Substituting stock with time: the effect of delivery spare time on safety stock

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Abstract

The aim of this paper is to explore some operative aspects of *virtual safety stock management* and specifically how the parameters of the traditional safety stock management model could be modified when the delivery due dates do no impose binding constraint on the production or the delivery pace. The presence of an extra-time for the delivery can be exploited to decrease the safety stock and/or to increase service level indeed.

The analysis has been carried out in the most general case in which safety stock is kept in order to protect from both the variability of the demand and the variability of supplier delivery lead times.

Introduction

As Krupp persuasively asserts, safety stocks are inevitable (Krupp, 1997). When inventory availability is measured in terms of the no-stockout probability per order cycle and the traditional safety factor approach to setting safety stock levels is employed, safety stocks are a function of the management-specified customer service level and the standard deviation of demand during delivery time. The customer service level is in this way translated into a safety stock level which is physically made available in the warehouse.

However, Clarke underlines the fact that by treating logistics systems in strict physical terms we impose constraints on them which can restrict their flexibility and can limit the utilisation of resources (Clarke, 1998). In a *virtual stockholding* environment, stocks should be treated in terms of their availability, not their identity or their physical form. Specifically, if high levels of service are to be met then high levels of safety stock need to be retained since demand can be very variable. The cost of holding safety stock, however, can be very high, and some safety stock might never get used. Actually, safety stock does not need to physically reside in the warehouse as far as it will be available when needed. Despite Clarke's important contribution to the literature and the interest that nowadays has been raised on the *virtual supply chain* management, it does not appear that virtual stockholding issues has been treated yet in a detailed operative manner and this concept is still seen in 2002 as a futuristic intuition.

When dealing with manufacturing supply chain indeed, it may happen that the delivery due dates, requested by the customers, do no impose binding constraint on the production or the delivery pace: ordered products, ready to be shipped, are kept in the warehouse even for one or two days before being loaded on the carriers. This situation may occur when the logistic chain to reach the customer is particularly short or fast, or when the carrier performs the deliveries only in certain day of the week. In this cases, when the order arrives, the product does not need to be physically available, as it will not be immediately loaded on the carrier.

The aim of this paper is to explore some operative aspects of *virtual safety stock management*, and specifically how all the parameters of the traditional safety stock management models are affected by the previously described opportunity. The presence of an extra-time for the delivery indeed can be exploited to decrease the safety stock and/or to increase service level and some heuristics to quantify this effect will be proposed.

Stock and time trade-offs

Companies approach the Safety Stock (SS) problem with the traditional trade-off between stockholding-cost and service level targets or stock-out costs: once a preliminary solution is reached, some more improvement are searched considering the opportunities of backorders, if possible. In this way however, stock availability is very much treated in black and white: let's consider a typical situation in which a retailer buys products from the supplier and sells them to the final customer, as shown in the figure 1 below:

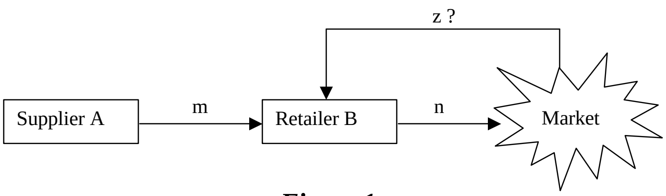

Figure 1

The market express a variable and non-deterministic demand. The delivery lead time from the supplier to the retailer is indicated as *m* in the example, while the delivery lead time from the retailer to the market is indicated by *n*. Z represents the delivery time requirements, that means that the customer is expecting the product to be delivered to destination within z time-units from his order. We shall identify four cases:

- $z > (m + n)$
- $z < n$
- $z = n$
- $n < z < (m + n)$

In the first case, the retailer will implement a pure pull-system. Neither safety stock nor cycle stock is kept in the retailer warehouse. This case is already well known in the literature.
In the second case, the retailer cannot comply with service requirements. Probably some solution to diminish the *n* time through a different logistic system or a compromise with customer's needs would be searched.

In both the last two cases, the retailer has to hold stock, either cycle stock or safety stock in case of variable demand or delivery lead times. The traditional SS theory however fails to efficiently distinguish them: if we do not consider the latter case, we would only deal with physical stock. On the contrary, the retailer has the possibility to wait $Y = (z - n)$ time units before loading the product on the carrier for the final delivery and this flexibility must have some influence on the possibility to lower SS level; specifically we will demonstrate that time Y can be intended as a *Virtual Safety Stock*.
The analysis will be carried out in the most general case in which safety stock is kept in order to protect from both the variability of the demand and the variability of supplier delivery lead times; anyway at first the two problems will be treated separately.

