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BRIEF COMMUNICATIONS

Lévy–Laplace Operators in Functional Rigged Hilbert Spaces

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New results on analytical properties of the Lévy–Laplace operator (the Lévy Laplacian) and the semigroups generated by this operator are established in this paper. To this end, we use the methods developed earlier for the Lévy–Volterra operators (all necessary definitions are given below); the corresponding results are similar in both cases. The analogy arises because Hilbert space is replaced by a rigged Hilbert space; in fact, Hilbert space corresponds to a degenerate case (cf. [1, 2]): it is just in this case that the Lévy Laplacian possesses "unusual" properties (cf. [3, 4]). Our approach develops the techniques proposed in [1, 5–8].

1. DEFINITIONS AND PRELIMINARY FACTS

In this and in the following sections we describe the relationship between the Laplace operators and derivations. Unless indicated otherwise, only real vector spaces are considered. If E and G are locally convex spaces (LCS), then $L(E,G)$ denotes the space of all linear continuous maps from E to G; if $G=\mathbb{R}^1$, then E^* stands for $L(E,G)$. A map $F\colon E\to G$ is called *differentiable* (*in the Hadamard sense*) at the point $x\in E$ if there exists an element $F'(x)\in L(E,G)$, called the *derivative* of F at the point x, such that if $r_x(h)=F(x+h)-F(x)-F'(x)h$, then $t_n^{-1}r_x(t_nh_n)\to 0$ as $n\to\infty$ for any converging sequence $(h_n)\subset E$ and any sequence $(t_n)\subset\mathbb{R}$ converging to zero. Higher-order derivatives are defined by induction, and the spaces $L(E,G)$, $L(E,L(E,G))$, etc., are equipped with the topology of convergence on sequentially compact subsets. A map $F\colon E\to G$ is called a *C^n-map* ($n\in\mathbb{N}$), if F is n-times differentiable and F, as well as all the maps $F^{(k)}\colon E\to L(E,\dots,L(E,G),\dots)$, $k=1,2,\dots,n$, are continuous. The vector space of all C^n-maps acting from the space E to G is denoted by the symbol $C^n(E,G)$; if $G=\mathbb{R}^1$, then we shall write $C^n(E)$ instead of $C^n(E,G)$; the vector space of all real functions on E is denoted by $\mathcal{F}(E)$. The next definition is the motivation for defining the Laplace operator.

Definition 1 [1]**.** Let S_0 be a linear function defined on the vector subspace $\operatorname{dom} S_0$ of the vector space $L(E,E^*)$. A *homogeneous linear differential operator of second order* (corresponding to the functional S_0) in the functional space E is a linear map acting from a subspace of the space $C^2(E)$ to the space $\mathcal{F}(E)$; we denote it by symbol Δ_{S_0} and define its action by the following rule: $\operatorname{dom}\Delta_{S_0}=\{g\in C^2(E)\colon \forall x\in E\ g''(x)\in\operatorname{dom}S_0\}$; if $f\in\operatorname{dom}\Delta_{S_0}$, then $(\Delta_{S_0}g)(x)=S_0(g''(x))$.

Nonhomogeneous linear differential operators of arbitrary order acting in $C^n(E)$ can be defined in a similar way [9].

Any linear map acting from a vector subspace of the space $\mathcal{F}(E)$ to $\mathcal{F}(E)$ so that its restriction to $C^n(E)$ is a differential operator, is also called a *differential operator*.

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Example 1. Let $A \in L(E, E)$ and $S_0^A(B) = \operatorname{tr}(BA)$ for a suitable choice of $B \in L(E, E^*)$. Then $\Delta_{S_0^A}$ is called a *Laplace–Volterra operator*. Moreover, it is usually assumed that E is a Hilbert space; it was precisely this case that was considered by L. Gross and Yu. L. Daletskii.

