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The impact of population ageing on household portfolios and asset returns

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10.1

Introduction

The objective of this chapter is to present the evolution of the literature on household portfolios and its research perspectives. In doing this, the viewpoint assumed is not that of an institution such as a pension fund, but that of an individual household decision maker who has to take consumption and portfolio decision over her life[3] In this specific connection, the chapter highlights the modelling requirements necessary to analyse the impact of population ageing on household portfolios, and ultimately on asset returns, with special focus on the financial segment of the portfolios. Recent demographic trends have given impetus to the literature with contributions that are diversified according to the approach (theoretical and/or empirical) and the perspective taken (micro vs. macro).

The possible impacts of ageing on household portfolios rest on the observed heterogeneity in portfolio allocations, which, beside other factors (e.g. income, wealth, education, family size etc.), is determined by the age of the household taking the financial decisions. Such portfolio heterogeneity, which is apparent both in terms of stock market participation and asset allocations, has been

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[3]In other Chapters of this book the institutional viewpoint is taken and asset-liability management models are illustrated: see Chapters 7, 8, 10, 17, 18, 19.

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studied both in empirical literature, providing detailed evidence on the issue, and in theoretical literature, aiming to find an explanation for it.

We believe that a useful approach to the understanding of the whole issue is first to look at the empirical regularities emerging from the data and then to review theories of portfolio choices developed to account for the observed portfolio features. In fact, most recent theoretical models have been progressively extending the seminal Merton-Samuelson model with the aim to explain empirical regularities which are at odds with model predictions as stressed by many (e.g. Curcuru et al. (2009) and Gomes and Michaelides, 2005).

This evolution of the literature suggests the structure of this chapter. In Section 2 we review the empirical studies: we first take a microeconomic perspective and look at the evidence on household portfolio allocations and then briefly recall the literature that takes a macroeconomic perspective and looks at the possible impact of ageing on financial markets and hence on asset returns. In Section 3 we overview theoretical models that provide microeconomic foundation and normative implications for the household portfolio decisions. The last section concludes and points out some open research questions.

10.2

The empirical evidence

Since the mid 1990s, the nexus between age and finance has inspired a lively debate, which has given rise to an increasing number of empirical studies highly diversified in terms of methodology, data and results.

In this section we review this literature distinguishing two streams, microeconomic or macroeconomic. The studies with a microeconomic perspective, i.e. those assessing the link between age and financial portfolio choices[4] are our main focus and are reviewed first and more extensively (Section 2.1). The evidence reported by most of these studies, if considered at an aggregate level, is the foundation for what financial markets fear the most as for ageing population, i.e. the Asset Meltdown Hypothesis (AMH): a larger working-age cohort, such as that of *baby boomers*, first drives up the demand for financial assets (accumulated to finance retirement) thereby exerting an upward pressure on asset prices, and then, upon the arrival of the following smaller cohort of working-aged to sell their assets to, puts downward pressure on asset prices. Hence, in Section 2.2 we will review some of the studies that, taking a macroeconomic perspective,

[4] Attention has also been given to the life-cycle pattern in savings and wealth accumulation, which goes beyond the scope of this chapter. For reviews of this literature see e.g. Bosworth et al. (1991) and Browning and Lusardi (1996).

look at the possible impact of ageing on financial markets and hence on asset returns.

10.2.1

The empirical evidence in a micro-perspective

In this section we discuss some empirical studies aimed at assessing the impact of age on household financial portfolio choices. When estimating the average portfolio allocation as a function of age, an important identification problem is faced, because financial choices are simultaneously affected by three different but related effects, namely age, time and cohort one. The time effect refers to the particular moment in which the decision is taken. As an example, a favorable (negative) period for stock market returns not only increases (decreases) the average financial wealth of households, but may also modify its average allocation. The cohort effect concerns those consequences that the date of birth may have on individual financial choices. As Poterba (2001) puts it:

> *individuals born prior to the Great Depression may have a greater desire to save than those born later, reflecting their greater experience with economic hardship and the loss of financial wealth.*

Finally, the age effect captures the life-cycle effect on financial wealth allocation. The identification problem can be thus summarized as follows: at any time t a person born in year c is a_t years old, where $a_t = t - c$. Being age (a_t), time (t) and cohort effect (c) a linear combination of each other, they cannot be separately identified.[5] The solution is to rule out one of these three effects and try to assess the two remaining, recalling that time-series or single cross-section data allows depicting only one of them at time, while panel data or repeated cross-sections allow the separate estimation of any two out of three of these effects.[6]

We now review the contributions considering the intensive margin of portfolio decisions, i.e. how much of financial wealth to allocate to risky assets (allocation decision), and then those investigating also the extensive margin, i.e. whether to hold or not risky assets at all (participation decision).

[5] For an investigation of this issue, see Ameriks and Zeldes (2004).

[6] In studies investigating the role of age on household portfolio choices (micro level), such as those described in this Section, panel data are collected along time and across different households. While in data used to investigate the effect of the ageing population on financial market (aggregate level, Section 2.2), the additional dimension besides time generally refers to different countries.

All of these contributions are basically framed into the life-cycle theory, stating that the optimal allocation of financial portfolio should vary with age, and share the following features: i) they are quite recent, as the availability of the data on which they are based (mainly survey) has recently increased; ii) to some extent, they all find evidence of a relevant effect of age on household portfolio decisions.

Yoo (1994a) is one among the first investigating the role of age in portfolio selection and uses to this end data from 1962 Survey of the Financial Characteristics of Consumers and from the 1983 and 1986 Surveys of Consumer Finances (SCF). He finds that the percentage of individuals seeking riskier (safe) assets progressively diminishes (increases) along with age and the average portfolio allocation in cash, bonds and stocks is different across the 5 different age classes analyzed (25-34, 35-44, 45-54, 55-64, 65+): the share held in cash diminishes throughout the working life and increases thereafter, while those of bonds and stocks display an inverse pattern, increasing until retirement and decreasing thereafter. To isolate the effect of age from that of other household characteristics, the following tobit regression is estimated.

$$\begin{aligned}\alpha_i = \quad & \beta_0 + \beta_1 Pop_i^{25-34} + \beta_2 Pop_i^{35-44} + \beta_3 Pop_i^{45-54} \\ & + \beta_4 Pop_i^{55-64} + \beta_5 Pop_i^{65+} + \beta_6 Kids_i + \beta_7 Adults_i \\ & + \beta_8 Male + \beta_9 White + \beta_{10} Married_i + \beta_{11} HS_i + \beta_{12} Col_i \\ & + \beta_{13} Y_i + \beta_{14} W_i + \epsilon_i \end{aligned} \tag{10.1}$$

where α_i is the portfolio share held in each of three assets (cash, bonds and equities) and ϵ_i represents the error term. Explanatory variables include dummies for age-class ($Pop_i^{25-34}, \ldots, Pop_i^{65+}$), gender (Male), marriage status (Married), High School (HS) and College (Col) education, as well as the number of children (Kids) and adults (Adult) in the household, the income (Y) and the wealth (W). The results suggest that age is a significant factor in determining portfolio composition and that, consistently with the life-cycle model, the share held in equities increases while working and decreases after retirement.

Bodie and Crane (1997) use a different dataset, namely the 1996 Teachers Insurance and Annuity Association-College Retirement Equities Fund (TIAA-CREF), which allows the analyses to be performed at an individual rather than at a household level. The authors create four net worth and four age categories, for a total of 16 groups, and observe the pattern of four kinds of assets (cash, tax-exempt bonds, taxable bonds, and equity). They find that the fraction of equities varies systematically and inversely with age. This result is further supported by a OLS multiple regression in which the fraction held in equity is regressed on a set of variables, including age, net worth, home ownership, job category, education, gender and marital status. The results suggest that, at a 95% level of significance, for each additional year of age, the share held in risky assets reduces by 0.6%, which is consistent with generally accepted investment principles according to which the equity fraction should decline by 1% for each year of age (see, e.g. Malkiel, 1996).

Tin (1998) also uses US data, taken from the 1991 Survey of Income and Program Participation (SIPP), to investigate the household demand for financial assets in a life-cycle framework. The author estimates via OLS the following regression

$$\log m_i = \beta_0 + \beta_1 \log w_i + \beta_2 \log \pi_i + \sum_{n=1}^{N} \delta_n \log \pi_{ni} + \alpha S_i + \epsilon_i \quad (10.2)$$

where m_i is the quantity of asset demanded by household i, w_i is either labor income, wealth, or net worth of the household, π_i is the opportunity cost of holding the asset, similarly π_{ni} is the price or user cost of the n-th asset other than m_i, S_i is a set of socio-demographic control variables and ϵ is the error term. The regressions are run for several assets (including non-interest and interest-earning checking accounts, money market deposits, certificates of deposits, municipal or corporate bonds, stocks and mutual funds) and for three households categories: young (under 35); middle-age (35 to 59); and elderly (60 and over). As the author puts it:

> *the results show that the propensities to hold financial assets differ substantially among young, middle-age, and old householders . . . The life-cycle hypothesis generally holds as far as the relation between labor income and asset demand is concerned.*

The contributions by James Poterba are among the most influential in the age-finance literature. In Poterba (2001), the author analyses the implications of a simplified OLG model describing the link between demographic variables and capital price. In this framework, he examines the age-profile of corporate stock holdings, net financial assets and net-worth for individuals in different age-classes, as from the US 1995 SCF. It emerges that the asset holdings reach their apogee in the age-classes between 30 and 60 and then slightly decrease, although "there is only a limited downturn in average asset holdings at older ages". Next, the analysis is moved from single to repeated cross-section data (1983, 1986, 1989, 1992 and 1995 SCF) to estimate the following regression

$$y_{it} = \sum_{j=1}^{13} \alpha_j Age_{it} + \sum_{1=1}^{12} \gamma_i Cohort_{ijt} + \epsilon_{it} \quad (10.3)$$

where the dependent variable is the level of either common stocks, net financial assets or net worth held by investor i at time i, Age_{it} is a dummy for j different 5-year age-groups (from 15-19 to 75 and over) and $Cohort_{ijt}$ is specific for 5-year birth-cohorts (from 1971-1975 to before 1925). In this way the author focuses on age and cohort effects and implicitly assumes no time-effect. For the coefficients Poterba (2001) reports that "has a surprisingly small impact on the estimated age structure of asset holdings". In fact, the values for late middle-aged and retired are not that different: around \$32,500 for the former against around \$28,000- 25,000 for the latter. Furthermore, for net financial assets "there is virtually no decline in old age".

