

Cost efficiency of Italian Commercial Banks: a Stochastic Frontier analysis.

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Abstract

During 90's, the Italian banking system faced a new competitive environment both widening the dimensional scale and pursuing a rationalization process. Some insights could be drawn through efficiency analysis by estimation of a stochastic cost frontier for the period 1993-2004. Benchmark analysis not only highlights the contribution of the main factors that affect efficiency, but also allows evaluation of efficiency dynamics through time, determining the presence of technical progress and scale economies. However, such measure is significant if the sample of firms is homogeneous hence, accounting for heterogeneity of the units involved is then a goal of our analysis.

Key words: Italian banks, Stochastic Frontier, Cost Function, Scale Economies, Technical Progress, Heterogeneity

JEL Classification: C13, D24, G14, G21, G28.

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1. Introduction

At the beginning of 90's, Italy adopted the Second EEC Banking Directive (89/646/EEC) on the coordination of laws, regulations and administrative provisions relating to the taking up and pursuit of the business of credit institutions, which represented the main step towards deregulation and the creation of the European Common Banking Market. Until 80's, Italian credit market was highly regulated, with portfolio constraints, credit constraints, limits to foreign exchange positioning and trading, the imposition of high reserve requirements, supervisory restrictions at the opening of branches in the national territory. Deregulation allowed the banking system to meet the requirements connected to the innovation process of financial institution, the development of IT-based production processes, the market globalization and the competition of specialized intermediaries. Further, the prevailing public property of the control rights was an issue. The Government's choice to privatize during 90's was simply down to the need to raise capital, due to Maastricht constraints on public deficit and debt levels. Subsequently, once the major banks were privatized, it became obvious that greater efficiency and profitability were necessary conditions to become more competitive in a larger European Union context. So, the subsequent the banking system reform process aimed at several goals: higher competition in the markets and consolidation of the whole sector while fostering the privatization process. Today, these targets turns out only partially achieved, because the Banking Foundations are the main shareholders of the major commercial banks, and for the presence of hybrid subjects, like the Cooperative Banks (Banche Popolari). However, in a new competitive environment, the banking system privileged the rationalization of production

processes and ownership structure, usually increasing the dimensions, through merger and acquisition (M&A) operations, thanks also to low interest rates (leverage), that substantially affected the ownership structure, the degree of sector concentration, the geographical coverage and the number of branches. The final outcome will be a progressive change of the banking model. The process of concentration of the Italian banking system during 90's interested nearly half of the whole sector, in terms of asset values¹, and involved a drastic reduction of the number of banks. Banks showed similar dynamics, with a clear reduction (around 10%) of the number of subjects operating in Italy.

Table 1 – Merger and Acquisitions (1)

Year	M&A operations between Italian banks		Acquisitions of Italian banks		Acquisitions of foreign banks by Italian banking groups	
	Number of operations	Assets(2)	Number of operations	Assets	Number of operations	Assets
Total 1996-2000	159	5.99	113	32.12	20	1.88
Total 2001-2004	79	0.36	38	8.79	15	0.99

(1) M&A operations between Italian banks and between Italian and foreign banks, excluding the among group operations

(2) Individual assets of the undergoing banks (% of total assets of the Italian units)

Source Bank of Italy Bollettino Economico

After 2000, only a small number of M&A operations interested the domestic market: 37% of total asset was acquired by Italian banks between 1996 and 2000, 9% from 2001 to 2004. Therefore the weight of foreign acquisitions has progressively grown². However, such reduction in the number of banks does not imply necessarily benefits in terms of efficiency of operating units, neither less degree of competition, which stems from the distribution of branches, the development of alternative channels of distribution, especially in innovative

¹ From 1996 to 2004 there have been 287 M&A operations.

² Equity holdings of the first ten commercial banks, in the archives held from the Consob (the Regulatory Authority for the Italian securities market) show that, as a result of such operations, the average capital share hold by foreign subjects, weighted with the value of consolidated assets of the operating units in Italy, was around 20%. The amount of capital left over, the banking foundations held an average share of 16%, while 56% was market flow.

sectors (such as asset management and structured finance). Table and diagrams below show the dynamics of some dimensional and efficiency indicators.

Table 2 – Major Italian banks

Banks	Gross bank product	Cost Income ratio	Number of workers / number of branches	Net profit / Gross Bank Product
IntesaBCI	511739	1.54	17.50	0.10%
Sanpaolo Imi	437468	1.94	20.37	0.14%
Unicredito Italiano	363704	1.41	16.65	0.19%
Monte dei Paschi di Siena	206519	1.50	17.51	0.12%
Capitalia	196326	1.71	17.34	-0.04%
Banca Nazionale del Lavoro	194669	1.80	29.89	0.02%
Popolare di Verona - S.Geminiano e S.Prospiero	72135	1.24	12.26	0.18%
Banca Antoniana - Popolare Veneta	58400	1.34	12.13	0.11%
Bipiemme - Banca Popolare di Milano	56853	1.38	15.64	0.09%
Banca Lombarda	56851	1.20	10.87	0.17%
Banca Popolare dell'Emilia Romagna	52890	1.20	12.86	0.13%
Creditizio Bipielle	48192	1.53	10.10	0.07%
Deutsche Bank	40655	1.38	16.91	0.19%
Credito Emiliano - Credem	34536	1.32	11.83	0.15%
Cassa di Risparmio di Firenze	30619	1.34	15.56	0.14%
Carige	27039	1.11	12.60	0.16%
Banca Sella	22828	0.98	12.64	0.06%
Banca Popolare di Sondrio	19010	1.21	11.27	0.15%
Banca Popolare di Vicenza	18099	1.08	10.22	0.22%
Banca delle Marche	17031	1.36	11.55	0.16%
Credito Valtellinese	15684	0.99	11.45	0.08%
Banco Desio	10012	0.87	14.98	0.14%
Etruria	8819	1.18	12.18	0.18%
Cassa di Risparmio di Ferrara	6819	0.98	10.47	0.16%
Cassa di Risparmio di Ravenna	6579	1.17	9.43	0.11%
Banca Popolare di Intra	5690	1.25	13.29	0.17%
Cassa di Risparmio di San Miniato	5534	1.44	14.49	0.07%
Banca Popolare Pugliese	3601	1.18	14.11	0.14%
Banca Agricola Popolare di Ragusa	3558	1.01	12.47	0.33%
Cassa di Risparmio della Provincia di Teramo	3091	1.01	10.69	0.20%

Figure 1 – Labour cost and number of branches (Commercial banks)

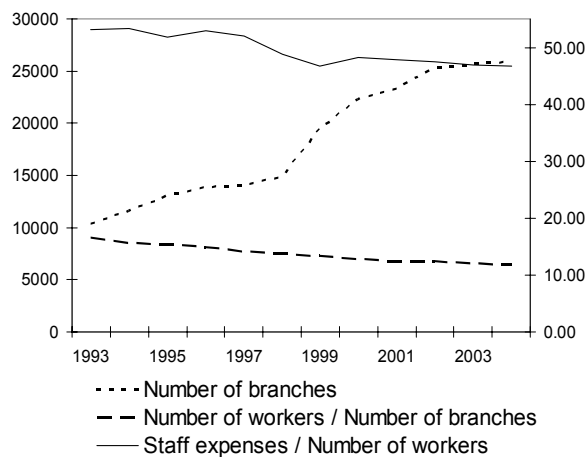
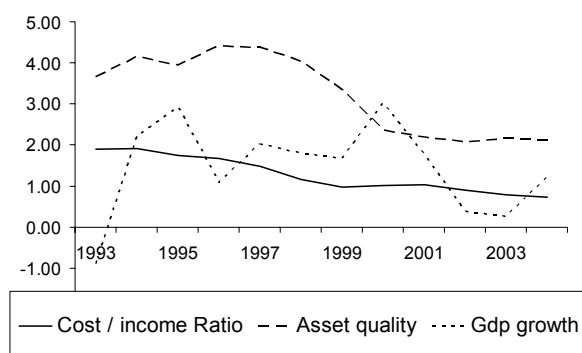


Figure 2 – Costs and asset quality (Commercial banks)



The considerable increase of the number of branches and the huge decline in the number of employees led to a sizable reduction of the labour cost (in real terms) in connection with the number of workers and of the cost-income ratio (total costs/overall business margin). Beside the cuts in expenditures and the labour input rationalization, credit quality showed major improvements, with a considerable reduction, from 1997 onwards, of the non-performing loans (NPL) to total loans ratio. The contraction of the volume of NPL showed a light reversal in 2002, when doubtful loans increased 1,9% y/y.

Asset quality regained speed during the following years: in 2004 the flow of assets turned into NPL stopped around 0.9% of total customer loans. Despite a prolonged economic slowdown, financial stability of business firms, in connection with progress in credit risk management techniques and improvement in selection and appraisal of loans, mostly contributed to such outcomes. However, it should be stressed that, in 2001-2003, contraction of NPL has to be ascribed to securitization³, particularly for large banks. Securitization allows only a partial risk reduction, with the exception of the junior class (the

³ Securitization of non performing loans amounted to 7,1 billions euros in 2001, 1,3 billions euros in 2002 and 1 billion euros in 2003.

speculative grade component of the ABS was taken up by originator, while the Investment Grade ones took place at institutional investors), although the meaningful retrenchment of doubtful loans substantially contributed to reduce credit risk exposure.