2. THE LAPLACE OPERATORS

Let H be a separable Hilbert space. We suppose that H is identified with the dual Hilbert space H^*. Let E be a locally convex topological space which is a dense vector subspace of H; moreover, we suppose that the canonical embedding of E to H is continuous. Let the space E^* be equipped by a locally convex topology compatible with the duality between E^* and E. Then the map $H^*(=H) \to E^*$, dual to the embedding $E \to H$, is continuous and injective and its range is dense in E^*. The previously introduced objects define the rigged Hilbert space $E \subset H = H^* \subset E^*$. Note that if $x \in E$, $g \in H \subset E^*$, then $(x, g)_H = \langle g, x \rangle$ $(\equiv g(x))$ in the natural notation. Let $\mathbf{e} = (\mathbf{e}_n)$ be an orthonormal basis in H consisting of elements from E. Further, we assume that the linear hull $E_{\mathbf{e}}$ of the basis $\mathbf{e}$ is dense in E.

Proposition 1. *The series $\sum_{n=1}^{\infty} a_n \mathbf{e}_n$ converges to an element $f \in E^*$ in the topology $\sigma(E^*, E_{\mathbf{e}})$ if and only if $a_n = f(e_n)$ for all n.*

Corollary 1. *The map $A\colon E^* \ni f \mapsto (f(\mathbf{e}_n)) \in \mathbb{R}^\infty$ is a homeomorphism between the space $(E^*, \sigma(E^*, E_{\mathbf{e}}))$ and its image in $\mathbb{R}^\infty$.*

Corollary 2. *The map A from Corollary 1 is a continuous bijection from E^* to $A(E^*)$.*

In the sequel, the symbol $\sum_{n=1}^{\infty} a_n \mathbf{e}_n$ denotes the unique element f from the space E^* such that $f(e_n) = a_n$, provided such an element exists.

Remark 1. If $f \in E^*$, $g \in E$, then in general, the series $\sum_{n=1}^{\infty} f(\mathbf{e}_n) g(\mathbf{e}_n)$ can diverge (see Example 2).

Remark 2. The following are equivalent:
(1) for all $f \in E^*$, $g \in E$, the series $\sum_{n=1}^{\infty} f(\mathbf{e}_n) g(\mathbf{e}_n)$ converges;
(2) for all $f \in E^*$ the series $\sum_{n=1}^{\infty} f(\mathbf{e}_n)\mathbf{e}_n$ converges in the topology $\sigma(E^*, E)$;
(3) for all $g \in E$ the series $\sum_{n=1}^{\infty} g(\mathbf{e}_n)\mathbf{e}_n$ converges in the topology $\sigma(E, E^*)$.

Example 2. Set $H = L_2(-1, 1)$, $E = C[-1, 1]$. Then E^* is the space of all Borel measures (of alternating sings) on $[-1, 1]$. Let

$$\mathbf{e}_n(t) = c_n \cos\Big(\frac{\pi}{2} nt\Big) \qquad \text{for all} \quad n = 0, 1, 2, \dots,$$

and let $f \in C[-1, 1]$ be a function whose Fourier series diverges at the origin, i.e., a function f for which the series

$$\sum_{j=1}^{\infty} f(\mathbf{e}_j)\delta(\mathbf{e}_j) = \sum_{j=1}^{\infty} f(\mathbf{e}_j)\mathbf{e}_j(0)$$

diverges.

Example 3. Suppose that D is a strictly positive Hilbert–Schmidt operator in H, $(\mathbf{e}_n)$ is an (orthonormal) basis in H consisting of the eigenvectors of the operator D, $E = \bigcap_n D^n H$, and E is endowed by the topology determined by the family of Hilbert norms

$$\|\cdot\|_p, \qquad (\|h\|_p)^2 = (D^{-p}h, D^{-p}h)_H.$$

Then E is a reflexive nuclear Fréchet space, and for each $g \in E^*$, the series $\sum_{n=1}^{\infty} \langle g, \mathbf{e}_n \rangle \mathbf{e}_n$ converges in E^* even in the strong topology.

Now let $\mathcal{F}_e(E)$ be the set of all functions $f \in \mathcal{F}(E)$ which have two derivatives along vectors from the basis $\mathbf{e}$. Let S be a positive linear functional defined on a vector subspace $\operatorname{dom} S$ of the space of sequences containing the space ℓ_1.