By contrast, Bellante and Green (2004) find age to be a significant determinant of portfolio allocation. The authors test the life-cycle risk-aversion hypothesis specifically for elderly. Using a single cross-section dataset on a subset of US households whose one or more members are 70 or over in 1993-1994, the authors estimate several OLS regressions including:

$$\begin{aligned}\alpha_i = \quad & \beta_0 + \beta_1 \ln(NW)_i + \beta_2 [\ln(NW)]^2 + \beta_3 Age_i \ln(W)_i \\ & +\beta_4 Female_i + \beta_5 Male_i + \beta_6 Health_i + \beta_7 Non - White_i \\ & +\beta_8 HS_i + \beta_9 College_i + \beta_{10} Kids_i + \epsilon_i \end{aligned} \tag{10.4}$$

where α is the share of risky assets in the financial portfolio of household i, $\ln(NW)$ the log of net wealth, Age the age of the head-of-the-household minus 65, $Kids$ the number of children, $Female$ and $Male$ are dummies for single-female or single-male households, and $Health$, $Non - White$, HS and $College$ are dummies respectively for poor health status, non-white race and highest education level. Overall, all coefficients but β_5 and β_{10} are significant and display the expected sings. In particular, the age-coefficient is negatively signed and highly significant, suggesting that the share held in risky assets tends to reduce with age, i.e. that ageing is typically associated with a stronger relative risk-aversion.

The studies reported so far focus mainly on the allocation decision. Some studies however distinguish the intensive margin of portfolio decision, i.e. the decision concerning how much of financial (or total) asset to hold in risky assets, from the extensive margin, i.e. the decision to hold or not risky assets at all (participation decision).

One example is Poterba and Samwick (2001) that, focussing on age and cohort effects, investigate the relationship between age and portfolio structure for US households analyzing both the decisions of holding risky assets as well as on the fraction of net worth allocated to risky assets. They use data from the 1983, 1989 and 1992 SCF to estimate the regression

$$y_i = \alpha + \sum \beta_n Age_{in} + \sum \gamma_m Cohort_{im} + \epsilon_{ij} \tag{10.5}$$

where Age_{in} is a dummy variable for whether the household head is in the relevant age class, $Cohort_{im}$ is a dummy variable for the relevant cohort the household head belongs to and y_i denotes the holdings by household $iofasset$j (taxable equity, all equities, tax-exempt bonds, taxable$bonds$, tax-deferred accounts, bank accounts, other financial assets), being either a dummy for whether the household has positive amounts of the asset category (probit estimate) or the share of the household's total financial assets held in each category j (tobit estimate). For directly-held equities, the authors find statistically significant age effects and cohort effects: as for the former, both the probability of holding and the share held in risky assets evolve according to an humped-shaped pattern along the life-cycle; as for the latter, older cohorts seem to be more likely to hold stocks, so that all in all, both the participation and the allocation decision seem to point towards and increasing risk-attitude of elderly with respect to young people.

Ameriks and Zeldes (2004) carry out a further distinction, by analysing separately: i) equity ownership; ii) equity portfolio share; and iii) equity portfolio share conditional of equity ownership. For each of these portfolio decisions, the authors perform an analysis of the age-cohort-time effect identification problem based on US data from SCF (1989, 1992, 1995, and 1998) and TIAA-CREF (1987-1999), by means of a set of regressions (probit for i and OLS for ii and iii) in which either no cohort-effect is assumed, i.e. explanatory variables include only time and age dummies, or no time-effect is assumed, i.e. only age and cohort dummies are included as independent variables. Their result is summarized as follows. Assuming no time-effect, basically all of the three decisions considered display an increasing pattern along with age (whereby the extensive margin decision is increasing only up to 58 and flat after retirement) which is at variance with the typical view of financial advisors. By contrast, if no cohort-effect is assumed, the decision to whether or not to hold risky assets as well as the unconditional equity portfolio shares display humped-shaped patterns along with age, while the age effect seems to disappear only if equity portfolio shares conditional on equity holding are considered. As a result, the evidence in terms of age-effect on portfolio allocation depends on the identifying assumptions. Nevertheless, based on parsimony and plausibility arguments, the authors conclude that time effects should always be considered in this kind of analyses, suggesting therefore that cohort-effect should be excluded instead. The conclusion reached is thus that age strongly affects portfolio decisions at the extensive margin but, once the decision to hold risky assets has been taken, age does not really matter in determining how much of financial wealth to allocate to these assets.

The analysis by Coile and Milligan (2006) further contributes to the investigation of US household average portfolios by specifically focussing on its evolution after retirement, once again distinguishing between participation and allocation decision. Five asset categories are considered: (i) principal residence; (ii) vehicles; (iii) financial risky assets, including bonds, Individual Retirement Accounts (IRAs) and stocks; (iv) bank accounts; and (v) business and other real estate. The authors use six waves of the US Health and Retirement Study (HRS) spanning over the period 1992-2002 and estimate the panel regression

$$\alpha_{it} = \beta_0 + \beta_i Age_{it} + \beta X_{it} + \gamma_t + \epsilon_{it} \tag{10.6}$$

where the dependent variable is either a dummy for the household holding the asset class (participation decision), the share of total assets held in each asset-class or the dollar amount held in each asset-category (allocation decision). Explanatory variables include the wave dummies (γ_t), the age of the older member of the household (Age_{it}) and a set of control variables (X_{it}), including marital status, region of residence, religion, race, being US born, and educational category. The age-coefficients are overall significant and negatively signed for all assets but bank accounts: i.e. older households have generally lower ownership probability and hold lower shares/dollar amounts of all assets but bank accounts. Coile and Milligan (2006) note that

> *bank accounts are dominated by other assets on a risk-return basis, yet there is an increasing proportion of household assets devoted to them with age. ... It may be a transitory result as windfalls of insurance money, pension lump sums, or proceeds from the sales of housing pass through bank accounts on their way to other asset classes. Alternatively, it may be that the complexity of financial arrangements leads seniors, particularly those with diminished mental or physical capacity, to select portfolios that are easier to manage.*

Based on this evidence, the authors conclude in favor of sizeable effects of age on asset allocation over the long-term.

The impact of age on household portfolio choices has been recently studied also for countries other than the US.

Guiso et al. (2002) collect a number of relevant papers referred to several countries, namely Alessie et al. (2002) for the Netherlands, Banks and Tanner (2002) for UK, Bertaut and Starr-McCluer (2002) for US, Eymann and Börsch-Supan (2002) for Germany and Guiso and Jappelli (2002) for the Italian case. All empirical studies basically follow the same methodology. First, financial assets are grouped into three broad risk-categories: "safe" (e.g. bank accounts), "fairly safe" (e.g. government bonds) and "risky" (e.g. stocks), whereby the definitions may slightly change from country to country. Then, for both extensive and intensive margin portfolio decisions, the authors carry out both explorative and econometric analyses. As for the former, the authors examine the average household portfolios across different age-classes. Except for the Dutch and the German cases, a clear humped-shaped age-profile is observed, at least with regards to the participation decision. For instance, for the US case Bertaut and Starr-McCluer (2002) report that the average share invested in risky assets peaks for households aged between 45 and 64 while declines moving towards younger or older investors. Similarly, Guiso and Jappelli (2002) observe that the share of Italian households investing in risky assets increases from around 15% of those 30 years of age or lower to almost 20% of the middle-aged and then falls once again to around 10% for the 60-69 and to less than 7% for the over 70. By contrast, Eymann and Börsch-Supan (2002) for Germany and Alessie et al. (2002) for the Netherlands report that elderly seem more willing to hold risky assets. For the econometric analyses, national survey data are used to estimate either cross-sectional and/or panel regressions in which the dependent variable is either a binary variable (participation decision) and/or the share of financial wealth invested in some assets (allocation decision). Explicative variables include all other household features which could play a role in shaping portfolio choices, including age, net wealth, income, gender and level of education, although the exact specification varies across countries. For instance, Guiso and Jappelli (2002) for the Italian case estimate (with both cross-section and panel data)

$$\begin{aligned}\alpha_i = \quad & \beta_0 + \beta_0 A + \beta_2 \tfrac{A^2}{1000} + \beta_3 Y + \beta_4 \tfrac{Y^2}{1000} + \beta_5 W + \beta_6 \tfrac{W^2}{1000} \\ & +\beta_7 Size + \beta_8 Kids + \beta_9 Married + \beta_{10} Male \\ & +\beta_{11} South + \beta_{12} Edu + \beta_{13} u + \beta_{14} Bank + T + \epsilon \end{aligned} \tag{10.7}$$

where the dependent variable is in turn the dummy variable or the share invested in risky assets and explanatory variables include the age of the family head (A), household income (Y) and wealth (W), both in linear and quadratic terms, the family size ($Size$) and the number of children ($Kids$), dummy variables for the marital status ($Married$), gender ($Male$), geographic zone of residence ($South$), level of education of the family head (Edu), as well as the average unemployment rate (u) and the index of bank diffusion ($Bank$) in the province of residence together with year dummies (T). Bertaut and Starr-McCluer (2002) for the US case estimate

$$\begin{aligned}\alpha_i = \quad & \beta_0 + \beta_1 A_1^{<35} + \beta_2 A_i^{55-64} + \beta_3 A_i^{65+} + \beta_4 \ln(Y_i) + \beta_6 NonWhite_i \\ & +\beta_7 HS_i + \beta_8 Col_i + \beta_9 Married + \beta_{10} Female \\ & +\beta_{11} Self + \beta_{12} DBPens_i + \beta_{13} RET_i + \beta_{14} u_i + T + \epsilon \end{aligned} \tag{10.8}$$

where the dependent variable is again either a dummy for the participation decision or the share invested in risky assets for the allocation one and explanatory variables include dummies variables for age-class ($A_1^{<35}$, A_i^{55-64}, A_i^{65+}), log income ($\ln(Y)$) and wealth ($\ln(W)$), and dummies for being non-white ($NonWhite$), for the highest level of education (HS for High School and Col for College), for the marital status and the gender ($Married$ and $Female$), for being self-employed ($Self$), retired (RET) or owner of a defined-benefit pension fund ($DBPens$), as well as the average unemployment rate (u) in the state of residence. In all studies but the Dutch, the results confirm that age plays a significant role in determining whether or not to hold risky assets, although once this decision is taken this factor only slightly affects the final portfolio allocation. For instance, Guiso and Jappelli (2002) in the probit regression for participation report that age coefficients are statistically different from zero and quite high in magnitude (probability of holding risky assets increasing by 4% between age 25 and 40 and declining by 8% between age 40 and 70) while in the allocation-decision regression, despite being still significant, they are sensibly smaller in magnitude. Banks and Tanner (2002) report similar results for UK since the probability of holding risky-asset progressively increases with age-classes up to 60-69 age-class and then falls.

