Kinetic energy recovery system for sailing yachts

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Abstract:

SEAKERS (SEA Kinetic Energy Recovery System) is a research project, funded within the 7th EU Framework Programme and officially started on January 1st, 2011, whose goal is to develop an innovative device consisting in a kinetic energy recovery system for sailing yachts based on the conversion of boat oscillations (heave, pitch and roll) caused by the sea into electric energy by means of a linear generator.

Therefore, SEAKERS addresses a well known unsatisfied requirement of yacht owners, since energy is a resource of primary importance in a boat, especially in a sailing one: it is well known that during a one day cruise, electricity consumption has to be carefully managed (for instance the refrigerator is switched off), so as not to be short of energy at night. It often happens that, after one day of sail cruise, it is necessary to recharge the batteries through the onboard generator, which means keeping it on for hours, producing very annoying noise, smoke and pollution.

The device that is going to be developed aims at recovering as much kinetic energy as possible from the natural movements of a sailing yacht on the sea, therefore taking the view of a boat as a moving wave energy converter with energy harvesting capacity. The boat's motions can be vertical oscillations due to the buoyancy in the presence of sea waves, both when the boat is still or sailing, and rolling and pitching motions originated both by sailing in wavy waters and by the normal boat dynamics due to the sails' propulsion. Linear generators will convert kinetic energy into electrical energy to be used as "green" electricity for any possible application on board.

Preliminary calculations show that a properly configured system could be able to recover 100-400 W under most sea conditions, which can be an extremely attractive result since an electric energy availability of 1-2 kWh on a sailing yacht is of significant interest.

Keywords:

Wave Energy Recovery, Linear Generator, Sail Yacht.

1. Introduction

This paper presents some preliminary results obtained in the SEAKERS project, whose aim is to design and test a kinetic energy recovery system to be used on board of sail yachts in order to recover energy from the wave-induced boat's vertical motion.

Such a system is able to recover actual free energy, as opposed to other devices, already commercially available, that subtracts energy from the propulsion offered by the wind's lift on the sails, as in the case of micro-wind turbines installed on the boat, which are set into motion by the apparent wind originating from the yacht's motion.

In practical terms, the SEAKERS device is intended to be a linear oscillator, with a mass oscillating vertically inside a prismatic guide and gaining kinetic energy; if the mass is the moving element of a linear generator, the resulting mechanical energy can be extracted and converted into electricity. The oscillating mass incorporates permanent magnets which, moving in proximity of stator windings, generate electric power due to electromagnetic induction.

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The idea of a linear generator originates from work carried out at the University of Uppsala [1-4], where such devices have been designed and tested in order to recover wave energy from a buoy, oscillating on the sea surface, connected to a rope that makes a piston move inside a generator placed on the seafloor. In the SEAKERS project, the oscillating mass is set into motion not directly by the sea waves but by its inertia as the yacht is subject to heave, pitch and roll motions.

In order to design the test-bed for testing the generator, it is necessary to set up a reliable model of different sea conditions that could be of practical interest for a normal cruise on a sail yacht (thus there is no need to consider extreme, stormy waves) and of the ship motion due to such sea states. Furthermore, it is interesting to find out, by means of a very simple mechanical model of the linear generator, how much power could be extracted under these simplifying assumptions, in order to decide whether the project's outcome could in principle be commercially viable, and quickly to provide data against which results from more detailed analytical models and experimental tests could later be compared.

This paper presents the results obtained in this first stage of the project, detailing first the model of sea conditions (section 2), then the outcome of simulations on the yacht's motion carried out by means a commercial software (section 3), and finally the results of the analysis carried out on a linear mechanical system located on the boat (section 4).

2. Wave excitation

2.1. Wave spectra

The main characteristic of sea waves is randomness. Indeed, by checking even a short a time series, two characteristics arise: height and period of a wave are different from height and period of another wave. For this reason, the free surface elevation of sea waves is modelled as a stochastic process and is assumed to be a random, Gaussian, ergodic process in the time domain [5-8].

Mathematically, sea elevation can be reconstructed in one dimension as a Fourier series as follows:

$$\zeta(x,t) = \sum_{j=1}^{n} Z_j \cos\left(\omega_j t - k_j x + \vartheta_j\right) \qquad (1)$$

In this equation, Z_j is the wave amplitude for the j-th wave form of circular frequency ω_j, k_j its wave number (dependent on ω_j through the dispersion relation) and ϑ_j its phase shift. The *dispersion relation* defines the relationship between wave frequency and wave number; in deep water it is expressed as [5-8]:

$$\omega^2 = kg \qquad (2)$$

where g is the acceleration of gravity. (It may be useful to recall that wave number and wave length are mutually dependent: $k = 2\pi / \lambda$).

Given a sea elevation time pattern ζ for a given spatial coordinate x, the amplitudes Z_j of its Fourier series may be evaluated as Fourier transforms of ζ:

$$Z_j = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \zeta(t)\exp\left(-i\,2\pi\,\omega_j t\right)\mathrm{d}t \qquad (3)$$

The most meaningful representation from a statistical point of view of a particular sea state is given in the frequency domain by means of the wave spectrum $S(\omega)$, which is defined as:

$$S(\omega_j)\mathrm{d}\omega = Z_j Z_j^* = \left|Z_j\right|^2 \qquad (4)$$

Z_j^* being the complex conjugate of Z_j. Therefore, the spectrum $S(\omega_j)$ is proportional to the energy content of the j-th wave form of circular frequency ω_j, while the area under the spectrum s proportional to the overall energy content of the sea state described by sea elevation $\zeta(t)$:

$$\int_{-\infty}^{\infty} S(\omega)\mathrm{d}\omega = \sum_{j=1}^{n} \left|Z_j\right|^2 \qquad (5)$$

Since $S(\omega)$ is an even function, that is, $S(-\omega) = S(\omega)$, and taking also into account that negative angular frequencies have no physical meaning beyond that of the corresponding positive values, it is frequently adopted an alternative definition of the energy spectrum (S_ζ), which is defined for positive angular frequencies only:

$$2\int_{0}^{\infty} S_\zeta(\omega)\mathrm{d}\omega = \int_{-\infty}^{\infty} S(\omega)\mathrm{d}\omega \qquad (6)$$

By virtue of equation (5), the energy spectrum is correlated to the overall energy content of the sea state, because the energy content of a single sinusoidal wave is proportional to the square of its height. Furthermore, statistical data that can be gleaned from the energy spectrum correspond to important parameters for the description of a sea state. Of particular importance is the 0-th spectrum moment, which is equivalent to the area under the wave spectrum curve:

$$m_0 = \int_{0}^{\infty} S_\zeta(\omega)\mathrm{d}\omega \qquad (7)$$

For a narrow band spectrum, it can be demonstrated that the root mean square (RMS) wave amplitude is given by $\sqrt{m_0}$, and the RMS value of wave height (crest to trough) is therefore:

$$H_{RMS} = 2\sqrt{m_0} \qquad (8)$$

One of the most useful parameter to represent the sea state is the *significant wave height*, which is the mean of highest third wave heights, and for narrow band spectrum it is given by [5-8]:

$$H_s = 4\sqrt{m_0} \qquad (9)$$

Significant wave amplitude is by definition half the corresponding wave height:

$$\zeta_{0s} = 2\sqrt{m_0} \qquad (10)$$

2.2. Simulation assumptions

The simulations that will be presented in the following sections were carried out taking into account statistical wave data for the Mediterranean Sea, with particular reference to the measurements taken at Capo Linaro (Civitavecchia, Italy)[1].

In the case of random waves, it is possible to find a particular set of parameters that make the JONSWAP spectrum suitable to represent sea conditions in the location of interest (the above mentioned Capo Linaro near Civitavecchia).

The JONSWAP spectrum was developed from extensive field measurements in the context of the Joint North Sea Wave Project [5-8]. This formulation is suitable for wind-generated waves in fetch limited locations. The inputs are the wind speed and the fetch length. The mathematical formulation is given by equation:

[1] Personal communications with Prof. Felice Arena, University of Reggio Calabria, 2011.

$$S_\zeta(\omega) = \alpha g^2 \omega_p^{-5} \exp\left[-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right] \exp\left\{\ln(\gamma) \exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right]\right\} \quad (11)$$

In the above equation, $\omega_p = 2\pi/T_p$ is the peak circular frequency, α is the Phillips' parameter given by $\alpha = 0.0076(gx/\overline{U}^2)^{-0.22}$, where x is the fetch length and $\overline{U}$ the mean wind speed, and γ is the peak-shape parameter. For practical applications, σ can be assumed equal to 0.08 in the whole frequency domain.

In deep water, wave period and length are correlated by the dispersion relation (2), which may be rewritten as follows:

$$\lambda = gT^2/(2\pi) \quad (12)$$

Wave velocity is given by:

$$c = \lambda/T = gT/(2\pi) \quad (13)$$

Furthermore, wave period is also related to the significant wave height through parameters α and γ.

$$T_p = f(\alpha,\gamma)\sqrt{H_s/g} \quad (14)$$

Thus, higher waves are longer (12), propagate faster (13) and are less frequent (14). The JONSWAP spectrum is completely defined when the significant wave height H_s and parameters α and γ are specified.

In order to represent correctly sea conditions at Capo Linaro, values of α, γ, f and T_p are chosen according to the following table. The corresponding wave spectra are illustrated in fig. 1.

Table 1. Parameters used to represent random sea waves at Capo Linaro near Civitavecchia, Italy.