Table 3 – Market Risks: Capital Absorption

(end of period data; percent of supervisory capital)

	Market risk	Debt	including: generic risk	including: specific risk	Equity	including: generic risk	including: specific risk	including: other risks
1996	2.83*	2.23	1.59	0.64	0.21	0.14	0.08	0.06
1997	3.59*	2.34	1.63	0.71	0.32	0.22	0.10	0.50
1998	4.06*	2.64	1.87	0.77	0.45	0.28	0.16	0.66
1999	5.70	3.70	2.40	1.30	0.60	0.40	0.20	1.10
2000	6.20	3.60	2.40	1.20	1.10	0.50	0.60	1.10
2001	6.79	3.80	2.28	1.52	1.38	0.92	0.46	1.20
2002	5.38	3.36	2.09	1.27	0.47	0.20	0.27	1.24
2003	5.39	3.00	1.84	1.16	0.55	0.25	0.30	1.55
2004	5.45	3.02	1.57	1.45	0.59	0.28	0.31	1.63
2005	5.60	2.87	1.68	1.18	0.92	0.46	0.45	1.54

* Total asset requirements do not include 3rd level subordinated liabilities

Thanks to securitization banks have met three results: a sharp reduction of NPL; a lower degree of risk exposure of loans portfolio; the recovery of free capital, which naturally increases the operating ability of credit institutions. As Bank of Italy 2003 Annual Report states: *“Between 1999 and 2003 banks transferred loans amounting to €71.9 billion to the market through securitizations, including €26.4 billion of bad debts. Securitizations of bad debts essentially came to an end in 2002. The risk on the securitized credits was virtually all taken up by Italian or foreign institutional investors. In the case of securitizations of bad debts, the originator banks not only charged the writedowns to income but also took up the riskiest securities”*.

Moreover, we have to remark the continuous improvement of capital adequacy, the strict control of the solvency ratio and the most efficient use of capital. After 2000’s IT bubble, more efficient and liquid financial markets, together with more effective risk attitude of the investors enabled credit institutions to improve their capital conditions, both with equities

and subordinated debt; at present, supervisory capital closely meets Basel requirements, while there's further room for systemic consolidation.

Table 4 – Supervisory Capital and Solvency Ratio (Commercial banks)

	Supervisory Capital	Solvency Ratio
1996	97135	12.3
1997	102012	11.4
1998	107299	11.3
1999	110532	10.6
2000	118625	10.1
2001	129229	10.4
2002	134385	11.2
2003	139829	11.4
2004	149157	11.6

Our data turns out rather homogenous. The size of production processes shows a stable and positive relation with costs and outputs: the correlation coefficients between the size variable (number of branches) and variables representing costs and outputs are next to 0,8.

So far, the empirical literature related to the Italian experience provided not always unambiguous results on the extent of product mix efficiency of the Italian banking system. Resti (2000), Focarelli et al. (1999) Girardone, Molyneux and Gardener (2004) point out that the mean level of cost inefficiency of Italian banks places between 13% and 15%, with a slight trend towards reduction over time. Recent evidence (Casu and Girardone 2007) confirms that average overall efficiency score, in 2005 vary between 62.04% in DEA estimations and 80.47% (down from 84.13% in 2000) in SFA estimations.

In a recent paper, Drummond, Maechler and Marcelino (2007) assess the degree of banking competition and efficiency in Italy. They find competition in the Italian banking sector has intensified in loan and deposit markets in recent years, but banks still operate in a high-cost,

high-income system, and efficiency gains have yet to fully materialize. Persistently high operating profits, coupled with high revenues and/or high costs, are frequently associated with non competitive behavior. The pricing data suggest relatively high costs of banking.

In addition, many of these authors find important economies of scale; the more efficient credit institutions are placed in the northern part of the country, while dimensional scale is less significant.

This paper aims at investigating the performance of the Italian banking system from 1993 to 2004, determining the contribution of the main factors that characterize banks efficiency. Our analytical instrument is a stochastic cost frontier, which allows us to determine both a measure of technical efficiency and a ranking of productive units (based on the distance from the production contour), by comparing each unit with the case of full efficiency. The dynamic evolution of efficiency through time is also tested as well as the presence of technical progress and economies of scale.

The specification of a cost function shared by the whole banking sector requires input and output indicators common to all firms (since multifunctional groups accede to the same technology, sharing the same factors), in order to obtain a meaningful benchmark; further, we include credit quality and riskiness indicators to account for the pronounced variations in credit assessment and performance and the development of new business models, which could affect the efficiency levels.

Deviations from the optimal path can be ascribed to inefficient input mix chosen by the management or to exogenous factors (random disturbances). But when panel data are non-homogeneous, firms can turn out to be simply different because of their size, their geographical position, their branch dispersion and their main business activity. Moreover,

we provide further contribution to efficiency assessment, allowing for the influence of environmental heterogeneity on performances of Italian banking institutions. The paper is organized as follows. In section 2 we illustrate the methodology, data and estimation results. Chapter 2.1 shows panel estimates. Section 3 analyzes heterogeneity and its effects on efficiency. Economies of scale and technical progress are tested in chapter 3.1; chapter 3.2 describes the hypothesis of time variability of the efficiency measure. Section 4 concludes and remarks.

2. Methodology, data and estimation results

A valuable element on the debate on the banking system, for both decision making of executives, and regulating and supervision activity of government and authorities, comes from estimates of relative efficiency (which allow cost efficiency comparisons of banks through time) and scale economies (on the basis of the existing technology). Technical efficiency regards the ability to obtain the maximum level of output from given inputs, or the minimum amount of input factors which realize an output target level⁴.

The development of an efficiency measure, based on a production or cost frontier, obtained by DEA non parametric technique, has been developed starting from the initial contribution of Charnes, Cooper and Rhodes (1978). The DEA approach, although more flexible, with no a priori constraints on data, does not discriminate among inefficiency and random disturbances, as any deviation from the deterministic frontier is interpreted as inefficiency, with no regard for other elements (as data measurement errors for instance). Alternatively,

⁴ Output level improvement or inputs saving measures coincide only in case of constant returns to scale.

the parametric methodology of *Stochastic Frontier Analysis* (SFA), introduced by Aigner, Lovell and Schmidt (1977) and Meeusen and Van den Broeck (1977) and recently extended by Kumbhakar and Knox Lovell (2000) leads to a (stochastic) *best-practice* frontier, by comparison of performances of all units of the economic system; such methodology allows discrimination between the effect caused by stochastic disturbances or by technical inefficiency, which can simultaneously characterize the deviations from the *best-practice* frontier. Nonetheless, such parametric approach implies some restrictions as well, by imposing a specific functional form to the error distribution and requiring ad hoc hypothesis on the efficiency component distribution. Compared to DEA, SFA studies lead, on average, to higher efficiency values and smaller dispersion. In Berger and Humphrey (1997), 24 SFA applications, as referred to the efficiency of the United States banking system, showed an 84%⁵ average level of efficiency, although variation is rather high (from 61% to 95%). Non-parametric techniques reduce the mean value of efficiency back to 72%, (with an average level of inefficiency equal to 39%), while dispersion turns out excessively large (from 31% to 97%). In summary, our study adopts the Stochastic Frontier approach with respect to Data Envelopment Analysis (DEA), mainly because DEA tends to over-estimate inefficiencies.

However, benchmark analysis should compare firms that are similar enough to make comparisons meaningful and take scores reflecting effectiveness of firms as significant. Firms may deviate from benchmark and show poor performances not only because they are typically inefficient units (in case of some ineffectiveness of management) but also for

⁵ Inefficiency is, on average, equal to 19%. In fact, if only 84% of the resources used by a firm would be sufficient to produce the same amount of output, then the average firm needs 19% more resources to produce the same output as the most efficient one. I.e. $84\% = 1 / (1 + 19\%)$ or $19\% = (1 - 84\%) / 84\%$.

some inaccuracy due to random noise or just because intrinsically “different” from the common reference. In other terms, the analysis should discriminate between these sources of deviation and account for the impact of heterogeneity on efficiency scores; the volatility of estimates and efficiency rankings is thus probably due to missed heterogeneity elements. In the light of the integrated nature of production in the banking industry, we adopt a mixed methodology, which includes both intermediation and also distinctive features of the production and services supply approach⁶. Neither approach individually accounts for the whole complexity of the functions performed by banks, whose activity ranges from the usual credit and monetary functions, to the contribution to the monetary policy transmission mechanism, to the production and distribution of national income, to the service function. Banks offer to customers a much wider complementary and collateral supply service range, both of banking type (asset management, treasury management, collateral management, lead managing and underwriting in primary markets for bonds and equity IPO markets, advisory bent) and non-banking ones (leasing, factoring). Our choice is oriented to mitigate these various features of bank activity, and aims at evaluating the total efficiency of multipurpose banks, rather than single aspects of activity.

The analytical structure of our study is based on a second order approximation of the “true” cost function by means of a flexible translog cost function, which simultaneously measures the degree of inefficiency both from the input and output standpoint, handling multiple outputs while preserving the typical properties of symmetry and curvature of the frontier.

