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A NEW ALGEBRA IN THE STOCHASTIC APPROXIMATION FOR THE MODEL OF A PARTICLE INTERACTING WITH A QUANTUM FIELD

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When the stochastic approximation is used to calculate correlation functions in the model of a particle interacting with a quantum field, a new algebra with temperature-dependent commutation relations appears. This algebra generalizes the free (Boltzmann) algebra.

1. Introduction

We consider the model of a particle interacting with a quantum field. Such models have been extensively studied in elementary particle physics, in solid-state physics, in quantum optics, etc. [1–4]. We investigate correlation functions using the stochastic approximation method of Van Hove and Friedrichs. Accardi, Frigerio, and Lu applied this method to quantum optic models [5]. One of these models was analyzed [6] in the dipole approximation. The method consists in using a scaling limit, where the asymptotic behavior of the correlation function is considered at large times and small coupling constants. Then, the limiting dynamics are integrable, in a sense, for a series of problems, and explicit expressions can be obtained for the correlation functions [6]. The limit is called "stochastic" because free correlators become "δ-correlated" in time in this limit. (That is, we have the white-noise random process.)

Our main result is that in the temperature stochastic limit, a new mathematic structure arises in the model of particle interacting with a quantum field. We call this structure the *free temperature algebra*. It is a Boltzmannian algebra.

We consider correlation functions for special operators (the collective variables). In the stochastic limit, the theory is simplified and is described by the free temperature algebra; the correlators correspond to some states of the free temperature algebra. Further investigation of this Boltzmannian algebra, which governs the limiting dynamics, is interesting in itself. The stochastic limit of this model at zero temperature was considered in [7–9].

The simplest Boltzmannian algebra is generated by the relations

$$B(k)B^{\dagger}(k') = \delta(k-k'), \qquad k,k' \in R^d.$$

Such relations have been investigated in mathematics [10–15]; they were obtained in the stochastic limit of the model of a particle interacting with a quantum field [7] and in the large-N limit of quantum chromodynamics with the gauge group $SU(N)$ [16].

The free temperature algebra below can be schematically described as the Boltzmannian algebra with the generators $b(k)$, $b^{\dagger}(k)$, and $\mathbf{p}$, which satisfy the relations

$$b(k)b^{\dagger}(k') = \frac{\delta(\widetilde{\omega}(k)+k\mathbf{p})}{1-e^{-\beta\omega(k)}}\delta(k-k'),$$
$$b(k)\mathbf{p} = (\mathbf{p}+k)b(k),$$

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where $\omega(k)$ is the one-particle energy of the quantum field and β is the reciprocal temperature. This algebra is a deformation of the Boltzmann algebra.

In Sec. 2, we give the general description of the stochastic limit. In Sec. 3, we formulate Theorem 1 on the stochastic limit and present examples of calculating two-point and four-point correlation functions. In Sec. 4, we provide Theorem 2, which describes a free temperature algebra. The complete proof of Theorem 1 is in Sec. 5.

2. The stochastic limit

The stochastic limit is a scaling limit in quantum theory. We consider a system with the Hamiltonian

$$H = H_0 + \lambda H_{\mathrm{I}}.$$

The evolution operator $U_t^{(\lambda)} = e^{itH_0}e^{-itH}$ satisfies the Schrödinger equation in the interaction representation,

$$\frac{\partial}{\partial t}U_t^{(\lambda)} = -i\lambda H_{\mathrm{I}}(t)U_t^{(\lambda)}, \qquad U_0^{(\lambda)} = 1, \tag{1}$$

where $H_{\mathrm{I}}(t) = e^{itH_0}H_{\mathrm{I}}e^{-itH_0}$. We take λ to be a small constant and consider the accumulated influence of small perturbations over a long time interval by studying the limiting transitions $\lambda \to 0$, $t \to \infty$ and making the replacement $t \to t/\lambda^2$ in the evolution equation,

$$\frac{\partial}{\partial t}U_{t/\lambda^2}^{(\lambda)} = -\frac{i}{\lambda}H_{\mathrm{I}}\left(\frac{t}{\lambda^2}\right)U_{t/\lambda^2}^{(\lambda)}.$$

If the limits

$$\lim_{\lambda\to 0} U_{t/\lambda^2}^{(\lambda)} = U_t, \tag{2}$$

$$\lim_{\lambda\to 0} \frac{1}{\lambda} H_{\mathrm{I}}\left(\frac{t}{\lambda^2}\right) = H_t \tag{3}$$

exist, then the limiting operator U_t satisfies the equation

$$\partial_t U_t = -iH_tU_t, \qquad U_0 = 1. \tag{4}$$

If all the corresponding correlators are regular in this limit, then the limit exists. In the limit $\lambda \to 0$, many problems become exactly solvable. The replacement $t \to t/\lambda^2$ with the subsequent limiting transition $\lambda \to 0$ corresponds to the simultaneous limiting transitions $\lambda \to 0$ and $t \to \infty$ while keeping $\lambda^2 t = \text{const}$. Here, $\lambda^2 t$ is a "slow" time scale. This limit describes the leading contribution to the dynamics in the weak-coupling regime at large times, i.e., the effect of accumulated weak perturbations. The physical idea is that the quantum field behaves as a chaotic object—the quantum white noise—at the "slow" time scale. This quantum white noise is the δ-correlated quantum random process $b(t,k)$, which is also called the "master field," and we seek the commutation relations for this object.

We consider a quantum-mechanical system to be the triple of an observation algebra $\mathcal{A}$, a space of states, and an evolution operator. Furthermore, we assume that the state space is the Hilbert space of the Gelfand–Neimark–Segal (GNS) representation generated by a state $\langle\cdot\rangle$ of the observation algebra. Either the vacuum or the mean temperature can be chosen as such a functional. The evolution operator is the operator $U_t^{(\lambda)}$ (an evolution operator in the interaction representation).