H_s [m]	α	γ	$f(\alpha,\gamma)$	T_p [s]
0.5	0.016	1.0	13.2	2.98
1.0	0.008	2.0	14.9	4.75
1.5	0.010	0.5	15.5	6.06
2.0	0.008	0.5	16.4	7.40

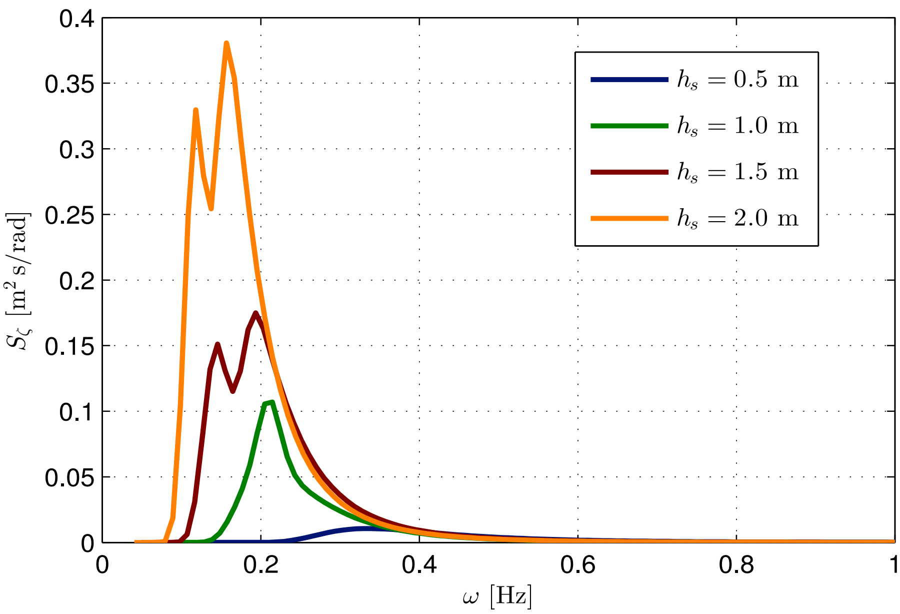

Fig. 1. Wave spectra representing sea conditions at Capo Linaro near Civitavecchia, Italy.

3. Yacht's response

3.1. Encounter frequency

Due to its forward speed V, the wave spectrum for the ship is different than for a fixed observer. When studying the ship's response it is therefore necessary to take into account the frequency at which it actually encounters the waves (*encounter frequency*). The encounter frequency depends on wave velocity and ship speed and relative direction with respect to waves μ. Angle μ is defined between the forward directions of wave and ship: thus for bow waves $\mu = \pi$, for transverse waves $\mu = \pi/2$, and for aft waves $\mu = 0$.

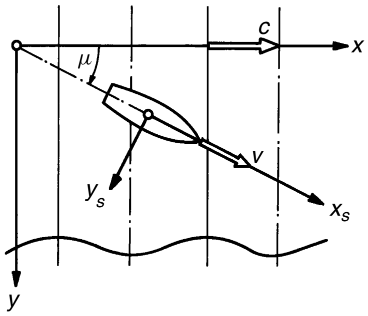

Fig. 2. Definition of angle of encounter [5].

Encounter frequency[2] must be evaluated taking into account the velocity component of the ship in the direction of the waves, subtracting wave velocity c. The relative velocity is given by:

$$V_{rel} = c - V\cos\mu \tag{15}$$

Thus the encounter period is:

$$T_e = \frac{\lambda}{V_{rel}} = \frac{\lambda}{c - V\cos\mu} \tag{16}$$

The encounter frequency is thus given by:

$$\omega_e = \frac{2\pi}{\lambda}\left(c - V\cos\mu\right) \tag{17}$$

For seakeeping purposes, the assumption of deep water may be applied; in this case, taking into account the dispersion relation (2), the encounter frequency can be finally derived as:

$$\omega_e = \omega - \frac{\omega^2 V}{g}\cos\mu \tag{18}$$

The wave energy spectrum must also be modified according to the encounter frequency (it is practically a Doppler shift of the spectrum). Since the energy content of a spectrum must be the same for any observer, fixed or moving with the ship, the 0-th momentum must be the same:

$$m_0 = \int_0^\infty S(\omega)\mathrm{d}\omega = \int_0^\infty S_e(\omega_e)\mathrm{d}\omega_e \tag{19}$$

Therefore, the relation between wave spectrum and encounter spectrum is the following:

$$S_e(\omega_e)\mathrm{d}\omega_e = S(\omega)\mathrm{d}\omega \tag{20}$$

[2] In this paper, the term "frequency" will be used indifferently to identify both frequency f, measured in Hz, or angular (circular) frequency ω, measured in rad/s.

which becomes, taking into account that $\mathrm{d}\omega_e = \left|1 - 2\omega V \cos\mu / g\right| \mathrm{d}\omega$:

$$S_e(\omega_e) = \frac{S(\omega)}{\left|1 - 2\frac{\omega V}{g}\cos\mu\right|} \tag{21}$$

3.2. Response amplitude operators

The ship response is usually described in terms of transfer functions (RAO, Response Amplitude Operator), which give the normalised amplitude of the resulting ship's motion for a sinusoidal excitation of frequency ω_e, the normalization factor being the wave amplitude ζ_0 for linear motions, the wave slope $k\zeta_0 = 2\pi\zeta_0 / \lambda$ for angular motions and the wave acceleration $\omega_e^2\zeta_0$ for accelerations:

$$\mathrm{RAO}_z(\omega_e) = \frac{z_0}{\zeta_0} \tag{22}$$

$$\mathrm{RAO}_\vartheta(\omega_e) = \frac{\vartheta_0}{k\zeta_0} \tag{23}$$

$$\mathrm{RAO}_a(\omega_e) = \frac{a_0}{\omega_e^2\zeta_0} \tag{24}$$

Obviously, equally important are the phase shifts α of each motion with respect to the wave excitation. With the knowledge of RAOs and phase shifts, it is possible to reconstruct heave (z), pitch (ϑ) and roll (η) motions from a sinusoidal wave excitation $\zeta(t) = \zeta_0 \exp(i\omega_e t)$as follows:

$$z(t) = z_0 \exp[i(\omega_e t + \alpha_z)] \tag{25}$$

$$\vartheta(t) = \vartheta_0 \exp[i(\omega_e t + \alpha_\vartheta)] \tag{26}$$

$$\eta(t) = \eta_0 \exp[i(\omega_e t + \alpha_\eta)] \tag{27}$$

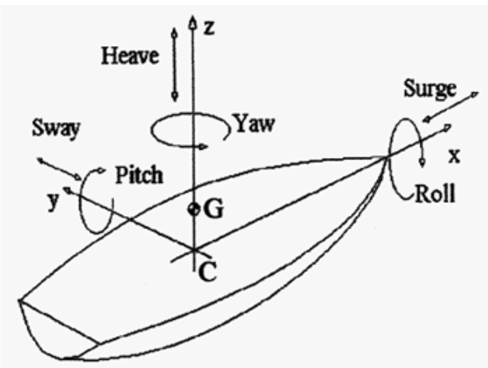

Figure 3. Coordinate system and definition of motion [9].

Therefore, vertical oscillations for any point on the ship may be calculated as follows (fig. 3):

$$y(t) = z(t) - L\sin(\vartheta(t)) + B\sin(\eta(t)) \tag{28}$$

where L and B are the longitudinal and lateral distance of the point of interest from the center of gravity. Since angular motions (pitch and roll) are usually small, it is possible to approximate the above expression:

$$y(t) \cong z(t) - L\vartheta(t) + B\eta(t) \qquad (29)$$

Therefore, being the sum of harmonic motions (phasors), the vertical oscillation $y(t)$ is also represented by a harmonic oscillation:

$$y(t) = y_0 \exp\left[i\left(\omega_e t + \alpha_y\right)\right] \qquad (30)$$

and it is possible to define a RAO for the particular point of interest on the ship:

$$\mathrm{RAO}_y\left(\omega_e\right) = \frac{y_0}{\zeta_0} \qquad (31)$$

In case of a random wave excitation, with the assumption that the response is a linear function of wave amplitude and applying the superposition principle, vertical motion can be reconstructed as:

$$y(t) = \sum_{j=1}^{n} y_{0,j} \cos\left(\omega_{e,j} t + \alpha_{y,j}\right) \qquad (32)$$

where each oscillation amplitude $y_{0,j}$ is a function of frequency and amplitude of the j-th harmonic, according to (31).

Furthermore, it is possible to demonstrate that the ship's response energy spectrum is given by the product of the square of the RAO and the wave energy spectrum. Thus, heave motion's energy spectrum is:

$$S_z\left(\omega_\mathrm{e}\right) = \mathrm{RAO}_z^2\left(\omega_e\right) S_\zeta\left(\omega_e\right) \qquad (33)$$

and analogous equations hold for the other motions, while for any point on the ship the energy spectrum associated to its wave-induced motion is:

$$S_y\left(\omega_\mathrm{e}\right) = \mathrm{RAO}_y^2\left(\omega_e\right) S_\zeta\left(\omega_e\right) \qquad (34)$$

3.3. Simulation results

The foundation for the commercial software package *Seakeeper*[3], which was used to carry out the computation of the yacht's motions under different wave conditions, is the linear strip theory based on the work of Salvesen [10], which is used to calculate the coupled heave and pitch response of the vessel; the roll response is calculated using linear roll damping theory [11].