Demand variability

Let us start considering demand (D) variability and fixed supplier delivery lead times (DT): in the traditional SS analysis, an increase in the expected demand results in a higher number of product requested at the end of the period; for this reason safety stock is kept indeed. However for sure, an increase of the demand will always result in the exhaustion of cycle stock in advance with respect to when it was expected; Figure 2 shows the inventory level for a generic retailer in a supply chain and the normal distribution which describes the variability of the demand:

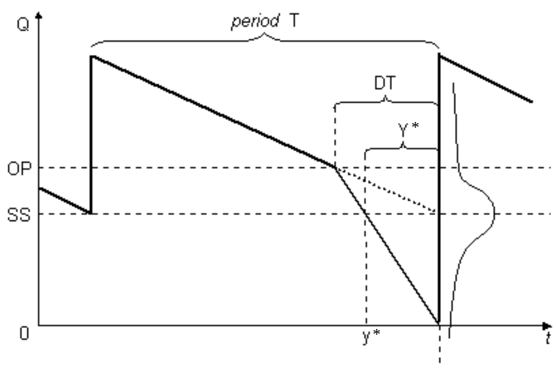

Figure 2

The result of a sudden increase in the demand – after the purchase order has been launched in the Order Point (OP) – is that the crossing point between the inventory line and the SS level will be in point (y^*) instead at the end of the DT. In the situation shown in the figure, the SS is anyway adequate to fulfill the demand while obviously a further increase in the demand would have resulted in a stockout. If we make the hypothesis of linear demand, we can say that if our safety stock has been chosen to assure that specific service level, we expect that we would not run out of cycle stock *before* time (y^*).

An optimal situation would be if we can fulfill the orders arrived after time (y^*) with the products of the new lot of cycle stock which will be available at the end of the DT. In this situation there would not be any need of SS, but we would need an *delivery spare time* (DST) which goes from (y^*) to the arrival of the new lot. Now we will perform a simple calculus focusing on Figure 3.

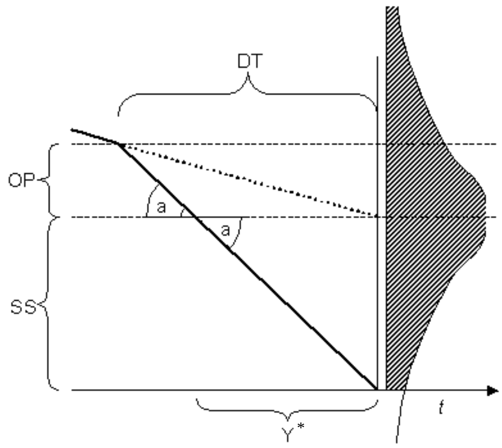

Figure 3

If we indicate with Y^* the time from y^* to the arrival of the new lot, we have:

$$Y^* \cdot tg(a) = SS_d$$

and

$$(DT - Y^*) \cdot tg(a) = OP = d \cdot DT$$

where d is the demand rate, hence it stands

$$Y^* = \frac{1}{1/DT + d/SS_d} = Y_d^*$$

that means that, for a certain service level, we are translating an information which concerns stock in an information which concerns time. In other words, we could have provided the same service level without any safety stock but only with a DST of ($Y=Y^*$) time-units, with respect to the variability of the demand and being fixed the delivery lead time. If we know that our aim is to fulfill any demand providing the cycle stock is not exhausted prior to time (y^*) – and that is equal to a certain service level with the hypotheses of linear demand, as it has been previously stated – we can substitute the entire safety stock with a delivery policy in which the ordered product is being loaded on the carrier Y^* days after the order is receipt.

For this reason we can say that the DST provides a *Virtual Safety Stock* (VSS). On the contrary we will refer to the traditional SS as *Physical Safety Stock* (PSS).

What will then happen to the inventory level in the next period? For sure in the first time there will be a demand increase which is inherited from the previous period, and it could happen that the OP will be now reached in a shorter time, or that we will need to increase the next supply order. But the inherited demand increase comes from the stochastic consumption of safety stock which has to reflect – in theory – a normal distribution over an infinite time horizon; thus we can expect that each demand increase will be compensated by a demand decrease in future periods, so that the eventual corrections to the lot-sizing order of each period would be minimal.

