Growth, cycles, and stabilization policy

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This paper presents an analysis of the joint determination of growth and business cycles with the view to studying the long-run implications of short-term monetary stabilization policy. The analysis is based on a simple stochastic growth model in which both real and nominal shocks have permanent effects on output due to nominal rigidities (wage contracts) and an endogenous technology (learning-by-doing). It is shown that there is a negative correlation between the mean and variance of output growth irrespective of the source of fluctuations. It is also shown that, in spite of this, there may exist a conflict between short-term stabilization and long-term growth depending on the type of disturbance. Finally, it is shown that, from a welfare perspective, the optimal monetary policy is that policy which maximizes long-run growth to the exclusion of stabilization considerations.

1. Introduction

A long-standing tradition in macroeconomics—at both theoretical and empirical levels—is the separation of the study of growth from the study of business cycles. The basis of this dichotomy is the presumption that aggregate time series can be decomposed into long-term trends and short-term fluctuations which are determined independently of each other. This presumption runs counter to the implications of endogenous growth theory, according to which any type of shock—be it temporary or permanent, real or nominal—can have a permanent effect on output so long as it changes the amount of resources on which productivity improvements depend (e.g., Stadler, 1990; Pelloni, 1997; Fatas, 2000). Under such circumstances, there is no *a priori* reason to distinguish between low-frequency and high-frequency variations in economic activity, and the presence of stochastic trends is to be explained not by some arbitrary, exogenous impulse process (e.g., non-stationary productivity shocks), but rather by endogenous responses of technology to changes in the current state of the economy.

Recently, the question of precisely how cyclical fluctuations (booms and recessions) might affect secular trends (long-run growth) has been the subject

of an expanding body of literature. Broadly speaking, one may distinguish between two contrasting approaches with the potential to generate different conclusions based on alternative assumptions about the mechanism responsible for engendering endogenous technological change.[1] According to one class of models—models based on re-allocation effects, whereby the mechanism entails deliberate actions (purposeful learning) which substitute for production activities—recessions are events which have a positive impact on growth by reducing the opportunity cost of diverting resources away from manufacturing towards productivity improvements (e.g., Aghion and Saint-Paul, 1998a, 1998b). According to another class of models—models based on externality effects, whereby the mechanism reflects non-deliberate actions (serendipitous learning) which are complements to production activity—recessions are episodes which have a negative influence on growth by lowering factor employment through which expertise, knowledge and skills are acquired and disseminated (e.g., Martin and Rogers, 1997, 2000). Within the context of each of these frameworks, attention has also been given to the question of how growth may be affected by the precise structure of business cycles in terms of their amplitude, frequency and persistence. Of particular interest has been the relationship between growth and volatility with different analyses reaching different conclusions (a positive or negative relationship) depending on what type of model is employed, what values for parameters are assumed and what types of disturbance are considered (e.g., Canton, 1996; Smith, 1996; Martin and Rogers, 1997, 2000; de Hek, 1999; Jones *et al.*, 1999; Aghion and Saint-Paul, 1998a, 1998b; Blackburn and Galindev, 2003; Blackburn and Pelloni, 2004).[2]

Significantly, all of the above analyses are based on purely real models of the economy and there are very few investigations that explore the role of monetary factors. An exception is the recent contribution by Dotsey and Sarte (2000) who develop a stochastic monetary growth model in which agents are subject to a cash-in-advance constraint, while operating a simple *AK* production technology with a fixed amount of labour. It is shown that an increase in the volatility of monetary growth, coupled with an increase in average monetary growth, may lead to either an increase or a decrease in average output growth due to offsetting effects through

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[1] In addition to the references that follow, see Ramey and Ramey (1991) and Caballero and Hammour (1994) for related contributions.

[2] This conflict in results at the theoretical level is matched by a similar conflict in evidence at the empirical level. In both cross-section and time series studies, the correlation between the average growth rate of output and the variability of output growth is found sometimes to be positive (e.g., Kormendi and Meguire, 1985; Grier and Tullock, 1989; Caporale and McKiernan, 1996), sometimes to be negative (e.g., Ramey and Ramey, 1995; Martin and Rogers, 2000; Kneller and Young, 2001), and sometimes to be zero (e.g., Dawson and Stephenson, 1997; Speight, 1999; Grier and Perry, 2000). In addition there is evidence to suggest that the correlation is sensitive to the level of disaggregation. For example, Imbs (2002) reports estimates of a significantly negative correlation when using aggregate data across countries, but a significantly positive correlation when using disaggregated data across sectors.

precautionary savings and inflation taxes.[3] One objective of the present paper is to conduct a similar analysis but within the context of a quite different monetary growth model that allows for endogenous labour, multiple shocks, learning-by-doing, and nominal rigidities. The last of these features, encapsulated in the form of one-period wage contracts, does not appear in any of the above literature. Our analysis provides a further illustration of the joint determination of growth and business cycles.

A second concern of the paper relates to an important, but largely neglected, issue in the formulation and evaluation of macroeconomic policy. This is the extent to which policies designed to stabilize short-run fluctuations might also affect the long-run performance of the economy. The existence of a relationship between growth and volatility has an obvious bearing on this issue: depending on whether this relationship is negative or positive, there is the presumption that successful stabilization would also entail either an improvement or deterioration in growth prospects. The potential significance of this is self-evident, especially given the fact that it takes only small changes in the growth rate to produce substantial cumulative gains or losses in output. As yet, however, there are very few analyses that deal with the issue explicitly. Two recent contributions that do so are those of Blackburn (1999) and Martin and Rogers (1997). The former presents a model of imperfect competition with nominal rigidities in which monetary stabilization policy has a negative effect on long-run growth. The latter consider a real model of the economy with perfect competition in which fiscal stabilization policy has a positive effect on long-run growth. In both cases sustainable growth occurs due to learning-by-doing which provides the only source of intrinsic dynamics, there being no capital accumulation or any other propagation mechanism under the direct control of agents. The models are also highly stylized in a number of other respects. In Blackburn (1999) there is no explicit optimization by either agents or policy makers so that the normative aspects of policy are eschewed. In Martin and Rogers (1997) agents solve only a purely static optimization problem from which employment is determined as a zero-one variable due to linear preferences. The analysis that follows is based on a more fully-specified dynamic general equilibrium model in which both capital accumulation and wage determination reflect the optimal decision rules of intertemporally maximizing agents, and in which both the growth and welfare effects of monetary stabilization policy may be evaluated explicitly.