The stochastic limit of the observation algebra is as follows. The free evolution $A(t) = e^{itH_0}Ae^{-itH_0}$ corresponds to a given element A of the observation algebra. We find elements A_i for which expressions of the type

$$\lim_{\lambda\to 0}\left\langle \frac{1}{\lambda}A_1\left(\frac{t_1}{\lambda^2}\right)\cdots\frac{1}{\lambda}A_k\left(\frac{t_k}{\lambda^2}\right)\right\rangle$$

have nontrivial limits. If we find an algebra $\mathcal{B}$ (whose elements are denoted by B_i) and a state $\langle\langle\cdot\rangle\rangle$ from $\mathcal{B}$ such that the equality

$$\lim_{\lambda\to 0}\left\langle \frac{1}{\lambda}A_1\left(\frac{t_1}{\lambda^2}\right)\dots\frac{1}{\lambda}A_k\left(\frac{t_k}{\lambda^2}\right)\right\rangle = \langle\langle B_1(t_1)\dots B_k(t_k)\rangle\rangle$$

holds, then the algebra $\mathcal{B}$ is called the stochastic limit of the observation algebra $\mathcal{A}$. The GNS representation of this algebra is generated by the state $\langle\langle\cdot\rangle\rangle$. For investigating the evolution determined by Eq. (1), it is sufficient to calculate the stochastic limit of the observation algebra elements that enter the interaction Hamiltonian H_{I}.

Analyzing the perturbation series (see below), we conclude that by virtue of the Wick theorem, a normally ordered perturbation series can be represented as a diagram series. For some models, only semiplanar diagrams contribute in the stochastic limit. An algebra of free creation–annihilation operators corresponds to semiplanar diagrams. In the considered model, the deformation of the Boltzmann algebra we call the free temperature algebra arises in the stochastic limit.

3. The model of a particle interacting with a quantum field

We study the model of a particle interacting with the quantum field at a nonzero temperature. The coordinate $q = (q_1, \dots, q_d)$ and momentum $p = (p_1, \dots, p_d)$ operators of a quantum particle satisfy the commutation relations

$$[q_m, p_n] = i\delta_{mn}.$$

The quantum field is described by bosonic operators (operator-valued distributions)

$$a(k) = (a_1(k), \dots, a_d(k)), \qquad a^\dagger(k) = (a_1^\dagger(k), \dots, a_d^\dagger(k)), \qquad k \in R^d,$$

with the commutation relations

$$[a_j(k), a_h^\dagger(k')] = \delta_{jh}\delta(k-k').$$

The Hamiltonian of the considered system is

$$H = H_0 + \lambda H_{\mathrm{I}} = \int \omega(k)a^\dagger(k)a(k)\,dk + \frac{1}{2}p^2 + \lambda H_{\mathrm{I}},$$

where ω is a positive function on R^d, e.g., $\omega(k) = |k|$, and H_{I} determines the interaction between a free particle and the quantum field. The particle–field interaction is expressed via the potential $\mathcal{A}(x)$ of the quantum field acting on a particle with the coordinate $x \in R^d$. The interaction potential is

$$H_{\mathrm{I}} = p\mathcal{A}(q) + \mathcal{A}(q)p, \tag{5}$$

where

$$\mathcal{A}(q) = \int dk\,\{g(k)e^{ik\cdot q}a^\dagger(k) + \overline{g}(k)e^{-ik\cdot q}a(k)\}. \tag{6}$$

The time dependence of $H_{\mathrm{I}}(t)$ is determined by the operator

$$a_\lambda(t,k) = \frac{1}{\lambda}e^{i\frac{t}{\lambda^2}H_0}e^{-ikq}a(k)e^{-i\frac{t}{\lambda^2}H_0} = \frac{1}{\lambda}e^{\frac{-i(\widetilde{\omega}(k)+kp)t}{\lambda^2}}e^{-ikq}a(k), \tag{7}$$

where $\widetilde{\omega}(k) = \omega(k) + k^2/2$.

We investigate the limit of temperature correlation functions,

$$\lim_{\lambda\to 0}\langle a_\lambda^{\epsilon_N}(t_N,k_N)a_\lambda^{\epsilon_{N-1}}(t_{N-1},k_{N-1})\dots a_\lambda^{\epsilon_1}(t_1,k_1)\rangle.$$

The means

$$\langle a(k)a^\dagger(k')\rangle = \frac{\delta(k-k')}{1-e^{-\beta\omega(k)}},$$
$$\langle a^\dagger(k')a(k)\rangle = \frac{\delta(k-k')}{e^{\beta\omega(k)}-1}$$

are temperature bosonic correlators (other correlators can be calculated from the Wick theorem), and $\varepsilon \equiv \{\epsilon_N,\dots,\epsilon_1\} \in \{1,0\}^N$, $\epsilon \in \{1,0\}$ ($\epsilon = 0$ for a and $\epsilon = 1$ for $a^\dagger$). The temperature mean $\langle\cdot\rangle$ is the Gibbs mean w.r.t. the field degrees of freedom that does not include the quantum particle degrees of freedom:

$$\langle X\rangle = \frac{\mathrm{tr}\big(Xe^{-\beta\int\omega(k)a^\dagger(k)a(k)\,dk}\big)}{\mathrm{tr}\big(e^{-\beta\int\omega(k)a^\dagger(k)a(k)\,dk}\big)}.$$

The mean $\langle\cdot\rangle$ w.r.t. the quantum particle degrees of freedom is the conventional expectation, i.e., $\langle pX\rangle = p\langle X\rangle$.

For $N = 2n$ and for equal numbers of creation and annihilation operators, we consider the partition $\sigma(\varepsilon)$ of the sequence ε into pairs from zero and unity, which corresponds to the Wick expansion

$$\big\langle a_\lambda^{\epsilon_N}(t_N,k_N)a_\lambda^{\epsilon_{N-1}}(t_{N-1},k_{N-1})\dots a_\lambda^{\epsilon_1}(t_1,k_1)\big\rangle$$

into the creation–annihilation operator pairs. Any such partition corresponds to a Wick diagram. We are interested in partitions that correspond to semiplanar nonintersecting diagrams. We call such partitions nontrivial.

A Wick diagram can be constructed from a partition as follows. We place the indices 0 and 1 of the sequence ε on an interval in the increasing-number order. We then connect indices for which the corresponding pair exists in the partition $\sigma(\varepsilon)$ with arcs. If all arcs of the resulting diagram can be placed without intersections on one half-plane w.r.t. the interval (line), then we call such a diagram a semiplanar nonintersecting diagram and the corresponding partition $\sigma(\varepsilon)$ a nontrivial partition.