The main purpose of the kinematic model presented is to provide reasonable data about the response of a generic yacht to different sea conditions, in order to have reliable information on the motion which the SEAKERS device is subjected to. Since the project does not address a particular yacht model, nor even a specific size of boat, there was no point in developing a focused *in-house* software: hence the choice of adopting a commercial software that has a proven record of reliability, using it to simulate the response of a yacht of adequate length included in the extensive library provided.

The yacht's model used in the numerical simulations is one of the library models that can be found in Seakeeper's library, since it has geometric and mass properties comparable to those of commercial sail yachts of interest for the SEAKERS project.

The most relevant hydrostatic properties of this yacht are given in table 2. The generator considered in the simulations presented is placed at bow on the longitudinal axis (B = 0) at a distance L = 5.17 m from the center of gravity.

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[3] *Seakeeper* is a software by Formation Design Systems Pty Ltd (trading as FormSys); website: http://www.formsys.com/maxsurf/msproducts/seakeeper.

Figure 4 gives an overview of 21 two-dimensional sections used in the Seakeeper software to evaluate sectional hydrodynamic masses, damping coefficients, and all other data needed in the context of the strip theory [5-9].

Table 2. Yacht's hydrostatic properties.

	Value	*UoM*
Displacement	6.531	t
Volume (displaced)	6.372	t
Overall length	11.5	m
Draft amidships	2.475	m
Immersed depth	3.054	m
Waterline length	10.64	m
Max beam on waterline	2.866	m
Max section area	1.213	m^2
Waterplane area	21.21	m^2
Prismatic coefficient (Cp)	0.494	
Block coefficient (Cb)	0.068	

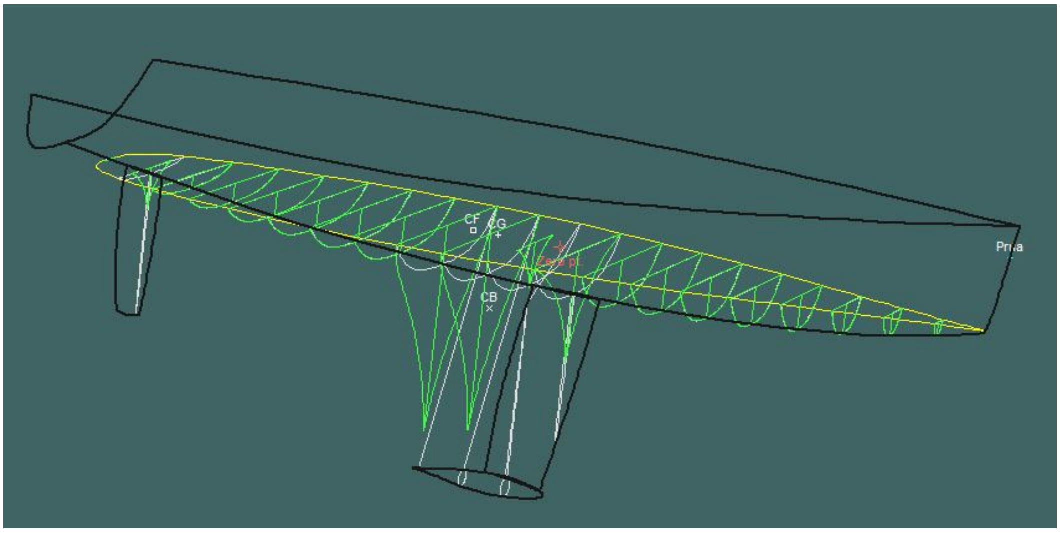

Fig. 4. Yacht's mapped sections used to evaluate hydrodynamic coefficients in the equations of motion by the Seakeeper software.

As illustrated in section 3.2, the ship's response is defined by means of Response Amplitude Operators (RAO) and phase shifts, with reference to a sinusoidal wave excitation. Figures 5 and 7 show values of RAO for each motion (heave, pitch and roll) for two different speeds ($V = 5$ knt and $V = 8$ knt respectively), while figs. 6 and 8 show the phase shifts, as obtained by means of the *Seakeeper* software. For the roll motion, the default value of non-dimensional damping factor proposed by the software has been taken into account.

The response to random waves is illustrated in figs. 9 and 10 in terms of energy spectra of the vertical oscillations (34). The significant oscillation amplitudes are obtained from these spectra in the same way as the significant wave amplitude (10) is calculated from the wave spectrum:

$$y_{0s} = 2\sqrt{m_{0y}} = 2\sqrt{\int_0^\infty S_y(\omega_e)\,\mathrm{d}\omega_e} \qquad (35)$$

Values of significant vertical oscillation amplitudes, corresponding to the energy spectra of figs. 9 and 10, are given in table 3.

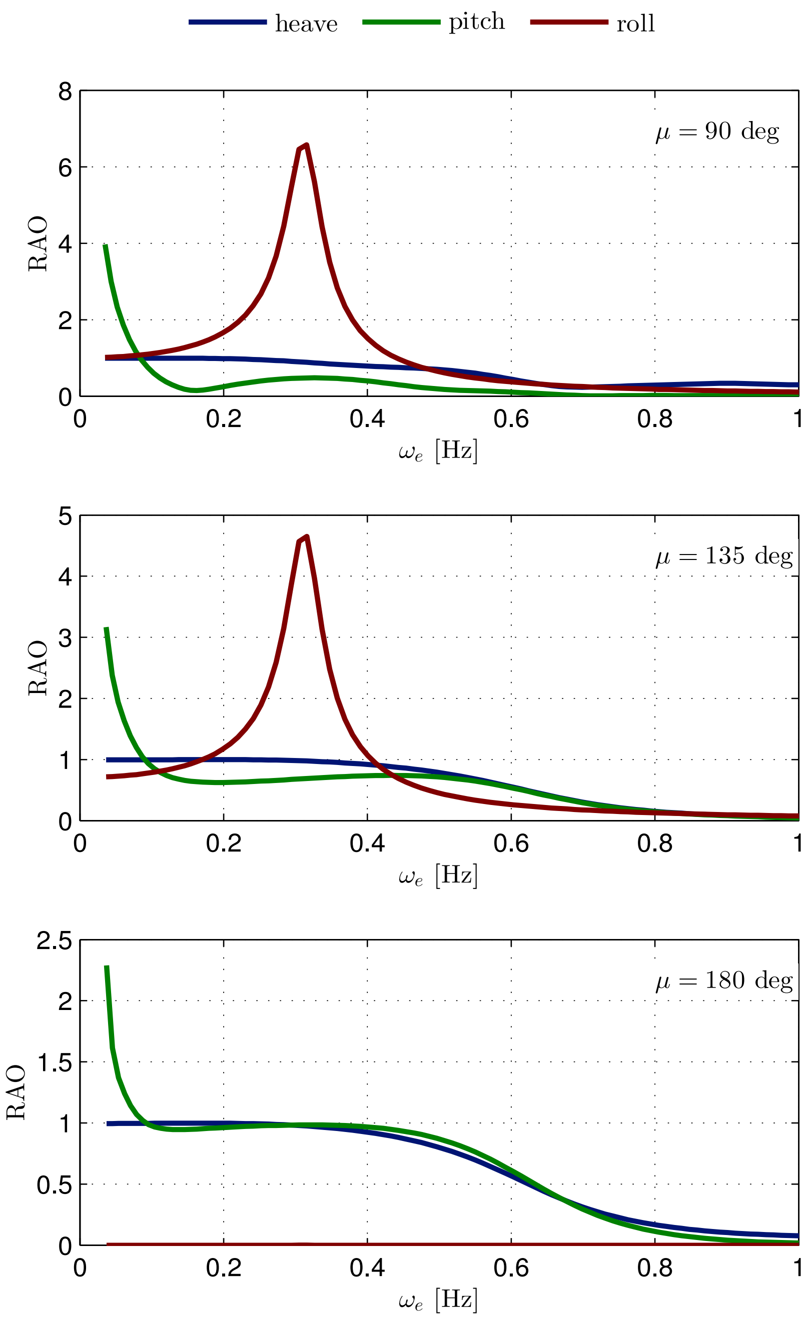

Fig. 5. Yacht's response: response amplitude operators (RAO) at speed V = 5 knt.