What about the case in which $Y < Y^*$? In this situation the DST is not big enough to substitute the entire safety stock so a combination of physical and virtual safety stock is needed. Figure 4 below shows an example:

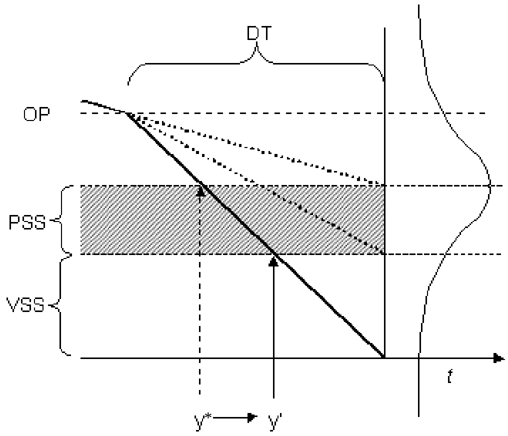

Figure 4

It is possible to see that the presence of a physical safety stock layer shifts the (y^*) point to the position (y'), that means that, as far as the virtual safety stock is concerned, we would need a shorter $Y=Y'<Y^*$ delivery spare time in order to reach a certain service level.
Now, being

$PSS_d = SS_d - VSS_d$

and

$VSS_d = Y' \ tg(a)$

since

$$tg(a) = \frac{d \cdot DT + k\sigma_d \sqrt{DT}}{DT} = d + \frac{k\sigma_d}{\sqrt{DT}}$$

we have

$$VSS_d = Y' \cdot d + Y' \frac{k\sigma_d}{\sqrt{DT}}$$

hence

$$PSS_d = k\sigma_d \sqrt{DT} - Y' \left[d + \frac{k\sigma_d}{\sqrt{DT}} \right]$$

In this way the SS is reduced with a factor Y'. With deterministic lead times it could be reasonable, in case the demand have increased over its expected value, to use firstly the physical stock and only when this is extinguished, to rely on the virtual safety stock.

Delivery lead time variability

Now we will consider variable delivery time while the demand will be fixed. Obviously a delay in the delivery of the new lot will be translated in a stockout if no SS if kept. If we consider a fixed demand, the cycle stock will be exhausted exactly at the end of the period T while the new lot will be still traveling towards our warehouses. Figure 5 shows the inventory level for a generic retailer in a supply chain and the normal distribution which describes the variability of the delivery lead times:

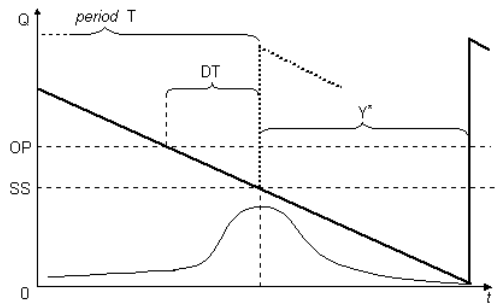

Figure 5

In this scenario the opportunity to exploit the DST is much easier. It would be sufficient that $Y=Y^*$ was equal to the maximum delivery delay that we wanted to bear and the Virtual Safety Stock would entirely substitute the physical safety stock. In formulas,

$Y^* \cdot d = SS_{DT}$

where *d* is the demand rate, hence it simply stands

$$Y^* = \frac{1}{d/SS_{DT}} = k \cdot \sigma_{DT} = Y^*_{DT}$$

from the comparison of this result with that obtained in the analysis of the case in which the demand was variable and the delivery lead time was not, it is possible to notice that always stands

$Y^*_d < Y^*_{DT}$

which means that the Virtual Safety Stock is always more effective when the demand is variable with respect to the case when the delivery lead time is variable.
Now we will analyze the combination of physical and virtual safety stock when the lead

time is variable and the demand is not, which is showed in the Figure 6 below.