Theorem 1. *The limit of the temperature correlation functions always exists. Moreover,*

1. *if the number of creation operators differs from the number of annihilation operators, then the correlator is zero (even before the limiting transition);*

2. *if the numbers of creation and of annihilation operators are equal, then the limit of the correlator*

$$\lim_{\lambda\to 0}\big\langle a_\lambda^{\epsilon_{2n}}(t_{2n},k_{2n})a_\lambda^{\epsilon_{2n-1}}(t_{2n-1},k_{2n-1})\dots a_\lambda^{\epsilon_1}(t_1,k_1)\big\rangle \tag{8}$$

is the sum over nontrivial partitions

$$\sum_{\sigma(\varepsilon)}\prod_{h=1}^{n}\delta(k_{m_h'}-k_{m_h})c_{m_hm_h'}(k_{m_h})2\pi\delta(t_{m_h'}-t_{m_h})\times$$
$$\times\,\delta\Big(\widetilde{\omega}(k_{m_h})+k_{m_h}p+\sum_\alpha(-1)^{\epsilon_\alpha}\chi_{(m_\alpha,m_\alpha')}(m_h)k_{m_\alpha}k_{m_h}-\varepsilon_hk_{m_h}^2\Big), \tag{9}$$

where $\{(m_j',m_j);\ j=1,\dots,n\}$ are partitions of the set $\{1,\dots,2n\}$ that correspond to the partitions $\sigma(\varepsilon)$. The quantities m_h' and m_h correspond to the annihilation and creation operators respectively,

$$c_{m_hm_h'}(k) = \frac{1}{1-e^{-\beta\omega(k)}},\quad m_h' > m_h,$$
$$c_{m_hm_h'}(k) = \frac{1}{e^{\beta\omega(k)}-1},\quad m_h' < m_h,$$

$(-1)^{\varepsilon_h} = 1$ *for* $m'_h > m_h$ *and* $(-1)^{\varepsilon_h} = -1$ *for* $m'_h < m_h$. *The function* $\chi_{(m_\alpha, m'_\alpha)}$ *is the indicator (the characteristic function) of the interval* (m_α, m'_α), *i.e., the sum over* α *goes over pairs* (m_α, m'_α) *such that* m_h *is placed between* m_α *and* m'_α.

We prove Theorem 1 in Sec. 5. Here, we illustrate Theorem 1 by calculating two-point and four-point correlators.

The two-point correlator is

$$\lim_{\lambda\to 0}\langle a_\lambda(t,k_1)a^\dagger_\lambda(\tau,k_2)\rangle = \lim_{\lambda\to 0}\frac{1}{\lambda^2}\, e^{-it/\lambda^2(\widetilde{\omega}(k_1)+k_1p)}e^{-iq(k_1-k_2)}e^{i\frac{\tau}{\lambda^2}(\widetilde{\omega}(k_2)+k_2p)}\langle a(k_1)a^\dagger(k_2)\rangle. \tag{10}$$

Using the formulas for bosonic two-point correlators, we obtain

$$\lim_{\lambda\to 0}\frac{1}{\lambda^2}\, e^{-i\frac{t-\tau}{\lambda^2}(\widetilde{\omega}(k_1)+k_1p)}\frac{\delta(k_1-k_2)}{1-e^{-\beta\omega(k_1)}}.$$

Using the formula

$$\lim_{\lambda\to 0}\frac{1}{\lambda^2}\, e^{\frac{it}{\lambda^2}(\widetilde{\omega}(k)+kp)} = 2\pi\delta(\widetilde{\omega}(k)+kp)\delta(t), \tag{11}$$

we obtain the following expression for the two-point correlation function limit:

$$\lim_{\lambda\to 0}\langle a_\lambda(t,k_1)a^\dagger_\lambda(\tau,k_2)\rangle = 2\pi\delta(t-\tau)\delta(\widetilde{\omega}(k_1)+k_1p)\frac{\delta(k_1-k_2)}{1-e^{-\beta\omega(k_1)}}. \tag{12}$$

We now consider the two-point correlation function

$$\lim_{\lambda\to 0}\langle a^\dagger_\lambda(\tau,k_2)a_\lambda(t,k_1)\rangle = \lim_{\lambda\to 0}\frac{1}{\lambda^2}\left\langle a^\dagger(k_2)e^{ik_2q}e^{i\frac{\tau}{\lambda^2}(\widetilde{\omega}(k_2)+k_2p)}e^{-i\frac{t}{\lambda^2}(\widetilde{\omega}(k_1)+k_1p)}e^{-ik_1q}a(k_1)\right\rangle. \tag{13}$$

Using the Weyl operator commutation relations

$$e^{i\alpha p}e^{i\beta q} = e^{i\beta q}e^{i\alpha p}e^{i\alpha\beta},$$

where $[p,q] = -i$, we obtain

$$\lim_{\lambda\to 0}\frac{1}{\lambda^2}\frac{\delta(k_2-k_1)}{e^{\beta\omega(k_1)}-1}\, e^{-i\frac{t-\tau}{\lambda^2}(\widetilde{\omega}(k_1)+k_1p-k_1^2)}$$

for correlation function (13). Using formula (11), we obtain

$$\lim_{\lambda\to 0}\langle a^\dagger_\lambda(\tau,k_2)a_\lambda(t,k_1)\rangle = 2\pi\delta(t-\tau)\delta\big(\widetilde{\omega}(k_1)+k_1p-k_1^2\big)\frac{\delta(k_2-k_1)}{e^{\beta\omega(k_1)}-1}. \tag{14}$$

We calculate the limit of the four-point correlation function

$$\lim_{\lambda\to 0}\big\langle a_\lambda(t_1,k_1)a_\lambda(t_2,k_2)a^\dagger_\lambda(t'_2,k'_2)a^\dagger_\lambda(t'_1,k'_1)\big\rangle. \tag{15}$$