⁶ In the production approach banks provide multiple services to customers. Efficiency is measured by comparing the amount of services supplied with the quantity of resources used. In the intermediation approach banks allocate resources from units in surplus (by raising funds through short and long term debt instruments) to units in deficit (granting loans to corporations). In this latter case, provision’s measures and their costs are added to the traditional physical inputs (labour and capital), while outputs are represented by loans, securities and other income-generating activities.

Thus we obtain both a numerical value of efficiency (the X-efficiency) and a ranking of production units, which is more robust if production units gather themselves around the average values. Unlike Cobb-Douglas, a translog function allows adequate handling of multiple outputs, while preserving the typical properties of symmetry and curvature of the frontier. Furthermore, increasing distortions of efficiency levels could originate from excessive simplification of the production processes, if significant factors are excluded from analysis.

We use accounting data to analyse the production process of Italian commercial banks. The dataset for the period between 1993 and 2004 includes 30 commercial banks and is based upon ABI (Italian Banking Association) “Bilbank analysis” database, which supplies data for BankScope (a widely used database published by Bureau Van Dijk) and contains information on the balance sheet, the income statement (profit/loss and cash flow) and the supplementary note to yearly and half-yearly accounts of Italian banks. Variables included in our analysis regard costs, the amounts of output and input prices. Total costs are the sum of interests paid and assimilated burden plus the administration fees in the profit and loss account.

Output items are:

- i) Two variables from the income statement: interest income plus dividends; non interest income (trading, services and others);
- ii) Two variables from the balance sheet: loans (credits to customers) and asset securities (bonds and other debt securities, stocks, shares and other capital instruments).

Inputs include:

- i) The labour cost (the ratio between staff expenses and the average number of employees);
- ii) The average price of funding (the ratio between interests paid and total liabilities);
- iii) Other administration expenses;
- iv) The cost of capital, calculated as the ratio between the supervisory capital and the gross bank product (rather than current assets)⁷.

The empiric literature has generally neglected the effect on the efficiency measure stemming from the exclusion of non-traditional activities, which have instead characterized the evolution of banks' productive processes during the last few years. Their absence could cause an incorrect specification of the cost function and twist the economies of scale estimate. Clark and Siems (2002) include an *asset equivalent off-balance-sheet* activity measure⁸, as proposed by Boyd and Gertler (1994), to quantify non-interest income, assessing that the impact of such activities on bank efficiency could be important. Also, following the intuitions of Clark (1996) and Berger and Mester (1997), we believe that the inclusion of capital stock among output, and a measure of credit quality (to check for riskiness as well) can improve the efficiency estimate. The lack of a capital stock measure,

⁷ The cost of capital for banks is quite different from the cost of capital for non financial firms (i.e. industrial), in many respects. First, banks are more highly leveraged than industrial firms, so the cost of equity assumes decisive importance; second, capital assets are usually not significant in banking industry and lastly, if the required capital ratios are bound at the margin, than the equity needed for project capital is easily quantified. The working capital (equity plus subordinated debt) is therefore not only regulatory capital but also a factor in business development. Moreover, capital funding is subject to market risk, especially during funding phase, so our definition of cost of capital already includes the price of capital.

⁸ This technique compares the typical items of the assets side with off-balance sheet assets. Our efficiency measure is thus consistent in terms of interest margin volatility, linked at the use of derivatives for hedging strategies.

for instance, would involve a significant scale distortion, since the largest firms would show higher efficiency levels simply as a consequence of the process of capital accumulation through time.

Our maintained hypothesis is that Italian banks i ($i=1, \dots, 30$) aim at minimizing a cost function in order to produce 4 outputs Q , using 4 inputs X for given prices⁹ w_k under a common production function constraint. Hence:

$$\min C = \sum_{j=1}^k w_j x_j(Q, W)$$

$$\text{Sub } F(Q, X) \leq 0$$

By replacing the optimum input demand functions $x^*(Q, W)$ obtained from the constrained optimization process, into the cost function, we get the minimum cost level, i.e. the benchmark against which the cost of other production units is compared with.

Estimation of the translog functional form in the Stochastic Frontier framework needs an additional composed error term $\varepsilon_{it} = (v_{it} - u_{it})$. The first component of the error term v_{it} , is a symmetric disturbance capturing the effect of noise, while the second component u_{it} , is a one-sided non-negative disturbance reflecting the effect of inefficiency¹⁰.

The first model is a multiproduct translog cost function (*cross-section*), often used to analyse efficiency of several production units. Impact of output and prices on costs should be positive, while we also include time trend variables to capture the potential changes in technology, and two control variables that indicate asset quality and the typical bank credit function.

⁹ Commercial banks behave as price-takers when demanding the x inputs, as we assume perfect competition on the inputs market.

¹⁰ In case of a cost function, inefficiency involves an excessive input use for given prices, and higher costs; therefore the u_{it} component is strictly positive.

Thus, the translog cost function is:

$$\begin{aligned}
\ln TC = & \alpha_0 + \sum_{i=1}^4 \alpha_i \ln Q_i + \sum_{i=1}^4 \beta_i \ln W_i + \lambda_1 \text{ASSQ} + \lambda_2 \text{CRED_ATT} + \\
& + \frac{1}{2} \left(\sum_{i=1}^4 \sum_{j=1}^4 \delta_{ij} \ln Q_i \ln Q_j + \sum_{i=1}^4 \sum_{j=1}^4 \gamma_{ij} \ln W_i \ln W_j + \lambda_{11} (\text{ASSQ})^2 + \lambda_{22} (\text{CRED_ATT})^2 + \right. \\
& + \sum_{i=1}^4 \sum_{j=1}^4 \rho_{ij} \ln Q_i \ln W_j + \sum_{i=1}^4 \lambda_{1Q} \ln Q_i * \text{ASSQ} + \sum_{i=1}^4 \lambda_{1W} \ln W_i * \text{ASSQ} + \\
& + \sum_{i=1}^4 \lambda_{2Q} \ln Q_i * \text{CRED_ATT} + \sum_{i=1}^4 \lambda_{2W} \ln W_i * \text{CRED_ATT} + \sum_{i=1}^4 h_{it} \ln Q_i * t + \\
& \left. + \sum_{i=1}^4 k_{it} \ln W_i * t + \lambda_{1it} \text{ASSQ} * t + \lambda_{2it} \text{CRED_ATT} * t + k_t * t + k_{tt} * t^2 + \varepsilon \right)
\end{aligned}$$

ln TC	= log of total costs (interests paid and assimilated burden plus labour costs and other administration expenses).
Q _i	= ith output
Q ₁	= Interest income plus dividends
Q ₂	= Non interest income (trading, services and others)
Q ₃	= Loans (credits to customers)
Q ₄	= Asset securities (bonds and other debt securities, stocks, shares and other capital instruments)
W _i	= ith input price
W ₁	= Labour cost (ratio between staff expenses and average number of employees)
W ₂	= Average price of funding (ratio between interests paid and total liabilities)
W ₃	= Other administration expenses
W ₄	= Cost of capital (ratio between supervisory capital and gross bank product)
ASSQ	= credit quality indicator (ratio between NPL and total loans to customers)
CRED_ATT	= ratio between credit to customers and total assets

As usual in literature, (Okuda and Hashimoto 2004 and Lang and Welzel 1996 among others), we impose a few constraints, regarding price homogeneity¹¹, monotonicity of quantities and prices, symmetry between the partial derivatives and concavity of prices. To keep a sufficient number of degrees of freedom (and simplify the process of estimation and convergence of the likelihood) we further assume (as in Okuda and Mieno 1999) that the cost function is separable between prices and outputs. The estimation process, reflecting judgments about whether a relationship is likely to be quadratic versus linear, does not

¹¹ The hypothesis of linear price homogeneity is implicit when estimating a dependent variable normalized to a price component (in our case the labour factor price).

follow a standard second order Taylor series expansion. In addition, given that some interaction terms proved to be highly correlated with other explanatory variables, it was decided to exclude them from the equation to avoid problems with multicollinearity. Therefore, the benchmark model leads to a cross-section like:

$$\log(TC / w_1) = \alpha_0 + \sum_{i=1}^4 \beta_i \log q_i + \sum_{j=2}^4 \gamma_j \log(w_j / w_1) + \frac{1}{2} \sum_{l=2}^4 \sum_{j=2}^4 \gamma_{lj} \log\left(\frac{w_j}{w_1}\right) + \sum_{j=2}^4 \kappa_j \log(w_j / w_1) * T + \sum_{l=1}^4 \sum_{j=2}^4 h_{lj} q_l \log\left(\frac{w_j}{w_1}\right) + \alpha T + \beta T^2 + \lambda_1 ASSQ + \lambda_2 CRED - ATT + \varepsilon_{it}$$