Definition 2. By the *Laplace operator* on $\mathbf{F}_e(E)$ defined by the functional S, we mean the map $\Delta_S\colon \operatorname{dom}\Delta_S \to \mathcal{F}(E)$ acting by the rule

$$(\Delta_S f)(x) = S((f''_{jj}(x))_{j=1}^{\infty}),$$

where

$$\operatorname{dom}\Delta_S = \{f \in \mathcal{F}_{\mathbf{e}}(E) : (f''_{jj}(x)) \in \operatorname{dom} S\} \qquad \left(\text{and} \quad f''_{jj}(x) = \frac{d^2}{dt^2}\bigg|_{t=0} f(x + t\mathbf{e}_j)\right).$$

If, moreover, $\ell_1 \subset \ker S$, then the operator Δ_S is called the *Lévy–Laplace operator*; if $S((x_n)) = \sum a_n x_n$, where $a_n \geq 0$, then Δ_S is the *Laplace–Volterra operator*.

We will show that the well-known relationship between the Laplace–Volterra operator and the standard definition of a trace (see Example 1) is similar to the relationship between the Lévy–Laplace operator and an analogous object called the Lévy trace.

Let the functional S_c act as follows:

$$\operatorname{dom} S_c = \left\{(a_n) \in \mathbb{R}^{\infty} : \exists \lim_n \frac{1}{n}\sum_{j=1}^{n} a_j\right\}, \qquad S_c(\{a_n\}) = \lim_n \frac{1}{n}\sum_{j=1}^{n} a_j$$

(i.e., S_c is a Cesáro mean of the corresponding sequence). Further, we call the operator Δ_{S_c} a *classical Lévy Laplacian* and denote it by $\Delta_{\mathcal{L}}$.

Remark 3. In a similar way, one can define the Cesáro mean for elements of the space G^{∞} (where G is a locally convex topological space), and then, by using this procedure, one can introduce the classical Lévy Laplacian for maps from E to G.

Definition 3. By a *Lévy trace* (relative to the basis $\mathbf{e}$) we mean the functional $\operatorname{tr}_{\mathcal{L}}$ defined on $L(E, E^*)$ (or, maybe, on its part) by the equality $\operatorname{tr}_{\mathcal{L}} A = S_c((\langle A\mathbf{e}_j, \mathbf{e}_j\rangle))$.

The Lévy trace on $L(E, L(E, G))$ is defined analogously. If $f \in C^2(E)$, then

$$\Delta_{\operatorname{tr}_{\mathcal{L}}} f(x) = \Delta_{S_c} f(x) = \Delta_{\mathcal{L}} f(x) = \operatorname{tr}_{\mathcal{L}} f''(x).$$

Definition 4. The *Lévy inner product* $\langle\,\cdot\,,\,\cdot\,\rangle_{\mathcal{L}}$ (relative to the basis $\mathbf{e}$) is defined as follows:

$$\operatorname{dom}\langle\,\cdot\,,\,\cdot\,\rangle_{\mathcal{L}} = \{(a, b) \in E^* \times E^*; \quad (\langle a, \mathbf{e}_j\rangle \cdot \langle b, \mathbf{e}_j\rangle) \in \operatorname{dom} S_c\}.$$

If $(a, b) \in \operatorname{dom}\langle\,\cdot\,,\,\cdot\,\rangle\mathcal{L}$, then

$$\langle a, b\rangle_{\mathcal{L}} = S_c((\langle a, \mathbf{e}_j\rangle \cdot \langle b, \mathbf{e}_j\rangle)).$$

Remark 4. The domain of the Lévy inner product is not necessarily a vector space; moreover, even the set E_0^* of $f \in E^*$ for which Lévy scalar square $\langle f, f\rangle$ exists need not be a vector space (see Example 4).