The Wick temperature theorem implies

$$\begin{aligned}\langle a(k_1)a(k_2)a^\dagger(k'_2)a^\dagger(k'_1)\rangle = {}&\frac{1}{1-e^{-\beta\omega(k_1)}}\,\frac{1}{1-e^{-\beta\omega(k_2)}}\times\\ &\times\big(\delta(k_2-k'_2)\delta(k_1-k'_1)+\delta(k_1-k'_2)\delta(k_2-k'_1)\big).\end{aligned} \tag{16}$$

Formula (16) for the bosonic correlation function $\langle a(k_1)a(k_2)a^\dagger(k_2')a^\dagger(k_1')\rangle$ contains two terms, which are proportional to δ-functions and correspond to the Wick diagrams. The result for the first term $\mathcal{L}_1$ proportional to $\delta(k_1-k_1')\delta(k_2-k_2')$ is

$$\mathcal{L}_1 = \lim_{\lambda\to 0}\frac{1}{1-e^{-\beta\omega(k_1)}}\,\frac{1}{1-e^{-\beta\omega(k_2)}}\delta(k_1-k_1')\delta(k_2-k_2')\times \\ \times\frac{1}{\lambda^4}e^{-i\frac{t_1-t_1'}{\lambda^2}(\tilde{\omega}(k_1)+k_1p)}e^{-i\frac{t_2-t_2'}{\lambda^2}(\tilde{\omega}(k_2)+k_2p)}e^{-i\frac{t_2-t_2'}{\lambda^2}k_1k_2}. \tag{17}$$

From formula (11), we obtain

$$\mathcal{L}_1 = (2\pi)^2\frac{1}{1-e^{-\beta\omega(k_1)}}\,\frac{1}{1-e^{-\beta\omega(k_2)}}\delta(k_1-k_1')\delta(k_2-k_2')\delta(t_1-t_1')\delta(t_2-t_2')\times \\ \times\,\delta\big(\tilde{\omega}(k_1)+k_1p)\delta(\tilde{\omega}(k_2)+k_2p+k_1k_2\big). \tag{18}$$

The second term $\mathcal{L}_2$ of the correlation function is proportional to $\delta(k_1-k_2')\delta(k_2-k_1')$, and formula (11) implies

$$\mathcal{L}_2 = \lim_{\lambda\to 0}\frac{1}{1-e^{-\beta\omega(k_1)}}\,\frac{1}{1-e^{-\beta\omega(k_2)}}\delta(k_1-k_2')\delta(k_2-k_1')\times \\ \times\frac{1}{\lambda^4}e^{-i\frac{t_1-t_2'}{\lambda^2}(\tilde{\omega}(k_1)+k_1p)}e^{-i\frac{t_2-t_1'}{\lambda^2}(\tilde{\omega}(k_2)+k_2p)}e^{-i\frac{t_2-t_2'}{\lambda^2}k_1k_2}=0.$$

Therefore, formula (18) is the four-point correlation function.

4. The free temperature algebra

We want to construct the algebra $\mathcal{B}$ and the states $\langle\langle\cdot\rangle\rangle$ of this algebra. Then, the correlation functions of the algebra $\mathcal{B}$ w.r.t. the state $\langle\langle\cdot\rangle\rangle$ must reproduce the stochastic limits of the temperature correlation functions of the observation algebra $\mathcal{A}$, i.e., the identities

$$\lim_{\lambda\to 0}\langle a_\lambda^{\epsilon_N}(t_N,k_N)a_\lambda^{\epsilon_{N-1}}(t_{N-1},k_{N-1})\dots a_\lambda^{\epsilon_1}(t_1,k_1)\rangle = \langle\langle b^{\epsilon_N}(t_N,k_N)b^{\epsilon_{N-1}}(t_{N-1},k_{N-1})\dots b^{\epsilon_1}(t_1,k_1)\rangle\rangle$$

must be valid.

The correlation function from Theorem 1 that corresponds to the index sequence ε is equal to the sum over semiplanar diagrams corresponding to partitions $\sigma(\varepsilon)$. The contribution of each semiplanar diagram is the product of arc terms. An arc term consists of the momentum δ-function with the temperature weight (this comes from the bosonic correlation function), the $2\pi\delta$-function w.r.t. the corresponding times, and the phase δ-function. The phase is the sum of the arc factor and the factors of all arcs that lie inside the given arc. This simple structure permits the correlation function to be expressed as the state of the algebra $\mathcal{B}$ whose structure is given by the following theorem.

Theorem 2. *Correlation functions of Theorem 1 are reproduced if the quantity $b(t,k)$ is the sum of two independent random quantities*

$$b(t,k) = b_1(t,k) + b_2^\dagger(t,k), \tag{19}$$

which satisfy the free temperature algebra relations, and if the functional $\langle\langle\cdot\rangle\rangle$ is equal to the mean vacuum for the fields $b_i(t,k)$. The generators $\{p, b_i(t,k), b_i^\dagger(t,k),\ i=1,2\}$, of the free temperature algebra satisfy the relations

$$b_1(t,k_1)b_1^\dagger(\tau,k_2) = 2\pi\delta(t-\tau)\delta(\tilde{\omega}(k_1)+k_1p)\frac{\delta(k_1-k_2)}{1-e^{-\beta\omega(k_1)}}, \tag{20}$$

$$b_2(t,k_1)b_2^\dagger(\tau,k_2) = 2\pi\delta(t-\tau)\delta(\tilde{\omega}(k_1)+k_1(p-k_1))\frac{\delta(k_1-k_2)}{e^{\beta\omega(k_1)}-1}, \tag{21}$$

$$b_1(t,k_1)b_2^\dagger(\tau,k_2) = b_2(\tau,k_2)b_1^\dagger(t,k_1) = 0, \tag{22}$$

$$b_1(t,k)p = (p+k)b_1(t,k). \tag{23}$$

$$b_2(t,k)p = (p-k)b_2(t,k). \tag{24}$$

Proof. The proof is by direct calculation. We expand the correlation function

$$\langle\langle b^{\epsilon_N}(t_N,k_N)b^{\epsilon_{N-1}}(t_{N-1},k_{N-1})\dots b^{\epsilon_1}(t_1,k_1)\rangle\rangle$$

in the sum of correlation functions of monomials in the generators $b_i^{\epsilon}(t,k)$. We use relations (20)–(22) for canceling in the monomial correlation functions. We push the obtained δ-functions between the $b_i^{\epsilon}(t,k)$ using relations (23) and (24). We continue this procedure until the monomial is normally ordered. Because the functional $\langle\langle\cdot\rangle\rangle$ is the mean vacuum, the correlation function contains only the δ-function contribution.