There are three sources of stochastic fluctuations in our model—a preference shock, a technology shock, and a monetary growth shock. The central bank operates a feedback rule for determining how monetary policy responds to each of these shocks. The first main result of our analysis is that, *ceteris paribus*,

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[3] The assumption of a positive correlation between the mean and variance of monetary growth (or inflation) is justified by appealing to empirical evidence.

an increase in the variance of each shock causes both an increase in the variance and a decrease in the mean of output growth. The latter effect occurs because of the increase in uncertainty about the state of the economy, in general, and the response of monetary policy, in particular. Workers react to this greater uncertainty by setting a higher contract wage at the cost of lower average employment. This leads to a lower average rate of capital accumulation and, with it, a lower average growth rate of output. In this way, the model predicts a negative correlation between short-run (cyclical) volatility and long-run (secular) growth. Our second main result is that, depending on the type of shock, a policy that reduces volatility may either increase or decrease growth. This difference is observed when considering, for example, monetary growth and technology shocks. In the case of the former, output may be stabilized by an appropriate (offsetting) response of monetary policy which mitigates varia-tions in the money supply; this has the effect of reducing nominal uncertainty which, for reasons just given, leads to higher average employment and higher average growth. In the case of the latter, output may also be stabilized by an appropriate policy response, but this now injects fluctuations into the money supply; as such, nominal uncertainty rises, causing average employment and growth to fall. A corollary of these findings is that the optimal policy which minimizes volatility may not only differ from, but may also conflict with, the optimal policy which maximizes growth. Finally, our third main result is that the growth-maximizing policy is also the policy that maximizes welfare. This accords with the view that the welfare effects of business fluctuations are trivial compared to the welfare effects of growth.

Section 2 contains a description of the model. Section 3 presents the solution of the model. Section 4 turns to the analysis of growth, volatility and stabilization policy. Section 5 concludes.

2. The model

We consider an artificial economy in which there are constant populations (normalized to one) of identical, immortal households and identical, competitive firms. The basic structure of this economy is described by a standard stochastic, monetary growth model. Naturally, whilst the model is more general than some others, it is constructed in such a way as to focus and simplify the analysis, and is not meant to provide a complete account of the mechanisms underlying aggregate fluctuations. In particular, since our intention is to illustrate without having to resort to numerical simulations, we adopt the usual specifications of preferences and technologies that admit closed-form solutions.[4] The exogenous shocks in the model are also chosen to provide clear and simple examples of how different

[4] That is, we assume logarithmic utility functions, Cobb-Douglas production functions and 100% rates of depreciation (e.g., Benassy, 1995; Gali, 1999).

conclusions may be reached on the basis of different assumptions about the sources of stochastic fluctuations.

2.1 Firms

The representative firm combines N_t units of labour with K_t units of capital to produce Y_t units of output according to

$$Y_t = \Psi_t (Z_t N_t)^\alpha K_t^{1-\alpha} \tag{1}$$

$$\Psi_t = \Psi \exp(\psi_t) \tag{2}$$

$$\Psi > 0; \quad \alpha \in (0, 1)$$

The term ψ_t represents a technology shock which is assumed to be identically, independently and normally distributed with mean zero and variance σ_ψ^2. The term Z_t denotes an index of knowledge which is freely available to all firms and which is acquired through serendipitous learning-by-doing. This provides the mechanism of endogenous growth in the model. Following convention, we approximate the stock of disembodied knowledge by the aggregate stock of capital which is taken as given by each firm so that learning takes the form of a pure externality.

Labour and capital are hired from households at the real wage rate W_t/P_t and real rental rate R_t, respectively, where W_t is the nominal wage and P_t is the price of output. Profit maximization implies

$$\frac{W_t}{P_t} = \alpha \Psi_t Z_t^\alpha N_t^{\alpha-1} K_t^{1-\alpha} \tag{3}$$

$$R_t = (1 - \alpha)\Psi_t Z_t^\alpha N_t^\alpha K_t^{-\alpha} \tag{4}$$

2.2 Households

The representative household derives lifetime utility, U, according to

$$U = \sum_{t=0}^{\infty} \beta^t \left[\Gamma \log(C_t) + \Theta \log\left(\frac{M_t}{P_t}\right) - \Lambda_t N_t^\eta \right] \tag{5}$$

$$\Lambda_t = \Lambda \exp(\lambda_t) \tag{6}$$

$$\beta \in (0, 1); \quad \Gamma, \Theta, \Lambda > 0; \quad \eta > 1,$$

where C_t denotes consumption and M_t denotes nominal money balances. To generate a demand function for money, we adopt the familiar short-cut

device of introducing money directly into the utility function, rather than specifying explicitly a separate transactions technology.[5] The specification of the labour term is another feature that our model shares with several others and encompasses the linear case ($\eta = 1$) associated either with the assumption of constant marginal utility of leisure, or with a reduced-form preference ordering under circumstances where labour is indivisible and individuals choose employment lotteries in the manner of Hansen (1985) and Rogerson (1988). The quantity λ_t represents a preference (or taste) shock, being an identically and independently distributed normal random variable with mean zero and variance σ_λ^2.[6]

The budget constraint facing the household is given by

$$C_t + \frac{M_t}{P_t} + A_{t+1} = \frac{W_t}{P_t} N_t + \frac{M_{t-1}\Phi_t}{P_t} + R_t A_t \qquad (7)$$

where A_t denotes real asset holdings and ϕ_t is a proportional monetary transfer.[7] The right-hand-side of this expression gives the household's total real resources in each period, being comprised of labour income, previously accumulated money balances (augmented by the nominal transfer) and previously accumulated assets (augmented by interest payments). The left-hand-side shows the allocation of these resources between consumption, further additions to money holdings and further additions to asset holdings.