Cerny et al. (2005) structure an OLG model in which rational and forward-looking households optimize the allocation between risky and safe financial assets and housing. After having calibrated the model on the UK economy, the authors perform simulations under different scenarios and find in all cases that the optimal portfolio composition substantially varies with age: portfolios of older households should generally show decreasing portion of real assets and increasing importance of financial ones, whereby within the latter safe assets should tend to progressively increase with respect to risky ones along with the age of the household.

Brunetti and Torricelli (2007) examine the Italian household portfolios using data taken from the Bank of Italy Survey of Household Income and Wealth (SHIW) for 1995 - 2004. The authors first group financial assets into comparable credit and market risk-categories and then examine the average portfolio by dividing households first by age-classes, in order to depict a possible age-effect on allocation choices, and then by both age-classes and NW quartiles, in order to test whether the age-effect persists even under different economic conditions. In addition, the 5% richest households are separately studied. The results show that, despite the several changes occurred over the decade 1995-2004, the average Italian household portfolios are allocated consistently with the life-cycle theory: middle-aged hold riskier portfolios, while older ones tend to disinvest risky financial instruments and turn to safer assets. As the age-effect is depicted also across all NW quartiles, with the sole exception of 5% richest households, the authors conclude that the age-effect on financial choices is robust to both economic conditions and to the market changes occurred during the decade under analysis.

Frijns et al. (2008) introduce a model that links various behavioral finance concepts to individual investor's portfolio choice and collect survey data (on university students and employees) to test inter alia how socio-demographic factors such as age and gender affect portfolio choices. Both market conditions (e.g. risk-return trade-off) and individual factors play a significant role in portfolio choices, whereby specifically for the latter they find evidence that older investors tend to invest more in risky assets.

To sum up, with very few exceptions (e.g. Poterba, 2001), age is found to be among the most relevant determinants of portfolio decisions, although the shape of portfolio decision along life-cycle arising by data might slightly differ (i.e. humped-shaped rather than linear) depending on the decision on which the study is focused (participation vs allocation decision) as well as the dataset used.

10.2.2

The empirical evidence in a macro-perspective

Provided that age affects portfolio choices, which is what many studies reviewed in the previous section suggest, a change in the age-structure of the population might translate into different aggregate demand for certain financial assets with direct consequences on their prices and returns. This fear, which is referred to as the Asset Meltdown Hypothesis (AMH), implies that financial markets might be significantly affected by the ongoing population "greying".

From an empirical point of view, this issue has been mainly addressed by means of regression analyses aimed at assessing the degree of correlation between some measures of financial asset prices or returns and a set of demographic measures. Financial variables however are generally quite volatile, while demographic ones are typically slow-moving: as a result, the statistical power of the analyses is reduced and the observations required for reliable estimates need to be increased.[7)] Some contributions face this problem by using time-series as longest as possible (accordingly to data availability). Others instead turn for their analyses to panel datasets, by pooling observations from different countries.

The studies are highly diversified not only in terms of datasets, and hence estimation techniques used, but also in terms of model specifications: the dependent variable used might be either asset prices, asset returns or equity risk premium; the explanatory variables might include only demographic variables or both demographic and economic and financial measures; finally, the demographic measures chosen can also differ, with some studies employing the average age of working population or old-dependency ratios, while others preferring the proportion of different age-classes over the total population. The empirical evidence reported is not conclusive: some contributions, which will be reviewed at first, find only weak, if any, relationship between population ageing and financial asset returns, while others report evidence which substantiates the concerns for the AMH.

James Poterba has extensively investigated the link between population ageing and financial asset returns. In Poterba (2001), the author examines the age-wealth profiles in US based on several waves of SCF (1983, 1986, 1989, 1992

[7)] For the econometric problems entailed by low frequency financial data see, e.g., Campbell et al. (1997).

and 1995) merging them with the projected evolution of US population over the period 2000-2050 to forecast the future aggregate asset demand of financial assets. He concludes that "the aging of the baby boom cohort does not result in a significant decline in asset demand" thereby disproving the AMH. Poterba (2001) also examines the historical relationship between population age-structure and asset prices and returns, by estimating the regression

$$r_t = \beta_0 + \beta_1 Demo_t + \epsilon_t \tag{10.9}$$

where the dependent variable is the real return on either T-Bills, long-term Government Bonds or Stocks and $Demo$ represents several demographic measures, namely median age, average age of adult population (i.e. aged 20 or over) and the ratios of aged between 40 and 64 over total, adult and retired (i.e. aged 65 or over). The regressions are estimated using data for three countries and spanning over different time periods: US (full sample, 1926-1999, and post-war period, 1947-1999), Canada (1961-1997 for stocks and 1950-1997 for fixed-income instruments) and UK (1961-1996 for equities and 1950-1996 for fixed-income assets). For US, the author finds weak evidence of a link between asset returns and demographic structure, whereby the best results are obtained for fixed-income markets and using the ratio of aged 40-64 over the total population as demographic measure. The estimated coefficients suggest that "an increase in the fraction of the population in the key asset-accumulating years . . . lowers observed returns", although with implausibly large effects, so that omitted-variable problems are insinuated. For Canada, the coefficients are statistically significant only for fixed-income assets and point towards a positive relation between real returns and middle-aged. By contrast, the results for UK show negatively signed and generally non-significant coefficients, which "further weaken the claim that demographic structure and asset returns exhibit systematic linkages". For the US, Poterba (2001) also studies the relationship between demographic variables (Demo) and stock prices normalized by corporate dividend (P/D), by estimating the following regression both in levels and in first differences

$$\left(\frac{P}{D}\right)_t = \beta_0 + \beta_1 Demo_t + \epsilon_t \tag{10.10}$$

In levels, several demographic variables are significant, although the possibility of "spurious regression" cannot be excluded. By contrast, the analysis in first differences originates statistically significant coefficients in only two out of five cases.

Poterba (2004) presents new findings on the historical correlation between population age structure and asset prices and returns, by estimating different specifications of (10) using US data spanning over the period 1926-2003. As in the previous study, the results are provided both over the whole sample period and over the post-war period (i.e. 1947- 2003), but the demographic measures considered are: (i) the share of the total population between ages 40 and 64; (ii) the share of the total population over age 65; (iii) the share of the adult population between the ages of 40 and 64; and (iv) the share of the adult population over the age of 65. Most of the estimated coefficients turn out to be non-significant, providing evidence against the relationship between asset returns and population age structure, at least over the last seventy-year period. However, results are slightly different when asset prices rather than asset returns are used as dependent variable, i.e. when the estimated model is

$$\left(\frac{P}{D}\right)_t = \beta_0 + \beta_1 Demo_t + \alpha Z_t + \epsilon_t \qquad (10.11)$$

where Z_t represents a set of additional variables included as control variables, i.e. the real interest rate and the economic growth rates. The coefficients of demographic variables are generally found significant and correctly signed, but when the same regression is run in first differences the coefficients become non-significant again, casting serious doubts on the robustness of previous results.

Bellante and Green (2004) report a significant effect of age on portfolio decisions (see previous Section), as for AMH they agree with Poterba (2001, 2004) and are skeptical about the eventual consequences that might occur at an aggregate level. In their words

> *the concern for a securities market 'meltdown' may be grossly exaggerated, since as retirees age, they do not seem greatly inclined to sell off their risky assets*
> $[and]$
> *concerns for a dramatic shift in the market returns to risky assets may be unwarranted.*

Basically the same conclusion is reached in GAO (2006), in which the possible impact of ageing on financial market prices is examined by means of a twofold analysis. First, the plausibility of dramatic dis-savings by retirees is checked, based on data from both SCF (over the period 1992-2004) and HRS (1994-2004). Inter alia, the explorative analyses show that retirees does not really liquidate their assets and, if any, asset dissaving occurs quite gradually due to bequest motive and the need to hedge longevity risk. Furthermore, almost two thirds of the baby-boomers' financial assets are concentrated in the portfolios of the 10% richest households, which are traditionally less sensitive to age-affect on asset allocations (see e.g. Guiso et al. (2002) or Brunetti and Torricelli, 2007). Based on this evidence the authors conclude that the AMH is not likely valid. Then, the historical link between financial and demographic dynamics is further tested by estimating the regression

$$r_t = \beta_0 + \beta_1 DY_{t-1} + \beta_2 TS_{t-1} + \beta_3 DefSpread_t + \beta_4 \Delta IP_{t+1} + \beta_5 Demo + \epsilon_t \qquad (10.12)$$

where r_t is the real annual return on stocks while explanatory variables include: the dividend yield (DY_{t-1}), the term spread (TS_{t-1}), the default spread shock ($DefSpread_t$), the change in industrial production (ΔIP_{t+1}) and a demographic variable ($Demo$) defined either as the proportion of the population age

40-64 or as the "middle-aged-to-young' or 'MY' ratio, i.e. the ratio of aged 40-49 to those aged 20-29. The demographic coefficients are overall positively signed and statistically significant: yet, non-demographic variables can explain more variation in historical stock returns with respect to demographic ones, leading the authors to conclude that baby boomers' retirement is unlikely to have such a dramatic impact on financial asset prices.

Basically the same conclusion is reached by Börsch-Supan (2004), Oliveira Martins et al. (2005) and Saarenheimo (2005), who simulate the long-run effects of population ageing by means of simulations on OLG models. Despite different calibrations and different scenarios are considered, all contributions provide little support for the AMH.

If one strand of the literature provides reassuring evidence against the concrete realization of the AMH, other contributions, some of which are reviewed hereafter, find dissimilar results, pointing towards substantial and significant effects of population ageing on financial markets.

Yoo (1994b) empirically tests the relationship between the population age-structure and asset returns in US over the period 1926-1988. His results suggest that a 1% increase in the relative size of the 45-year-old group can reduce the asset returns up to around 2%. The robustness of the result is further tested by estimating

$$\begin{aligned} R_t = \quad & \beta_0 + \beta_1 Pop_t^{25-34} + \beta_2 Pop_t^{35-44} + \beta_3 Pop_t^{45-54} \\ & + \beta_4 Pop_t^{55-64} + \beta_5 Pop_t^{65+} + \epsilon_t \end{aligned} \tag{10.13}$$

where the dependent variable (R_t) is the total return on either equities, bonds, and T-bills, while the explanatory ones represent the share of US population in several age-classes. Once again, the evidence reported is of a statistically significant relationship between age distribution and the returns for several type of financial assets, especially in the post-war era.