By virtue of relations (20) and (21), the pairings $b_1(t_{m'_h},k_{m'_h})b_1^{\dagger}(t_{m_h},k_{m_h})$ and $b_2(t_{m_h},k_{m_h})b_2^{\dagger}(t_{m'_h},k_{m'_h})$ are

$$\delta(k_{m'_h}-k_{m_h})c_{m_h m'_h}(k_{m_h})2\pi\delta(t_{m'_h}-t_{m_h})\delta\Big(\widetilde{\omega}(k_{m_h})+k_{m_h}p-\varepsilon_h k_{m_h}^2\Big). \tag{25}$$

Relations (23) and (24) contribute

$$\sum_{\alpha}(-1)^{\varepsilon_\alpha}\chi_{(m_\alpha,m'_\alpha)}(m_h)$$

in the phase shift (the argument of the last δ-function in (25)), which arises when pushing the δ-functions through the $b_i^{\epsilon}(t,k)$. This completes the proof of Theorem 2.

The fields b_i of the free temperature algebra appear in the stochastic limit of the Araki–Woods construction, which permits the temperature boson state in its bosonic variant to be represented through the vacuum state of the bosonic pair.

We introduce two independent bosonic fields $c_1(k)$ and $c_2(k)$ with the commutation relation

$$\left[c_i(k),c_j^{\dagger}(k')\right]=\delta_{ij}\delta(k-k').$$

Each of the fields $c_i(k)$ acts in the Fock representation. Performing the Bogoliubov transformation

$$a(k)=\sqrt{m(k)}c_1(k)+\sqrt{m(k)-1}c_2^{\dagger}(k), \tag{26}$$

$$a^{\dagger}(k)=\sqrt{m(k)}c_1^{\dagger}(k)+\sqrt{m(k)-1}c_2(k), \tag{27}$$

we obtain

$$[a(k),a^{\dagger}(k')]=\delta(k-k').$$

For the mean vacuums, the relation

$$\langle a(k)a^{\dagger}(k')\rangle=m(k)\delta(k-k')$$

holds. If we take

$$m(k)=\frac{1}{1-e^{-\beta\omega_k}},$$

then the temperature state arises.

In the stochastic limit, the operator $a_\lambda(t,k)$ becomes the sum of two white noises, $b_1(t,k)+b_2^{\dagger}(t,k)$. Formula (26) then becomes

$$\begin{aligned}b(t,k)&=\lim_{\lambda\to 0}\frac{1}{\lambda}\,e^{-i\frac{t}{\lambda^2}(\tilde\omega(k)+kp)}e^{-ikq}a(k)=\\&=\lim_{\lambda\to 0}\frac{1}{\lambda}\,e^{-i\frac{t}{\lambda^2}(\tilde\omega(k)+kp)}e^{-ikq}\sqrt{m(k)}c_1(k)+\lim_{\lambda\to 0}\frac{1}{\lambda}\,e^{-i\frac{t}{\lambda^2}(\tilde\omega(k)+kp)}e^{-ikq}\sqrt{m(k)-1}c_2^{\dagger}(k)=\\&=b_1(t,k)+b_2^{\dagger}(t,k).\end{aligned}$$

5. The n-point correlation function

Here, we calculate stochastic limits of n-point correlation functions for a particle interacting with a nonrelativistic quantum field, i.e., we prove Theorem 1.

Using the Weyl-operator product rules

$$e^{i\alpha p}e^{i\beta q} = e^{i(\alpha p+\beta q)}e^{i\frac{1}{2}\alpha\beta},$$

we obtain the free energy of field operator (7)

$$a_\lambda^\epsilon(t,k) \equiv \frac{1}{\lambda}\exp\left[-i(-1)^\epsilon\left\{\frac{t}{\lambda^2}(\tilde{\omega}(k)+kp)+kq-\frac{1}{2}\frac{t}{\lambda^2}k^2\right\}\right]a^\epsilon(k). \tag{28}$$

For the index sequence $\varepsilon = \{\epsilon_{2n},\dots,\epsilon_1\}\in\{1,0\}^{2n}$, we obtain

$$\left\langle\prod_{j=1}^{2n}a_\lambda^{\epsilon_j}(t_j,k_j)\right\rangle = \prod_{j=1}^{2n}\left\{\frac{1}{\lambda}\exp\left[-i(-1)^{\epsilon_j}\left\{\frac{t_j}{\lambda^2}(\tilde{\omega}(k_j)+k_jp)+k_jq-\frac{1}{2}\frac{t_j}{\lambda^2}k_j^2\right\}\right]\right\}\left\langle\prod_{h=1}^{2n}a^{\epsilon_h}(k_h)\right\rangle, \tag{29}$$

while

$$\left\langle\prod_{h=1}^{2n}a^{\epsilon_h}(k_h)\right\rangle = \sum_{m'_h\neq m_h}\prod_{h=1}^{n}\delta(k_{m'_h}-k_{m_h})c_{m_h m'_h}(k_{m_h}), \tag{30}$$

which is equal to the sum of pairings of all creation–annihilation operators. The operators are assumed to be ordered from right to left in these products; therefore, we obtain

$$\left\langle\prod_{j=1}^{2n}a_\lambda^{\epsilon_j}(t_j,k_j)\right\rangle = \prod_{j=1}^{2n}\left\{\frac{1}{\lambda}\exp\left[-i(-1)^{\epsilon_j}\left\{\frac{t_j}{\lambda^2}(\tilde{\omega}(k_j)+k_jp)+k_jq-\frac{1}{2}\frac{t_j}{\lambda^2}k_j^2\right\}\right]\right\}\times$$
$$\times\sum_{m'_h\neq m_h}\prod_{h=1}^{n}\delta(k_{m'_h}-k_{m_h})c_{m_h m'_h}(k_{m_h}). \tag{31}$$