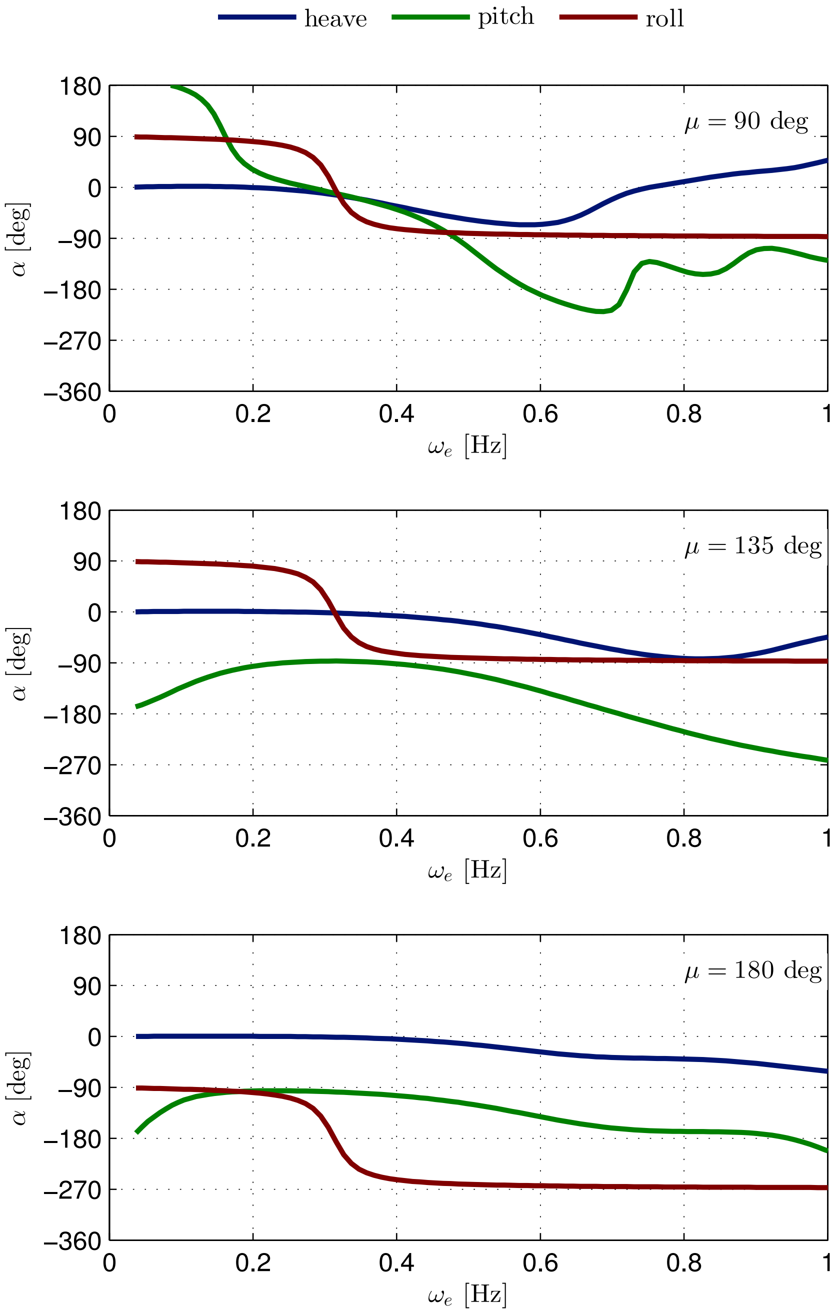

Fig. 6. Yacht's response: phase shifts at speed V = 5 knt.

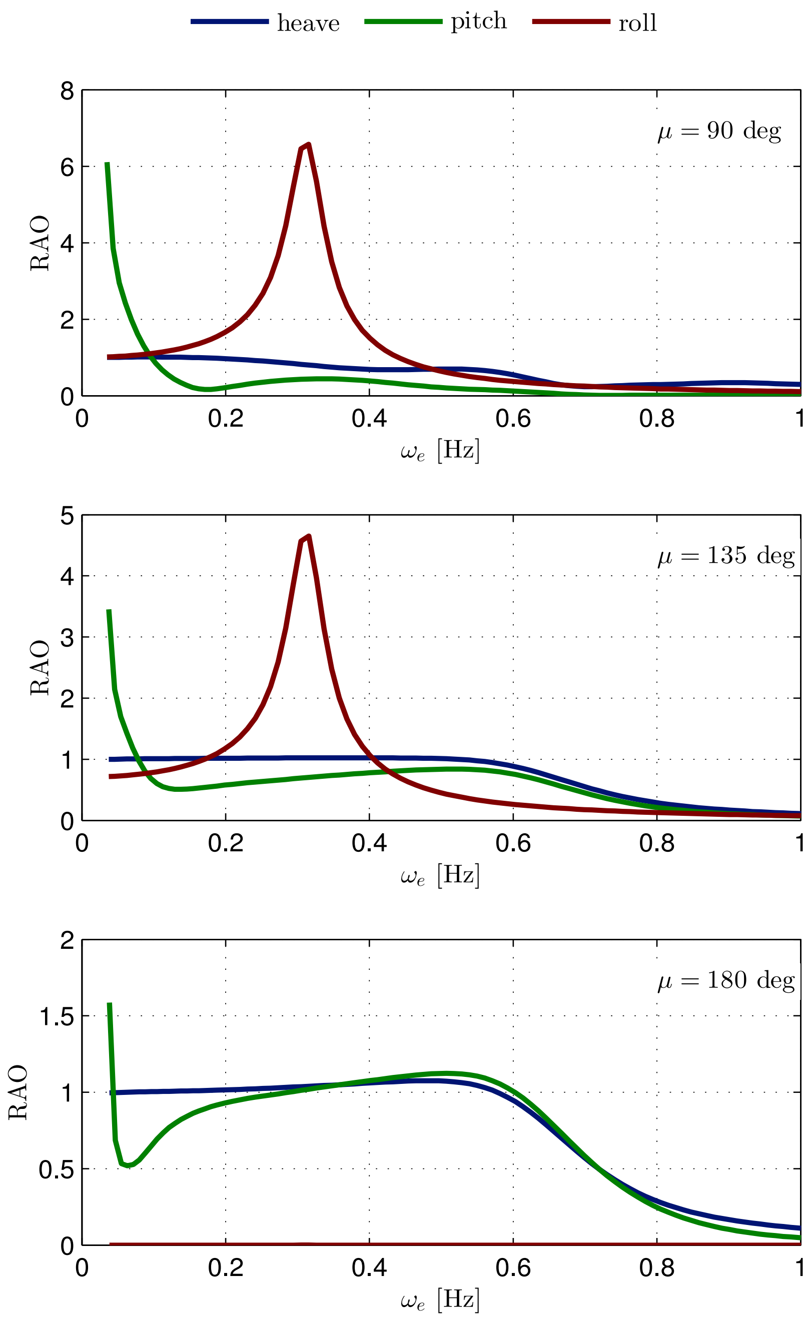

Fig. 7. Yacht's response: response amplitude operators (RAOs) at speed V = 8 knt.

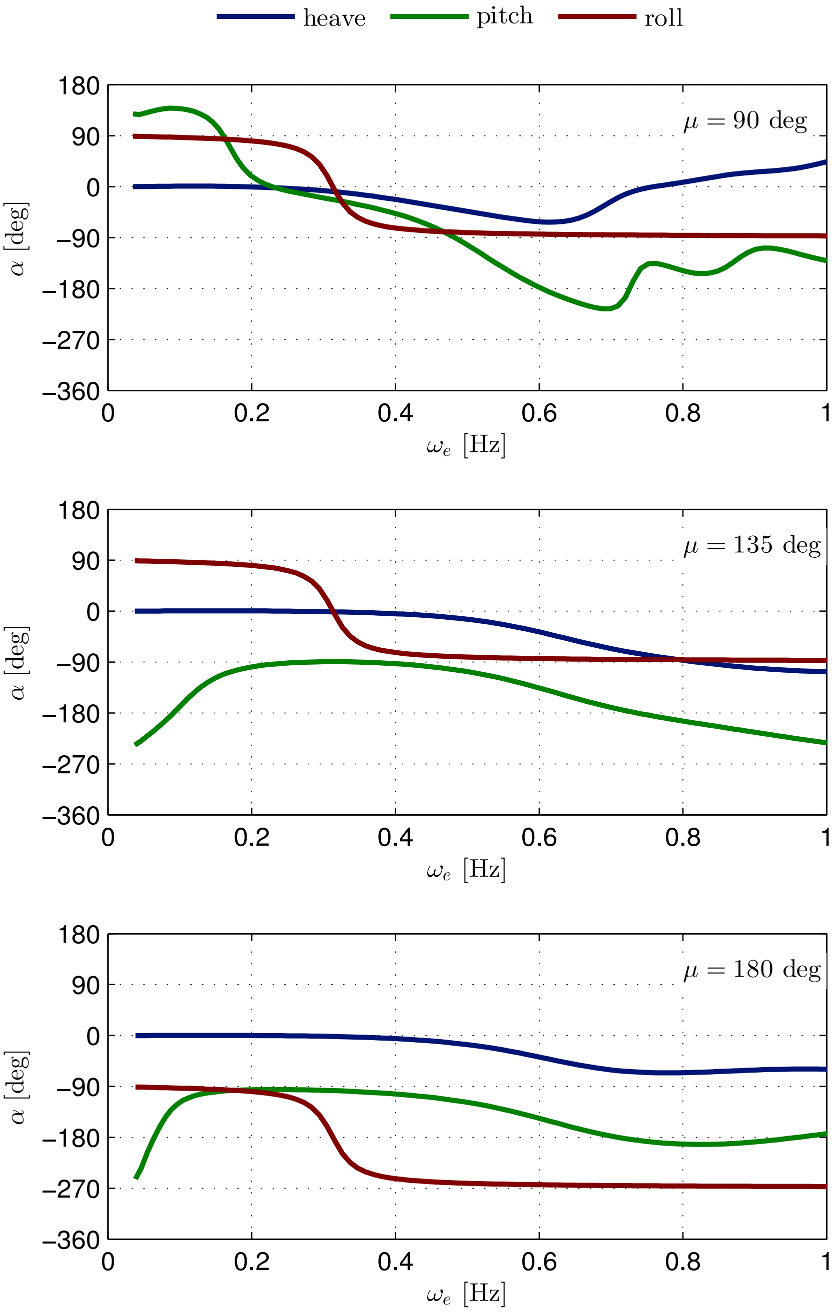

Fig. 8. Yacht's response: phase shifts at speed V = 8 knt.