Bakshi and Chen (1994) also perform several analyses on US data, referring to the period 1900-1990 and preferring the average age of adult population (aged 20 or over) as demographic measure. The rationale is twofold: first, people younger than 20 generally do not play a determinant role in economic decision making; second, in the authors words

> *the fraction of persons 65 and older can increase, but this does not necessarily mean that population is ageing, because the fraction of young persons may increase at the same time*

so that they believe the average age measures more correctly the real ageing process. The authors observe a strong co-evolution of the demographic variable with stocks markets prices, especially between 1946-1966 and 1981-1990. This result is further confirmed by a forecasting exercise aimed to assess whether and to what extent demographic variables might predict future risk premiums (RP_{t+1}), i.e.

$$RP_{t+1} = \beta_0 + \beta_1 \Delta \bar{A}_t^{20+} + \beta_2 \Delta C_t + \beta_3 DivY_t + \beta_4 Term + \beta_5 + \epsilon_{t+1} \quad (10.14)$$

where explanatory variables include the percentage change in average age and in real per capita consumption, as well as the dividend yield on S&P500 and the term premium. Several specifications of (14) are estimated by means of OLS and results over the post-war period imply that a progressively older population might substantially affect capital markets by increasing the equity risk-premium.

Erb et al. (1997) utilize panel data to empirically assess the relationship between real equity returns and average age. They use data spanning over the period 1970-1995 and referring to 18 countries.[8)] First time-series and cross-section regressions are run. As for the former, the authors report a positive relationship between the proportion of middle-aged and equity returns. As for the latter, a positive relationship is detected between real equity returns and demographics across all countries. Then, data are pooled and the same forecast exercise as in Bakshi and Chen (1994) is implemented, i.e.

$$r_{it+k} = \beta_0 + \beta_1 \Delta \bar{A}_{it} + \beta_2 LE_{it} + \beta_3 \Delta Pop_{it} + \epsilon_{it} \quad (10.15)$$

Where the expected rate of return over the next K periods is regressed on three demographic measures: change in average age, current life expectancy and current population growth. Either considering one or five-year horizons, only $\bar{A}_{it}$ displays significant (and positive) coefficients. The authors find consistent results including in the sample 27 additional emerging countries[9)] and provide further evidence of the forecast power of demographic variables on long-run expected returns.

Brooks (2000, 2002) are among the most representative simulation-based studies aimed to assess the implications of ageing on financial market. Both papers are based on a closed-economy OLG model with rational and forward-looking agents but they substantially differ for the demographic changes simulation. Brooks (2000) explores the effects that ageing might have on the rates of return of both safe and risky assets. He assumes no bequest motive and studies the effects of two demographic shocks, namely a baby-boom and a following baby-bust. The results point towards significant impact of these demographic changes,

8) Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, Netherlands, Norway, Singapore, Spain, Sweden, Switzerland, UK and US.

9) Argentina, Brazil, Chile, China, Colombia, Finland, Greece, Hungary, India, Indonesia, Ireland, Jordan, Malaysia, Mexico, New Zealand, Nigeria, Pakistan, The Philippines, Poland, Portugal, South Africa, South Korea, Sri Lanka, Thailand, Turkey, Venezuela and Zimbabwe.

whereby the baby-boom exerts a downward pressure on asset prices while the opposite occurs during the baby-bust. Accordingly, the equity premium first decreases and then increases, making the author conclude that, even accounting for rationality and forward-looking behavior, demographic changes may affect financial markets. The same experiment is repeated in Brooks (2002), using US real demographic data are used (period 1870-2020), so that the following demographic changes are considered: (i) the pre-war baby bust in the 1940s; (ii) the post-war baby boom in the 1960s; and (iii) the post-war baby bust in the 1980s. in fact, the rate of return changes in response to each of these demographic changes. More specifically, the first baby bust leads to a reduction of both risk-free and risky rates of return, while during the baby-boom risk-free returns increase as well as risky assets return, although the latter are projected to increase less than the former, so that the baby boom translates into a lower equity risk premium. Hence, results are consistent with Brooks (2000) and point towards relevant repercussions of demographic dynamics on financial markets.

Davis and Li (2003) use 1950-1999 data for 7 OECD countries (US, UK, Japan, Germany, France, Italy and Spain) to empirically test the relationship between demographics and equities, which is at first investigated by means of the panel regression

$$\begin{aligned}\Delta\ln(P^e)_{it} = \quad & \beta_0 + \beta_1 Pop^{20-39}_{it} + \beta_2 Pop^{40-64}_{it} + \beta_3 \Delta GDP^{HP}_{it} \\ & + \beta_4 \Delta(GDP - GDP^{HP})_{it} + \beta_5 LR_{it} + \beta_6 Vol(P^e)_{it} \\ & + \beta_7 DY_{it-1} + \epsilon_{it}\end{aligned} \tag{10.16}$$

where the dependent variable is the log difference of real equity prices. Explanatory variables include the shares of population aged 20-39 and 40-64, the trend GDP growth rate, the output gap, the long-term real interest rate, the average equity prices volatility and the lagged dividend yield. In all the specifications tested, both demographic variables are strongly significant and positively signed, although the coefficient for is lower in magnitude with respect to . Cross-country data are then aggregated (using annual GDP as weights) and a time-series regression is run (which differs from (16) only for the absence of country-subscripts i). Here quite different results are obtained since most non-demographic variables are not significant and among demographic ones only has a (positive) significant coefficient. Yet, non-demographic and demographic variables together can explain overall up to 30-50% of the real equity prices variation, which is much more than what other studies report (e.g. Yoo (1994b) reports a maximum of 15%). Davis and Li (2003) also study the link between demographics and long-term government bond yields, i.e.

$$LR_{it} = \quad \beta_0 + \beta_1 Pop^{20-39}_{it} + \beta_2 Pop^{40-64}_{it} + \beta_3 \Delta SR_{it} \tag{10.17}$$

$$+\beta_4(LR - SR)_{it-1} + \beta_5 \Delta \ln(CPI)_{it-1} + \beta_6 \Delta\Delta \ln(CPI)_{it}$$
$$+\beta_7 \Delta GDP_{it}^{HP} + \beta_8 \Delta(GDP - GDP^{HP})_{it} + \epsilon_{it}$$

where the dependent variable is the bond yield and explanatory variables include, besides the ones defined above, the first difference of the short rate, the lag of the term structure differential and both lag and acceleration of inflation. Once again the authors find that both demographic variables are significant and results suggest a positive (negative) relation between young (middle-aged) generation size and bond yields. Davis and Li (2003) thus conclude that demographic changes (especially those concerning the most financially active part of the population, i.e. aged 40-64) can have a significant impact on both stock prices and bond yields, even in presence of additional non-demographic explanatory variables.

As Yoo (1994) and Poterba (2001, 2004), Goyal (2004) studies the link between age-structure and stock market returns in an OLG framework and empirically test its implications by means of econometric techniques. US data spanning over almost all twentieth century are used to estimate, both in levels and in first differences

$$RP_{t+1} = \beta_0 + \beta_1 \bar{A}_t^{25+} + \beta_2 Pop_{it}^{25-44} + \beta_3 Pop_{it}^{45-64} + \beta_4 Pop_{it}^{65+} + \beta_5 DY_t + \beta_6 Flows_t + \epsilon_t \quad (10.18)$$

where RP_{t+1} is the next-year excess stock returns and explanatory variables include several current demographic variables, namely the average age of the population above 25 and the proportions of people aged between 25-44, 45-64, and 65+ as well as two control variables, i.e. the dividend yield and net outflows from the stock market. In both levels and first differences regressions, estimated coefficients are significantly different from zero and point towards a positive (negative) relationship between the proportion of middle-aged (retired) and stock prices. Overall demographic variables show a strong explanatory power, particularly if used in first differences. Next, the analysis is extended to a multi-year forecast framework by estimating

$$\sum_{i=1}^{K} RP_{t+1} = \beta_0 + \beta_1 \Delta Pop_{it}^{25-44} + \beta_2 \Delta Pop_{it}^{45-64} + \beta_3 Pop_{it}^{65+} + \beta_4 DY_t t, K + \epsilon_{t+K} \quad (10.19)$$

where the dependent variable is the sum of next K-periods excess returns while explanatory variables include the dividend yield and the projected percentage change in the proportions of population of aged 25-44, 45-64 and 65 or over between now and K-periods ahead. Results are presented for three- and five-year horizons. In both cases the demographic variables are jointly highly significant and their signs suggest an inverse (direct) relation between the proportion of middle-aged (retired) and stock returns.

Most recent works agree on the fact that the evidence of a relationship between ageing population and financial markets might be substantially different across countries.

Geanakoplos et al. (2004) develops two versions of a closed-economy OLG model: one "stochastic", including business cycle shocks, and one "deterministic", in which the size of the generations, dividends and wages are set in accordance with historical data for the US. Assuming either myopic or forward-looking agents, the model predicts that stock prices vary in response to the ratio of middle-aged to young people. The issue is further investigated by linear regression analyses. First, the regression is run

$$y_t = \beta_0 + \beta_1 \Delta \left(\frac{N_{MA}}{N_Y} \right)_t + \epsilon_t \tag{10.20}$$

wherey_t is either the S&P500 rate of return or the real short-term interest rate and the independent variables is the ratio between the number of middle-aged (N_{MA}) and the young people (N_Y). While the link with the short-term interest rate seems weak, the ratio of middle-aged to young people can explain up to 14% of the variability of stock market returns. Consistent results are obtained regressing the price-earnings ratio of the S&P500 firms on the ratio of middle-aged to young people, since a positive and highly significant coefficient is found. Some international evidence is also provided, using data for Germany, France, Japan and UK spanning over the period 1950-2001, by estimating:

$$P_t = \beta_0 + \beta_1 Demo_t + \epsilon_t \tag{10.21}$$

where P_t is the real stock price index and $Demo_t$ is either (N_{MA} / N_Y) or the size of the 35-59 cohort of the country under analysis. They find mixed results: no relationship at all in the UK, a weak relationship in Germany, a relatively significant relationship in France and quite strong relationship in Japan, which thus results one of the countries for which the demographic changes might more soundly affect financial markets.