The distributional assumptions regarding the composed error term $\varepsilon_{it} = (v_{it} - u_{it})$ was derived from Weinstein (1964). Error component v_{it} represents random disturbance, while $u_{it} \geq 0$ represents time-invariant cost (in)efficiency, including both technical and allocative inefficiency. Assuming v_{it} as i.i.d. $(0, \sigma_v^2)$ and uncorrelated with regressors, the equation can be estimated with a fixed effects approach, obtaining firm-specific intercepts. The firm with the smallest intercept is regarded as the most efficient, while other firms' inefficiency scores are estimated as the distance from the firm with the minimum estimated fixed effect¹². A convenient parameterization needs variance decomposition as¹³ $\sigma^2 = (\sigma_u^2 + \sigma_v^2)$ e $\lambda = \sigma_u / \sigma_v$. If $\lambda \rightarrow +\infty$, we get the deterministic frontier. If $\lambda \rightarrow 0$, it turns out that there is no inefficiency in disturbances, every firm lays on the frontier, and the model can be estimated by means of Ols methods. As we know, a deterministic frontier involves that any shift from the frontier (both from random noise or mis-specification of the functional form or data errors) is treated as inefficiency. Thus, the error term contains cost volatility (albeit

¹² u_i estimate comes from $u_i = \alpha_{0i} - \alpha_0$, efficiency from $CE_i = \exp[-u_i]$

¹³ Alternatively Battese and Corra (1977) assume $\sigma^2 = \sigma_v^2 + \sigma_u^2$ e $\gamma = \sigma_u^2 / (\sigma_v^2 + \sigma_u^2)$. γ parameter significance is crucial: if null hypothesis $\gamma = 0$ is true, then σ_u^2 is also equal to zero and u_{it} term should be removed, leading to Ols methods instead of SFA ones.

temporary) of the production units: the *best-practice* frontier is then stochastic and depends on various random occurrences, not all under the direct control of managers.

The composed error term implies one component following a symmetrical standard-normal distribution and the inefficiency term modelled with an asymmetric distribution, usually half-normal; indeed, inefficiency values (widely affected from distributional assumptions) are supposed non-negative and must conform to a truncated distribution. Both components should be orthogonal with respect to inputs, outputs, or to other variables included in the specification.

The standard half-normal hypothesis on the distribution of the inefficiency term (null on average), is rather strict, since most firms are bound to be close to full efficiency (or, vice versa, that very inefficient values are less probable).

Other distributional hypothesis could, in principle, suit our analysis. For instance, the truncated-normal distribution¹⁴, where the one-sided error term, u_i , is obtained truncating at zero the distribution of a non-zero mean variable, although standard errors estimates and convergence procedure could be distorted. However, it appears less suitable to discriminate between accidental disturbances and inefficiency, being close to a symmetrical distribution (as the one applied to the error term). Alternatively, Meeusen and Van den Broeck (1977) introduced an exponential distribution, which differ from half-normal, in that distribution values thicken mainly around zero, even if the estimated inefficiency values do not change meaningfully¹⁵.

¹⁴ The truncated-normal distribution, according to Berger and DeYoung, (1996), leads to similar results, with respect to half-normal one.

¹⁵ The normal gamma model gives an extension to the normal-exponential model by assuming that u follows a gamma distribution, which generalizes the one-parameter exponential distribution by introducing an additional

Various studies, applying several distributional hypotheses, verify the robustness of the proposed specification, by means of parameters and efficiency levels comparison. The coefficients are statistically significant and of the expected sign (in particular the price coefficients, whose significance is sometimes disregarded as opposed to obtaining a relevant efficiency measure).

Table 5 – Stochastic cost frontier under several error distribution assumptions

Variable	Half-normal		Truncated-normal		Exponential	
	Coefficient	t value	Coefficient	t value	Coefficient	t value
Constant	3.0810	42.27	3.0804	46.59	3.0084	44.13
LQ1	0.2769	12.85	0.3002	15.95	0.2626	12.17
LQ2	0.0580	11.34	0.0553	10.67	0.0580	10.53
LQ3	0.4786	20.74	0.4634	22.46	0.4909	20.94
LQ4	0.0274	3.36	0.0286	3.74	0.0222	2.65
W2N1*	0.0232	13.63	0.0229	15.44	0.0296	24.99
W3N1*	0.0002	13.36	0.0002	14.05	0.0002	12.87
W4N1*	0.0247	3.10	0.0230	3.29	0.0429	9.15
W23	0.0000	-1.60	0.0000	-1.78	0.0000	-2.06
W24	-0.0019	-3.86	-0.0019	-4.39	-0.0025	-11.21
W34	0.0000	6.09	0.0000	7.04	0.0000	10.11
TW2	0.0024	6.37	0.0022	6.57	0.0016	8.28
TW3	0.0000	-3.16	0.0000	-3.19	0.0000	-4.41
TW4	-0.0054	-4.68	-0.0050	-4.74	-0.0098	-14.99
CP1	0.0000	0.76	0.0000	1.04	0.0000	0.38
CP2	0.0000	4.20	0.0000	4.34	0.0000	4.92
CP3	0.0000	4.09	0.0000	4.40	0.0000	4.07
CP4	0.0000	0.02	0.0000	0.04	0.0000	-0.47
T	-0.0202	-3.89	-0.0199	-4.21		
T2	0.0006	2.02	0.0005	1.85		
ASSQ	0.7230	4.67	0.6904	4.97	0.8159	5.59
CRED_ATT	-0.5524	-10.44	-0.4924	-10.28	-0.6710	-13.32
μ^{**}			25.1031	0.0200		
<i>Variance parameters for compound error</i>						
λ	1.9191	13.91	23.6724	0.04	0.0375	
σ	0.1155	23.44	1.2608	0.04	0.0045	
$\ln L$	794.2984		810.6139		780.7321	
σ_v	0.0534		0.0532		0.0669	19.49
σ_u	0.1025		1.2596		0.0025	
θ					19.9789	8.55
* normalized through the labour input price						
** half-normal model has mean $\mu = 0$						
<i>Descriptive Statistics for technical inefficiency $E[u_i \varepsilon_i]$</i>						
	Mean		Std. Deviation		Min	
Half-normal	0.0801		0.0495		0.0107	
Truncated-normal	0.1164		0.0651		0.0123	
Exponential	0.05		0.0352		0.0117	

parameter to be estimated, and so provides a more flexible representation of the pattern of technical efficiency. However, the gamma density function has the exact shape of an exponential density when such additional parameter $P = 1$. Our estimate of the gamma model shows that parameter P is just equal to one, thus we display results for the exponential model only.

Raising non performing loans cause an increase in total costs. The parameters σ and λ are both significant in the half-normal specification. In particular, the significance of the parameter λ indicates that deviations from the frontier do not depend entirely on random noise, but on technical inefficiency as well. Log-likelihood increases when adopting a truncated-normal distribution hypothesis, but the mean value parameter is not significant, neither σ and λ turns out significant; this suggests some kind of distortion in the methodology. Model specification seems robust with respect to distributional assumptions: coefficients do not differ much, for any hypothesis on error. Parameter σ is instead much different in the truncated-normal model (from 10 to 20 times bigger than other hypotheses), which could involve some degree of distortion. The value of parameter $\mu = 25,1031$ is rather high and could influence the inefficiency estimation¹⁶ as well. Estimates turn out lightly different, in case of exponential distribution, especially for asset quality parameters and credit to total assets ratio. The mean value of inefficiency, estimated according to the various distributional hypotheses, is shown in table 5. There is a consistent increase in inefficiency levels in the truncated-normal case and a reduction of inefficiencies in the exponential case, where values are more concentrated around the best-practice point.

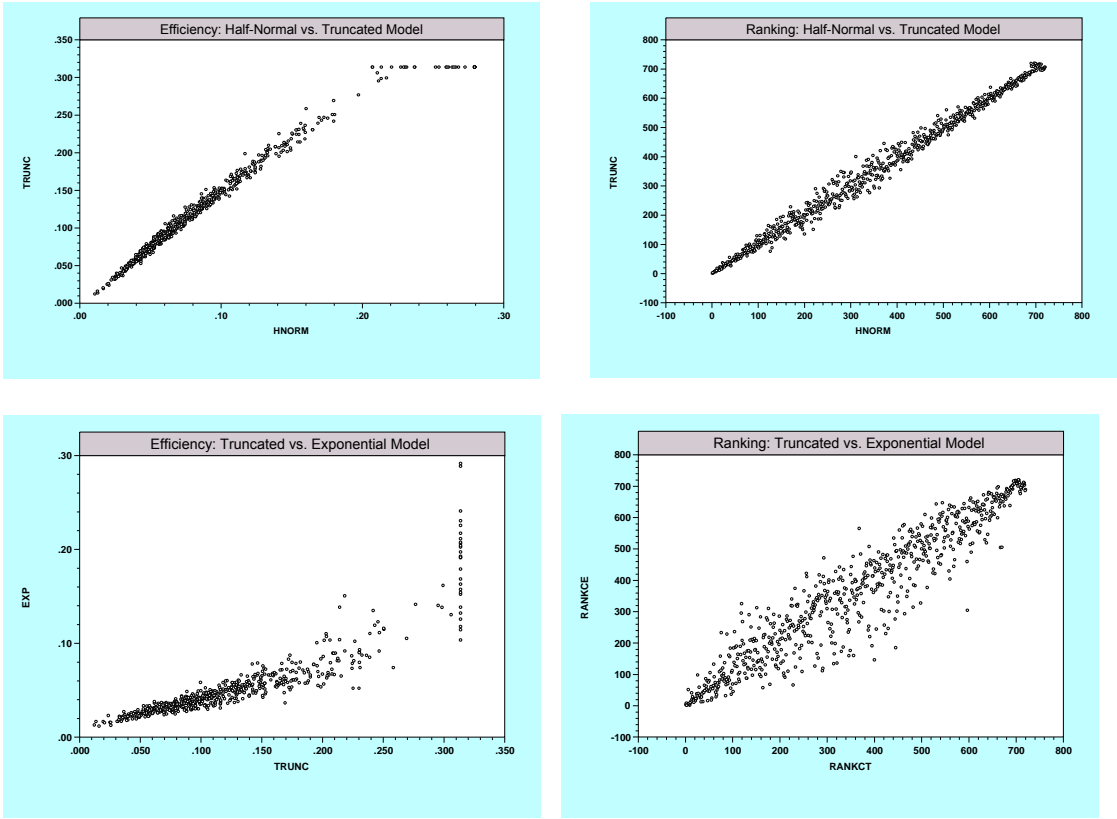
In accordance with Greene (2008), parameter estimates of the truncated-normal model look peculiar and could considerably alter the efficiency scores. The estimate of σ is 1,2608 compared to 0,1155 for the half-normal model, more than ten-fold increase; moreover the estimate of μ is very large (25,1031), suggesting a big impact on the inefficiency estimates