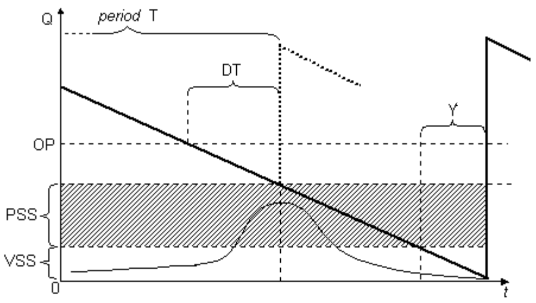

Figure 6

Even in this case the presence of a physical safety stock layer shifts the (y^*) point to the position (y'), that means that, as far as the virtual safety stock is concerned, we would need a shorter $Y=Y'<Y^*$ delivery spare time in order to reach a certain service level.
Now, being

$$PSS_{DT} = SS_{DT} - VSS_{DT}$$

and

$$VSS_{DT} = Y' \cdot d$$

simply stands

$$PSS_{DT} = k \cdot \sigma_{DT} \cdot d - Y' \cdot d$$

Even in this case the SS is reduced with a factor Y'. It is possible to see that, in accordance to what have been previously said, the physical safety stock needed to reach a certain service level in conjunction with virtual safety stock is bigger when the lead time is variable than when the demand is variable.

A simple heuristic to consider simultaneous demand and delivery time variability

Now we will analyze the case in which both demand and delivery lead time are variable. That means that we have two components which independently contribute to the system variability in an independent way. From the traditional analysis of the use of the SS in operations management it is known that the effect of the demand variability and the effect of the DT variability are compensative. In other words, we have that:

$$SS < SS_d + SS_{DT}$$

where SS_d represents the safety stock when only the demand is variable and SS_{DT} represents the safety stock when only the delivery time is variable. It is possible to identify a precise amount of safety stock which can be intended as shared among the two variability components; in the Figure 7 are shown three components of the safety stock:

- SS(DT) indicates the amount of stock which can be intended to be dedicated to the delivery time variability. Obviously it stands $SS(DT) < SS_{DT}$
- SS(d) indicates the amount of stock which can be intended to be dedicated to the demand variability. Obviously it stands $SS(d) < SS_d$
- SS(shared) is the amount of stock which can be used to protect from both demand or DT variability. It stands:

$$SS=SS(shared)+SS(d)+SS(DT)< SS_d+ SS_{DT}$$

These three components are shown in dependence to service level.

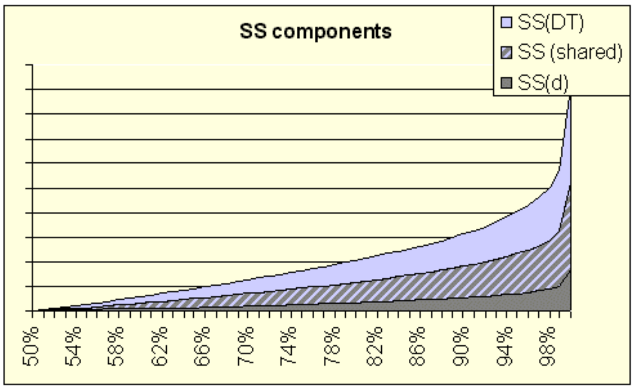

Figure 7

The same approach could be used to determine the percentage of physical safety stock needed in the case of demand and delivery time variability starting from the physical safety stock needed in the two separate cases. However this would lead to the definition of binary and non-continuous functions which

would not be so easy to use and evaluate in practice.
For this reason a simpler heuristic is proposed: we considered the presence of compensation factors only on the virtual safety stock while we considered that the physical safety stock amounts in the two variability case should simply be summed. Under this hypothesis we decided to analyze the effectiveness of the sum of the two physical safety stock, which means we define:

$$PSS_{TOT} = PSS_d + PSS_{DT}$$

with

$$PSS_d = k\sigma_d\sqrt{DT} - Y'\left[d + \frac{k\sigma_d}{\sqrt{DT}}\right]$$

$$PSS_{DT} = k \cdot \sigma_{DT} \cdot d - Y' \cdot d$$

In lot of real cases, the amount of PSS_{TOT} is less of the amount of traditional safety stock. For example Figure 8 shows the amount of PSS_{TOT} compared to the amount of traditional SS in dependence to service level, in a case in which:

d = norm(1000,200);

DT = norm(0.5,0.1);

That means that the demand and the delivery time are both stochastic variables that follow a gaussian distribution.