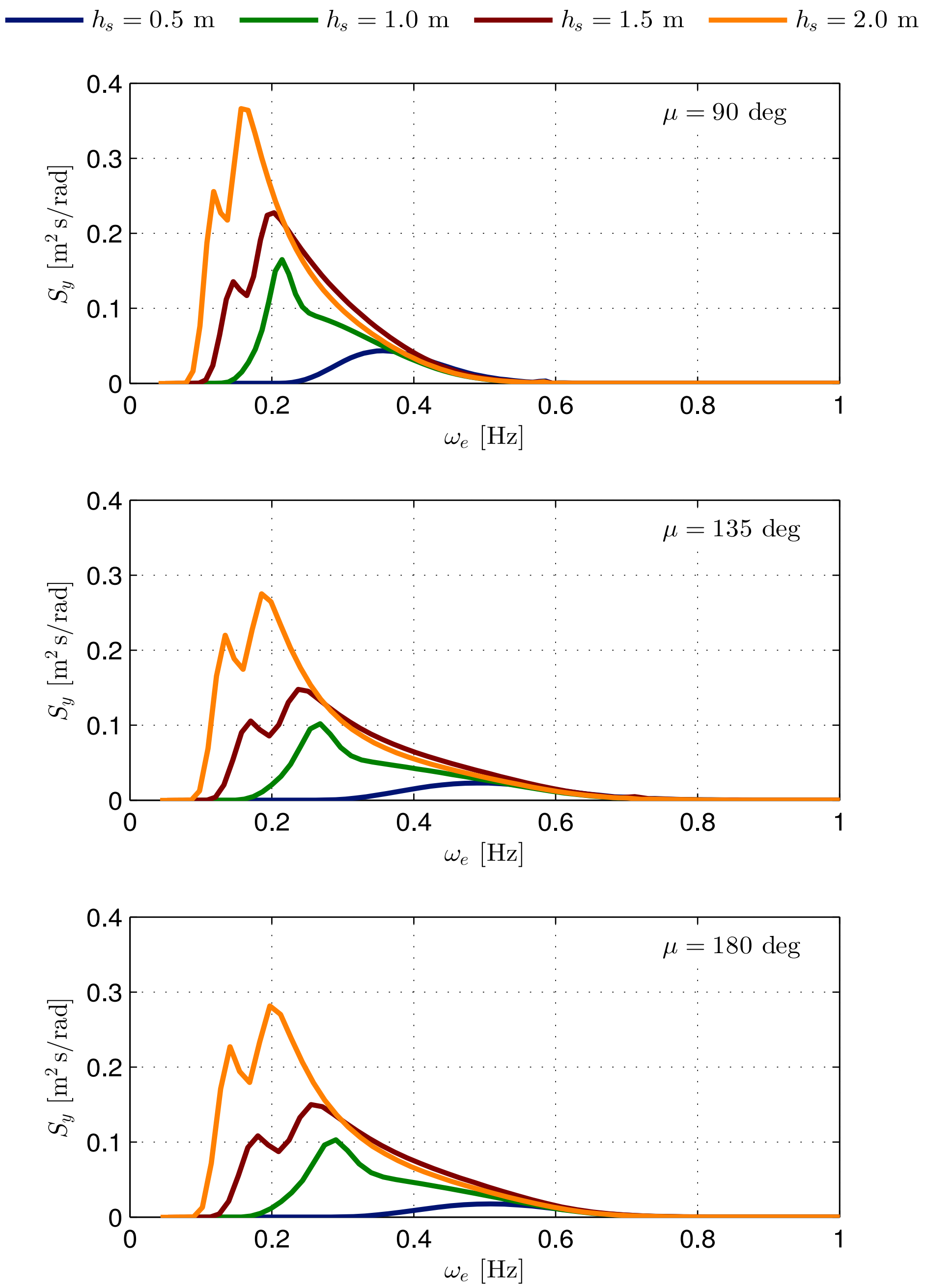

Fig. 9. Energy spectrum of vertical oscillations at the generator's location at speed V = 5 knt.

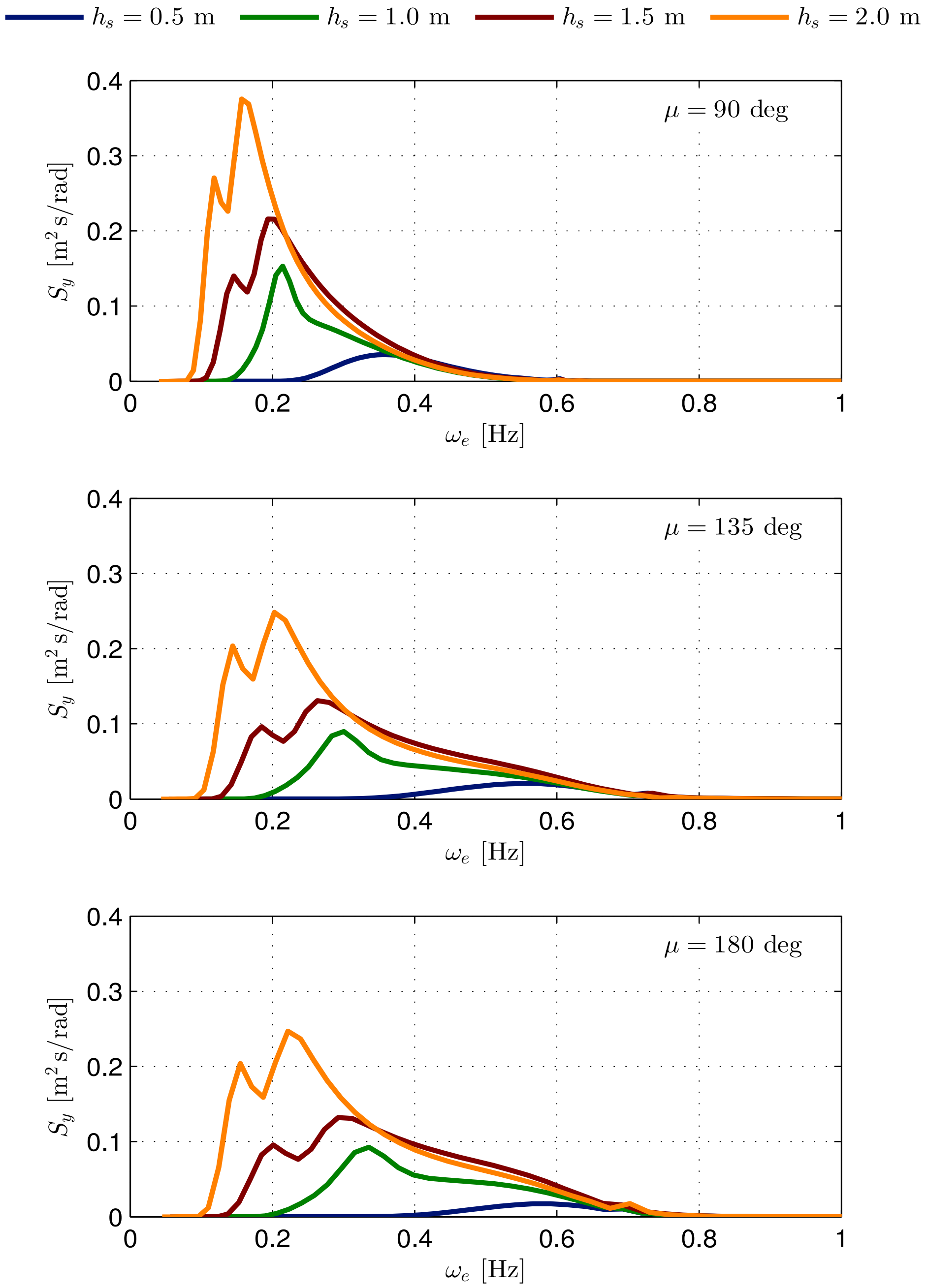

Fig. 10. Energy spectrum of vertical oscillations at the generator's location at speed V = 8 knt.

Table 3. Yacht's significant vertical oscillation amplitudes at the bow generator's location.

	y_{0s} [m]					
	V = 5 knt			V = 8 knt		
H_s [m]	μ = 90 deg	μ = 135 deg	μ = 180 deg	μ = 90 deg	μ = 135 deg	μ = 180 deg
0.5	0.433	0.383	0.322	0.405	0.366	0.318
1.0	0.721	0.684	0.688	0.680	0.697	0.732
1.5	0.980	0.943	0.967	0.937	0.964	1.024
2.0	1.147	1.127	1.175	1.120	1.152	1.233

4. Linear oscillator

4.1. General remarks

The linear generator that will be used in the SEAKERS device to recover energy from the wave-induced motions of the yacht is analysed and approximated in this paper as a simple linear mechanical oscillator, where the damping element represents a linear approximation of the effect of the electromagnetic force exerted by the generator as it provides a voltage difference proportional to the square of its relative velocity with respect to its basement, and the spring represents the stiffness of the generator's support. It is further assumed that the damping coefficient can be dynamically varied depending on sea conditions: this could be achieved in the final system by means of a variation of some electrical parameters in the associated circuit. The equation of motion is thus:

$$m\ddot{x} + c(\dot{x} - \dot{y}) + K(x - y) = -mg + F_s \qquad (36)$$

where m is the generator's mass, x its position in an inertial frame of reference, y is the basement's position, c is the damping coefficient, K the spring's stiffness, g the acceleration of gravity, F_s a static force provided by the support in order to balance the weight mg such that $F_s = mg$.