and, being the ratio $\frac{\sigma_u^2}{\sigma^2} = 0,998$, such impact is negligible. We found further evidence

¹⁶ However, since $E[u|\varepsilon]$ depends on the linear combination $[a(-\varepsilon) + (1-a) \mu]$, where $a = \sigma_u^2 / \sigma^2$, in the normal-truncated case $a = 0,99905$, therefore μ does not affect the inefficiency estimate.

about it, by comparing kernel density for truncated-normal and the other distribution models, which almost appears identical. Light changes emerge in case of the exponential distribution, for which we have omitted the trend terms to make estimation more stable. Descriptive statistics and kernel density estimators could underestimate the variation of the expectation of u , but, given that the extent of the bias widens as λ decreases, in our example the estimate of λ seems encouraging. Finally, inefficiencies and score values, under half-normal and truncated-normal assumption, show high correlation coefficients, as expected. Exponential distribution displays a lower value of the correlation coefficient, in particular for technical inefficiency measure (0,89).

Figure 3 – Scatter diagrams comparing different distribution ranking



Results do not change significantly limiting our analysis on specific banks, identified both on a dimensional basis, and a geographical positioning. Parameters are quite similar, while in general likelihood values are much lower due to the narrow sample. Inefficiency values are also lower.

2.1 Benchmark analysis through panel data

The analysis of data in panel format represents a methodological¹⁷ improvement with respect to cross-section analysis, and allows researchers to study dynamic relations, verify the presence of heterogeneity and reduce the distortion due to omitted variables. Therefore it is possible to check more complex behavioural hypotheses. Moreover, panel estimators prove to be more efficient, since the increase in the number of observations (from N to NT) reduces colinearity between variables (thanks to greater individual variability) and leads to adequate causality relations.

However, panel data strength could be overestimated, in particular if the sample is narrow, or the data selection process could be twisted when the choice of units does not take place according to a random sampling. As regards the inefficiency estimate, in the SFA model it is necessary to assume that the level of inefficiency of the single firm is not correlated with the input levels, and this is not always the case. The hypotheses of time invariance of inefficiency is much relevant with panel data, because the longer the sample the more robust the estimators would be (if constant through time), but over a prolonged time period it would be all the more difficult to consider as constant the efficiency of the single

¹⁷ Schmidt and Sickles (1984) highlight that, for cross-section estimators, the variance of expected inefficiency does not vanish neither asymptotically.

production unit. Panel estimate highlights the log-likelihood growth and, generally, a smaller significance of the variables added to the specification. At the same time, the mean inefficiency estimate turns out to be much higher than the cross-section one; it would follow that the adoption of a panel estimate could account for allocative inefficiency.

Table 6 – Panel data Stochastic Cost Frontier

Panel				
Variable	Coefficient	t value		
Constant	3.0107	13.10		
LQ1	0.2758	3.73		
LQ2	0.0217	1.89		
LQ3	0.5042	8.64		
LQ4	0.0269	1.25		
W2N1*	0.0227	3.41		
W3N1*	0.0003	10.50		
W4N1*	0.0303	1.01		
W23	0.0000	-0.11		
W24	-0.0020	-0.74		
W34	0.0000	0.67		
TW2	0.0026	2.82		
TW3	0.0000	-0.46		
TW4	-0.0039	-0.72		
CP1	0.0000	0.21		
CP2	0.0000	0.74		
CP3	0.0000	1.20		
CP4	0.0000	-0.06		
T	-0.0246	-1.85		
T2	0.0007	1.31		
ASSQ	0.4258	1.62		
CRED_ATT	-0.5623	-10.52		
* normalized through the labour input price				
<i>Variance parameters for compound error</i>				
λ	1.9090	1.68		
σ	0.1119	2.43		
Ln L	972.1411			
σ_v	0.0586			
σ_u	0.1119			
<i>Descriptive Statistics for technical inefficiency $E[u_i \varepsilon_i]$</i>				
	Mean	Std.	Min	Max
Panel	0.2109	0.0253	0.1758	0.3028

Here, a twofold combination of elements characterizes the inefficiency resulting from a cost function: technical inefficiency (the shift from optimum amount of output for given inputs), and allocative inefficiency, which instead descends from sub-optimal choice of inputs for given prices and outputs. Telling apart these two factors is identified as the Greene problem (Greene 1997; 2003); up to now, this theoretical problem lacks a satisfactory practical solution.

3. Heterogeneity impact on efficiency

The high variability of efficiency and ranking estimates shown in many studies highlights the uncertainties to obtain a proper classification measure with respect to an optimum benchmark. Taking into account that bank efficiency depends heavily on the specification of the error term, deviations from the efficient frontier could be ascribed not only to managerial deficiency in choosing the input mix, but also to inappropriate comparison of non-homogeneous units, that do not conform to the “ideal” model. We believe that the case of Italian banks requires such extension, given that increased deregulation, dimensional changes deriving from M&A operations and the rapid swing of firm attitudes in reply to the business and strategic environment have emphasized wide differences among operators.

Heterogeneity can be categorised in observable and unobservable heterogeneity. We do not take into account unobserved heterogeneity, which enters the model in the form of ‘effects’ and might reflect missing variables in the model. Observable heterogeneity, in contrast, is reflected in measured variables and would include specific shift factors that operate on the cost function. Systematic differences inside the sample of production units (*group specific heterogeneity*) foster a dual effect on the Stochastic Frontier, causing a parallel shift (when they enter the regression function) or systematic deviations (when they enter in the form of heteroskedasticity) from the frontier, or some combination of both. Moreover, evaluating the importance of the representative heterogeneity variables and the impact of those variables on the production technology or efficiency is still under debate.

The random residual in a Stochastic Frontier model contains a specific shift-factor v_{it} for each firm. The model then is thought to be “homogeneous” if we assume that firms only

deviate from one another because of such single factor. However, it is possible that deviations from the frontier can depend on other components (not included among outputs and inputs of the cost function), or otherwise the residuals could contain unexplained heteroskedasticity in the structural model. Our analysis will include the potential effects of “observable” heteroskedasticity, as explained by exogenous variables.

In the classical linear regression model, if the error term is heteroskedastic, estimates remain consistent and unbiased, but no longer efficient; in a Stochastic Frontier approach, each error component could turn out to be heteroskedastic, thus affecting the parameter estimates, the efficiency estimates or both.

If heteroskedasticity lies in the symmetric component v_i , our parameter estimates are unbiased (except the constant, whose estimate is downward biased); the technical inefficiency estimate is now affected, as there are two sources of variation: the first given by the random residual, the second from the weight of the residual which has an error component with non-constant variance. For this reason, if two producers have the same residual, their estimated efficiency will be different unless they have also the same error variance. Bos et al. (2005) observe that, usually, $\sigma^2_{v_i}$ directly changes with the firm scale and therefore average efficiency for small firms will result upward distorted, while bigger firms will be distorted downward, as heteroskedasticity is improperly attributed to technical inefficiency. To solve such puzzle, a parameter estimate β is obtained through generalized/weighted least squares, whose residuals replace Ols ones, and later they enter the efficiency estimate.

If instead, heteroskedasticity appears in the u_i component, both the cost function parameter estimates and the technical efficiency estimates will be affected. Hence the u_i

heteroskedasticity component produces biased estimates of intercepts and technological parameters for every firm. Technical efficiency estimates thus contain two sources of variability, but the variability of the weight attributed to the residuals now acts in inverse order, affecting downward efficiency estimates of small firms and upward those of bigger ones. Naturally σ_{ui}^2 cannot be estimated for every producer in a single cross-section, therefore σ_{ui}^2 must be expressed according to specific, time invariant exogenous variables (z_i) representing observable heterogeneity not related to the production structure, which captures firm or unit specific effects, using a proper maximum likelihood (ML) technique¹⁸. If heteroskedasticity appears in both components v_i and u_i , the distortion of efficiency estimate depends on the ratio ($\sigma_{vi}^2/\sigma_{ui}^2$): if such ratio is constant among units, then the efficiency estimate is unbiased, otherwise it is necessary to use ML methods. In summary:

- i) if heteroskedasticity is on v_i , the cost function parameter estimates are unbiased, while estimates of technical efficiency are biased;
- ii) if heteroskedasticity is on u_i , both cost function parameter estimates and technical efficiency estimates are biased;
- iii) if heteroskedasticity is on both error components, this causes distortions in opposite direction, and (luckily enough) on average, distortions could also decrease.