Each household confronts the problem of maximizing the expected value of intertemporal utility in (5) subject to the sequence of budget constraints in (7). This problem is solved, in part, by choosing plans for consumption, money balances and asset holdings that satisfy the following conditions:

$$\frac{1}{C_t} = \beta E_t \left(\frac{R_{t+1}}{C_{t+1}} \right) \qquad (8)$$

$$\frac{1}{P_t C_t} = \frac{\Theta}{\Gamma M_t} + \beta E_t \left(\frac{\Phi_{t+1}}{P_{t+1} C_{t+1}} \right) \qquad (9)$$

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[5] The quantity M_t is understood to represent nominal cash balances at the end of period t. Our analysis would be unchanged if one was to replace this with beginning-of-period money holdings, as in some other models.

[6] As in some other analyses, the model could be extended to allow the other preference parameters, Γ and Θ, to be stochastic as well (e.g., Ireland, 1997; McCallum and Nelson, 1999). We choose not to do so for the sake of simplicity and brevity.

[7] The assumption that monetary transfers are proportional (rather than lump-sum) is made largely for analytical convenience (e.g., Benassy, 1995).

where E_t denotes the expectations operator.[8] Each of these conditions has the usual interpretation of equating the current marginal costs and (expected) future marginal benefits of foregoing a unit of consumption in favour of an additional unit of savings (either assets or money). Plans for the number of hours to work are governed by circumstances in the labour market which we treat as being imperfectly competitive and imperfectly flexible. We adopt a standard monopoly union model of wage determination, whereby households (or unions) set nominal wages and firms determine the level of employment. We also make the assumption, familiar in business cycle analysis but less so in growth theory, that wage setting takes place prior to the realizations of shocks on the basis of one-period contracts. Accordingly, the economy displays nominal rigidities, as in the early contracting models of Gray (1976) and Fischer (1977), as well as those of a more recent vintage (e.g., Benassy, 1995; Cho and Cooley, 1995). In contrast to these models, however, we suppose that the contract wage is chosen so as to maximize households' expected utility (e.g., Hairault and Portier, 1993; Rankin, 1998), rather than to satisfy some *ad hoc* criterion, such as the maximization of other union objectives or the requirement that the labour market is expected to clear. When making this choice, workers take account of the response of labour demand, as expressed in (3). Given this, the optimal wage set at the end of period $t-1$ for period t is found to satisfy[9]

$$\eta E_{t-1}\left(\Lambda_t N_t^\eta\right) = \alpha \Gamma W_t E_{t-1}\left(\frac{N_t}{P_t C_t}\right) \tag{10}$$

The maximizing behaviour of the representative household is now characterized fully by the first-order conditions in (8), (9), and (10), the budget constraint in (7) and the transversality conditions $\lim_{\tau\to\infty}\beta^\tau E_t\left(M_{t+\tau}/P_{t+\tau}C_{t+\tau}\right) = \lim_{\tau\to\infty}\beta^\tau E_t\left(A_{t+\tau+1}/C_{t+\tau}\right) = 0$.

2.3 Monetary policy

We assume that monetary policy is governed by a feedback rule through which the central bank exercises imperfect control over the aggregate money supply. This feedback rule dictates how the central bank responds to the occurence of exogenous shocks in the economy. The imprecision in monetary control reflects the assumption that the central bank's policy instrument is the growth rate of the monetary

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[8] Multiplying these expressions through by Γ produces the term Γ/C_t which is understood to be the marginal utility of consumption, or shadow value of wealth, being equal to the Lagrange multiplier attached to (7).

[9] That is, W_t is chosen so as to maximize the expected value of (5) subject to (7) and the condition that $N_t = (W_t/P_t \alpha \Psi_t Z_t^\alpha K_t^{1-\alpha})^{1/(\alpha-1)}$ from (3).

base to which the growth rate of the money supply is imperfectly (randomly) related.[10] Formally, we have

$$H_t = H_{t-1}\Phi_t \tag{11}$$

$$\Phi_t = F_t \exp(\phi_t) \tag{12}$$

$$F_t = F \exp(\rho_\phi \phi_t + \rho_\lambda \lambda_t + \rho_\psi \psi_t) \tag{13}$$

$$F > 1; \quad \rho_\phi, \rho_\lambda, \rho_\psi \gtreqless 0$$

where H_t denotes the money supply, F_t denotes the feedback rule and ϕ_t is the control error which we assume to be identically, independently and normally distributed with mean zero and variance σ_ϕ^2. If $\rho_\phi = \rho_\lambda = \rho_\psi = 0$, then monetary policy is completely unresponsive to changes in the state of the economy and the money supply grows at the exogenous, stochastic rate $F\exp(\phi_t)$. If any of these feedback parameters are non-zero, however, then monetary policy responds systematically to the realizations of the shocks.[11]

3. General equilibrium

The solution of the model is a stochastic dynamic general equilibrium which describes the aggregate behaviour of the economy based on the optimal decision rules that solve firms' and households' maximization problems. The equilibrium is computed by combining the relationships obtained so far with the market clearing conditions $C_t + K_{t+1} = Y_t$ (for goods), $K_t = A_t$ (for capital), and $M_t = H_t$ (for money). Given the structure of the model, we may proceed in two stages, the first of which entails determining the solutions for consumption, capital and money holdings for a given level of employment, and the second of which involves establishing the solution for employment, itself. Details of the derivations are relegated to an Appendix.