Using the Weyl-operator product rules, we find

$$\prod_{j=1}^{2n}\left\{\frac{1}{\lambda}\exp\left[-i(-1)^{\epsilon_j}\left\{\frac{t_j}{\lambda^2}(\tilde{\omega}(k_j)+k_jp)+k_jq-\frac{1}{2}\frac{t_j}{\lambda^2}k_j^2\right\}\right]\right\} =$$
$$= \exp\left\{-\frac{i}{2}\sum_{1\le j<l\le 2n}(-1)^{\epsilon_j+\epsilon_l}k_jk_l\frac{t_j-t_l}{\lambda^2}\right\}\times$$
$$\times\left(\frac{1}{\lambda}\right)^{2n}\exp\left[-i\sum_{j=1}^{2n}(-1)^{\epsilon_j}\left\{\frac{t_j}{\lambda^2}(\tilde{\omega}(k_j)+k_jp)+k_jq-\frac{1}{2}\frac{t_j}{\lambda^2}k_j^2\right\}\right]. \tag{32}$$

Because the indices m'_h and m_h range over disjoint halves of the $2n$ indices l, $(-1)^{\epsilon_{m'_h}}=1$, and $(-1)^{\epsilon_{m_h}}=$

−1, we obtain the following expression for the phase factor entering formula (32):

$$-\frac{i}{2}\sum_{l=1}^{2n}\sum_{j<l}(-1)^{\epsilon_j+\epsilon_l}k_jk_l(t_j-t_l) =$$

$$= -\frac{i}{2}\sum_{h=1}^{n}\left\{\sum_{1\le j<m'_h}(-1)^{\epsilon_j}k_jk_{m'_h}(t_j-t_{m'_h}) - \sum_{1\le j<m_h}(-1)^{\epsilon_j}k_jk_{m_h}(t_j-t_{m_h})\right\} =$$

$$= -\frac{i}{2}\sum_{h=1}^{n}\left\{\sum_{\alpha}^{m'_\alpha<m'_h} k_{m'_\alpha}k_{m'_h}(t_{m'_\alpha}-t_{m'_h}) - \sum_{\beta}^{m_\beta<m'_h} k_{m_\beta}k_{m'_h}(t_{m_\beta}-t_{m'_h}) - \right.$$
$$\left. - \sum_{\gamma}^{m'_\gamma<m_h} k_{m'_\gamma}k_{m_h}(t_{m'_\gamma}-t_{m_h}) + \sum_{\delta}^{m_\delta<m_h} k_{m_\delta}k_{m_h}(t_{m_\delta}-t_{m_h})\right\} \equiv$$

$$\equiv -\frac{i}{2}\sum_{h=1}^{n}(\mathrm{I}_h+\mathrm{II}_h). \tag{33}$$

Here, we have used $k_{m_h} = k_{m'_h}$. Unifying the first term with the third term and the second term with the fourth term, we obtain

$$\mathrm{I}_h = \sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}(t_{m'_\alpha}-t_{m'_h}) - \sum_{\gamma}^{m'_\gamma<m_h} k_{m_\gamma}k_{m_h}(t_{m'_\gamma}-t_{m_h}) =$$
$$= \sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}(t_{m'_\alpha}-t_{m_h}) + \sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}(t_{m_h}-t_{m'_h}) - \sum_{\gamma}^{m'_\gamma<m_h} k_{m_\gamma}k_{m_h}(t_{m'_\gamma}-t_{m_h}) =$$
$$= \sum_{\alpha}^{m_h<m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}(t_{m'_\alpha}-t_{m_h}) + \sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}(t_{m_h}-t_{m'_h})$$

for $m'_h > m_h$ and

$$\mathrm{I}_h = -\sum_{\alpha}^{m'_h<m'_\alpha<m_h} k_{m_\alpha}k_{m_h}(t_{m'_\alpha}-t_{m_h}) + \sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}(t_{m_h}-t_{m'_h}) + k_{m_h}k_{m_h}(t_{m_h}-t_{m'_h})$$

for $m'_h < m_h$. The sum of the second and fourth terms is

$$-\mathrm{II}_h = \sum_{\beta}^{m_\beta<m'_h} k_{m_\beta}k_{m_h}(t_{m_\beta}-t_{m'_h}) - \sum_{\delta}^{m_\delta<m_h} k_{m_\delta}k_{m_h}(t_{m_\delta}-t_{m_h}) =$$
$$= \sum_{\beta}^{m_\beta<m'_h} k_{m_\beta}k_{m_h}(t_{m_\beta}-t_{m'_h}) - \sum_{\delta}^{m_\delta<m_h} k_{m_\delta}k_{m_h}(t_{m_\delta}-t_{m'_h}) - \sum_{\delta}^{m_\delta<m_h} k_{m_\delta}k_{m_h}(t_{m'_h}-t_{m_h}) =$$
$$= \sum_{\beta}^{m_h<m_\beta<m'_h} k_{m_\beta}k_{m_h}(t_{m_\beta}-t_{m'_h}) + \sum_{\delta}^{m_\delta<m_h} k_{m_\delta}k_{m_h}(t_{m_h}-t_{m'_h}) + k_{m_h}k_{m_h}(t_{m_h}-t_{m'_h})$$

for $m'_h > m_h$ and

$$-\mathrm{II}_h = -\sum_{\beta}^{m'_h<m_\beta<m_h} k_{m_\beta}k_{m_h}(t_{m_\beta}-t_{m'_h}) + \sum_{\delta}^{m_\delta<m_h} k_{m_\delta}k_{m_h}(t_{m_h}-t_{m'_h})$$

for $m'_h < m_h$. We now have

$$\begin{aligned}\mathrm{I}_h + \mathrm{II}_h = &\sum_{\alpha}^{m_h < m'_\alpha < m'_h} k_{m_\alpha} k_{m_h} (t_{m'_\alpha} - t_{m_h}) + \sum_{\alpha}^{m'_\alpha < m'_h} k_{m_\alpha} k_{m_h} (t_{m_h} - t_{m'_h}) - \\ &- \sum_{\beta}^{m_h < m_\beta < m'_h} k_{m_\beta} k_{m_h} (t_{m_\beta} - t_{m'_h}) - \sum_{\beta}^{m_\beta < m_h} k_{m_\beta} k_{m_h} (t_{m_h} - t_{m'_h}) - \\ &- k_{m_h} k_{m_h} (t_{m_h} - t_{m'_h})\end{aligned} \tag{34}$$