In order to reduce the degree of mis-specification, some authors estimated production frontiers taking into account the heterogeneity of technical choices, by rating observations according to different categories using exogenous variables. The technology estimate can be derived following alternative methodologies: variables representing heterogeneity can be

¹⁸ Kumbhakar and Knox Lovell (2000) show several estimation methods.

included in the deterministic component of the model, directly affecting the cost function and modifying the frontier, or, indirectly, they can be used as regressors of the efficiency levels obtained from a traditional cost function. In the first case, parameters can be estimated with the usual techniques and, by ML methods, the independent variables are assumed to leave firm's efficiency¹⁹ unaffected.

The second methodology imposes a two step procedure to explain the inefficiency variation: firstly, sample observations are divided into separate clusters, based upon an a priori information set (ownership structures, geographical distribution), or from cluster analysis results on input/output relationships; then a cost function is estimated by applying ML or GLS methods. Inefficiency values, after normalization, are regressed on exogenous variables possibly correlated with the levels of inefficiency of the initial regression. Since inefficiency ranges between zero and one, it is necessary to use a limited dependent regression model. In this case, the frontier remains unchanged for banks, while the inefficiency deviation distribution is affected: the exogenous factor influence now prevails on the capacity of the single production unit to reach the frontier, restraining the role of the (common) technology which characterizes each unit.

The two-step procedure has been much criticized (see Wang and Schmidt 2002). The hypothesis of u_{it} independent and identical distribution in the first step clashes with environmental variables z_{it} , which reflect heterogeneity, affecting inefficiency, in the second step. Furthermore, if x_{it} 's are correlated with z_{it} 's, the ML estimator is biased, since first regression omits z_{it} 's variables, altering also the inefficiency estimates. Moreover, since $u_{it} = f(z_{it})$, then u_{it} 's contain an error which is correlated with z_{it} 's, so that z_{it} estimates

¹⁹ For an application of this methodology see Filippini, Wild, Kuenzle (2001).

in the second step are downward biased. Given these remarks, we directly model the inefficiency components and the explanatory variables in a single regression.

In the sample period, a process of re-arrangement of bank returns followed deep variations in the business model of banks. The non-interest income component raised above the traditional income sources, represented by net interest margin. This process substantially regarded the whole banking system, characterized by a similar production function, which highlights the multifunctional nature of single firms. However, some differences arise when observing the ratio between loans to customers and total assets, which represents the weight of the credit function with respect to the financial function in the banking system: over 12 years, the growth of this indicator was much higher for small banks than for bigger ones.

Table 7 - Credit quality and banks business model (1993-2004)

	Credit Quality	Securities / Assets	Loans / Assets	Non interest income / Overall business margin	Interest Margin / Overall business margin
Southern and Central Italy					
Average*	0.05	0.13	0.59	0.42	0.64
Variation**	-0.03	0.03	0.13	0.11	-0.10
Northern Italy					
Average	0.02	0.14	0.62	0.47	0.59
Variation	-0.01	0.01	0.14	0.10	-0.08
Small Banks					
Average	0.03	0.14	0.60	0.45	0.61
Variation	-0.02	0.01	0.15	0.10	-0.08
Large Banks					
Average	0.04	0.12	0.64	0.46	0.61
Variation	-0.02	0.04	0.09	0.12	-0.10

* in percent

** between subperiods 1993-98 and 1999-2004

Moreover, in various studies reflecting peculiar realities, interesting differences in the production technology among big and small banks typically emerge. The integration process of the banking system often appears adequate, while major uncertainties are still related to the consolidation process and to the lack of competitiveness. More relevant differences arise at geographic level. While the production function is quite similar for

northern and southern situated banks, the overall business margin is much more affected by non-interest income for northern banks, while southern banks show a clear prevalence of the traditional business model. The credit quality trend turns out to be consistent with such dynamics. A bigger interest margin weight accounts for the higher risk-premium of southern banks, which generally suffer from a higher riskiness, due to historical development delays, as reflected in more NPL and doubtful loans. We do not emphasize any differentiation on the basis of ownership structure, as, in general, we deal with multifunctional groups, which perform similar activities; furthermore, as governance is concerned, Italian laws allow some peculiarities for a number of subjects, which much relax the shareholders and market surveillance/control, as in case of Foundations being in corporate capital or in per capita voting case in the Cooperative Banks.

Therefore, the variables representing heterogeneity (multiplied for the northern banks dummy) are:

- i) The interest margin to overall business margin ratio,
- ii) The total loans to total assets ratio.

These two parameters show the different environment in which Italian banks operate, and do not depend on the choices of the single production units. These parameters are therefore shift factors on the cost function.

The inclusion of heterogeneity factors improves the cost frontier specification and leads to a more significant measure of efficiency as derived from a random effects model. Each model, represented in table 8, comes from Pitt and Lee's (1981), which is slightly different from the conventional random effects model in that the individual specific effects are assumed to follow a half-normal distribution.

Table 8 -Stochastic cost frontier under various heterogeneity hypotheses

Variable	A		B		C	
	Coefficient	t value	Coefficient	t value	Coefficient	t value
Constant	2.9849	6.11	2.9915	9.88	2.9813	9.79
LQ1	0.2676	2.38	0.2755	2.83	0.2800	2.86
LQ2	0.0320	0.79	0.0224	1.76	0.0220	1.26
LQ3	0.5096	4.79	0.5071	6.53	0.5125	7.08
LQ4	0.0223	0.56	0.0275	1.00	0.0210	0.71
W2N1*	0.0227	1.64	0.0229	2.99	0.0218	2.29
W3N1*	0.0002	9.11	0.0003	8.43	0.0003	12.54
W4N1*	0.0289	0.47	0.0309	0.91	0.0304	0.65
W23	0.0000	0.06	0.0000	-0.13	0.0000	-0.18
W24	-0.0021	-0.35	-0.0020	-0.68	-0.0020	-0.45
W34	0.0000	0.26	0.0000	0.61	0.0000	0.48
TW2	0.0026	1.79	0.0025	2.56	0.0024	2.18
TW3	0.0000	-0.36	0.0000	-0.34	0.0000	-0.38
TW4	-0.0034	-0.31	-0.0041	-0.65	-0.0040	-0.50
CP1	0.0000	0.21	0.0000	0.12	0.0000	0.11
CP2	0.0000	0.34	0.0000	0.60	0.0000	0.52
CP3	0.0000	0.80	0.0000	0.84	0.0000	0.99
CP4	0.0000	-0.07	0.0000	-0.06	0.0000	-0.14
T	-0.0247	-0.90	-0.0241	-1.72	-0.0251	-1.53
T2	0.0007	0.67	0.0007	1.15	0.0007	1.23
ASSQ	0.5458	1.64	0.3733	1.38	0.3609	2.14
CRED_ATT	-0.5587	-9.88	-0.5523	-8.04	-0.5562	-7.22
DNOM_M	0.1238	1.33				
DNOCRAT	-0.0827	-1.73				
μ						
Constant			-6.1130	-0.04		
DNOM_M			-14.7858	-0.05		
DNOCRAT			9.5569	0.05		
Heteroscedasticity in symmetric component (v)						
DNOM_M					4.8054	2.63
DNOCRAT					-3.1095	-2.18
Variance parameters for compound error						
λ	1.9068	1.83	9.1688	0.02	2.0787	1.55
σ	0.1104	2.64	0.5412	0.04	0.1100	2.18
Ln L	980.6398		971.2388		983.8349	
σ_v	0.0579		0.0590		0.0529	
σ_u	0.1104		0.5412		0.1100	
A = ln cost function						
B = ln the distribution average of u_i						
C = ln the distribution variance of u_i						
* normalized through the labour input price						
Descriptive Statistics for technical inefficiency $E[u_i \epsilon_i]$						
	Mean		Std. Deviation		Min	Max
A	0.2076		0.0262		0.1774	0.3029
B	0.2942		0.0357		0.2468	0.4237
C	0.2069		0.0277		0.1578	0.3017

Parameters estimates are substantially stable under any specification. If heterogeneity is included in the cost frontier, the asset quality parameter increases very much, with respect to the panel estimate (0,55 against 0,43), highlighting a possible correlation with the exogenous components, whose significance is much limited. When exogenous components are included in the mean distribution of u_i they show non-significant values. Both the σ_u

