ELECTRICAL SIMULATION OF A CANTILEVER BEAM

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Cantilever beams are interesting transducers for chemical, physical and biological sensors. The loadings of these devices, caused by adsorption/absorption processes, can be detected by measuring the variation of the resonance frequency. The sensitivity of these transducers are related essentially to the first natural resonant mode. Damping vibration strategies have also proved to be of paramount relevance in all those cases where vibrations represent disturbances.
Knowledge of the dynamic behavior of cantilevers, whatever the specific application may be, is mandatory for better development of the system itself.
For this reason a study of an appropriate model of the cantilever system has been conducted for an accurate prediction of its dynamics.

1. Introduction

Cantilever beams are often integrated into microsystems as part of sensors.
The simulation of electro-mechanical systems can be difficult because usually the simulators evaluate only the electrical or mechanical behavior of the device.
In order to avoid this problem we have used an electronic model for the cantilever beam and the transducer, so that it is possible simulate the whole system on an electrical simulator.

2. The experiment

As shown in Figure 1, the system under study is a steel cantilever beam (250mm length, 40mm width and 1.5mm thick) with piezoelectric wafers applied on both sides [1]. The piezoelectric wafers have been used to force the cantilever and to detect its vibration. Its role is to link the mechanical system to the electrical system. The parameters used for the electronic simulation of the cantilever beam have been evaluated from the sample data of the free damping vibration and force vibration.
The resonance frequencies of a local point of the cantilever have been measured by an oscilloscope connected to the upper piezoelectric wafer, while a sinusoidal wave generator was connected via an amplifier to the bottom piezoelectric wafer

(actuator). The resonance frequency was estimated by evaluating the amplitude of the response signal (its maximum level corresponds to the resonance.

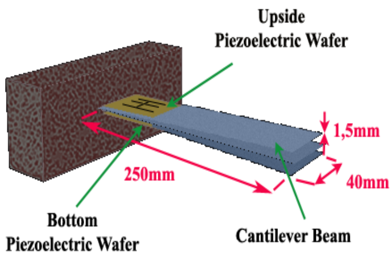

Figure 1 – The cantilever beam.

Table 1 – Cantilever beam resonance frequency

Actuator peak to peak voltage (Vpp)	1.7	[V]
1^{st} resonance freq.	21.1	[Hz]
Phase (input-output)	65	[degree]
Sensor Vpp (output)	2.3	[V]
3dB band pass	~ 0.2	[Hz]
2^{nd} resonance freq.	127.7	[Hz]
Phase (input-output)	90	[degree]
Sensor Vpp	.8	[V]
3dB band pass	~ 0.9	[Hz]

3. The electrical equivalent model

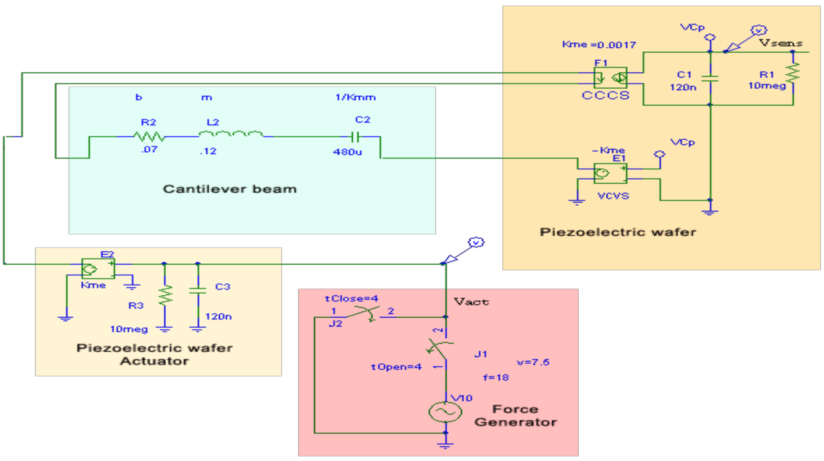

Figure 2 – The electrical model of the system.

The system of mechanical and piezoelectric device equations can be written as:

$$\begin{cases} m\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + k_{mm}x + k_{me}V(t) = F(t) \\ k_{me}x + k_{ee}V(t) = -q \end{cases} \qquad 1)$$

where

x :	one dimensional displacement	[m]
m :	mass of the structure (lever)	[kg]
b :	coefficient of viscous damping	[N s m^{-1}]
F :	force	[N]
t :	time	[s]
V :	piezoelectric voltage	[Volt]
q :	charge	[Coulomb]
K_{ij} :	Piezoelectrical parameter: $\begin{cases} \text{m : mechanical} \\ \text{e : electrical} \end{cases}$	

Considering the equation of a RLC series electrical circuit[1], it is possible to rewrite the equations (1) as [2]:

$$\begin{cases} m\dfrac{di}{dt} + bi + k_{mm}\int idt + k_{me}V_1(t) = V(t) \\ k_{me}i + C_1\dfrac{dV_1(t)}{dt} = -i_1 \end{cases} \qquad 2)$$

where there is a mathematical equivalence, with no physical meaning, between the mechanical parameters and the electrical parameters shown in Table 2.