Consistent results are found in Ang and Maddaloni (2005) , where the predictive power of demographic changes on future equity risk premium is examined using use two distinct datasets. The first spans over the whole twentieth century and refers to France, Germany, Japan, US and UK. The second includes monthly data over a smaller sample period (1970-2000) for 15 developed countries.[10] For the econometric analysis, the authors first study the predictability of the excess return k-periods ahead () over three different forecast horizons (k = 1, 2 and 5 years) estimating the following regression with GMM for each country separately

$$RP_{t+K} = \beta_0 + \beta Z_t + \epsilon_{t+K} \tag{10.22}$$

[10] Australia, Austria, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, UK and US.

where Z_t represents the set of explanatory variables including both demographic and control variables. For the latter, the authors choose delayed consumption growth and term spread since both are recognized predictors of equity returns and have data available over quite long sample periods. For the demographic measures, the authors use the average age of the adult population, the proportion of adults over 65 and the proportion of population in working age, i.e. aged between 20 and 40, all used in first differences rather than in levels. The results are sensibly different by country. The US has only a weak positive relationship between demographics and excess returns. The same holds for UK, for which the only demographic measure robust to control variables across all forecast horizons is Pop^{65+}. In France and Germany all demographic variables are significant, although only at the 10% level and over short time-horizons, displaying in addition negative signs in contrast to US and UK cases. Finally, in Japan all demographic variables show strongly significant and negative coefficients, at least at 1-year horizon. Next, the authors pool the data and estimate (22) across all 5 countries simultaneously. The results for the panel analysis confirm that Pop^{65+} is the only demographic variable that maintains significant predictive power across all forecast horizons, also when control variables are included. Finally, the authors extend the analyses to the 15 countries and re-estimate (22) using the monthly dataset. Also in this case results confirm the significant and negative correlation between Pop^{65+} and excess returns across all forecast horizons. Ang and Maddaloni (2005) thus conclude that, despite some cross-countries differences[11)], demographics play an important role in predicting excess returns.

Country differences also matter in Brooks (2006), where the author constructs a long-run (1900-2005) panel dataset referred to 16 developed countries[12)] and estimates the linear regression

$$y_{it} = \lambda_i + \beta_t + \gamma_1 Z_{1it} + \gamma_2 Z_{2it} + \gamma_3 Z_{3it} + \epsilon_{it} \tag{10.23}$$

with

$$Z_{Nit} = \sum_{j=1}^{J} j^N p_{ijt} - \frac{1}{J}\sum_{j=1}^{J} j^N, \quad N = 1, 2, 3$$

where y_{it} is either the price or the total return on stocks, stock indices, bonds and T-Bills, while the explanatory variables include two dummies, λ_i for the country and β_t for the year, and indirectly $p_{1it}, p_{2it}, \ldots, p_{Jit}$, i.e. the shares over the entire population of people in the J age group. The econometric specification proposed by Brooks (2006), as in Yoo (1994a), includes the entire

11) In particular, the authors observe more significant and higher coefficients for countries with more generous social security systems and less developed financial markets.

12) Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland, UK and US.

age distribution into the regression, thereby avoiding an arbitrary partition of the population. Nonetheless, he demographic variables are not explicitly included into the model, so that the coefficients $\gamma_1, \gamma_2, \gamma_3$ are not directly interpretable: yet, the implicit age-coefficients, capturing the sensitivity of asset price and returns to age distribution, can easily be recovered. The results are presented for both financial asset prices and returns. Indeed different pictures arise: with asset prices, a significant effect of age is reported whereby the older the population, the lower the price of stock relative to T-Bill. On the other hand, using real financial asset returns only little evidence of a link between demographics and financial markets is found and only with regards to fixed-income market, consistently with Poterba (2001, 2004) and opposite to Davis and Li (2003) and Brunetti and Torricelli (2008), as described in what follows. Furthermore, when (23) is estimated allowing the coefficients $\gamma_1, \gamma_2, \gamma_3$ to vary across countries, the results point towards a substantial heterogeneity across countries. In particular, Brooks (2006) observes that "English-speaking economies . . . do not conform well the life-cycle hypothesis", thereby confirming the little evidence of decumulation during retirement already reported by other studies (e.g. Poterba (2001) and GAO, 2006). By contrast, countries such as Italy, Finland, Sweden, Norway and Japan exhibit the more familiar life-cycle pattern, which the author brings back to the limited household participation to the equity markets that characterizes these countries. Hence, also in this case, the conclusion drawn sensibly depends on the country considered.

The evidence reported in Brunetti and Torricelli (2008) implicitly supports the same argument. In this contribution, the empirical connection between financial asset returns and population ageing dynamics is investigated using annual data over 1958-2004 referred to Italy, taken as a case study based on its pronounced ageing, which is much steeper than the US one. The authors first follow Poterba (2001, 2004) and estimate regressions such as

$$r_t = \alpha + \beta D_t + \epsilon_t \tag{10.24}$$

where r_t is the real return on either stocks, long-term government bonds or short-term government bonds (*Buoni Ordinari del Tesoro*, BOT), $\mathbf{D_t}$ is the vector of demographic variables, basically represented by the shares of different age-classes (20-40, 40-64, over 65) over total or adult population. Consistently with Poterba (2004), the results are not robust across the variants examined, leading to the conclusion that demographic variables alone cannot satisfactorily explain the dynamics of financial asset returns. Next, observing that the purely demographic specification might be affected by omitted-variable problems the authors follow Davis and Li (2003) and estimate

$$r_t = \alpha + \beta D_t + \gamma F_t + \epsilon_t \tag{10.25}$$

where $\mathbf{F_t}$represents a vector of financial variables, differently defined depending on the dependent variable (e.g. dividend-yield, stock prices volatility for stocks, and term spread, inflation, GDP grow rate and output gap for bonds). From the extended specification a different picture emerges: the results are largely consistent across variants and point towards a significant role of demographic variables in affecting financial returns, especially on stock market for which it appears that a higher share of early (late) working-aged is associated with an increase (decrease) of real returns on stocks. Hence, based on a comparative analysis of the evidence obtained for Italy and that reported by Poterba (2001, 2004) and Davis and Li (2003) for US, the authors notice that the evidence for Italy, experiencing a stronger population dynamics, is more clear-cut providing therefore evidence in support the importance of the country-specific age-dynamics in explaining the impact of demographics on financial markets.

10.3

Models for portfolio choices and life-cycle asset allocations

Portfolio selection models for the household are based on classical portfolio theory and have closely followed its evolution which, as for the application to the household, has also been fostered by the empirical evidence surveyed in Section 2.1. The present Section cannot - and by no means aspire to - provide an exhaustive survey of a theory which is vast, manifold and still growing. Rather, by concentrating on models that analyse both consumption/saving and asset allocation decisions, the purpose here is twofold:

i. to recall the evolution of the household portfolio theory from the early models to the more realistic ones highlighting how the basic assumptions have been progressively, and often alternatively, released – in order to capture empirical facts and life-time effects (see Sections 3.1 and 3.2);

ii. to focus on life-time asset allocation models that account for the effect of age and attain consistency with the empirical findings; in this connection preference is accorded to models which rest on assumptions that better depict two relevant sources of risk for households: labour income and the length of life. In order to better illustrate the state of the art, two main models will be taken as representative and illustrated in more detail (see Sections 3.3 and 3.4).

In other words, preference is here given to models where the causes of heterogeneity are not so much related to external factors (e.g. returns distribution, tax frictions) as rather to household-specific features (e.g. objectives and/or human capital).

10.3.1

The Seminal Models

The basic model for the optimal portfolio selection is the static one-period model based on the maximization of a concave Von Neumann-Morgenstern utility function of end-of-period wealth. Under specific assumptions on absolute risk tolerance or risk aversion (being one measure the reciprocal of the other) and/or the asset return distribution, this model produces a fundamental result in the theory of finance known as the two-fund separation theorem. The theorem, essentially states that all agents choose a portfolio made of two funds: the risk-free asset and a fund comprising all other risky assets. The Markowitz (1952) mean-variance formulation is by far the most well-known one. Its main implications, i.e. the risk-return trade off and portfolio diversification, hinge on either a quadratic expected utility or normally distributed returns.

More generally, the basic static model mainly suffers of two major drawbacks that are particularly apparent in connection with household portfolios: empirically is not supported by the data since real portfolios do not appear to be as diversified as predicted by the model and, theoretically, being a static model, it is not appropriate for the analysis of the life-cycle asset allocations. If the first limitation could be overcome also within the static framework by making the model setup more realistic (e.g. including a form of background risk such as uninsurable labour income), the second one calls for an extension of the model to a multiperiod setting.[13)]

In the late 1960s multiperiod versions of the basic model were proposed in the pioneering papers by Mossin (1968), Merton (1969, 1971) and Samuelson (1969). As for the modelling setup, the common feature of these papers is to solve a dynamic optimization problem for a risk-averse household which maximizes expected utility subject to a budget constraint. The models can differ for the setup (discrete vs. continuous), for the specification of the utility function and the assumption on the asset return process, whereby explicit solutions can be in some cases derived depending on the utility function and the asset return process assumed.

Additionally, the models differ according to their main focus. In fact, the multiperiod setup immediately calls for consideration of optimal decision rules not only for portfolio selection, but also for consumption. However some models (e.g. Mossin, 1968) abstract from the consumption/saving problem and aim to find the optimal portfolio which is specific to retirement (i.e. there is

[13)]Gollier (2002) provides an excellent survey on the classical theory of household portfolio, where also the static problem is discussed in detail. In particular, the author highlights the dual interpretation of the very same as either a static portfolio selection problem under uncertainty (Arrow-Debreu portfolio problem) or a lifetime consumption-saving problem under certainty and clearly illustrates the condition for the validity of the two-fund separation theorem (i.e. all agents must have linear absolute risk tolerance with the same slope).

no consumption from portfolio before retirement). As summarized by Brandt (2009) both in the discrete and in the continuous time case, it can be shown that the optimal multiperiod portfolio consists of two components: a myopic portfolio, which is equal to the sequence of one-period optimal portfolios and a hedging portfolio, which serves to hedge against changes in the investment opportunities. It follows that the optimality of the myopic strategy depends on assumed features for the household objectives (i.e. the utility function) and returns process (i.e. the investment opportunities). Specifically, the myopic strategy is optimal with Constant Relative Risk Aversion (CRRA) utility - under which the optimization problem is non recursive and utility is homothetic in wealth - if excess returns are independent of the innovations in the state variables.[14)]

However, by considering intermediate consumption results change since the decision maker can attain smoothing in wealth shocks by means of variations in consumption. In this connection, it is useful to follow Gollier (2002) and stress three effects that play a role in determining the optimal consumption and portfolio rules. The first is the time diversification effect: it means that, ceteris paribus, longer horizons allow to better smooth shocks and hence younger household should take up more risk. The second one is the wealth effect and accounts for the role of wealth in connection with the time horizon: if the wealth level in each period changes with the horizon, so does consumption per period (e.g. lower for younger households that face longer horizons) and the overall effect on risk taking depends on the relationship between risk tolerance and consumption.[15)] Finally, the repeated risk effect captures the idea that, in a multiperiod setting, taking risk today can affect risk -taking in the future: this again depends on risk tolerance.