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[10] Aside from its realism, this assumption serves as a convenient device for incorporating nominal shocks into the model. An alternative approach which yields identcal results is to introduce money demand (or velocity) shocks by allowing the preference parameter Θ to be stochastic (e.g., Ireland, 1997).

[11] As in other models, we suppose that the monetary authority is able to respond to the contemporaneous realizations of disturbances so that monetary policy can have real effects in the presence of one period wage contracts (e.g., Ireland, 1997; Gali, 1999). More generally, we would assume that the reaction lag in monetary policy is shorter than the length of contracts. Observe that the monetary authority can do no better than to respond directly to the underlying shocks if it is able to observe these shocks: any other policy rule (e.g., a rule based on broad economic aggregates) would imply that information is being wasted. Given this, then our analysis may be viewed as demonstrating how policy conflicts may arise even when policy decisions are based on detailed, accurate information.

3.1 Consumption, capital accumulation, and cash balances

After appropriate substitutions, we are able to write (8) and (9) as

$$\frac{K_{t+1}}{C_t} = \beta(1-\alpha) + \beta(1-\alpha)E_t\left(\frac{K_{t+2}}{C_{t+1}}\right) \tag{14}$$

$$\frac{M_t}{P_tC_t} = \frac{\Theta}{\Gamma} + \beta E_t\left(\frac{M_{t+1}}{P_{t+1}C_{t+1}}\right) \tag{15}$$

Each of these expressions defines a stochastic expectations difference equation which is solved by imposing the relevant transversality condition. Doing this, and exploiting our other relationships, we obtain the following results:

$$C_t = (1-\omega)Y_t \tag{16}$$

$$K_{t+1} = \omega Y_t \tag{17}$$

$$\frac{M_t}{P_t} = \Omega Y_t \tag{18}$$

where $\omega = \beta(1-\alpha)$ and $\Omega = (1-\omega)\Theta/(1-\beta)\Gamma$. These expressions show that the equilibrium levels of consumption, capital and real money balances are all proportional to the level of output. This is a direct consequence of our assumptions about preferences and technologies which allow us to obtain simple, closed-form solutions. The solutions imply that, for a given level of employment, consumption, and capital (as well as output) are independent of monetary factors. As we shall now see, however, employment is not invariant with respect to changes in nominal variables, meaning that monetary shocks and monetary policy can have real effects in the economy.

3.2 Employment

According to our description of the labour market, households supply whatever labour is demanded by firms at the optimally chosen contract wage implied by (10). After various manipulations, a precise expression for this wage may be obtained as

$$W_t = \left(\frac{\alpha}{\Omega}\right)\left[\frac{\eta(1-\omega)}{\alpha^2\Gamma}\right]^{1/\eta}\left[E_{t-1}\left(M_t^\eta \Lambda_t\right)\right]^{1/\eta} \tag{19}$$

Thus the nominal wage depends on expectations about the money supply, M_t, and the preference shock, Λ_t. Given the processes governing these variables, it is possible to compute the value of expectations as

$E_{t-1}(M_t^\eta \Lambda_t) = (M_{t-1}F)^\eta \Lambda \exp\{(1/2)[\eta^2(\rho_\phi + 1)^2\sigma_\phi^2 + (\eta\rho_\lambda + 1)^2\sigma_\lambda^2 + \eta^2\rho_\psi^2\sigma_\psi^2]\}$. In general, therefore, the nominal wage increases with an increase in the variance of each of the shocks.

Having established the above, we may now proceed to derive an expression for the equilibrium level of employment. This turns out to be

$$N_t = \left[\frac{\alpha^2\Gamma}{\eta(1 - \omega)}\right]^{1/\eta} \frac{M_t}{\left[E_{t-1}(M_t^\eta \Lambda_t)\right]^{1/\eta}} = \left[\frac{\alpha^2\Gamma}{\eta\Lambda(1 - \omega)}\right]^{1/\eta} \frac{\exp(\epsilon_t)}{\exp(\sigma^2)} \qquad (20)$$

$$\epsilon_t = (\rho_\phi + 1)\phi_t + \rho_\lambda\lambda_t + \rho_\psi\psi_t \qquad (21)$$

$$\sigma^2 = \frac{\eta(\rho_\phi + 1)^2}{2}\sigma_\phi^2 + \frac{(\eta\rho_\lambda + 1)^2}{2\eta}\sigma_\lambda^2 + \frac{\eta\rho_\psi^2}{2}\sigma_\psi^2 \qquad (22)$$

Accordingly, we arrive at the well-known result that, in the presence of nominal rigidities, fluctuations in employment can occur because of (unanticipated) fluctuations in money. This result would not arise were we to abandon the notion of wage contracts and assume, instead, that wages are chosen contingent on the realization of M_t. Under such circumstances, this variable would vanish from (20) and employment would deviate from its perfectly competitive level only by a constant factor of proportionality, α, reflecting the pure ineffiency effect of monopolistic wage setting (i.e., a downward bias to employment associated with an upward bias to wages). As it is, the existence of pre-determined wages in our model means that monetary fluctuations are non-neutral, having real effects on the economy through variations in employment, consumption, investment, and output. The final expression for employment is seen to depend on both the realizations and variances of the exogenous shocks. Note that the preference shock, λ_t, and the technology shock, ψ_t, disappear from this expression if monetary policy is unresponsive to them (i.e., if $\rho_\lambda = \rho_\psi = 0$). In the case of the former this is because of the fact that employment is completely demand-determined under our assumption of wage contracting. In the case of the latter, it is because of offsetting income and substitution effects under our specifications of preferences and technologies. Notwithstanding these observations, a positive realization of any of the shocks leads to an increase in the money supply, an increase in the demand for goods and an increase in the demand for labour. Conversely, larger variances of the shocks lead to higher nominal wages and a lower demand for labour.[12]

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[12] To the extent that larger variances of the shocks lead to greater volatility in the money supply, this result is consistent with the findings obtained from some other models (which provide different explanations) and may be interpreted as saying that employment is negatively related to the degree of nominal uncertainty (e.g., Evans, 1989; Sorensen, 1992; Rankin, 1998).

