PROBLEMS

11404. *Proposed by Raimond Struble, North Carolina State at Raleigh, Raleigh, NC.* Any three non-concurrent cevians of a triangle create a subtriangle. Identify the sets of non-concurrent cevians which create a subtriangle whose incenter coincides with the incenter of the primary triangle. (A *cevian* of a triangle is a line segment joining a vertex to an interior point of the opposite edge.)

11405. *Proposed by Ovidiu Furdui, Campia Turzii, Cluj, Romania.* Let P be an interior point of a tetrahedron $ABCD$. When X is a vertex, let X' be the intersection of the opposite face with the line through X and P. Let XP denote the length of the line segment from X to P.
(a) Show that $PA \cdot PB \cdot PC \cdot PD \geq 81 PA' \cdot PB' \cdot PC' \cdot PD'$, with equality if and only if P is the centroid of $ABCD$.
(b) When X is a vertex, let X'' be the foot of the perpendicular from P to the plane of the face opposite X. Show that $PA \cdot PB \cdot PC \cdot PD = 81 PA'' \cdot PB'' \cdot PC'' \cdot PD''$ if and only if the tetrahedron is regular and P is its centroid.

11406. *Proposed by A. A. Dzhumadil'daeva, Almaty, Republics Physics and Mathematics School, Almaty, Kazakhstan.* Let $n!!$ denote the product of all positive integers not greater than n and congruent to n mod 2, and let $0!! = (-1)!! = 1$. Thus, $7!! = 105$ and $8!! = 384$. For positive integer n, find

$$\sum_{i=0}^{n} \binom{n}{i} (2i-1)!!\,(2(n-i)-1)!!$$

in closed form.

11407. *Proposed by Erwin Just (Emeritus), Bronx Community College of the City University of New York, New York, NY.* Let p be prime greater than 3. Does there exists a ring with more than one element (not necessarily having a multiplicative identity) such that for all x in the ring, $\sum_{i=1}^{p} x^{2i-1} = 0$?

11408. *Proposed by Marius Cavachi, "Ovidius" University of Constanţa, Constanţa, Romania.* Let k be a fixed integer greater than 1. Prove that there exists an integer n greater than 1, and distinct integers $a_1, a_2, \ldots, a_n$, all greater than 1, such that both $\sum_{j=1}^{n} a_j$ and $\sum_{j=1}^{n} \phi(a_j)$ are kth powers of a positive integer. Here ϕ denotes Euler's totient function.

11409. *Proposed by Paolo Perfetti, Dept. Math, University "Tor Vergata", Rome, Italy.* For positive real α and β, let

$$S(\alpha, \beta, N) = \sum_{n=2}^{N} n \log(n)(-1)^n \prod_{k=2}^{n} \frac{\alpha + k \log k}{\beta + (k+1)\log(k+1)}.$$

Show that if $\beta > \alpha$, then $\lim_{N\to\infty} S(\alpha, \beta, N)$ exists.

11410. *Proposed by Omran Kouba, Higher Institute for Applied Sciences and Technology, Damascus, Syria.* For $0 < \phi < \pi/2$, find

$$\lim_{x\to 0} x^{-2} \left(\frac{1}{2} \log\cos\phi + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \frac{\sin^2(nx)}{(nx)^2} \sin^2(n\phi) \right).$$