

Exchange options with stochastic volatility

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ABSTRACT. In a recent paper , [AS], the problem of pricing plain vanilla options under quite general stochastic dynamics for the volatility of the underlying asset was approached in a novel way. Namely, using the idea of developing the option price in a power series of the correlation coefficient between the asset price and the volatility processes around zero correlation, the authors in [AS], were able to prove the convergence of such a series under mild assumptions. Moreover it was shown that the first coefficients of the series are computable (at least in an approximate way) and this allows for good estimates of the true option prices. In the present work we perform a first attempt to extend such a technique to a multidimensional setting. Namely, we consider the case of an exchange or Margrabe option, [M], written on two assets whose processes are assumed to be of log-normal type but with stochastic volatilities. Therefore the market is incomplete. In our model, as in [M], the two assets S^1 and S^2 are correlated with correlation coefficient ρ . The volatilities of the assets, respectively V^1 and V^2 , follow each an Ornstein-Uhlenbeck dynamics. We first solve the option pricing problem when $\rho = 0$ and then develop an expansion in powers of ρ around zero in order to find an approximation of the pricing formula for the case of general correlation values.

Keywords: Margrabe Options, Stochastic volatility, SDE's

References

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