4. Stochastic endogenous growth and monetary policy

We are now in a position to address the main issues of interest to us—namely, the extent to which there are linkages between the cyclical and secular properties of aggregate fluctuations, and the implications of such linkages for monetary policy. We do this by solving for the growth rate of output, from which the growth rates of other non-stationary variables (consumption and capital) may be inferred. These growth rates are both stochastic and endogenous. It is recalled that we account for the latter property on the basis of learning-by-doing, formalized by approximating the stock of disembodied knowledge available to firms by the aggregate stock of capital: that is, $Z_t = K_t$ in (1). As shown by others, the main implication of this is to make it possible for the level of output (and, with it, the levels of other variables) to depend on the accumulated realizations of any type of shock, whether real or nominal, temporary or permanent. We show this to be true in the present framework. More significantly, we establish the result that the average rate of growth of output is a function of the variances of the shocks, implying a relationship between secular growth and cyclical volatility from which we draw further implications regarding the growth and welfare effects of monetary stabilization policy.

4.1 The output process

Substitution of (17) and (20) into (1) yields

$$\frac{Y_t}{Y_{t-1}} = \Psi\omega\left[\frac{\alpha^2\Gamma}{\eta\Lambda(1-\omega)}\right]^{\alpha/\eta}\frac{\exp(\alpha\epsilon_t + \psi_t)}{\exp(\alpha\sigma^2)} \tag{23}$$

Defining $y_t = \log(Y_t)$, we then have

$$y_t - y_{t-1} = \delta - \alpha\sigma^2 + \varepsilon_t \tag{24}$$

$$\varepsilon_t = \alpha(\rho_\phi + 1)\phi_t + \alpha\rho_\lambda\lambda_t + (\alpha\rho_\psi + 1)\psi_t \tag{25}$$

where $\delta = \log(\Psi\omega) + (\alpha/\eta)\log[\alpha^2\Gamma/\eta\Lambda(1-\omega)]$. Hence

$$y_t = y_0 + (\delta - \alpha\sigma^2)t + \sum_{j=1}^{t}\varepsilon_j \tag{26}$$

Expression (24) shows that output follows a non-stationary stochastic process that is integrated of order one. This process is described by a random walk with drift, implying that the economy displays stochastic and sustainable growth. In turn, this means that the level of output is permanently affected by the occurrence of exogenous shocks, as indicated by (26). Accordingly, the model provides another example of how endogenous technological change (in our case, learning-by-doing)

can generate unit roots and stochastic trends in macroeconomic time series without having to assume unit root stochastic processes for exogenous shocks (in particular, technology shocks).[13] Since the growth rate of technology is $z_t - z_{t-1} = y_t - y_{t-1}$, we have the standard result of learning-by-doing models that technological change is pro-cyclical. In addition, since output (like employment) depends positively on monetary surprises, we have the other standard result of such models that positive (negative) demand shocks have positive (negative) effects on the growth rate. Of greater interest to us, however, is the fact that the drift term in (24), $\delta - \alpha\sigma^2$, depends on the variances of all of the underlying disturbances, as reflected in the term σ^2. This is indicative of a relationship between growth and volatility, a matter to which we now turn.

Given (24) and (25), together with the definition of σ^2, we may compute the mean and variance of output growth as, respectively,

$$\text{Mean}(y_t - y_{t-1}) = \delta - \frac{\alpha\eta(\rho_\phi + 1)^2}{2}\sigma_\phi^2 - \frac{\alpha(\eta\rho_\lambda + 1)^2}{2\eta}\sigma_\lambda^2 - \frac{\alpha\eta\rho_\psi^2}{2}\sigma_\psi^2 \qquad (27)$$

$$\text{Var}(y_t - y_{t-1}) = \alpha^2(\rho_\phi + 1)^2\sigma_\phi^2 + \alpha^2\rho_\lambda^2\sigma_\lambda^2 + (\alpha\rho_\psi + 1)^2\sigma_\psi^2 \qquad (28)$$

These expressions show that, in general, an increase in the variance of any of the shocks causes an increase in $\text{Var}(y_t - y_{t-1})$ but a decrease in $\text{Mean}(y_t - y_{t-1})$. This is the sense in which the model generates a negative correlation between long-run (secular) growth and short-run (cyclical) volatility.

To gain some insight into the above result, begin by considering the mechanism through which the growth rate of output, as summarized by (24) and (25), is affected by each type of disturbance, recalling that nominal wages are fixed at the level in (19), that employment is demand-determined at the level in (20) and that monetary policy operates through the feedback parameters ρ_ϕ, ρ_λ, and ρ_ψ in (13). In the case of the nominal disturbance, ϕ_t, output growth varies because of variations in employment induced by both the shock, itself, and any response of monetary policy to it. If $(\rho_\phi + 1)\phi_t > 0$, then monetary growth increases, causing output and employment to increase as prices rise and real wages fall. In the case of the preference shock, λ_t, it is also true that output growth fluctuates because of changes in employment, but, as indicated previously, these changes occur only to the extent that monetary policy reacts to the shock. If $\rho_\lambda\lambda_t > 0$, then there is an increase in monetary growth which raises employment and output for the reasons just given. In the case of the technology shock, ψ_t, output growth varies both directly as a result of the shock, and indirectly as a result of any employment variations caused by monetary policy.

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[13] This is true for even the simplest version of the model, where the only disturbance is the technology shock and where monetary growth occurs at a constant, exogenous rate. Setting $\phi_t = \lambda_t = 0$, together with $\rho_\psi = 0$, yields $y_t - y_{t-1} = g + \psi_t$ which confirms that output follows a non-stationary (unit root) stochastic process, even though ψ_t is a purely temporary disturbance.