In sum, in a multiperiod model, while the time-diversification effect is quite clear-cut, the latter two effects depend on the utility function taken to represent household preferences. An interesting benchmark to illustrate the joint working of these three effects is that of CRRA utility, which is a special case of Hyperbolic Absolute Risk Aversion (HARA) utility displaying linear risk tolerance: in this case, the time diversification effect offsets the wealth effect, while taking risk today does not influence risk taking in subsequent period. It follows that the

[14)]Hakansson (1971) discussed Mossin (1968) results and provided weaker conditions for the optimality of myopic strategies. Specifically, log utility for general asset return distributions. In Mossin, with power utility, the assets must be independent to have the myopic property. Brandt (2009) summarizes and argues three cases where the myopic investment strategy is optimal, i.e.: constant investment opportunity set, stochastic but unhedgable investment opportunities and logarithmic utility.

[15)]Specifically, it is the risk-tolerance degree of homogeneity with respect to consumption that determines whether the time diversification effect prevails on the wealth effect (see Gollier, 2002).

myopic strategy is the optimal one[16], a result that prevails in the early models. In fact, Merton (1969) confirms in continuous time an important result proved by Samuelson (1969) in discrete time: for isoelastic marginal utility (i.e. CRRA) and a geometric Brownian motion for returns, the portfolio-selection decision is independent of the consumption decision and, for Bernoulli logarithmic utility, the separation goes both ways. Moreover, under the assumption of log-normally distributed asset prices (implied by the geometric Brownian motion hypothesis for the returns), a mutual-fund theorem result holds and Merton (1971) extends these results to more general utility functions and asset price assumptions thus showing that the classical Markowitz mean-variance rules hold without the hypotheses of the static case (i.e. quadratic utility and normal prices).

Therefore, overall seminal portfolio models provide portfolio rules that are independent of wealth, but more importantly of age so that life-cycle implications cannot be analysed. It is thus not surprising that the predictions of the classical models are markedly at odds with the empirical evidence presented in Section 2.

To make a step towards more suitable models for household portfolios, it is necessary to consider a setup which is more household specific.

10.3.2

More Realistic Portfolio Models

The review in Section 2.1 has highlighted some empirical regularities in household portfolios, which although different in magnitude, characterize household portfolios in most countries over the world. Specifically, three main stylized facts are apparent: low stock market participation, scarce diversification and a life-cycle pattern in household portfolio choices (either participation or allocation) that in most cases display a hump shape, whereby the investment in stocks peaks at middle ages.

The predictions of the models recalled in the previous section are not consistent with these empirical facts and, as a consequence, a body of literature has been growing since the 90s to reconcile theory with evidence. In doing so, the more recent models, while departing from some classical assumptions, have extended and added features of realism to the model setup.

To summarize, three are the main routes the literature has taken to attain life-cycle patterns in household portfolios:

– the modelling of household preferences different from CRRA,

[16] Watcher (2002) shows that, assuming CRRA preferences, the possibility of intermediate consumption affects the hedging component of a portfolio only and not the myopic one, with the overall effect of shortening the time horizon of the investor.

– the predictability in asset returns

– the consideration of trading frictions and market incompleteness.

By contrast to the classical papers where the primary assumptions (i.e. stationarity of returns and CRRA preferences) allowed for closed form solutions, most realistic models are not so easily tractable, requiring numerical and approximate solution methods.

A fundamental issue in portfolio models is modelling of household risk preferences via the utility function. In the previous Section, the role of the CRRA utility assumption clearly emerged in determining both desirable results (e.g. analytic tractability, asset demands independent of wealth) and undesirable ones, such as the independence of asset allocations from time horizon. If follows that an important generalization has concerned preferences: some authors (e.g. Campbell and Viceira, 1999) use Epstein-Zin-Weil preferences (Epstein and Zin, 1989) which are a generalization of CRRA preferences based on recursive utility. They retain the wealth scale independence of power utility, but in contrast to CRRA, allow to distinguish risk aversion from the elasticity of intertemporal substitution in consumption:[17] an important feature given that these parameters have different effects on optimal consumption and portfolio choice.

A different departure from the CRRA preference structure consists in assuming that past consumption choices affect current consumption, as in the so called habit formation (or habit persistence) proposed by Costantinides (1990), who shows that this assumption helps in explaining the equity premium puzzle (Mehra and Prescott, 1985) and variation in returns.

Based on some empirical evidence, a strand of literature (e.g. Barberis (2000), Campbell and Viceira, 1999) has assumed the predictability of asset returns. Brandt et al. (2005) also consider the case of an investor that is uncertain about the parameters of the data generating process and learns from realized returns and dividend yields: in this setting they obtain that learning reduces the allocation to stocks due to parameter uncertainty.[18]

Early models generally neglected trading frictions, whose consideration in the dynamic framework makes the analysis quite complex. In particular the absence of transaction costs, and specifically of fixed costs, has been demonstrated to be a possible explanation for the low levels of stock market participation (e.g. Basak and Cuoco, 1998). By contrast, the effect of proportional transaction costs

[17] For CRRA utility functions the elasticity of intertemporal substitution is the inverse of the coefficient of absolute risk aversion. This implies an unrealistic connection between two distinct feature of household preferences: the willingness to substitute consumption intertemporally and the willingness to take up risk.

[18] See the Review of Financial Studies (Vl.21, 4, 2008) where several articles are centred on the return predictability debate, which is far from being resolved.

is less clear cut: e.g. Costantinides (1986) concludes they do not discourage stock holding, while Heaton and Lucas (1997) find that they shift portfolios towards assets with lower transaction costs. Some kind of taxes play a role similar to proportional transaction costs in that they may prevent from portfolio rebalancing (e.g. Dammon et al., 2004).

The consideration of market incompleteness is often based on liquidity constraints or short selling restrictions, which in fact impede intertemporal smoothing of consumption and portfolio return and can provide an alternative explanation for non participation. Frictions are also behind models that consider the role of uninsurable labour income in explaining life-cycle asset allocations.

In sum, three are the main issues in long-term portfolio choices: the stochastic opportunity set, the consideration of illiquid assets (and particularly labour) and the uncertainty in the length of life. As for the former issue, many paper have contributed especially from the late 90s by considering interest rate or inflation risk and time-varying risk premia, an issue which is more tractable for an institution than household decision-maker.[19] Given the focus of this Chapter on individual household decisions, the attention is restricted to the inclusion into life-cycle models of the latter two issues, which are more household specific and increasingly necessary to attain features of realism in the decision scenario of most households. For these reasons in the next Sections we will concentrate on the role of labour income risk and uncertainty in the length of life.

10.3.3

Life-cycle Asset Allocation Models with uninsurable labour risk

A crucial element in setting up a model for life-cycle asset allocation is to account for the main reasons that motivate wealth accumulation, i.e. the precautionary savings motive connected with background risks and the bequest motive.

The most important background risk for household is possibly labour risk, which has been in fact considered in connection with portfolio choices by a strand of literature starting with the Merton (1971). The author analyses the issue in a framework where labour income can be capitalized and hence the risk insured.

By contrast, in order to see the effect of labour risk on portfolio choices it is fundamental to assume incompleteness so that labour income risk has to be considered, more realistically, uninsurable. In this connection Cocco et al. (2005) provide a model for consumption and portfolio choices in the presence of uninsurable labour income risk, which is becoming a milestone in the analysis of

[19]Examples are: Campebell and Viceira (1999, 2001), Watcher (2002). More references are in the survey by Brandt (2009). See also the models presented in Part III of this book.

this issue.[20] The article provides life-cycle consumption and portfolio rules for a realistically calibrated model with non-tradable labour income and borrowing constraints.[21] Although in other and subsequent papers, the model has been extended, the main feature and implications are already present in Cocco et al. (2005).[22] For this reason, we believe it is worth to illustrate the major model assumptions and their implications in order to highlight what are the modelling characteristics that allow to capture features of realism of portfolio rules such as the dependence of asset allocations on age.

Intuitively, the main departure of Cocco et al. (2005) from classical models is the inclusion of uninsurable labour risk (i.e. markets are incomplete). Specifically, the authors maintain that moral hazard problems, via borrowing constraints, prevent household to capitalize future labour income and thus labour income is a risky asset in household portfolios. However, labour income risk has no or very low correlation with the other financial assets: it is thus a substitute of the risk-free asset and plays a role in terms of portfolio diversification: e.g. young households, who already own a sort of risk-free asset in the form of labour income, tend to hold more risky asset than older households, who by contrast have lower "risk-free" labour holdings. Overall, as the authors conclude,

> *the share invested in equities is roughly decreasing with age. This is driven by the fact that the labour income profile itself is downward sloping,*

and the result is attained with no need to rest on the predictability of asset returns.

To get more insight into the model, its limitations and extensions it is worth illustrating the benchmark model proposed in Cocco et al. (2005) where, beside the standard ingredients of an optimal portfolio choice problem (i.e. preferences, asset returns processes and various constraints), the modelling labour income risk plays an important role.

Let K be a deterministic and exogenous working age, T the uncertain length of life and p_t the probability that the household-investor is alive at date $t+1$.

20) Also Heaton and Lucas (1997) and Viceira (2001) analyse the effect of uninsurable income risk on portfolio composition, but they do it in a infinite-horizon setting and hence in a stationary setup which is less appropriate for the analysis of life-cycle pattern. More related to Cocco et al. (2005) is Bertaut and Haliassos (1997) who also assume a finite horizon.

21) On these issues see also Guiso et al (1996).

22) In Gomes et al. (2008) the authors allow for flexible labour supply but results remain overall qualitatively similar to Cocco et al. (2005) although the ability to increase labour supply represents an alternative to an increase in savings against future income uncertainty so that the portfolio pattern can be less conservative with respect to the case of fixed labour supply.