Example 4. Suppose that the assumptions of Example 2 are fulfilled, and let $n_1 < n_2 < n_3 \dots$ be a sequence of integers such that $n_{j+1}/n_j \to \infty$. Let $f_k = \sum_{j=1}^{\infty} a_j^k \mathbf{e}_j \in E^*$ $(k = 1, 2)$, where $a_j^1 = 1$ for even j and $a_j^1 = 0$ for odd j; $a_j^2 = 1$ for even $j \in [n_{2m}, n_{2m+1}]$, $a_j^2 = 1$ for odd $j \in [n_{2m-1}, n_{2m}]$ and $a_j^2 = 0$ for the remaining j. Then $\langle f_1, f_1\rangle_{\mathcal{L}} = \langle f_2, f_2\rangle_{\mathcal{L}} = 1/2$, but $\langle f_1 + f_2, f_1 + f_2\rangle_{\mathcal{L}}$ does not exist.

Example 5. If $a \in E^*$ and $A = a \otimes a \in L(E, E^*)$, then $\operatorname{tr}_{\mathcal{L}} A = \langle a, a \rangle_{\mathcal{L}}$.

As an example, let us consider a vector subspace $E_{\mathcal{L}}$ in E^* such that $E_{\mathcal{L}} \times E_{\mathcal{L}} \subset \operatorname{dom}\langle \cdot\,, \cdot \rangle_{\mathcal{L}}$. Here we suppose that the assumptions of Example 3 are fulfilled. The general case can be considered in a similar way. For each $\lambda \in (0, \pi)$, we set $s_\lambda = \sum \sin(\lambda n)\mathbf{e}_n \quad (\in E^*)$. Suppose that ν is a σ-additive Borel measure on $(0, \pi)$, and set

$$s_\varphi = \int_0^\pi \varphi(\lambda) s_\lambda \, \nu(d\lambda) \qquad (\in E^*)$$

for each function $\varphi \in L_1((0,1), \nu) \cap L_2((0,1), \nu)$. Finally, let S^ν be the image of the space $L_1(0, \pi)$ under the map $f \mapsto s_f$. Then $S^\nu \times S^\nu \subset \operatorname{dom}\langle \cdot\,, \cdot \rangle_{\mathcal{L}}$ and the restriction of $\langle \cdot\,, \cdot \rangle_{\mathcal{L}}$ to $S^\nu \times S^\nu$ is an inner product on S^ν (if the support of ν is unbounded, then $(S^\nu, \langle \cdot\,, \cdot \rangle_{\mathcal{L}})$ is incomplete). Note that

$$\langle s_\varphi, s_\psi \rangle_{\mathcal{L}} = \int_0^\pi \varphi(\lambda)\psi(\lambda) \, \nu(d\lambda).$$

3. ANALYTICAL PROPERTIES OF THE LÉVY LAPLACIAN

Proposition 2. *If $g \in C^2(E)$, $f \in C^2(\mathbb{R}^1)$, then*

$$\Delta_{\mathcal{L}}(f \circ g)(x) = f''(g(x))\langle g'(x), g'(x) \rangle_{\mathcal{L}} + f'(g(x))(\Delta_{\mathcal{L}} g)(x).$$

Proof. By the chain rule, we have

$$(f \circ g)''(x) = f''(g(x))(g'(x) \otimes g'(x)) + f'(g(x)) \cdot g''(x).$$

It simply remains to apply the map $\operatorname{tr}_{\mathcal{L}}$ to both sides of this equality. $\square$

Corollary 3. *Let the assumptions of the previous proposition be fulfilled. If $\Delta_{\mathcal{L}} g = 0$, then $\Delta_{\mathcal{L}}(f \circ g)(x) = f''(g(x))\langle g'(x), g'(x) \rangle_{\mathcal{L}}$. If $\langle g'(x), g'(x) \rangle_{\mathcal{L}} = 0$ (in particular, this is the case if $E = H = E^*$), then we obtain a formula which usually arises in papers on the Lévy Laplacian:*

$$\Delta_{\mathcal{L}}(f \circ g)(x) = f'(g(x))((\Delta_{\mathcal{L}} g)(x)).$$

Finally, if $\Delta_{\mathcal{L}} g = 0$ and $\langle g'(x), g'(x) \rangle_{\mathcal{L}} = 0$ simultaneously, then $\Delta_{\mathcal{L}}(f \circ g)(x) = 0$.