The relations:

$$k_{me}V_1(t) = \tilde{V}(t)$$
$$k_{me}i(t) = \tilde{i}(t)$$

can be modeled respectively as a VCVS (Voltage Controlled Voltage Source) and as a CCCS (Current Controlled Current Source). Figure 2 shows the equivalent electrical circuit where:

Table 2: corresponding parameters

Mechanical Parameters	Electrical Parameters
m	L_{series}
b	R_{series}
$1/k_{mm}$	C_{series}
$1/k_{ee}$	C_{sens}
dx/dt	*i*
F	*V*

- E1 and F1 are, respectively, the VCVS and the CCCS; their gain is equal to G.
- the cantilever beam is represented by R2=R_{series}, L2=L_{series}, C2=C_{series};
- the piezoelectric sensor is represented by E1, F1, C1=C_{sens} and R1=R_{sens};
- the piezoelectric actuator is represented by E2, C3 and R3;
- the force generator V10 is timing by the switches J1 and J2.

[1] The equation of a RLC series electrical circuit can be written as: $L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = V(t)$ [3]

The gain of the transducer can be estimated measuring, at the resonance frequency $f = f_1$, the input voltage of the piezoelectric actuator, V_{act}, and the output voltage of the piezoelectric sensor, V_{sens}.
The expression of the gain G, shown in the equation (3), is obtained evaluating the modulus of current, i , through the RLC series circuit :

a) $i \cong \frac{V_{sens}\omega C_{sens}}{G}$ from the analysis of the piezoelectric sensor circuit ($R_{sens} >> \frac{1}{\omega C_{sens}}$)

b) $i = \frac{G(V_{act} - V_{sens})}{Z_{series}}$ from the analysis of the RLC series circuit

where

$$Z_{series} = \sqrt{R_{series}^2 + \left(\frac{\omega^2 L_{series} C_{series} - 1}{\omega C_{series}}\right)}$$

comparing a) and b) :

$$G^2 \cong \frac{V_{sens}\omega C_{sens} Z_{series}}{V_{act} - V_{sens}} \qquad 3)$$

4. Results and discussion

Using electrical simulators it is possible to calculate the cantilever vibrational amplitude of a local point of its surface (Figure 3). The advantage is that by using programs like PSpice it is easy to simulate the behavior of the mechanical and the electrical system joined together.

Table 3 : Comparison between real and simulation data

Free damped oscillation			
	Real data	Simulation data	
Start amplitude	0.27	0.29	[Volt]
Frequency	21.1	20.10	[Hz]
ζ (damped coefficient)	0.002323	0.001935	
Force damped oscillation (beating)			
Start amplitude	2.72	2.70.	[Volt]
Beating frequency	2.37	2.30	[Hz]
ζ (damped coefficient)	0.008378	0.01018	

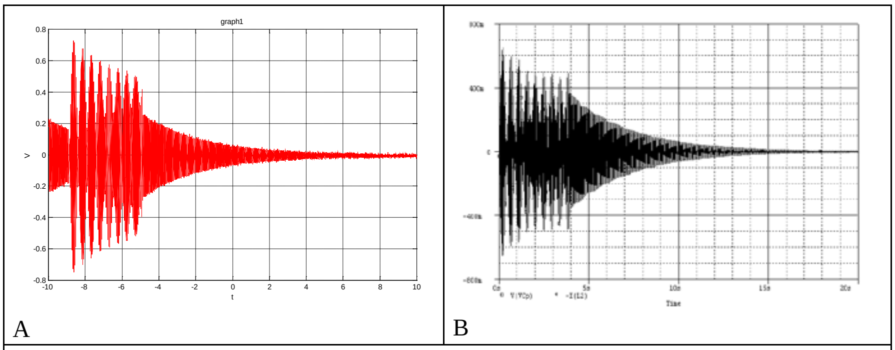

Figure 3 –The comparison between the sample data (A) and the simulation data (B)

Table 3 shows the comparison of the evaluated principal parameters [4] between the simulation data and the real data. The error can be reduced empirically by changing the parameter given by the equation model.

5. Conclusion

In this paper we have shown an electrical equivalent model for a cantilever beam connected to piezoelectric wafers.

The accuracy of the simulation depends of the goodness of the measurements on the physical device. The result of the simulation can then be improved empirically.

The limit of the electrical simulation is that it evaluates the behaviour of the cantilever beam where the piezoelectric wafer transducer is placed, i.e. only in a local area of the two dimensional cantilever.

6. References

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