for $m'_h > m_h$ and

$$\begin{aligned}\mathrm{I}_h + \mathrm{II}_h = -&\sum_{\alpha}^{m'_h < m'_\alpha < m_h} k_{m_\alpha} k_{m_h} (t_{m'_\alpha} - t_{m_h}) + \sum_{\alpha}^{m'_\alpha < m'_h} k_{m_\alpha} k_{m_h} (t_{m_h} - t_{m'_h}) + \\ &+ \sum_{\beta}^{m'_h < m_\beta < m_h} k_{m_\beta} k_{m_h} (t_{m_\beta} - t_{m'_h}) - \sum_{\beta}^{m_\beta < m_h} k_{m_\beta} k_{m_h} (t_{m_h} - t_{m'_h}) + \\ &+ k_{m_h} k_{m_h} (t_{m_h} - t_{m'_h})\end{aligned}$$

for $m'_h < m_h$.

We consider the following term in formula (32):

$$\left(\frac{1}{\lambda}\right)^{2n} \exp\left[-i \sum_{j=1}^{2n} (-1)^{\epsilon_j} \left\{ \frac{t_j}{\lambda^2} (\widetilde{\omega}(k_j) + k_j p) + k_j q - \frac{1}{2} \frac{t_j}{\lambda^2} k_j^2 \right\}\right] \sum_{m'_h \neq m_h} \prod_{h=1}^{n} \delta(k_{m'_h} - k_{m_h}) c_{m_h m'_h}(k_{m_h}).$$

Because

$$\begin{aligned}&\sum_{1 \le l \le 2n} (-1)^{\epsilon_l} t_l k_l = - \sum_{1 \le h \le n} (t_{m_h} - t_{m'_h}) k_{m_h}, \\ &\sum_{1 \le l \le 2n} (-1)^{\epsilon_l} k_l q = 0,\end{aligned} \tag{35}$$

the desired term becomes

$$\left(\frac{1}{\lambda}\right)^{2n} \exp\left[i \sum_{1 \le h \le n} \frac{t_{m_h} - t_{m'_h}}{\lambda^2} \left(\widetilde{\omega}(k_{m_h}) + k_{m_h} p - \frac{1}{2} k_{m_h}^2\right)\right] \sum_{m'_h \neq m_h} \prod_{h=1}^{n} \delta(k_{m'_h} - k_{m_h}) c_{m_h m'_h}(k_{m_h}).$$

After the change of variables

$$\begin{cases} u_{m_h} = t_{m_h}, \\ v_{m_h} = t_{m_h} - t_{m'_h}, \end{cases} \tag{36}$$

we can formulate the following lemma.

Lemma 1. *For the correlation function in Theorem* 1, *we have*

$$\begin{aligned}\left\langle \prod_{j=1}^{2n} a_\lambda^{\epsilon_j}(t_j, k_j) \right\rangle = &\exp\left[-\frac{i}{2} \frac{1}{\lambda^2} \sum_{h=1}^{n} \{\mathrm{I}_h + \mathrm{II}_h\}\right] \left(\frac{1}{\lambda}\right)^{2n} \exp\left[i \sum_{1 \le h \le n} \frac{v_{m_h}}{\lambda^2} \left(\widetilde{\omega}(k_{m_h}) + k_{m_h} p - \frac{1}{2} k_{m_h}^2\right)\right] \times \\ &\times \sum_{m'_h \neq m_h} \prod_{h=1}^{n} \delta(k_{m'_h} - k_{m_h}) c_{m_h m'_h}(k_{m_h}).\end{aligned} \tag{37}$$

The phase factor in (37) is

$$\sum_{\alpha}^{m_h<m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}(-v_{m_\alpha}+u_{m_\alpha}-u_{m_h})+\sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}v_{m_h}-$$
$$-\sum_{\beta}^{m_h<m_\beta<m'_h} k_{m_\beta}k_{m_h}(v_{m_h}+u_{m_\beta}-u_{m_h})-\sum_{\beta}^{m_\beta<m_h} k_{m_\beta}k_{m_h}v_{m_h}-k_{m_h}k_{m_h}v_{m_h} \tag{38}$$

for $m'_h > m_h$ *and*

$$-\sum_{\alpha}^{m'_h<m'_\alpha<m_h} k_{m_\alpha}k_{m_h}(-v_{m_\alpha}+u_{m_\alpha}-u_{m_h})+\sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}v_{m_h}+$$
$$+\sum_{\beta}^{m'_h<m_\beta<m_h} k_{m_\beta}k_{m_h}(v_{m_h}+u_{m_\beta}-u_{m_h})-\sum_{\beta}^{m_\beta<m_h} k_{m_\beta}k_{m_h}v_{m_h}+k_{m_h}k_{m_h}v_{m_h}$$

for $m'_h < m_h$.

The Riemann–Lebesgue lemma implies that the oscillator factors of type $\exp(ik^2u/\lambda^2)$ vanish in the limit $\lambda \to 0$. Therefore, the partition $\{(m_h, m'_h)\}$ in (8) survives in this limit iff for arbitrary fixed $h = 1, \ldots, n$ and for any α, either

$$m_h < m_\alpha < m'_h \Leftrightarrow m_h < m'_\alpha < m'_h,$$

or

$$m_h > m_\alpha > m'_h \Leftrightarrow m_h > m'_\alpha > m'_h$$

i.e., we have a partition corresponding to a semiplanar nonintersecting diagram. Correspondingly, only nontrivial partitions of the sequence $\varepsilon = \{\epsilon_{2n}, \ldots\ldots, \epsilon_1\} \in \{1,0\}^{2n}$ contribute to the correlation function in the limit. Letting $\{(m_h, m'_h)\}$ denote the partition, we find the corresponding phase factor (38),

$$\sum_{\alpha}^{m_h<m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}(-v_{m_\alpha}-v_{m_h})+\sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}v_{m_h}-\sum_{\beta}^{m_\beta<m_h} k_{m_\beta}k_{m_h}v_{m_h}-k_{m_h}k_{m_h}v_{m_h} \tag{39}$$

for $m'_h > m_h$ and

$$-\sum_{\alpha}^{m'_h<m'_\alpha<m_h} k_{m_\alpha}k_{m_h}(-v_{m_\alpha}-v_{m_h})+\sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}v_{m_h}-\sum_{\beta}^{m_\beta<m_h} k_{m_\beta}k_{m_h}v_{m_h}+k_{m_h}k_{m_h}v_{m_h}$$

for $m'_h < m_h$.