If $(\alpha\rho_\psi + 1)\psi_t > 0$, then there is an overall increase in output. Naturally, the greater are the variances of the shocks, the greater will be the extent to which output growth fluctuates, or the greater will be $\text{Var}(y_t - y_{t-1})$. But this is not all that happens— for our analysis also implies that there will be a reduction in the average growth rate, or a reduction in $\text{Mean}(y_t - y_{t-1})$. The mechanism in this case is as follows. As we have seen, a larger variance of each shock is associated with a higher average wage in (19) and a lower average level of employment in (20). The latter produces a lower average level of output and, with it, a lower average rate of capital accumulation. This general reduction in real economic activity is translated into a reduction in average growth by virtue of a fall in the rate of technological progress via the process of learning-by-doing.

A negative correlation between growth and volatility is a prediction of some other models, though the precise mechanism at work is different from that given above (e.g., Martin and Rogers, 1997, 2000; de Hek, 1999; Jones *et al.*, 1999; Dotsey and Sarte, 2000; Barlevy, 2002). Beyond the theoretical level, our analysis finds support in a number of empirical studies which, collectively, suggest that output growth is negatively related both to the amount of output variability and the degree of nominal uncertainty (e.g., Kormendi and Meguire, 1985; Grier and Tullock, 1989; Ramey and Ramey, 1995; Judson and Orphanides, 1996; Grier and Perry, 2000; Martin and Rogers, 2000; Kneller and Young, 2001; Imbs 2002).

4.2 Growth, stabilization and welfare

The existence of a relationship between growth and volatility adds a new dimension to the design and evaluation of macroeconomic policies aimed at stabilizing fluctuations. As indicated earlier, however, there are very few analyses that have so far attended to this issue. In the present framework stabilization policy is modelled explicitly in (13) as a feedback rule for monetary policy through which the central bank responds endogenously and systematically to realizations of each of the disturbances. The precise nature of the response in each case is summarized by the relevant feedback parameter—ρ_ϕ, ρ_λ, or ρ_ψ—which may be thought of as being chosen optimally by the bank according to its particular objectives.

Suppose that the central bank is concerned with both reducing short-term volatility, as given in (28), and enhancing long-term growth, as expressed in (27). In the case of the nominal shock, ϕ_t, there is no conflict between these objectives: from the perspective of either minimizing $\text{Var}(y_t - y_{t-1})$ or maximizing $\text{Mean}(y_t - y_{t-1})$, the optimal policy is the same, being to set $\rho_\phi = -1$. Such a policy eliminates completely the fluctuations and uncertainty that would otherwise arise from this disturbance, causing a higher level of real activity (because of lower nominal wages) and a higher rate of technological progress (via learning-by-doing).[14] This is an

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[14] Essentially, the central bank operates a policy which effectively gives it perfect control over the money supply. As indicated previously, the same policy would be optimal for the case in which the nominal disturbance is a money demand (rather than money supply) shock.

example of how monetary stabilization policy can be complementary to the promotion of growth. A counter-example is provided by consideration of the preference shock, λ_t: in this case it is optimal to set $\rho_\lambda = 0$ for minimizing $\text{Var}(y_t - y_{t-1})$, but $\rho_\lambda = -1/\eta$ for maximizing $\text{Mean}(y_t - y_{t-1})$. These results are explained by our earlier observation that employment depends only on the expectation, and not the realization, of this shock if monetary policy does not respond to it; but a negative response is called for if one wants to ensure that expectations remain low so that, on average, wages remain low and employment, output and output growth remain high. The opposite situation arises with respect to the technology shock, ψ_t: it is now the case that minimizing $\text{Var}(y_t - y_{t-1})$ entails setting $\rho_\psi = -1/\alpha$, while maximizing $\text{Mean}(y_t - y_{t-1})$ requires $\rho_\psi = 0$. By conditioning monetary policy on the realization of this shock, fluctuations in output can be stabilized, but only at the cost of raising expectations about the money supply (through greater nominal uncertainty) so that, on average, wages remain high and real activity remains low. Thus the model provides a simple illustration of how different scenarios may lead to different conclusions about the extent to which there may exist a trade-off between short-term stabilization and long-term growth.[15] In terms of optimizing such a trade-off, one might view the policy maker as maximizing the objective function

$$V = \text{Mean}(y_t - y_{t-1}) - \mu \, \text{Var}(y_t - y_{t-1}) \tag{29}$$

where $\mu \geqslant 0$ is the weight assigned to the stabilization objective relative to the growth objective. Under such circumstances, the optimal values for the feedback parameters are $\rho_\phi = -1$, $\rho_\lambda = -1/(\eta + 2\mu\alpha)$ and $\rho_\psi = -2\mu/(\eta + 2\mu\alpha)$, from which the values above may be obtained by considering the limiting cases of $\mu = 0$ and $\mu = \infty$.

Of course, minimizing volatility or maximizing growth may not be the same as maximizing welfare. To study the welfare effects of monetary policy, we compute the unconditional expected value of utility in (5), arriving at the expression

$$E(U) = U_0 - U_1 \left[\frac{\alpha\eta(\rho_\phi + 1)^2}{2}\sigma_\phi^2 + \frac{\alpha(\eta\rho_\lambda + 1)^2}{2\eta}\sigma_\lambda^2 + \frac{\alpha\eta\rho_\psi^2}{2}\sigma_\psi^2 \right] \tag{30}$$

where U_0 and U_1 are composite parameter terms, independent of monetary policy. A comparison of (30) with (27) reveals that welfare is maximized by choosing