Corollary 4. *Let $a \in E^*$ and $P_a \colon E \to \mathbb{R}^1$ be a homogeneous polynomial: $P_a(\xi) = \langle a, \xi \rangle^k$. Then $\Delta_{\mathcal{L}} P_a(\xi) = k(k-1)\langle a, a \rangle_{\mathcal{L}} \cdot (\langle a, \xi \rangle)^{k-2}$ (this means that both sides of the equality exist simultaneously, and in this case they are equal). In particular, if $\langle a, a \rangle_{\mathcal{L}} = 0$ (for example if $a \in H$), then $\Delta_{\mathcal{L}} P_a(\xi) = 0$ for all $\xi \in E$.*

Example 6. If $f \in E^*$, $F(\xi) = e^{\langle f, \xi \rangle}$, and the scalar square $\langle f, f \rangle_{\mathcal{L}}$ exists, then $(\Delta_{\mathcal{L}} F)(\xi) = \langle f, f \rangle_{\mathcal{L}} e^{\langle f, \xi \rangle}$. If $\Psi(\xi) = f((\xi, \xi)_H)$, then $(\Delta_{\mathcal{L}} \Psi)(\xi) = 2f'((\xi, \xi)_H)$.

Proposition 3 (The Leibniz formula). *If g and $f \in C^2(E)$, then*

$$\Delta_{\mathcal{L}}(f \cdot g)(x) = (g \cdot \Delta_{\mathcal{L}} f)(x) + (f \cdot \Delta_{\mathcal{L}} g)(x) + 2\langle f'(x), g'(x) \rangle_{\mathcal{L}}.$$

The proof is similar to the proof of Proposition 2.

Corollary 5. *If $E = H = E^*$, then $\langle f'(x), g'(x) \rangle_{\mathcal{L}} = 0$, and we have the well-known formula*

$$\Delta_{\mathcal{L}}(f \cdot g)(x) = (g \cdot \Delta_{\mathcal{L}} f)(x) + (f \cdot \Delta g)(x).$$

4. SOLUTION OF THE HEAT EQUATION WITH LÉVY LAPLACIAN

Proposition 4. *If $f \in E^*$ and $\langle f, f\rangle_{\mathcal{L}}$ exists, then the functions*

$$F_1(t,\xi) = e^{t\langle f,f\rangle_{\mathcal{L}}} e^{\langle f,\xi\rangle} \qquad \text{and} \qquad F_2(t,\xi) = e^{2t} e^{\langle \xi,\xi\rangle_H}$$

are solutions of the "Lévy heat equation"

$$\frac{\partial}{\partial t} F(t,\xi) = \Delta_{\mathcal{L}} F(t,\xi).$$

Theorem 1. *Suppose that G is a Borel subset of the set E_0^*, ν is a finite σ-additive measure on G, and*

$$\int e^{t\langle f,f\rangle_{\mathcal{L}}} e^{\langle f,x\rangle} \|\nu\|\,(df) < \infty \qquad \text{for each} \quad x,$$

where $\|\nu\|$ is the total variation of the measure ν. Then the function g defined by the equality

$$g(t,x) = \int e^{t\langle f,f\rangle_{\mathcal{L}}} e^{\langle f,x\rangle} \nu\,(df),$$

is a solution of the Lévy heat equation.

Remark 5. The function $\tilde{\nu}(x)$ defined by the equality $\tilde{\nu} = \int e^{\langle f,x\rangle} \nu\,(df)$ is called the *Laplace transform of the measure* ν, provided the integral in the right-hand side exists. The previous theorem states that the Laplace transform of the measure $e^{t\langle f,f\rangle_{\mathcal{L}}}\nu$ is a solution of the Lévy heat equation. By introducing an appropriate definition of the convolution of functions and measures, one can say that the solution is the convolution of the function $\tilde{\nu}(x)$ and the fundamental solution of the Cauchy problem; the inverse Laplace transform of the fundamental solution is the function $e^{t\langle f,f\rangle_{\mathcal{L}}}$. In fact, this result was established in [5] (cf. [10]), where the Fourier transform was used instead of the Laplace transform.