We investigate the obtained phase factor. For $m'_h > m_h$, we have

$$\sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}v_{m_h}=\sum_{\alpha}^{m_h<m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}v_{m_h}+\sum_{\alpha}^{m'_\alpha\le m_h} k_{m_\alpha}k_{m_h}v_{m_h}.$$

Because $m'_\alpha \neq m_h$,

$$\sum_{\alpha}^{m'_\alpha\le m_h} k_{m_\alpha}k_{m_h}v_{m_h}=\sum_{\alpha}^{m'_\alpha<m_h} k_{m_\alpha}k_{m_h}v_{m_h}.$$

Hence, the phase factor is

$$-\sum_{\alpha}^{m_h<m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}v_{m_\alpha} + \sum_{\alpha}^{m'_\alpha<m_h} k_{m_\alpha}k_{m_h}v_{m_h} - \sum_{\beta}^{m_\beta<m_h} k_{m_\beta}k_{m_h}v_{m_h} - k_{m_h}k_{m_h}v_{m_h}.$$

In the case $m'_h < m_h$, the condition for the absence of intersections implies

$$-\sum_{\alpha}^{m'_h<m'_\alpha<m_h} k_{m_\alpha}k_{m_h}(-v_{m_\alpha} - v_{m_h}) = -\sum_{\alpha}^{m'_h<m_\alpha<m_h} k_{m_\alpha}k_{m_h}(-v_{m_\alpha} - v_{m_h}).$$

Therefore, the phase factor is

$$\sum_{\alpha}^{m'_h<m'_\alpha<m_h} k_{m_\alpha}k_{m_h}v_{m_\alpha} + \sum_{\alpha}^{m'_\alpha<m'_h} k_{m_\alpha}k_{m_h}v_{m_h} - \sum_{\beta}^{m_\beta<m'_h} k_{m_\beta}k_{m_h}v_{m_h} + k_{m_h}k_{m_h}v_{m_h}.$$

Introducing the notation

$$\mathrm{I}_h + \mathrm{II}_h = \Phi_h - (-1)^{\varepsilon_h}k_{m_h}k_{m_h}v_{m_h},$$

where $(-1)^{\varepsilon_h} = 1$ for $m'_h > m_h$ and $(-1)^{\varepsilon_h} = -1$ for $m'_h < m_h$, we obtain the formulas for the phase factor,

$$\Phi_h = -\sum_{\alpha\in(m_h,m'_h)\text{ or }(m'_h,m_h)} (-1)^{\varepsilon_h}k_{m_\alpha}k_{m_h}v_{m_\alpha} - \sum_{\alpha:h\in(m_\alpha,m'_\alpha)\text{ or }(m'_\alpha,m_\alpha)} (-1)^{\varepsilon_\alpha}k_{m_\alpha}k_{m_h}v_{m_h},$$

$$\sum_{1\le h\le n}\Phi_h = -2\sum_{1\le h\le n}\ \sum_{\alpha:h\in(m_\alpha,m'_\alpha)\text{ or }(m'_\alpha,m_\alpha)} (-1)^{\varepsilon_\alpha}k_{m_\alpha}k_{m_h}v_{m_h} =$$

$$= -2\sum_{1\le h\le n}\sum_{\alpha}(-1)^{\varepsilon_\alpha}\chi_{(m_\alpha,m'_\alpha)}(m_h)k_{m_\alpha}k_{m_h}v_{m_h}.$$

Here, $\chi_{(m_\alpha,m'_\alpha)}$ is the indicator of the interval (m_α, m'_α) or (m'_α, m_α). Therefore, the following lemma is proved.

Lemma 2. *The contribution of nonintersecting diagrams in the correlation function is*

$$\left(\frac{1}{\lambda}\right)^{2n}\exp\left[i\sum_{1\le h\le n}\frac{v_{m_h}}{\lambda^2}\left((\tilde\omega(k_{m_h}) + k_{m_h}p) + \sum_{\alpha}(-1)^{\varepsilon_\alpha}\chi_{(m_\alpha,m'_\alpha)}(m_h)k_{m_\alpha}k_{m_h} - \right.\right.$$
$$\left.\left. - \frac{1}{2}k_{m_h}^2 + \frac{1}{2}(-1)^{\varepsilon_h}k_{m_h}^2\right)\right]\sum_{m'_h\ne m_h}\prod_{h=1}^{n}\delta(k_{m'_h} - k_{m_h})c_{m_hm'_h}(k_{m_h}).$$

Using Eq. (11) and keeping only nontrivial partitions, we obtain

$$\lim_{\lambda\to 0}\langle a_\lambda^{\varepsilon_{2n}}(t_{2n},k_{2n})a_\lambda^{\varepsilon_{2n-1}}(t_{2n-1},k_{2n-1})\dots a_\lambda^{\varepsilon_1}(t_1,k_1)\rangle =$$
$$= \sum_{m'_h\ne m_h}\prod_{h=1}^{n}\delta(k_{m'_h} - k_{m_h})c_{m_hm'_h}(k_{m_h})2\pi\delta(t_{m'_h} - t_{m_h})\times$$
$$\times\,\delta\left(\tilde\omega(k_{m_h}) + k_{m_h}p + \sum_{\alpha}(-1)^{\varepsilon_\alpha}\chi_{(m_\alpha,m'_\alpha)}(m_h)k_{m_\alpha}k_{m_h} - \varepsilon_hk_{m_h}^2\right),$$

where $\{(m'_j, m_j)\colon j = 1,\dots,n\}$ is the nontrivial partition $\{1,\dots,2n\}$ associated with ε. Theorem 1 is proved.

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