disk in the plane. On D we define functions $S_\alpha^{(1)}, S_\beta^{(2)}$, with values ± 1 , as follows: for any angle $\alpha \in [0, 2\pi)$, given the point p in the disk, one rotates this point clockwise of an angle α and defines $S_\alpha^{(1)}(p)$ to be +1 if the rotated p is in the upper half plane, -1 if it is in the lower half plane. $S_\alpha^{(2)}$ is defined by the **singlet condition**: $S_\alpha^{(1)} = -S_\alpha^{(2)}$.

First step: generate random points

an algorithm produces random points, p_1, \dots, p_N , uniformly distributed in the unit circle in the plane.

Second step: local choice of directions

Each observer (Obs 1, Obs 2 in the program) chooses independently of the other one which observable to measure.

Third step: local measurements

The central unit sends the sequence (p_j) of random points to the experimenters and the observer 1 (resp. 2) evaluates the function $S_a^1(p_j)$ (resp. $S_b^2(p_j)$), $j = 1, \dots, N$. The **evaluation must be done on the same point**, corresponding to the experimental fact that, in the EPR type experiments each measurement must be done on the same singlet pair. A measurement of the pair $(S_\alpha^{(1)}, S_\gamma^{(2)})$ is statistically indistinguishable from a measurement of the pair $(S_{\pi+\alpha}^{(1)}, S_{\pi+\gamma}^{(2)})$ so we always choose the pair $(S_\alpha^{(1)}, S_{\pi+\beta}^{(2)})$ with $0 \leq \alpha, \beta < \pi$.

Fourth step: exchange of information after the measurement

The central unit is informed of the results and calculates the 3 correlations involved in the Bell inequality:

$$\langle S_a^{(1)} S_b^{(2)} \rangle = \frac{1}{N} \sum_{j=1}^N S_a^{(1)}(p_j^{(1)}) S_b^{(2)}(p_j^{(1)}) \quad (1)$$

$$\langle S_b^{(2)} S_c^{(1)} \rangle = \frac{1}{N} \sum_{j=1}^N S_c^{(1)}(p_j^{(2)}) S_b^{(2)}(p_j^{(2)}) \quad (2)$$

$$\langle S_a^{(1)} S_c^{(2)} \rangle = \frac{1}{N} \sum_{j=1}^N S_a^{(1)}(p_j^{(3)}) S_c^{(2)}(p_j^{(3)}) \quad (3)$$

Notice that (**both in the real and in the simulation experiment**) the evaluation of the correlation (1) is done using the sequence $(p_j^{(1)})$ of results of experiment (1), similarly correlation (2) uses the sequence $(p_j^{(2)})$, and correlation (3) uses $(p_j^{(3)})$. It is not relevant in which order one computes these correlations, e.g. one can choose at random every time which of the three pairs (a, b) , (b, c) , (c, a) to measure (Aspect experiment): since the experiments are incompatible, the 3 sequences are uniquely determined.

Fifth step: computation of the empirical correlations

The central unit computes the 3 empirical correlations (1), (2), (3).

Sixth step: cleaning

In order to apply Bell's argument to the empirical data, it is **necessary** (cf. section (1) of [Ac99]) to *clean* these data as follows: **all the points $p_j^{(1)}$, $p_j^{(2)}$ such that**

$$S_b^{(2)}(p_j^{(1)}) \neq S_b^{(2)}(p_j^{(2)}) \quad (4)$$

have to be eliminated from the calculation of the empirical correlations (1), (2), (3).

An elementary argument of classical probability shows that, if the experiments are mutually independent this elimination does not alter significantly the empirical correlations. **It is however surprising that, in the vast literature on the Bell inequality, the necessity of this "cleaning operation" has not been sufficiently emphasized .**

Seventh step: computation of the cleaned correlations

After the "cleaning" one obtains two new sequences of points:

$$p_{j_1}^{(1)}, \dots, p_{j_{N_1}}^{(1)} ; p_{j_1}^{(2)}, \dots, p_{j_{N_1}}^{(2)} \quad (5)$$

with the property that

$$S_b^{(2)}(p_{j_\alpha}^{(1)}) = S_b^{(2)}(p_{j_\alpha}^{(2)}) ; \quad \alpha = 1, \dots, N_1 \quad (6)$$

The central unit computes the new correlations with the “cleaned” sequences and compares them with the original ones (1), (2). We have checked that **the difference between the original correlations and the “cleaned” ones is of orders of magnitude smaller than the violation of the inequality**. Therefore this violation cannot be attributed to the “cleaning” operation which, on the other hand is not an assumption related to our experiment, but only represents the explicitation of an assumption hidden in any comparison of the Bell inequality with the experimental data. For more informations on this point, cf. [AcRe99a].

Eigth step: checking the Bell inequality

According to the Bell inequality one should have

$$|\langle S_a^{(1)} S_b^{(2)} \rangle - \langle S_c^{(1)} S_b^{(2)} \rangle| \leq 1 + \langle S_a^{(1)} S_c^{(2)} \rangle \quad (7)$$

The program compares the difference

$$|\langle S_a^{(1)} S_b^{(2)} \rangle - \langle S_c^{(1)} S_b^{(2)} \rangle| - 1 - \langle S_a^{(1)} S_c^{(2)} \rangle \quad (8)$$

with zero. Violation of Bell’s inequality corresponds to a value of the difference (8) strictly greater than zero. The simulation experiments show that, if the directions a, b, c are chosen so that a coincides with the x -axis and the directions b, c intersect the second and fourth quadrant in such a way that $\widehat{ca} > \widehat{ac}$ then the difference (8) is strictly positive, **hence the Bell inequality is violated**.

The reason of the violation is the following: let us denote \widehat{ab} the sector of the $(-, -)$ concordance in the correlation $\langle S_a^{(1)} S_b^{(2)} \rangle$. Similarly we denote \widehat{cb} the sector of the $(-, -)$ concordance in the correlation $\langle S_c^{(1)} S_b^{(2)} \rangle$ and \widehat{ac} the sector of the $(-, -)$ concordance in the correlation $\langle S_a^{(1)} S_c^{(2)} \rangle$. With these notations the correlations (5), (6), (7) are given by:

$$\begin{aligned} \langle S_a^{(1)} S_b^{(2)} \rangle &= -1 + 4\widehat{ab} \\ \langle S_c^{(1)} S_b^{(2)} \rangle &= -1 + 4\widehat{cb} \\ \langle S_a^{(1)} S_c^{(2)} \rangle &= -1 + 4\widehat{ac} \end{aligned}$$

So the two sides of the Bell inequality are respectively:

$$\begin{aligned} |\langle S_a^{(1)} S_b^{(2)} \rangle - \langle S_c^{(1)} S_b^{(2)} \rangle| &= 4|\widehat{ab} - \widehat{cb}| = 4(\widehat{cb} - \widehat{ab}) \\ 1 + \langle S_a^{(1)} S_c^{(2)} \rangle &= 4\widehat{ac} \end{aligned}$$

and we are reduced to compare $\widehat{cb} - \widehat{ab} = \widehat{ca}$ and \widehat{ac} . Therefore if one chooses the axes so that $\widehat{ca} > \widehat{ac}$, it will follow that

$$|\langle S_a^{(1)} S_b^{(2)} \rangle - \langle S_c^{(1)} S_b^{(2)} \rangle| > 1 + \langle S_a^{(1)} S_c^{(2)} \rangle$$

which violates the Bell inequality (14).

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