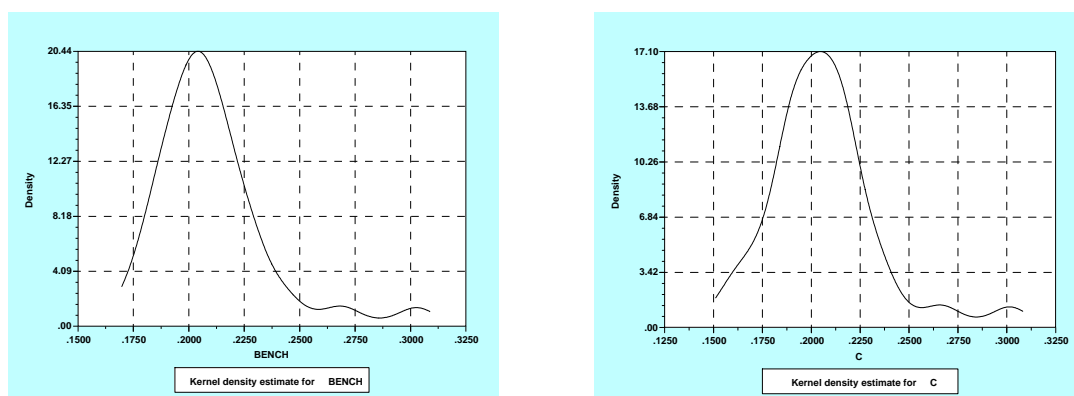
estimate and the parameter λ rise significantly and, therefore, the mean inefficiency value rises by about a third. Inefficiency estimates from model A and C display analogous mean values and standard deviations, while model B shows a much higher technical inefficiency. The most satisfactory specification is obtained when heterogeneity factors are inserted to explain the u_i distribution variance (model C), despite the lack of strong theoretical justifications. Our choice of the favourite model is based upon likelihood reaching the maximum value²⁰, and on the inefficiency distribution, which appears slightly less concentrated (as can be seen in figure 4).

However, based on the correlation matrix, the efficiency and score values depend heavily on the heterogeneity assumptions (similar result can be found in Greene 2004b).

Table 9 - Correlation coefficients within the benchmark model

Technical Inefficiency			Ranking		
Model A Efficiency	Model B Efficiency	Model C Efficiency	Model A Ranking	Model B Ranking	Model C Ranking
0.991	0.999	0.881	0.969	0.998	0.759

Figure 4 - Kernel density (benchmark vs. C model)



²⁰ Such insight is confirmed by significance (at 95% confidence level) of Likelihood Ratio test.

3.1 Scale economies and technical progress

The Italian banking sector is characterized by various sharing elements, as a heavy fixed costs structure and productivity factors common to several production units, therefore scale economies can basically result from an efficient behaviour, as already highlighted by Leland and Pyle (1977). The consolidation process of Italian commercial banks should promote effectiveness, by increasing the dimension of production processes and widening the range of activities and products offered to customers, through reorganization, rationalization and/or integration operations.

We would like to add our opinion that greater efficiency has been produced by the adoption of massive investment in Information Technology and instruments such as increased use of Outsourcing. We have not included specific test to demonstrate this, because this was not the primary scope of the paper.

The total elasticity of scale at time t displays scale economies when the total differential of the cost function with respect to output is less than 1. Our simulations show a value equal to 0,8286 in panel estimate case (0,8356 if we include the heteroskedastic component in the distribution variance of u_i); those values remain quite stable under various specifications, and also in time-varying inefficiency case. Therefore, we believe that the growth of dimensional scale improved efficiency in the banking sector. This conclusion is quite general, in that it encompasses returns from higher production levels (determined by fixed costs) “pure” scale economies (both operating at “system” and “organizational” level), human capital accumulation, knowledge, and built-in progress.

Beyond economies of scale, a measure of technical progress (output growth or cost reduction over time maintaining the factors of the production fixed), can offer further information on the managers performance in running the production unit. The process of operating costs reduction, which follows adjustment towards information technologies or widening range of new products, need a steady control, since capital stock increase often implies some kind of waste. Technical progress can be measured by the following partial derivative of the cost function with respect to time, in that the coefficients on the time trend terms (linear and non-linear trend plus interaction terms) are interpreted as measures of the impact of technical change:

$$\frac{\partial \ln TC}{\partial t} = \alpha + \beta t + \sum_{i=1}^r \varphi_i \ln Q_i + \sum_{h=1}^s \theta_h \ln P_h$$

From Baltagi and Griffin (1988), the partial derivative includes three components identifying the impact of:

- (i) pure technical change, $\alpha + \beta t$;
- (ii) scale augmenting technical change, φ_i ;
- (iii) non-neutral technical change, θ_h .

As in Altunbas, Goddard and Molyneux (1999) “*Pure technical change accounts for reductions in total cost achievable holding constant the efficient scale of production required to produce any specific mix of outputs, and the shares of each of the inputs in total cost. Scale augmenting technical change (unavailable in our setup due to colinearity reasons mentioned above) reflects changes in the sensitivity of total cost to variations in the efficient scale of production. If $\varphi_i < 0$ for all i , the scale of production which minimises average cost for a given output mix is increasing over time. Finally, non-neutral technical*

change accounts for the sensitivity of total cost to variations in unit input prices, so $\theta_h < 0$ implies that the share of the cost of input h in total cost is decreasing over time.”

In our case, under several specifications, the overall value of technical change turns out negative (-0,0259 in the panel model with heteroskedasticity), confirming the presence of technical progress for the Italian banks, with a lower than proportionate cost expansion over time. More than 90% of the global measure comes from pure technical change, while non-neutral technical change, as defined by the sum of interaction coefficients, turns out much lower and with most coefficients non significant, thus revealing a scarce sensitivity of total cost to variations in unit input prices.

Since the dependent variable is normalized through the labour input price, a labour-saving distortion is revealed, with labour productivity growing proportionally more than wages, as a consequence of the reorganization process of the sector.

However, the increase of labour productivity may be offset by inefficiency elements, since technical progress does not always imply cost reductions (in particular not including staff expenses): if we rule out operating from staff costs, in proportion to gross banking product, such measure slightly diminishes from 0,86 in 1998 to 0,83 in 2004, but registers a rising trend in the years 2000-2001-2002; indeed, we have to consider that the attempt to contract staff costs fostered by outsourcing low value added activities, pushed upwards other operating costs, and probably increased inefficiency in some areas. Therefore, the consolidation process of the banking sector during the second half of 90's, apart from achieving cost savings, aimed at multiple targets, capturing market shares in product/services segments, or creating bigger firms to collect the synergies, integrations and investments necessary in order to operate in more efficient way. The dimensional

increase in particular does not imply, at first, any gain in efficiency – with the transitory partnership of central and governance structures of acquired subjects or overlapping the business units (like branches) – but promotes the development of firms products/services oriented, and diverts infrastructures and huge professional skills investments in order to acquire competitive advantages. Some examples come from banks' owned asset management firms, the wide offer of specialized products and services to manage market risks, and investment banking activities. Our cost function does not lead to a measure of *economies of scope*, i.e. the synergetic effects from the joint production of various activities. However, the efficiency measure for single banks allows to estimate the effectiveness of the concentration process driven from M&A activities and joint-ventures, in search of an optimal dimension for the attainment of synergies that potentially improve the competitiveness profiles.

3.2 Time-variability of efficiency

An important hypothesis regarding efficiency estimate is the time-varying error term volatility. When time series span is long, the time invariance of inefficiency assumption of both fixed and random effects models is likely to be problematic. However, inefficiency scores are more stable over time (and reliable) when inefficiency is small relative to industry-wide cost changes that occur at the same time, or when technology dispersion is imperfect. In order to cope with time variability of inefficiencies, the stochastic frontier model could be slightly modified into a 'true' fixed or random effects formulation as proposed by Greene (2005). While the former allows for freely time varying inefficiency,

and allows the heterogeneity term to be correlated with the included variables, the latter includes a random (across firms) constant term.

Table 10 – Stochastic cost frontier - Time invariant vs. time varying inefficiencies