It is assumed here that the support can exert such a static force in order to balance the mass' weight; it can be seen that mechanical springs alone cannot play such a role, because the resulting stiffness would be too high for the typical forcing frequencies. Indeed, if the spring were to counterbalance the weight with a limited elongation at rest $l = 0.05$ m, the resulting natural frequency would be $\omega_n = \sqrt{g/l} \approx 2.2$ Hz, which is much larger than the forcing frequency of sea waves: as the following section explains, this would make the system too stiff, i.e. the mass would move rigidly with the basement, with no relative motion between the two and, thus, no power extracted.

Equation (36) can thus be rewritten eliminating all static forces and introducing the relative position $s = x\text{-}y$ of the mass in a frame of reference moving with the basement:

$$m\ddot{s} + c\dot{s} + Ks = -m\ddot{y} \qquad (37)$$

which becomes the well-known second order ordinary differential equation for an oscillating body:

$$\ddot{s} + 2\beta\omega_n\dot{s} + \omega_n^2 s = -\ddot{y} \qquad (38)$$

with the introduction of the natural frequency of the oscillator:

$$\omega_n = \sqrt{K/m} \qquad (39)$$

and of the damping ratio:

$$\beta = c/\left(2\sqrt{Km}\right) = c/(2m\omega_n) \qquad (40)$$

4.2. Response to sinusoidal waves

The steady-state response of the linear mechanical system to a harmonic forcing of the type $y(t) = y_0 \exp(i\omega_e t)$ is itself harmonic:

$$s(t) = \sigma_0 \exp(i\omega_e t) \qquad (41)$$

with a complex amplitude σ_0 given by:

$$\frac{\sigma_0}{y_0} = \frac{n^2}{n^2 - 1 - i\,2\beta\,n} \qquad (42)$$

where n is the ratio of forcing and natural frequency:

$$n = \omega_e/\omega_n \qquad (43)$$

The magnitude of σ_0 gives the amplitude s_0 of the harmonic motion of the generator (fig. 11, top), and its ratio with the ship's oscillation amplitude is the response amplitude operator for the generator's relative motion:

$$\frac{s_0}{y_0} = \mathrm{RAO}_s(\beta, \omega_n, \omega_e) = \frac{n^2}{\sqrt{(1-n^2)^2 + (2\beta n)^2}} \qquad (44)$$

while its argument α gives the phase of the generator's motion with respect to the forcing oscillation (fig. 11, bottom):

$$\alpha = \arctan \frac{2\beta n}{n^2 - 1} \qquad (45)$$

The resulting harmonic motion can therefore be expressed as:

$$s(t) = s_0 \exp(i\omega_e t + \alpha) \qquad (46)$$

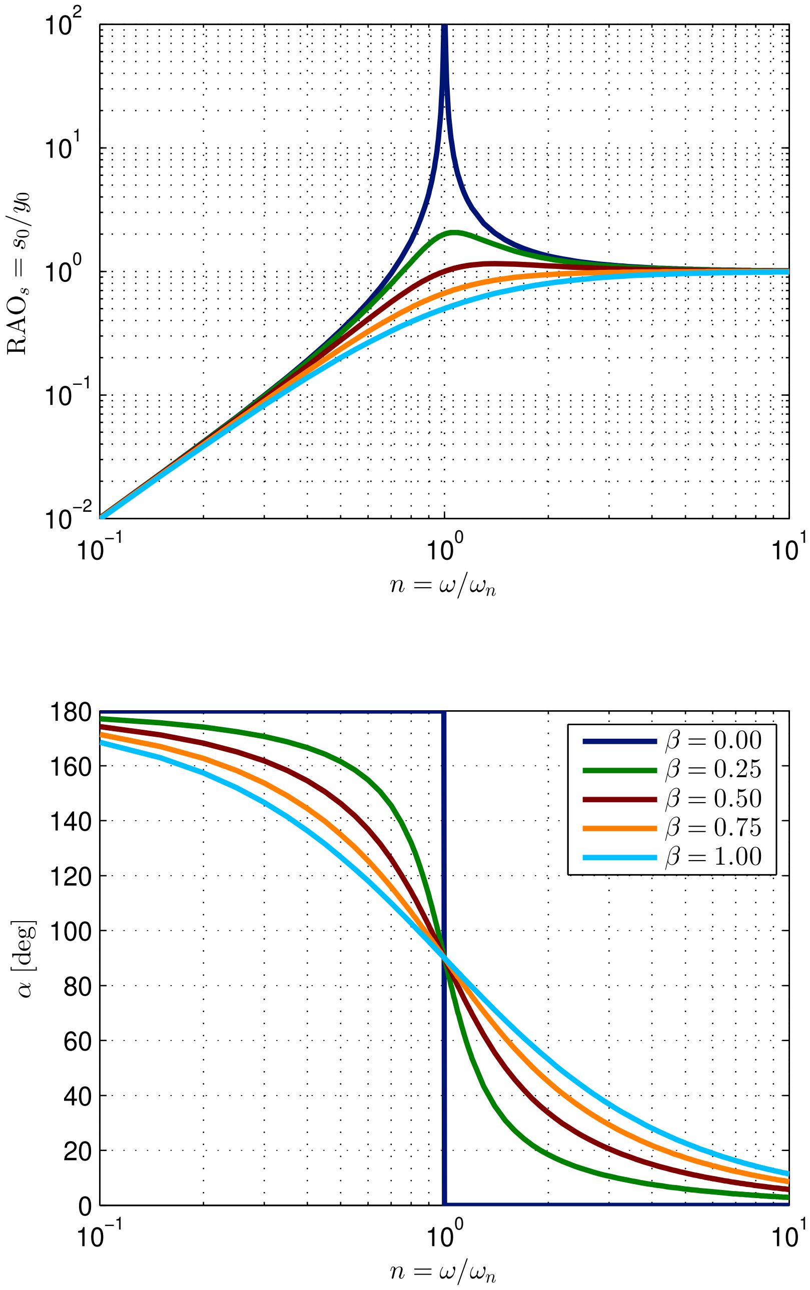

Fig. 11. Frequency response of the harmonic oscillator: amplitude (top) and phase (bottom).

In this model, damping the oscillations in the linear mechanical system provides the mean to extract energy from the wave excitation; thus, it is interesting to identify optimal values for the damping coefficient c in order to extract the maximum power. The power absorbed is given by:

$$P(t) = c\dot{s}^2 \tag{47}$$

and its average value over one cycle (which will be indicated as Π) is:

$$\Pi = \frac{1}{T}\int_0^T P(t)\mathrm{d}t = \frac{1}{2}c\omega_e^2 s_0^2 \tag{48}$$

the above expression, taking into account (40) and (44), becomes:

$$\Pi = m\omega_e^3 y_0^2 \frac{n^3\beta}{\left(1-n^2\right)^2 + \left(2\beta n\right)^2} \tag{49}$$

Since the oscillations are constrained by the size of the generator, two different scenarios must be considered. In the first one, let us imagine that the undamped oscillations do not reach the maximum range allowed $s_{\max}$: in this case, increasing the damping coefficient from 0 initially yields higher values of Π even though s_0 decreases according to (44), until a maximum for Π is reached, beyond which it decreases. The optimum value of β can thus be found when $\mathrm{d}\Pi/\mathrm{d}\beta = 0$:

$$\beta_{\mathrm{opt}} = \frac{1}{2}\frac{\left|1-n^2\right|}{n} \tag{50}$$

with corresponding optimum damping coefficient, maximum average power and oscillation amplitude given by:

$$c_{\mathrm{opt}} = m\omega\left|1-\frac{1}{n^2}\right| \tag{51}$$

$$\Pi_{\max} = \frac{1}{4}m\omega_e^3 y_0^2\left|1-\frac{1}{n^2}\right|^{-1} \tag{52}$$

$$s_0 = y_0\frac{1}{\sqrt{2}}\left|1-\frac{1}{n^2}\right|^{-1} \tag{53}$$

In the second case, the undamped oscillations would be larger than the maximum allowed range $s_{\max}$: then it is possible to extract more power by increasing the damping coefficient while the oscillation amplitude y_0 remains at its maximum permissible level $s_{\max}$. It is possible to show that the maximum power is obtained when the damping coefficient is such that the oscillation given by (44) is exactly equal to the stroke ($s_0 = s_{\max}$):

$$\beta_{\mathrm{opt}} = \frac{n}{2}\sqrt{\left(\frac{y_0}{s_{\max}}\right)^2 - \left(1-\frac{1}{n^2}\right)^2} \tag{54}$$

$$c_{\mathrm{opt}} = m\omega\sqrt{\left(\frac{y_0}{s_{\max}}\right)^2 - \left(1-\frac{1}{n^2}\right)^2} \tag{55}$$

and the corresponding maximum average power is:

$$\Pi_{\max} = \frac{1}{4}m\omega_e^3 s_{\max}^2\sqrt{\left(\frac{y_0}{s_{\max}}\right)^2 - \left(1-\frac{1}{n^2}\right)^2} \tag{56}$$

If the mechanical system is "tuned" to the forcing wave condition ($n \cong 1$), then the above expression can be simplified as follows:

$$\Pi_{\max} \cong \frac{1}{4} m \omega_e^3 s_{\max} y_0 \quad \text{if } n \cong 1 \tag{57}$$

It is possible to find out which wave excitations make the system reach its maximum stroke $s_{\max}$ by setting the oscillation amplitude given by (53) equal to $s_{\max}$, yielding:

$$\frac{y_0}{s_{\max}} = \sqrt{2}\left|1 - \frac{1}{n^2}\right| \tag{58}$$

Thus, for wave amplitudes originating boat oscillations lower than the limit set by the above equation, the system oscillates "freely" and equations (51)-(53) apply, while for higher waves more damping, and thus more power, is available, in order to constrain the system within the maximum stroke allowed, and (55)-(56) apply.