The household i is assumed to have time-separable CRRA preferences of the following type

$$\mathbf{E_t}\sum_{\mathbf{t=1}}^{\mathbf{T}}\delta_{\mathbf{t-1}}\left(\prod_{\mathbf{J=0}}^{\mathbf{t-2}}\mathbf{p_j}\right)\left\{\mathbf{p_{t-1}}\frac{\mathbf{C_{it}^{1-\gamma}}}{\mathbf{1-\gamma}}+\mathbf{b(1-p_{t-1})}\frac{\mathbf{D_{it}^{1-\gamma}}}{\mathbf{1-\gamma}}\right\} \quad (10.26)$$

where: $\delta < 1$ is the discount factor, $\gamma > 0$ is the coefficient of relative risk aversion, C_t and D_t are, respectively, the consumption level and the amount of bequest at time t.

The exogenous em labour income process, for $t < K$, is assumed to be the sum of a deterministic component that can capture the hump shape of earnings over the life-cycle and a stochastic one, which is made of a persistent part v_{it} and a transitory shock ϵ_{it}

$$\log Y_{it} = f(t, Z_{it}) + v_{it} + \epsilon_{it} \quad (10.27)$$

with

Z_{it} = vector of individual characteristics

ϵ_{it} distributed as $N(0, \sigma_\epsilon^2$

$$v_{it} = v_{it-1} + u_{it} \quad (10.28)$$

u_{it} it is uncorrelated with ϵ_{it} and distributed as $N(0, \sigma_\epsilon^2$.

While the transitory component ϵ_{it} it is uncorrelated across households, the permanent shock u_{it} it can be decomposed in an aggregate component and a transitory one, both normally distributed with zero mean and constant variance

$$u_{it} = \xi_t + w_{it} \quad (10.29)$$

The *retirement income*, for $t > K$, is assumed to be a constant fraction λ in the last working year

$$\log Y_{it} = \log(\lambda) + F(t, Z_{ik}) + v_{it} \quad (10.30)$$

In the financial markets, two assets exist: a risk-free asset and a risky one whose gross real *excess return* over the risk-free is

$$R_{t+1} - \bar{R}_f = \mu + \eta_{t+1} \quad (10.31)$$

where $\bar{R}_f$ is the risk-free return and the innovation η_{t+1} is assumed to be normally i.i.d. with constant variance, but correlated with the aggregate component of labour income with a coefficient π.

A crucial assumption is the borrowing constraint since it impedes the household to capitalize or borrow against future labour income or retirement wealth

$$B_{it} \geq 0. \quad (10.32)$$

It is justified by moral hazard/adverse selection arguments, which as the authors stress are particularly stringent in the early years of the household adult life.

The *short-selling constraint*

$$S_{it} \geq 0. \tag{10.33}$$

implies non negative allocation in equities at all dates. Borrowing and short-selling constraints imply that the proportion invested in equities is $\alpha_{it} \in [0, 1]$ and wealth is non-negative.

Against this setup, the household i in period t start with a wealth Wit and maximizes (26) subject to constraints (27)-(33) in order to obtain optimal consumption and portfolio rules

$$C_{it}(X_{it}, v_{it})$$

and

$$\alpha_{it}(X_{it}, v_{it})$$

which are a function of the state variables: time t, cash-on-hand $(X_{it} = W_{it} + Y_{it})$ and the stochastic persistent component of labour income, v_{it}.

Although the dimensionality of the problem can be reduced[23)], the model cannot be solved analytically and the authors obtain numerical solutions by backward induction, after appropriate calibration of the model to real data. Given the role of labour income in the model, particular attention is devoted to the calibration of the corresponding process which is done on PSID data (a longitudinal US Panel Study on Income Dynamics) and in line with the literature on the subject (e.g. Attanasio (1995), Hubbard et al., 1994).

Simulations results are presented for the benchmark case of the second education group (i.e. high school) but are shown to remain qualitatively unaltered for the other income groups: this is due to the fact that in this model the different groups are solely characterized by the age at which working age begins. A stronger characterization of the benchmark case lies in the correlation between labour income risk and the stock market, which is assumed to be absent (i.e. $\rho = 0$).

As for the life-cycle pattern of portfolio choices, the result is quite clear: the investment in stocks is roughly decreasing with age. To understand this result, recall that in the paper labour is essentially characterized as a bond-like asset and portfolio decisions are determined by the household labour income profile also in relation to wealth. Thus young household, who have a very steep labour income profile, display a rapidly increasing implicit riskless holding (represented by labour income) and diversify by investing in stocks. Later in life the labour income profile is not so steep and the portfolio rule is evaluated at higher wealth levels so that the portfolio moves away from stocks. This overall result rationalizes and supports professional advice suggesting to shift portfolio

[23)] The value function is homogeneous with respect to vit which can be normalized to one.

towards relatively riskless assets as the household ages[24], but, as discussed below, this is in contrast to most empirical evidence on the issue (see Section 2.1).

However, some extensions considered in the paper attain results which are closer to the empirical evidence. For example, an empirically calibrated small probability of disastrous labour income draw lowers stock holding and produces heterogeneity in young household portfolio choices, while endogenous borrowing can explain non participation decisions of young household. Moreover, the sensitivity analyses performed in many directions pave the way to many subsequent literature contributions. Worth mentioning is the sensitivity analysis with respect to labour income risk and in particular to its correlation with stock returns: a positive correlation has significant portfolio effects indicating a lower level of stock for young and a higher level for middle-aged households. This line of research is taken up by Benzoni et al. (2007) as described later.

A related paper is Gomes and Michaelides (2005) where the labour income process is calibrated as in Cocco et al. (2005), but preferences are Epstein-Zin and a fixed entry cost is assumed. The objective is to provide theoretical support for empirical findings on participation rates and asset allocations conditional on participation: heterogeneity in risk aversion seems explain both. In fact, on one hand household with small risk aversion and small elasticity of intertemporal substitution smooth earning shocks with little buffer wealth and this explain low participation, on the other more risk-averse household accumulate more wealth and hence participate in the stock market since young, but do not invest the whole portfolio in it.

As for the role of labour income risk, Cocco et al. (2005) and related extensions highlight that the assumption of null correlation with stock market risk impedes to obtain realistic portfolio rules. Based on previous evidence on the correlation between human capital and market returns, Benzoni et al. (2007) study, in a continuous time model, the optimal portfolio choice over life-cycle in a setup that is essentially the same[25] as the one just described for Cocco et al. (2005), but for the correlation assumption. Specifically, the authors assume that the aggregate component of labour income (the equivalent of ξ_t in eq. (29)) is cointegrated with aggregate dividends and hence with the stock returns. The cointegration assumption makes labour income more of a stock-like asset (than a bond-like one as in Cocco et al., 2005). The cointegration is modelled as a mean reverting process with k being the coefficient of mean reversion, whereby if $k = 0$ there is no cointegration. Its inverse $1/k$ provides the time necessary for cointegration to act. If the residual working life is long (i.e. young households),

[24] The typical reference here is the rule suggesting to place (100-age)% of wealth in a well-diversified stock portfolio (see Malkiel, 1996).

[25] Benzoni et al. (2007) do not explicitly model retirement income, but calibrate the bequest function so as to capture the saving necessary for consumption in the retirement years. This is equivalent to assuming that the household receive an annuity in retirement years.

the return on the household's human capital is highly exposed to stock market returns and labour income resembles more a risky asset than a risk-free one so that young households, who are already overexposed to stock market risk, find it optimal to go short in stocks or, in the presence of borrowing constraint, to invest the entire wealth in the risk-free bond. For middle-aged the cointegration has no time to act, labour income has bond-like features and the opposite is true. This is still true in the years before retirement, but for sufficiently short time before retirement a second effect prevails: due to lower future labour income, the value of the bond position implicit in human capital decreases and the household start reducing stocks in favour or risk-free assets. This is the hump-shape profile that Benzoni et al.(2007) obtain in the simulations of their model, which, in line with the literature is solved numerically by backward induction using standard difference method. Beyond robustness exercises, the authors also prove that the result still holds in the presence of return predictability.

In sum, by contrast to similar models (even micro-data calibrated ones) and conventional professional wisdom, Benzoni et al. (2007) attain a hump-shaped pattern for life-time portfolio that is consistent with most empirical evidence on the topic. The result provides an explanation to limited stock market participation that the authors stress to be different but complementary to those typically put forward in the literature:

> *In particular, our paper emphasizes that long-run cointegration between aggregate labor income and aggregate dividends has a first-order effect on the optimal portfolio decisions of an agent over the life cycle.*

However, it has to be stressed that the fundamental assumption for this result is questionable in that difficult to test. Evidence on the cointegration between human capital and market return is disparate if not weak, due to the well-known lack of power of cointegration tests. Moreover the results are very sensitive not only to the very same existence of a cointregation but also to the specific level[26].

10.3.4

Life-cycle Asset Allocation Models in the presence of annuities

The stochastic nature of a household investment horizon calls for the consideration of a further source of risk, which is known as longevity risk, i.e. the risk

[26] In fact, as the authors stress, it is econometrically very difficult even to distinguish between $k = 0$ and $k = 0.05$ and the peak of the hump-shape of the portfolio pattern depends on the value of the coefficient governing the cointegration k.

for the investor of living longer that predicted and hence run out of savings. To account for this issue, household portfolio models have to consider uncertainty in the length of life in connection with the role played by a particular type of assets, annuities, which in fact allow to transfer longevity risk to the insurer but are an illiquid instrument.[27)]

The question in a portfolio framework becomes the optimal investment in annuities. The seminal paper to answer this question is Yaari (1965), which ignores other sources of risks other than mortality and provides conditions for full annuitization, i.e. market completeness and no bequest motive, otherwise partial annuitization becomes optimal. Later on, portfolio models have analysed the case of constant life annuities (i.e. annuities providing a fixed payout) but they have often simplified the problem by either imposing full annuitization (e.g. Cairns et al., 2006) or disregarding other important features of the decision problem, such as the irreversibility of the annuities purchase, other sources of risks (e.g. Richard, 1975) or the impact of annuity markets during working life (e.g. Milevsky and Young (2007) consider the case of a retiree).

However, in their recent work Horneff et al. (2008)[28)] overcome some limitations of the previous literature and study the optimal consumption and saving strategy in the presence of constant life annuities, bonds and stocks in an incomplete market setting. The model allows for gradual purchase of annuities and considers the three main sources of risks faced by an household: risky stocks, untradable labour income during working life and stochastic time to death. In connection with the latter two features their model can be seen as extending those presented in the previous Section. In particular, the model shares many features of Cocco et al. (2005) and some of the parameters taken for the numerical solution of the optimization problem are borrowed from the former.