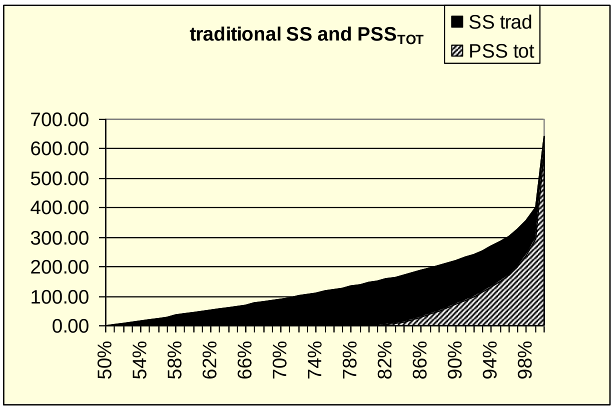

Figure 8

This is a case in which the traditional SS is always more than PSS_{TOT}, which means that the PSS_{TOT} strategy is effective for every chosen service level.
Indeed it is possible to see that only when both the standard deviation of the delivery time and the demand are very big in comparison to their average value, the required PSS_{TOT} would be more than the traditional SS: the Figure 9 shows as a 3-D surface the PSS_{TOT} strategy effectiveness, the three axis being:

[$(SS\text{-}PSS_{TOT})$; (σ_{DT}/DT); (σ_d/d)]

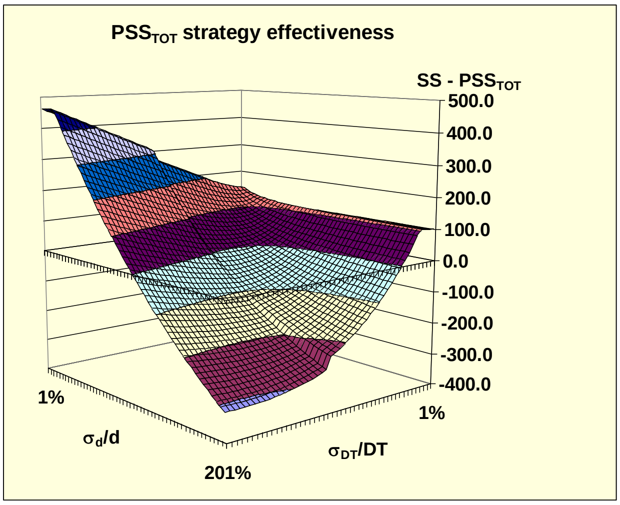

Figure 9

If we want to identify the couples $(\sigma_{DT}/DT; \sigma_d / d)$ for which the PSS_{TOT} strategy is effective we can look at the Figure 10 which describes the area in which stand:

[$SS - PSS_{TOT} > 0$]

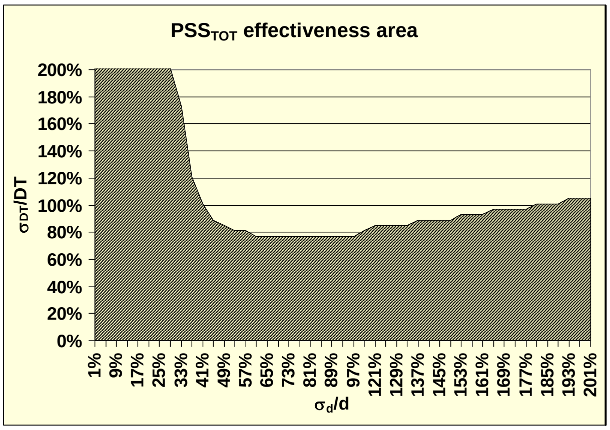

Figure 10

It is possible to see that unless

[$(\sigma_{DT}/DT) < 80\%$]

and

$[(\sigma_d / d) < 30\% \text{ and } (\sigma_{DT} / DT) < 200\%]$

the PSS_{TOT} strategy should always be preferred to the traditional SS formula.

Future research

This example shows how a simple heuristic which exploit the Virtual Safety Stock strategy, such as the PSS_{TOT} strategy, is anyway very effective in lots of real scenarios.
However the PSS_{TOT} strategy is clearly very little efficient, for the fact that does not consider the compensation factor among the physical stock for the variability of both the demand and the delivery time. This simplification results anyway in an unexpected increase in the service level.
Meanwhile, as it has been previously said, we should not forget that the implementation of the Virtual Safety Stock technique may lead to the generation of emergency orders to replenish the cycle stock in case the demand increases continuously for a relatively high number of periods; the cost for the launch of these emergency orders may overcome the savings coming from the decrease in the safety stock level, and this eventuality should be analyzed in future works through a simulative approach.
Lastly, there are much more opportunities, to reduce the safety stock, which originate from the Virtual Safety Stock technique and which go well beyond the results obtained with the PSS_{TOT} strategy; these opportunities may lead to new lines of research in which the stockholding costs in the supply chain are minimized and inventory management is optimized without the need of any major compromise or costly tradeoff with other company functions.

Main references

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