For $\beta > 0$, let us consider a finite σ-additive positive measure μ_β on the space S^ν (see Example 5) supported on the ball S_β of radius β, and suppose that $L_2(S_\beta)$ is the space of μ_β-square integrable complex functions on E^* (each function is defined μ_β-a.e. by its restriction to S_β), and let H_β be the space of the Laplace transforms (respectively H_β^F is the space of Fourier transforms) of the measures which are the products of functions from $L_2(S_\beta)$ and the measure μ_β. It is assumed that H_β (or H_β^F) has the structure of a Hilbert space generated by the inner product in $L_2(S_\beta)$. Let γ be a σ-finite nonnegative measure on $(0,\infty)$, and let $\mathbf{H}$ (or $\mathbf{H}^F$) be the continuous direct sum of the spaces H_β (or the spaces H_β^F, respectively) generated by this measure.

Proposition 5 (cf. [1, 2, 5]). *For each $\beta > 0$, the Laplace transform (the Fourier transform, respectively) of the measure μ_β is an eigenfunction of the Lévy Laplacian corresponding to the eigenvalue β^2 (or to the eigenvalue $-\beta^2$ respectively).*

Theorem 2. *The restriction of the Lévy Laplacian to the space $\mathbf{H}$ (to $\mathbf{H}^F$ respectively) defines an essentially self-adjoint operator $\Delta_{\mathcal{L}H}$ in this space; moreover, if $f(\cdot) \in \operatorname{dom}\Delta_{\mathcal{L}H}$, $f = \int_0^\infty f(\beta)\,\gamma(d\beta)$, then*

$$e^{t\Delta_{\mathcal{L}H}} f = \int_0^\infty e^{t\beta^2} f(\beta)\,\gamma(d\beta)$$

($e^{t\Delta_{\mathcal{L}H}} f = \int_0^\infty e^{-t\beta^2} f(\beta)\,\gamma(d\beta)$ respectively).

Consider a linear measurable function F on E^*, and let $\delta > 0$. Suppose that for each $\beta > 0$ we have $\mu_\beta\{x\colon |F(x)|^\delta = \beta^2\} > 0$, and for each $\lambda \in \mathbb{R}^1$, η_λ is a measure on S^ν supported by $S_{|\lambda|^\delta} \cap \{x\colon F(x) = \lambda\}$ and coinciding on this set with the restriction of the measure μ_β to the same set. Consider the space L_2^λ of complex η_λ-square integrable functions on E^*. Let γ_0 be a nonnegative measure on $\mathbb{R}^1$ and let $\mathbf{H}_0$ (respectively, $\mathbf{H}_0^F$) be the continuous direct sum of the Hilbert spaces H_λ (or H_λ^F) generated by this measure and defined as it was done previously.

Proposition 6. *The restriction of the Lévy Laplacian to* $\mathbf{H}_0$ (*respectively, to* $\mathbf{H}_0^F$) *defines an essentially self-adjoint operator* $\Delta_{\mathcal{L}H_0}$ *acting in this space*; *here, if*

$$f = \int_{-\infty}^{\infty} f(\lambda)\,\gamma_0(d\lambda) \in \operatorname{dom}\Delta_{\mathcal{L}H_0},$$

then

$$e^{t\Delta_{\mathcal{L}H_0}} f = \int_{-\infty}^{\infty} e^{t|\lambda|^{\delta}} f(\lambda)\,\gamma_0(d\lambda)$$

(*respectively,* $e^{t\Delta_{\mathcal{L}H_0}} f = \int_{-\infty}^{\infty} e^{-t|\lambda|^{\delta}} f(\lambda)\,\gamma_0(d\lambda)$).

Remark 6. If $\delta \in [1,2]$, then in order to compute the last integral, it suffices to find the expectation with respect to a stable distribution with parameter δ. Note that the measures η_λ can be constructed as surface measures generated by a smooth measure η on E^*. By choosing a suitable Gaussian measure as η, one can recover a number of results due to Saito (see [11, 12]), who used Hida's "white noise analysis."

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