More precisely, the problem differs from the one represented by equations (26)-(33) in the following:

i. Since Epstein-Zin preferences are assumed, equation (26) is replaced by the recursive formulation of intertemporal utility that allows to disentangle risk aversion γ from the elasticity of intertemporal substitution ψ.

ii. An incomplete annuity market is considered. Against the payment of an actuarial premium A_t, the annuitant receives a constant payment L until death

$$A_t = Lh_t \tag{10.34}$$

27) An overview on the treatment of longevity risk from the viewpoint of annuity providers and pension plans is provided by Biffis and Blake in Chapter 14 of this book.

28) Cocco and Gomes (2008) also consider longevity risk and the role of longevity bond in the optimal consumption/saving problem, but they do not study the optimal asset allocation.

with

$$h_t = (1+\delta)\sum_{s=1}^{T-t}\left(\prod_{u=t}^{t+s} p_u^a\right) R_f^{-s}$$

where δ = loading factor and p_u^a = survival probability used by the annuity provider, which is higher than the average survival probabilities p_u^s. The annuity provider hedge the guaranteed annuity payments by pooling mortality risks of annuitants: the funds of those who die are allocated among the living member of a cohort and this represent the source of the so called mortality credit, i.e. the excess annuity return over a bond. The market is incomplete because only life-long payouts are available and funds from annuity are invested in bonds only.

iii. Mortality is modelled by means of Gompertz's law. The force of mortality used by the provider and the subjective one are specified as the following function of the parameters m^i and b^i

$$\lambda_i^t = \frac{1}{b^t}\exp\left(\frac{t-m^i}{b^t}\right) \tag{10.35}$$

with

$$i = a, s$$

and the survival probability is

$$p_t^i = \exp\left(-\int_0^1 \lambda_{t+s}^i ds\right). \tag{10.36}$$

Additionally the subjective force of mortality is taken to be a linear transform of the one derived from average mortality tables.

It follows that wealth accumulation is determined by annuities as well and a borrowing constraint is placed on the annuities too $(A_t \geq 0)$. In this setup, beyond demand for stocks, bonds and consumption, there is one more choice variable that has to be determined: the optimal level of annuities in each period. Moreover all policy rules are a function of the same state variables as in Cocco et al. (2005) plus the annuity payouts form previously purchased annuities.

There is a trade-off between liquid financial savings and illiquid annuities which provide the mortality credit. The introduction of annuities thus poses a central question: is the mortality credit high enough to compensate for the illiquidity of the annuity? The irreversibility of the annuity purchase makes the consideration of labour income risk even more important, given the need for liquidity that income shocks normally bring by. Moreover extreme income shocks can also be interpreted as reflecting healthcare and nursing costs and hence the analysis in relation to annuities becomes even more important.

The model is first solved for the baseline case (no loads, no asymmetries between insurer's and anunuitant's beliefs, no bequest) so as to isolate the role of the annuities in the portfolio problem. As for investment in stocks and bonds results are qualitatively very similar to Cocco et al. (2005) given that the setup implies the same interpretation of labour income, which is more a bond-like asset than a stock-like one (as in Benzoni et al., 2007). However, the presence of annuities in most cases crowds out bonds thus indicating that mortality credit compensate for illiquidity of annuities. Moreover the optimal annuity holding increases over time in contrast to stock holdings and the explanation, as in Cocco et al. (2005) rests on the characterization of labour as a bond-like asset, whose holding decreases with age.

Sensitivity analyses allow to highlight important determinants of optimal annuities holdings. As expected the correlation between human capital and market risk increases the purchase of annuities at least until the retirement period (since retirement income is assumed uncorrelated with the market), while during retirement the purchase of annuities depends positively on the retirement income replacement ratio but does not vanishes even if the latter were set to one, a result that can help in the debate over the pension system.

10.4

Conclusions

In this Chapter the literature on household portfolio choices was analysed in view of population ageing by taking both a positive and a normative approach. The analysis starts with a recognition of the most important stylised facts emerging from the empirical literature surveyed in Section 2 and then shifts to models that can account for these facts or provide suggestions for optimal asset allocations. Since the literature has been extending in many directions in latter years, the objective here is not to provide an exhaustive survey and many issues had to be left out. Rather, by focussing on financial portfolio choices over both the working and the retirement period, the aim is to conclude with some research implications.

Since the mid of 1990s, the nexus between age and finance has inspired a lively debate, which has given rise to an increasing number of empirical studies highly diversified in terms of approach, methodology, data and results. According to the approach taken by the studies, we have distinguished two main streams in this literature: one assuming a micro-perspective and basically aimed at assessing the role of age as a determinant of the household portfolio decisions, and the other assuming instead a macro-perspective and basically aimed at investigating the possible implications, in terms of direction and magnitude, of ageing population on asset returns and ultimately on financial markets. The two strands of literature differ for the overall evidence reported: the works assuming a micro-perspective on the whole agree on age being an important determinant of portfolio decisions, although the specific pattern along the life-cycle results humped rather than linear depending primarily on whether participation or allocation decision is analyzed. At a macro-perspective, the evidence on the link between demographics and financial asset prices/returns is less uniform: some studies find only a weak, if any, relationship between demographic dynamics and financial variables, while others report significant effects of age and ageing on financial markets. The most recent contributions in this literature seem to agree on the fact that a role in explaining these differences might rely on the country analysed, and more specifically on its peculiarities in terms of both demographic dynamics and institutional settings, thereby encouraging further investigation on these issues.

Overall, three facts most strikingly emerge from the empirical literature: low stock market participation, scarce diversification and a life-cycle of household portfolios that display a hump shape, whereby the investment in stocks peaks at middle ages. This evidence contradicts popular financial advice suggesting investment in stock decreasing with age and most portfolio models. The survey over the theoretical literature highlights that, in order to capture real portfolio patterns, the dynamic nature of most recent models has to be coupled with an appropriate modelling of household specific assets, which share the feature of being illiquid. The most important among these is surely human capital, which materializes in the portfolio framework via labour income. This is why particular emphasis was devoted to Cocco et al. (2005) where the inclusion of labour income risk is made in an incomplete market setting so that labour income risk is uninsurable, an assumption that well represents real market conditions. However, an important source of background risk such as labour income risk alone is not sufficient to explain the hump-shaped pattern of observed portfolio choices, but a long-run cointegration between human capital and the stock market as in Benzoni at al. (2007) can account for it.

We believe this is the direction that future research should take. More specifically, models for household portfolio decisions should account for three main features affecting household decisions in a specific way: finite horizon together with longevity risk, borrowing constraints and non financial assets. As for the latter, it has to be highlighted that non financial wealth represents the major part of an household portfolio so that its features in terms of tradability, insurability and correlation with the stock market are of uttermost importance in determining financial choices.

The two most important component of non financial wealth are by large labour income and housing. A literature that addresses housing choices in connection with portfolio choices has been growing (e.g. Cocco (2004), Flavin and Yamashita (2002), Yao and Zhang, 2005). However, the different nature of housing with respect to other assets (consumption and investment) and its often leveraged status requires a specific focus and deserve a separate chapter, but its consideration is in some cases essential to explain some portfolio puzzles (e.g.

in Cocco (2004) housing crowds out stock holding). As for the link between labour income and financial markets, Benzoni et al. (2007) shows how the correlation between human capital and market returns can change the optimal household portfolio and suggests the inclusion of housing in their setup. This is interesting especially if evidence on the cointegration between the housing and the stock market is strong. However, since results under the cointegration assumption are very sensitive not only to the very same existence of a cointegration (which is also hard to test) but also to the specific level, very high on the research agenda is still the modelling of the link between labour income and financial markets. In particular, the long-run cointegration assumption essentially focus on the time dimension of the link between human capital and financial markets, whereby the idea is that in the long-run only young people will be alive and thus hit by the long-run cointegration. However, we believe that another important dimension to be considered in this connection is the type of human capital as captured for instance by the different education groups, which might deserve more differentiation in term of labour income risk (e.g. recall that in Cocco et al. (2005) education groups are solely characterized by the age at which working age begins). Also the modelling of the post-retirement period leaves room for further investigation: if in Cocco et al. (2005) retirement income is simply a constant fraction of labour income, in Benzoni et al. (2007) the post-retirement consumption and investment decisions are not explicitly modelled. Moreover there is a further element that differentiates the household in terms of income risk beside age: it is the connection between gender and marital status, which has been rarely considered in the portfolio setting (e.g. Bertocchi et al., 2009) and yet not in relation to life-cycle choices.

To conclude, two main challenges of household finance emerge from this Chapter overview, which, to say it with Campbell (2006), can be summarized as follows: empirical analyses that highlight how households do invest (i.e. positive household finance) have important measurement problems, while suggestions about how investments should be made (i.e. normative household finance) encounter modelling problems, which naturally bring about numerical solutions and calibration issues.

As far as measurements problems are concerned, most of the studies aimed at investigating household portfolio choices base their empirical analyses on micro-datasets, which could either be drawn from surveys (e.g. SCF for US, SHIW for Italy, etc..) or from (usually not publicly available) financial-institution datasets (e.g. bank or insurances data). Both sources however suffer from measurement problems. Survey data offer the advantages of being designed on representative samples and providing a comprehensive picture of household portfolios, including information about housing property, labor income and financial assets owning; by contrast, they are generally associated with low response rates and with generally unrealistic levels of diversification and riskiness of household portfolios due to the well known problems of non-reporting (i.e. the interviewed does not want to reveal the ownership of a certain financial asset) or under-reporting (i.e. the interviewed reports the presence of a certain asset in his/her portfolio, but does not know or reveal its amount). On the other hand, institutional datasets are more reliable as they do not suffer from these reporting issues, but besides being typically confidential, they have the relevant drawback of offering only a partial view of the portfolios (e.g. only financial assets held in that bank or insurance) thereby being not really suitable for a comprehensive investigation of the whole allocation choices implemented by the households.

As for some modeling problems (e.g. numerical solutions and calibration issues) implied by more realistic models, the household finance theory can benefit of a large and useful literature developed in connection with institutional portfolio choices (see e.g. Geyer and Ziemba (2008), Rudolf and Ziemba (2004), Zenios and Ziemba (2006) and contributions presented in other chapters in this book). In addition, as we have stressed above pointing out some research directions, more research on modelling is also needed to provide a source of financial advice and for economic policy considerations in view of current and perspective socio-economic scenarios increasingly characterized by more volatile labour income, important changes in the family structure, population ageing and less generous public pension systems. A further source of worry comes from recent financial market downturns (two in a decade) that cast doubts over the possibility of pension funds and annuity providers to cover, by relying on the market only, demographic risks typical of an ageing society.

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