It is worth to point out that in both cases the optimum value for the damping coefficient is directly proportional to the oscillator's mass and to the forcing frequency: the average power absorbed is therefore proportional to mass *m* and to the third power of forcing frequency (ω_e^3), as shown by (52) and (56).

In particular, the linear dependence on the oscillating mass *m* is, on the one hand, almost obvious because energy recovery depends on inertia and kinetic energy, but on the other hand it is an important property to be taken into account because it allows to design, test and prototype modular systems of relatively low mass, with the overall power extracted given by the sum of power available from different modules. For this reason, the results discussed in section 4.4 will be given with reference to a unit mass $m = 1$ kg.

4.3. Response to random waves

The frequency response of the harmonic oscillator can also be used when the external forcing is not harmonic (as in the case of real wave excitation): if $S_y(\omega_e)$ is the energy spectrum associated to the vertical oscillations of a particular point of interest, which can be evaluated from the wave energy spectrum by means of (34), then the energy spectrum associated to the relative motion *s* of a linear system such as the one described in the previous section is given by:

$$S_s(\beta, \omega_n, \omega_e) = \mathrm{RAO}_s(\beta, \omega_n, \omega_e)^2 S_y(\omega_e) \tag{59}$$

In the following considerations the dependence of relative motion and its spectrum on natural frequency ω_n will be implicitly assumed, so that $S_s(\beta,\omega_e) \equiv S_s(\beta,\omega_n,\omega_e)$.

The spectrum of relative motion allows the evaluation of the *significant oscillation amplitude* as follows:

$$s_{0s} = 2\sqrt{m_{0s}} = 2\sqrt{\int_0^\infty S_s(\beta, \omega_e)\,\mathrm{d}\omega_e} \tag{60}$$

Since power generation depends on the square of the generator's velocity (47), the energy spectrum related to relative velocity must be introduced. This is simply given by:

$$S_{\dot{s}}(\beta, \omega_e) = \omega_e^2 S_s(\beta, \omega_e) = \omega_e^2 \mathrm{RAO}_s(\beta, \omega_e)^2 S_y(\omega_e) \tag{61}$$

The 0-th moment of the velocity spectrum gives velocity's root mean square (RMS), which is related to average power generation:

$$\dot{s}_{\mathrm{RMS}}(\beta) = \sqrt{m_{0\dot{s}}} = \sqrt{\int_0^\infty S_{\dot{s}}(\beta, \omega_e)\,\mathrm{d}\omega_e} \tag{62}$$

$$\Pi_{\mathrm{RMS}}(\beta)=\frac{c}{m}\dot{s}_{\mathrm{RMS}}(\beta)^2=2\beta\omega_n\dot{s}_{\mathrm{RMS}}(\beta)^2 \qquad (63)$$

As in the case of sinusoidal waves, for a given natural frequency the generator's motion depends on the choice of the damping coefficient β, for which an optimum value is found by maximizing power output (63) with the constraint that the significant oscillation amplitude is lower than the maximum allowed range $s_{\max}$ (for this non-linear optimization procedure the MATLAB® function `fmincon` has been used):

$$\left.\frac{\mathrm{d}\Pi_{\mathrm{RMS}}}{\mathrm{d}\beta}\right|_{\beta_{\mathrm{opt}}}=0 \quad \text{with } s_{0s}\left(\beta_{\mathrm{opt}}\right)\leq s_{\max} \qquad (64)$$

4.4. Simulation results

In this section, results obtained with the mechanical model of the linear generator subject to random wave excitations for two different speeds (V = 5 knt and V = 8 knt) are discussed. As detailed in section 3.3, the yacht's oscillation is described by energy spectra represented in figs. 9 and 10.

The maximum stroke taken into consideration in the simulations is $s_{\max}$ = 0.5 m, since this value is about the highest possible on sail yachts of length from 10 to 14 m, which are the main target for the SEAKERS project. The natural frequency ω_n is taken as 0.25 Hz, in order to make the mechanical system almost resonant with most sea conditions that may be encountered.

Figures 12 and 13 show the energy spectra associated to the generator's relative motion and RMS power generation for different speeds and directions. Power generation values are also given in table 4, while table 5 gives calculated values of significant oscillation amplitudes.

From table 5 it is possible to see that for wave heights higher than 0.5 m, the generator's oscillation is always limited to the maximum range $s_{\max}$: in order to analyze the results, it is thus useful to consider the simplified equation for average power generation (57), which shows that, if the mechanical system is "tuned" to the forcing wave condition ($n \cong 1$), average power is proportional to the third power of the encounter frequency, and to the product of boat's vertical oscillation y_0 and maximum range $s_{\max}$.

Therefore, an increase in significant wave height gives rise to two opposite effects on power generation: on the one hand it increases due to its dependence on y_0, but on the other hand the wave energy spectrum shifts towards lower frequencies (fig. 1), resulting in a decrease in power generation. Clearly, this gives rise to a maximum power generation for a particular sea state, that under the assumptions taken into account in this paper correspond to a significant wave height of 1.5 m, as figs. 12 and 13, along with table 4 show.

In other words, even with high values of significant wave height, which correspond to rather low values of peak frequencies (table 1), if the full spectrum is taken into account significant contributions to the excitation can be found also at frequencies higher than the peak one, and these contributions increase average velocities and, consequently, power generation. Nonetheless, it is still possible to find that increasing wave heights beyond a certain threshold decreases the power output, because in this case significant contributions can indeed be found only at low frequencies.

Tables 6 and 7 report values of optimum damping coefficients and damping ratios as defined by the optimization procedure (64).

Table 4. Average power generation.

	Π_{RMS}/m [W/kg]					
	V = 5 knt			V = 8 knt		
H_s [m]	μ = 90 deg	μ = 135 deg	μ = 180 deg	μ = 90 deg	μ = 135 deg	μ = 180 deg
0.5	0.552	0.401	0.247	0.500	0.352	0.230
1.0	0.617	0.620	0.546	0.572	0.644	0.630
1.5	0.753	0.807	0.721	0.704	0.852	0.848
2.0	0.664	0.715	0.656	0.619	0.757	0.789

Table 5. Significant oscillation amplitudes for the linear mechanical system.

	s_{0s} [m]					
	V = 5 knt			V = 8 knt		
H_s [m]	μ = 90 deg	μ = 135 deg	μ = 180 deg	μ = 90 deg	μ = 135 deg	μ = 180 deg
0.5	0.500	0.332	0.281	0.477	0.303	0.263
1.0	0.500	0.500	0.500	0.500	0.500	0.500
1.5	0.500	0.500	0.500	0.500	0.500	0.500
2.0	0.500	0.500	0.500	0.500	0.500	0.500

Table 6. Optimum damping coefficients.

	c/m [N s/(m kg)]					
	V = 5 knt			V = 8 knt		
H_s [m]	μ = 90 deg	μ = 135 deg	μ = 180 deg	μ = 90 deg	μ = 135 deg	μ = 180 deg
0.5	1.522	2.709	2.724	1.469	3.142	3.228
1.0	2.410	2.729	2.816	2.232	3.036	3.413
1.5	3.096	3.665	3.851	2.894	4.139	4.725
2.0	3.107	3.732	4.017	2.934	4.223	4.922

Table 7. Optimum damping ratios.

	β					
	V = 5 knt			V = 8 knt		
H_s [m]	μ = 90 deg	μ = 135 deg	μ = 180 deg	μ = 90 deg	μ = 135 deg	μ = 180 deg
0.5	0.484	0.862	0.867	0.468	1.000	1.027
1.0	0.767	0.869	0.896	0.710	0.967	1.086
1.5	0.986	1.167	1.226	0.921	1.317	1.504
2.0	0.989	1.188	1.279	0.934	1.344	1.567