	Exponential *		Pitt-Lee *		Fixed Effects **		Random Effects **		Battese Coelli **		
Variable	Coefficient	t value	Coefficient	t value	Coefficient	t value	Coefficient	t value	Coefficient	t value	
Constant	2.9936	13.05	2.9813	9.79					3.0810	17.69	
LQ1	0.2755	3.45	0.2800	2.86	0.2705	15.07	0.2872	27.35	0.2769	9.00	
LQ2	0.0226	1.76	0.0220	1.26	0.0758	13.68	0.0193	5.66	0.0580	5.04	
LQ3	0.5068	7.81	0.5125	7.08	0.4684	20.54	0.4883	46.95	0.4786	15.03	
LQ4	0.0274	1.19	0.0210	0.71	0.0242	3.77	0.0231	7.29	0.0274	2.38	
W2N1	0.0229	3.89	0.0218	2.29	0.0221	16.73	0.0222	32.85	0.0232	7.96	
W3N1	0.0003	9.46	0.0003	12.54	0.0002	13.93	0.0003	42.97	0.0002	14.41	
W4N1	0.0309	1.28	0.0304	0.65	0.0166	3.59	0.0302	13.79	0.0247	2.38	
W23	0.0000	-0.14	0.0000	-0.18	0.0000	-2.96	0.0000	0.17	0.0000	-0.80	
W24	-0.0020	-0.94	-0.0020	-0.45	-0.0018	-10.00	-0.0021	-17.27	-0.0019	-2.53	
W34	0.0000	0.77	0.0000	0.48	0.0000	9.20	0.0000	8.31	0.0000	3.91	
TW2	0.0025	3.04	0.0024	2.18	0.0026	9.32	0.0025	15.33	0.0024	4.93	
TW3	0.0000	-0.42	0.0000	-0.38	0.0000	-5.39	0.0000	-6.25	0.0000	-0.87	
TW4	-0.0041	-0.83	-0.0040	-0.50	-0.0055	-6.96	-0.0036	-8.47	-0.0054	-3.38	
CP1	0.0000	0.15	0.0000	0.11	0.0000	1.03	0.0000	2.13	0.0000	0.50	
CP2	0.0000	0.76	0.0000	0.52	0.0000	6.81	0.0000	6.00	0.0000	0.88	
CP3	0.0000	1.03	0.0000	0.99	0.0000	4.40	0.0000	10.43	0.0000	2.40	
CP4	0.0000	-0.08	0.0000	-0.14	0.0000	-0.16	0.0000	-1.00	0.0000	0.00	
T	-0.0241	-2.05	-0.0251	-1.53	-0.0239	-6.33	-0.0239	-10.14	-0.0202	-3.16	
T2	0.0007	1.36	0.0007	1.23	0.0008	3.82	0.0007	5.90	0.0006	1.87	
ASSQ	0.3741	1.48	0.3609	2.14	0.8011	6.46	0.4903	8.31	0.7230	2.84	
CRED_ATT	-0.5509	-10.09	-0.5562	-7.22	-0.5569	-13.24	-0.5897	-33.25	-0.5524	-7.00	
Means for random parameters											
Constant							3.2270	101.10			
Scale parameters for distributions of random parameters											
Constant							0.0677	35.60			
Heteroscedasticity in symmetric component (v)											
DNOM_M			4.8054	2.63							
DNOCRAT			-3.1095	-2.18							
Variance parameters for compound error											
λ			2.0787	1.55	1.7634	24.66	0.0000	0.00	1.9191	7.28	
σ	0.0588	23.68	0.1100	2.18	0.1198	58.20	0.0582	79.93	0.1155		
θ		12.6910	1.44								
Ln L	970.8841		983.8349		895.8091		976.1029		893.2837		
σ_v	0.0588		0.0529		0.0591		0.0582		0.0534		
σ_u			0.1100		0.1042		0.0000		0.1025	115.39	
η parameter for time varying inefficiency									0.0100	1.01	
Implied std. Dev. Of random parameters								0.06771			
* Time invariant inefficiencies											
** Time varying inefficiencies											
Descriptive Statistics for technical inefficiency $E[u_i \varepsilon_i]$											
			Mean		Std. Deviation		Min		Max		
Exponential			0.0231		0.0066		0.0096		0.0404		
Pitt-Lee			0.2069		0.0277		0.1578		0.3017		
Fixed Effects			0.0757		0.0337		0.0131		0.2453		
Random Effects			NA		NA		NA		NA		
Battese Coelli			0.0797		0.0542		0.0096		0.2963		

Parameter estimates are consistent under various hypotheses (with the exception of credit quality), but efficiency values show high sensitivity to the time-varying volatility hypothesis. A scant relationship is displayed between time-invariant and time-varying

inefficiencies: dispersion is quite remarkable. Time-invariant estimates are very different among productive units, with much higher mean value (0,2069 in the case of Pitt and Lee heterogeneity adjusted model) as compared to time-varying estimates (0,0757 for the fixed effects model). In this instance, the differences seem more likely to be due to the presence of cross bank heterogeneity, rather than to the assumption of time invariance of the inefficiency estimates. The gap between these descriptive statistics is confirmed by the wide underlying disagreement between the two sets of estimates; they are indeed substantially uncorrelated (the correlation coefficient is 0,1443 with respect to fixed effects and 0,0289 with respect to random effects). Therefore, we conclude that while estimates seem quite robust as regards the distributive hypotheses and to the choice of fixed or random effects, (as already observed in Greene 2004a, 2004b, 2005), the hypothesis of time variability of inefficiency affects to a large extent the estimated model, and especially the inefficiency levels. In general terms, both the efficiency measures and the scores show high sensitivity to the methodological choice and to output variables that represent the production process of the banks. However, once a robust specification is achieved, the efficiency estimate shows some persistency, keeping rather stable over time, as already noted by Kwan and Eisenbeis (1994), Berger and Humphrey (1991) and Berg (1992). A possible explanation is that time-invariant estimates are typically affected by heterogeneity not connected to inefficiency. In particular, if data shows some heteroskedasticity, it remains awkward to separate the effects on the stability of efficiency estimates over time, resulting from economies of scale rather than inefficiency.

If panel data are collected over a longer period of time, the problem of time series stationarity can assume a major role. At first sight, the variables included in the cost

function are very likely to be non-stationary, but adopting a translog function can contribute to relax the problem.

Table 11 – Correlation Matrix for technical inefficiency

	Half-normal	Panel	A	B	C	Exponential	Pitt-Lee	Fixed Effects	Random Effects	Battese Coelli
Half-normal	1	0.6904	0.6828	0.6946	0.6497	-0.5824	0.6497	0.7517	0.6312	0.6955
Panel	0.6904	1	0.9909	0.9991	0.8807	-0.8929	0.8807	0.1168	0.0311	0.9576
A	0.6828	0.9909	1	0.9927	0.8373	-0.8693	0.8373	0.1121	0.0298	0.9433
B	0.6946	0.9991	0.9927	1	0.8804	-0.8850	0.8804	0.1195	0.0307	0.9606
C	0.6497	0.8807	0.8373	0.8804	1	-0.7568	1.0000	0.1443	0.0289	0.8813
Exponential	-0.5824	-0.8929	-0.8693	-0.8850	-0.7568	1	-0.7568	-0.0971	-0.0350	-0.8471
Pitt-Lee	0.6497	0.8807	0.8373	0.8804	1.0000	-0.7568	1	0.1443	0.0289	0.8813
Fixed Effects	0.7517	0.1168	0.1121	0.1195	0.1443	-0.0971	0.1443	1	0.8078	0.1134
Random Effects	0.6312	0.0311	0.0298	0.0307	0.0289	-0.0350	0.0289	0.8078	1	0.0162
Battese Coelli	0.6955	0.9576	0.9433	0.9606	0.8813	-0.8471	0.8813	0.1134	0.0162	1

In order to check our data we obtained (time-varying) inefficiency estimates with the Battese-Coelli methodology (in that those estimates are highly correlated with the invariant Pitt and Lee case); statistical tests, although biased by the short number of observations, confirmed stationarity of inefficiency values. This result probably implies that technical progress, if any, is rather weak. In a phase characterized by growing integration of the banking system, gathering all benefits connected to M&A operations can really take time. In the Italian case this is all the more true when considering that the business dimension increase did not always give rise to new players and new activities, but more often it has involved overlapping functions among various players, as well as the survival of the old governance structures, so that the Acquisition operations prevailed over Mergers.

4. Conclusions

During the last 15 years, the Italian banking system experienced a wide swing of the competitive environment, in part due to structural changes of financial conditions (the

downward convergence in short and long term interest rates, the withdrawal of a national monetary policy, the euro changeover) and in part to slackening in the potential output growth. The European integration process has moreover pushed authorities to implement liberalization and deregulation rules of credit markets, causing some rationalization operations and simplification of ownership structures of firms. As a result, on average, there are now few banks with a larger size.

Our analysis scope is to disclose how much these phenomena have had impact on cost and productivity efficiency of banking institutions. Our contribution confirms the presence of a certain degree of inefficiency in the Italian banking system that, in the period 1993/2004, tends to persist over time, although showing some narrowing. The mean value of inefficiency is slightly higher than other studies suggest and close to 20%, mainly because of an improper use of scale factors and of input congestion. Deviations from the efficient frontier could moreover be brought about not only from managerial deficiency in choosing the input mix, but also from unsuitable comparison of non homogenous firms, that do not conform their behaviour to the “ideal” model. In order to analyze the role of heterogeneity and to discriminate between environmental factors and inefficiency strictly speaking, we compared the benchmark specification with alternative cost frontiers, based on specifications that include variables capturing heterogeneity. Such parameters operate therefore as shift factors on the cost function. The more satisfactory specification is obtained when heterogeneity is included in the variance of the distribution of the residual u_i (capturing both technical and allocative inefficiency). Our result, although debatable on a pure theoretical basis, seems consistent in the light of likelihood reaching the maximum value and the lower concentration of inefficiency distribution. The inclusion of

“environmental” factors affects the estimated values, leading to different mean efficiency and ranking values, both uncorrelated with the benchmark specification. The different specifications of heterogeneity turn out meaningful, especially for rankings rather than the efficiency measures. However, the units ranked in extreme position do not endure excessive modifications: in fact, the best and the worst 5 units are substantially the same ones under the various efficiency specifications. Further analysis could try to explain whether heterogeneity estimates are significant and why relevant inefficiencies do persist on the banking system over time. In presence of a complete and meaningful set of data, SFA could be able to find useful application in single event studies too, like those connected to rationalization and/or reorganization in banking firms.

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