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[15] It may be noted that the policy rule which maximizes average growth is the same as the policy rule which maximizes average employment. From (24) we have $\text{Mean}(y_t - y_{t-1}) = \delta - \alpha\sigma^2$, while from (20)–(22) we have $\text{Mean}(n_t) = (1/\eta)\log[\alpha^2\Gamma/\eta\Lambda(1-\omega)] - \sigma^2$. Both of these expressions are maximized by setting $\rho_\phi = -1$, $\rho_\lambda = -1/\eta$ and $\rho_\psi = 0$, implying $\sigma^2 = 0$.

values for the feedback parameters which maximize growth: that is, $\rho_\phi = -1$, $\rho_\lambda = -1/\eta$ and $\rho_\psi = 0$. This implication of our analysis may be viewed within the context of the broader literature on the welfare costs of economic fluctuations. The seminal contribution is that of Lucas (1987) whose calculations based on the neo-classical growth model suggested that the welfare gains from eliminating business cycles are negligible compared to the welfare gains from maximizing growth. Significantly, this result has proved to be robust to a number of extensions, such as changes to the stochastic processes governing shocks (e.g., Obstfeld, 1994), generalizations of preferences and utility functions (e.g., Obstfeld, 1994; Pemberton, 1996; Dolmas, 1998; Otrok, 2001) and departures from a world of complete and perfectly functioning markets (e.g., Imrohoroglu, 1989; Atkeson and Phelan, 1994; Krusell and Smith, 1999; Beaudry and Pages, 2001; Storesletten *et al.*, 2001).[16] Intuitively, since agents can insure themselves (at least partially) against fluctuations in income, and since it takes only small changes in the growth rate to produce substantial cumulative changes in output, then any benefits that might accrue from lower volatility are eclipsed by the benefits that arise from higher growth. Given this, then our analysis may be seen as taking the literature a further step forward by showing explicitly how welfare is maximized through a policy designed solely to maximize growth, rather than a policy that is influenced in any way by stabilization objectives. At the same time, our analysis may also be viewed as providing a qualification to the established wisdom. Since the growth rate in our model is non-invariant with respect to changes in volatility, then business fluctuations may well have significant effects on welfare. Some recent calculations to support this are presented by Barlevy (2002) who obtains estimates of the costs of fluctuations that far exceed those obtained in previous studies based on the traditional dichotomy between growth and business cycles. Naturally, careful interpretation is needed here—for the costs of fluctuations arise not because of the effects of volatility *per se*, but rather because of the effects of volatility on growth. In any event, our analysis lends support to the views that, from a pure welfare perspective, it is growth, not volatility, that matters the most, and that it is growth, not volatility, to which macroeconomic policy should be directed. Indeed, our analysis suggests that policy makers may do rather well in relinquishing any concern about volatility and focusing, instead, on a simple growth objective.

5. Conclusions

Two of the most long-standing traditions in macroeconomics are the study of growth and the study of business cycles. Until recently, these traditions have been largely divorced from each other with little cross-fertilization of ideas between

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[16] For these reasons, we believe that our own results concerning welfare would also be robust to various extensions of the model.

them. With the emergence of endogenous growth theory, however, economists have began to question the validity of this dichotomy and there is now a growing body of research that seeks to explore the potential linkages between secular and cyclical activity. The present paper is intended as a further contribution to this new and important area of research.

Unlike most other contributions, our analysis allows a role for nominal factors—nominal shocks and nominal rigidities—in the joint determination of growth and business cycles. Together with an endogenous technology based on learning-by-doing, these factors are responsible for generating a stochastic and sustainable growth rate of output, the mean and variance of which are both dependent on the variances of both real and nominal shocks. In this way, the model is able to account for a negative relationship between long-run growth and short-run volatility in accordance with the findings of several empirical studies.

Another distinguishing feature of our analysis is the attention given to the role of policy—in particular, stabilization policy. Indeed, it is one of only a very few investigations that provide fully worked-out examples of how policies designed to mitigate the impact of exogenous shocks can have consequences for the long-run performance of the economy. The possibility of this adds a new dimension to the design and evaluation of such policies, the most important aspect of which may not be their stabilization qualities *per se*, but rather their potential to influence long-term growth prospects. According to our own investigation, monetary stabilization policy aimed at reducing stochastic fluctuations may work either for or against the promotion of long-run growth depending on the source of the fluctuations. This result is notable in itself and is made more notable by the fact that the optimal policy which maximizes welfare is not the policy which minimizes volatility but the policy which maximizes growth.

Our analysis is intended to be illustrative, being based deliberately on an analytically tractable framework for which closed-form solutions can be obtained from appropriate assumptions about preferences and technologies. The alternative approach would have been to use a more complicated model under more general assumptions and to conduct the analysis via numerical simulations. We have no reason to believe that the basic message of the paper would have been different had we followed this alternative which could have led one to lose sight of the precise objectives of the exercise and the intuition underlying the results. Nevertheless, it would be interesting to acquire an idea of the orders of magnitude of involved, and we intend to pursue this in later work by conducting a quantitative analysis of a more general, calibrated version of the model.

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Appendix

The results in (16), (17), and (18) may be computed as follows. Substitution of (4) into (8) delivers

$$\frac{1}{C_t} = \beta(1 - \alpha)E_t\left(\frac{Y_{t+1}}{C_{t+1}K_{t+1}}\right) \tag{A1}$$

which may be transformed into (14) by exploiting $Y_t = C_t + K_{t+1}$. Repeated substitution in (14) implies

$$\frac{K_{t+1}}{C_t} = (1 - \alpha)^\tau \beta^\tau E_t\left(\frac{K_{t+\tau+1}}{C_{t+\tau}}\right) + \sum_{i=1}^{\tau}[(1 - \alpha)\beta]^i \tag{A2}$$

Imposing the transversality condition $\lim_{\tau \to \infty} \beta^\tau E_t\left(K_{t+\tau+1}/C_{t+\tau}\right) = 0$ yields the solution

$$\frac{K_{t+1}}{C_t} = \frac{\beta(1 - \alpha)}{1 - \beta(1 - \alpha)} \tag{A3}$$