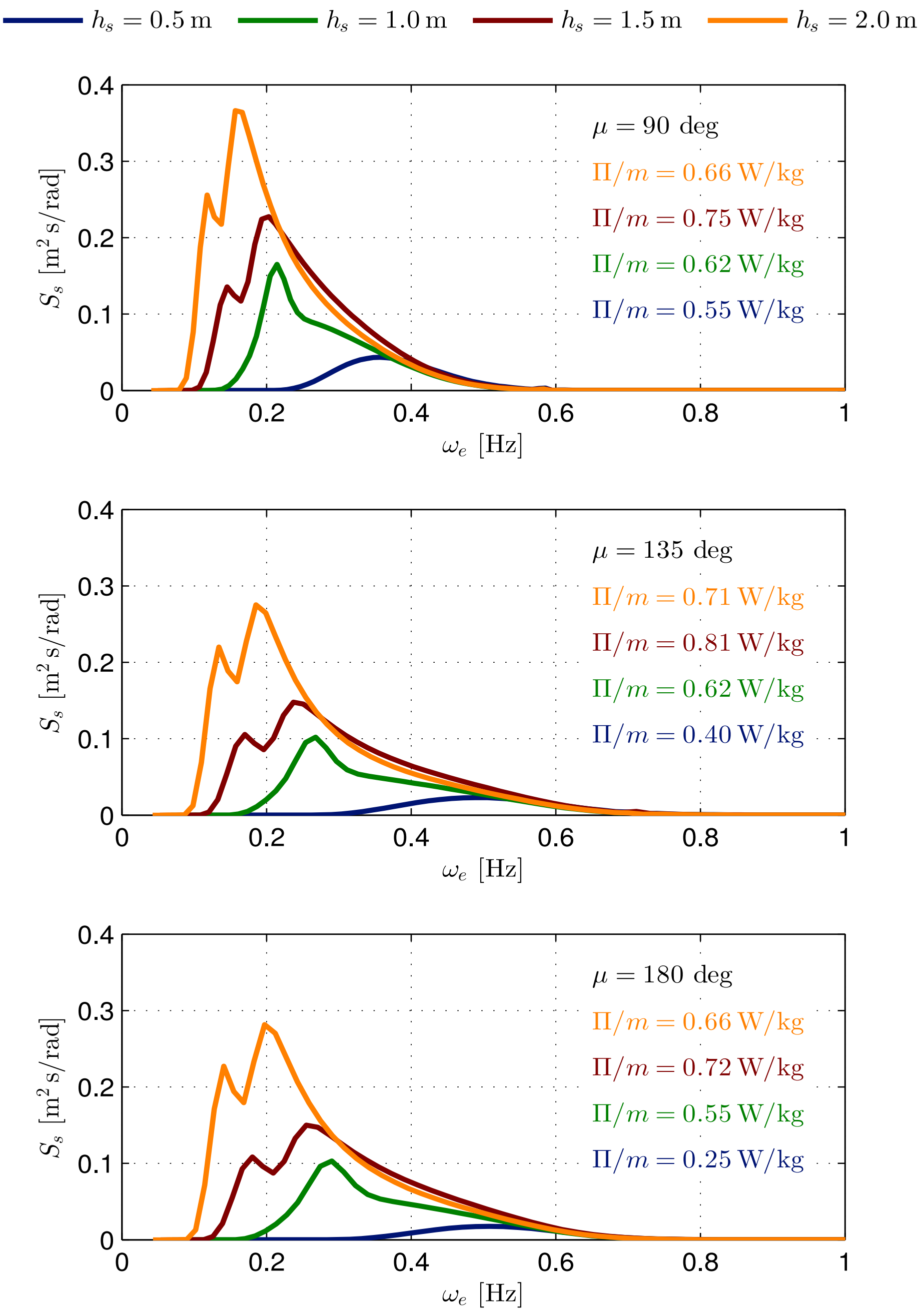

Fig. 12. Energy spectra associated to the linear mechanical system's motion at speed V = 5 knt.

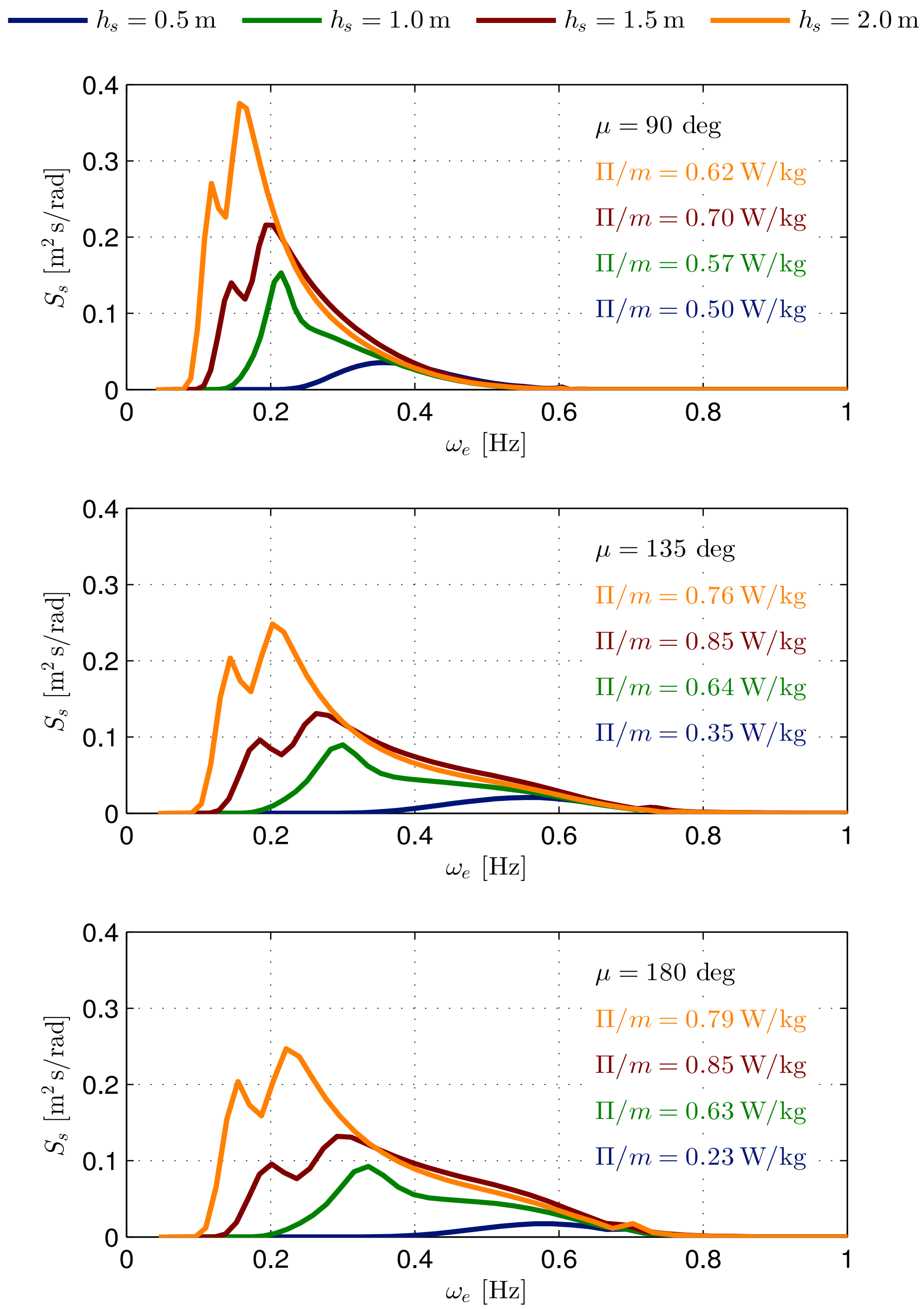

Fig. 13. Energy spectra associated to the linear mechanical system's motion at speed V = 8 knt.

5. Conclusions

This paper has provided an overview of the results obtained with the kinematic and mechanical models of the yacht's response to different wave excitations, and of the linear generator taken as a simple mechanical linear system which extracts power from the wave-induced motion by means of an ideal linear damping.

These preliminary evaluations, even though based on a rather simplified model of the generator, have produced some important insights on system dynamics and on the range of values to be assigned to several significant parameters, such as mechanical stiffness and damping ratio.

In particular, given the particular range of forcing frequencies, the mechanical stiffness must be chosen so as to obtain a natural frequency within the range of most forcing frequencies: a value of 0.25 Hz has been considered in this paper, with a resulting stiffness around 2.5 N/(m kg).

The choice of damping ratios is based on the maximisation of power output for a given natural frequency for different wave excitation conditions. Values of damping coefficients in the range 1.5-5.0 N s/(m kg) have been found, and this result will be useful in the definition of the electric circuit's physical parameters associated to the linear generator.

Power generation of up to 0.85 W/kg have been obtained in the most favourable sea conditions, and anyway values higher than 0.5 W/kg are available in most cases, which represent an interesting result for this particular application. Indeed, a total weight for the SEAKERS device of up to 200 kg can be considered acceptable on sail yachts with length in the range 12-14 m, especially in the case of a first equipment (as opposed to retrofitting an already existing yacht, because in this case many more design constraints should be addressed). With an optimized design, it is conceivable from preliminary evaluations that up to 50% of this weight (100 kg) could be allocated to the oscillating masses; in this case, an average power generation of almost 100 W is feasible, which could make possible to recover at least 1 kWh at the end of a day-long cruise. This amount of energy generation could indeed be interesting for this particular application.

Obviously, the issue related to the influence of this moving mass on sailing performance should be addressed: but specific calculations, which have not been reported here for brevity's sake, show that the inertial forces generated by the mass' motion are at least two orders of magnitude lower than the forces exerted by the sea on the boat. After all, this must be the case because the energy absorbed by the linear system is but a small fraction of the total energy of the incoming waves. Therefore, in all probability the impact of the added mass due to the generator on sailing performance can be safely deemed negligible.

It should be observed that the concept of "power availability" (or "availability factor") routinely used to appreciate the performance of a system based on renewable energy, is much less useful for this particular application, because the final goal is not the generation of electricity *per se* on a continuous basis but, rather, only on the particular occasion when the yacht is used for a cruise. An availability factor should therefore be considered only with reference to a single cruise, and it would take into account the time frequency of encountered wave height during a typical cruise of a sailing boat, because this is the main parameter influencing average power output. However, it is rather difficult to make any prediction about this probability, other than saying that sail cruises are most common when wind (and, consequently, wave) conditions are not extreme (i.e. with moderate winds and waves, while it is safe to assume that calm or stormy seas are avoided); furthermore, a leisure cruise usually requires stable weather conditions. In the end, it is reasonable to infer that in most cases sea conditions encountered on a leisure cruise are reasonably constant and marked by a significant wave height within the range 0.5-1.5 m. Under these assumptions, average power generation is expected to be almost constant during the whole cruise.

The next activities in the SEAKERS project will be focused on the implementation of a non-linear electro-mechanical model of the generator, in order to define suitable ranges for the most important electrical parameters in the system, and to narrow down the set of possible runs of the more detailed 2D and 3D models that have been set up.

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