EFFECTIVE COMPLEX CONDUCTIVITY OF SKELETAL MUSCLE IN THE RADIO-FREQUENCY RANGE

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INTRODUCTION

This work deals with the dielectric properties of biological tissues comprising tubular cells, such as skeletal muscle. This issue is of significance to many applications for noninvasive diagnosis and treatment, such as electrical impedance tomography, body composition, dialysis, radio-frequency hyperthermia and ablation [1].

The dielectric properties of tissues vary as a function of frequency. Experiments show indeed three dispersions, α, β and γ, mainly attributed to different relaxation processes: ionic diffusion, interfacial polarization and dipolar orientation, respectively [2]. The β dispersion, considered herein, takes place in the radio-frequency range and principally arises from the capacitive charging of cell membranes, known as Maxwell-Wagner effect. Different phenomenological relaxation models are available in the literature (*e.g.* [3]), as well as equivalent-circuit models (*e.g.* [4]), which however pose the problem of parameter identification.

In the present work, a micromechanical approach is used, which enables to derive the effective dielectric properties of the tissue from the properties of the constituent phases and to take into account microstructural details.

METHODS

The tissue is modelled as a two-phase fibrous composite material composed of a periodic hexagonal arrangement of conductive circular cylinders, representing the intra-cellular phases, embedded in a conductive matrix, modelling the extra-cellular phase (Fig.1). The interfaces between the cylinders and the matrix exhibit a capacitive impedance, taking into account the dielectric behaviour of cell membranes. The problem of the electrical conduction in this composite is analytically solved by employing the asymptotic homogenization method [5], whose central step is the solution of the so-called cell (or local) problem. In most cases, numerical techniques are needed; here it is solved in closed form, making use of Weierstrass elliptic functions [6].

RESULTS AND DISCUSSION

The above mentioned approach leads to a simple closed-form formula for the effective complex conductivity of the idealized tissue, which, to the authors' knowledge, is new in the literature. This formula has been validated by using finite-element solutions as a benchmark and a complete agreement has been obtained.

A comparison with the well known Pauly-Schwan (PS) [7] and Hanai-Asami-Koizumi (HAK) [8] models of the effective complex conductivity of cell suspensions is also presented. As shown in Figure 2, the present results are close to the ones supplied by the HAK-theory, which is believed to behave better than the PS theory at high values of fibre volume fractions, such as the value pertaining to skeletal muscle. Moreover, the number of relaxation processes accounted for by the present theory is discussed and their relative importance is investigated. The results show the prevalence of one relaxation process. This implies that electric interactions between fibres, here rigorously considered, can partially account for the experimentally observed broadening of the distribution of relaxation times in real tissues.

Eventually, a parametric analysis is performed, emphasizing the influence of microstructural parameters on the conductivity locus. For example, the curves in Figure 3 show a sharp dependence of the effective complex conductivity on the fibre volume fraction and, hence, they may be useful in estimating the latter quantity. This issue has clinical significance in situations implying changes in the body water content (*e.g.*, during dialysis).

Future research will consider different arrangements, uneven distribution of fibre size, irregular fibre shape, and 3D geometries.

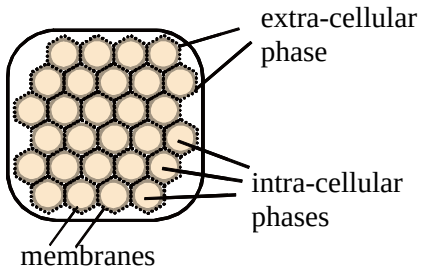

Figure 1: Geometry of the idealized tissue: cross section (the fibre size is exaggerated with respect to the sample size for illustrative purposes).

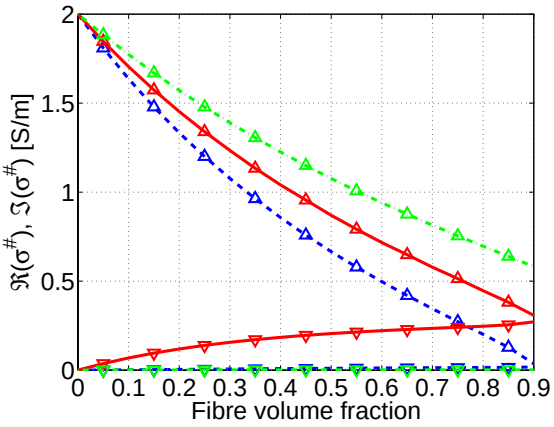

Figure 3: Effective complex conductivity $\sigma^{\#}$ vs. fibre volume fraction for different frequencies. Very low frequency: green dash-dotted; characteristic frequency: red solid; very high frequency: blue dashed. Real, $\Re$, [resp., imaginary, $\Im$,] part: triangles up [resp.,down].

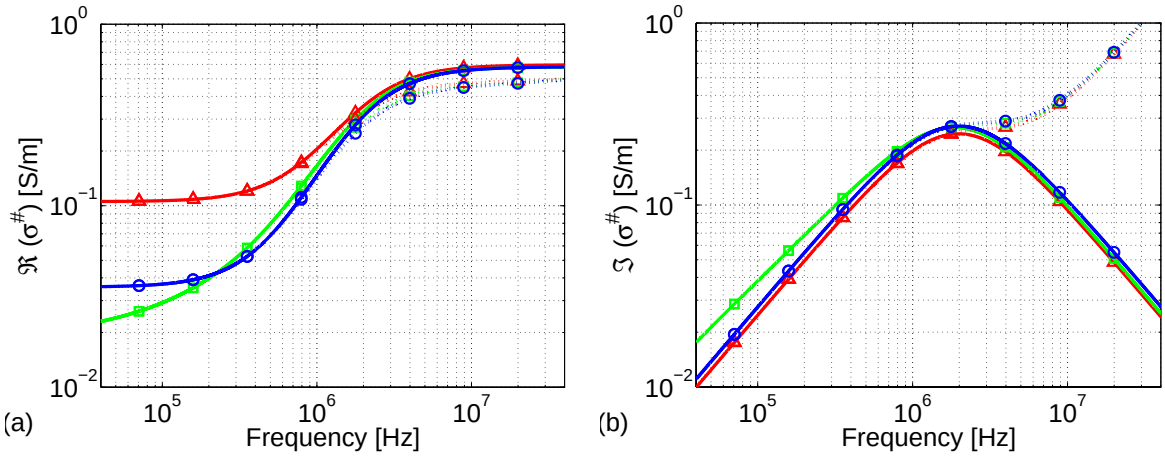

Figure 2: (a) Real part, $\Re$, and (b) imaginary part, $\Im$, of effective complex conductivity $\sigma^{\#}$ vs. frequency. The permittivities of the intra- and extra-cellular phases are neglected (solid lines) or taken into account (dotted lines). Present model: blue/circles; PS-type model: red/triangles up; HAK-type model: green/squares.

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