Combining (A3) with $Y_t = C_t + K_{t+1}$ gives the expressions in (16) and (17). Similarly, (11) in conjunction with $H_t = M_t$ implies that (9) may be converted to (15), for which repeated

substitution produces

$$\frac{M_t}{P_t C_t} = \beta^\tau E_t\left(\frac{M_{t+\tau}}{P_{t+\tau}C_{t+\tau}}\right) + \left(\frac{\Theta}{\Gamma}\right)\sum_{i=0}^{\tau-1}\beta^i \tag{A4}$$

Imposing the transversality condition $\lim_{\tau\to\infty}\beta^\tau E_t\left(M_{t+\tau}/P_{t+\tau}C_{t+\tau}\right) = 0$ gives the solution

$$\frac{M_t}{P_t C_t} = \frac{\beta\Theta}{(1-\beta)\Gamma} \tag{A5}$$

Together with (16), (A4) yields the expression in (18).

The results in (19) and (20) may be derived in the following manner. Substitution of (3) and (16) into (10) gives

$$\eta E_{t-1}\left(\Lambda t N_t^\eta\right) = \frac{\alpha^2 T}{1-\omega} \tag{A6}$$

In turn, (3) and (18) may be combined to obtain

$$N_t = \left(\frac{\alpha}{\Omega}\right)\left(\frac{M_t}{W_t}\right) \tag{A7}$$

which implies

$$\eta E_{t-1}\left(\Lambda_t N_t^\eta\right) = \eta\left(\frac{\alpha}{\Omega}\right)^\eta \frac{E_{t-1}(M_t^\eta\Lambda_t)}{W_t^\eta} \tag{A8}$$

Equating (A8) with (A6) yields the expression in (19). The value of expectations in this expression is computed by substituting for Λ_t and M_t using (6), (11), (12), and (13) to obtain $E_{t-1}(M_t^\eta\Lambda_t) = (M_{t-1}F)^\eta \Lambda E_{t-1}[\exp(\eta(\rho_\phi+1)\phi_t + (\eta\rho_\lambda+1)\lambda_t + \eta\rho_\psi\psi_t)]$, and exploiting the fact that $E[\exp(x)] = \exp(\frac{1}{2}\sigma^2)$ if x is a normally distributed random variable with mean zero and variance σ^2. The result in (20) is then derived by combining (19) with (A7) and making similar substitutions to form $M_t = M_{t-1}F\exp(\epsilon_t)$, where ϵ_t is defined in (21), and $[E_{t-1}(M_t^\eta\Lambda_t)]^{1/\eta} = M_{t-1}F\Lambda^{1/\eta}\{E_{t-1}[\exp(\eta(\rho_\phi+1)\phi_t + (\eta\rho_\lambda+1)\lambda_t + \eta\rho_\psi\psi_t)]\}^{1/\eta} = M_{t-1}F\Lambda^{1/\eta}\exp(\sigma^2)$, where σ^2 is defined in (22).

The result in (30) may be established along the following lines. Let $c_t = \log(C_t)$, $m_t = \log(M_t/P_t)$, $n_t = \log(N_t)$ and $k_t = \log(K_t)$. From (5), the unconditional expected value of utility may be written as

$$\begin{aligned}
E(U) &= E\left\{\sum_{t=0}^{\infty}\beta^t\left[\Gamma c_t + \Theta m_t - \Lambda_t N_t^\eta\right]\right\} \\
&= \Gamma\sum_{t=0}^{\infty}\beta^t E(c_t) + \Theta\sum_{t=0}^{\infty}\beta^t E(m_t) + \sum_{t=0}^{\infty}\beta^t E\left(\Lambda_t N_t^\eta\right)
\end{aligned} \tag{A9}$$

By virtue of (16), (18), and (24), $c_t - c_{t-1} = m_t - m_{t-1} = g + \varepsilon_t$. Hence,

$$c_t = c_0 + gt + \sum_{j=1}^{t} \varepsilon_j, \qquad m_t = m_0 + gt + \sum_{j=1}^{t} \varepsilon_j, \qquad t \geqslant 1 \tag{A10}$$

$$c_0 = \log[(1-\omega)\Psi] + \alpha n_0 + k_0, \qquad m_0 = \log(\Omega\Psi) + \alpha n_0 + k_0 \tag{A11}$$

with k_0 given. By virtue of (20) (using $\Lambda_t = \Lambda \exp(\lambda_t)$),

$$\Lambda_t N_t^{\eta} = \left[\frac{\alpha^2 \Gamma}{\eta(1-\omega)} \right] \frac{\exp(\eta \epsilon_t + \lambda_t)}{\exp[\eta \sigma^2]} \tag{A12}$$

$$n_t = \frac{1}{\eta} \log \left[\frac{\alpha^2 \Gamma}{\eta \Lambda (1-\omega)} \right] - \sigma^2 + \epsilon_t \tag{A13}$$

It follows from (A10), (A11), (A12), and (A13) that

$$E(c_t) = E(c_0) + gt \tag{A14}$$

$$E(m_t) = E(m_0) + gt \tag{A15}$$

$$E(c_0) = \log[(1-\omega)\Omega] + \alpha E(n_0) + k_0 \tag{A16}$$

$$E(m_0) = \log(\Omega\Psi) + \alpha E(n_0) + k_0 \tag{A17}$$

$$E\left(\Lambda_t N_t^{\eta}\right) = \frac{\alpha^2 \Gamma}{\eta(1-\omega)} \tag{A18}$$

$$E(n_0) = \frac{1}{\eta} \log \left[\frac{\alpha^2 \Gamma}{\eta \Lambda (1-\omega)} \right] - \sigma^2 \tag{A19}$$

These expressions provide all the necessary information by which to evaluate (A9). After some tedious algebra, the final result is (30).