# Investigation on pin-hole connection in flexible assembly

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## Resumen

En algunas tipologías de acoplamientos mecánicos el correcto montaje es conseguido también disfrutando la capacidad de deformación elastica de los elementos, más bien muchas veces esta característica garantiza el respeto de la "condición funcional" del mecanismo completo. En este caso particularmente difícil resulta el cálculo y la asignación de las tolerancias dimensionales y geométricas. En el diseño GT&D de las parte mecanicas en efecto, las mismas son consideradas rígidas y la acotación funcional del objeto no tiene en cuenta la suya capacidad de deformación elastica. La sola normativa relativa a esta situación es la UNI ISO 10579, que sin embargo abastece solamente indicaciones generales.

En esta situación, màs que las clásicas consideraciones y metodologías empleadas usualmente en la asignación de las tolerancias, también necesita tener en cuenta la deformablidad de los componentes, en relación a las tolerancias admisibles y a el nivel de los esfuerzos alcanzados por la deformación de las partes en el montaje.

En el presente trabajo se propone una metodología para estimar los efectos combinados de los errores geométricos y de las deformaciones admisibles, para asignar tolerancias que garanticen el respeto de las condiciones funcionales en el acoplamiento. A tal fin ha sido examinado un caso de acoplamiento simple de una plancha con dos pernos cilíndricos y una otra con dos agujeros para evaluar los efectos combinados de las tolerancias geométricas y la deformación de los pernos. Todos los posibles acoplamientos, en el respeto de la condición funcional, han sido calculados, antes, considerando los cuerpos como rígidos, y, siguientemente, considerando las deformaciones de los pernos admisibles. La evaluación de las partes desechadas ha sido calculada a través de un criterio estadístico que emplea el método Montecarlo y el análisis tensional a través de modelos FEM.

## Abstract

In some kind of mechanical products, the correct assembly is obtained considering also the deformations of every single element; sometimes this condition ensures the respect of design functionality. In this case, the calculation and allocation of dimensional and geometric tolerances is particularly difficult. In fact, with common GT&D method, the single parts are considered as rigid and the functional tolerancing method does not take into account deformations. The only reference standard is UNI ISO 10579 for flexible assembly. Moreover, its prescriptions are only general indications useful only for dimensional check. In such event, it must be held in consideration also the deformability of every part together with the imposed tolerances and together with the stress level due to the assembly process. This paper proposes a methodology to estimate the combined effects of geometric errors and admissible deformations, for assigning tolerances that ensure the respect of functional requirements of the assemblies.

A simple pin-hole connection has been proposed as an example; the case examined is composed of two cylindrical pins and their relative holes. The assessment of multiple effects of geometric tolerances and pin deformations, and the possibility of connections with respect to functional conditions are been investigated. First of all, all possible assembly configurations have been estimated considering the bodies as rigid. Subsequently, all not mountable assemblies have been analyzed including pin deformations. The percentage of reject parts has been carried out with a statistical method (Montecarlo method) while the stress analysis has been performed using Finite Elements Model.

**Key words**: Geometric and dimensional tolerances, localization errors, flexible assemblies.

## 1. Introduction

In order to have a correct assembly, geometric and dimensional errors, caused by the natural variability of the production process, have to be held under control. The studied assembly is made of two plate, one with the pins on its surface and the other with the relative holes. Localization errors of holes and pins centres have been observed and controlled with opportune indications about the tolerance range , and the shape of the pins will be also considered in relation to the parameters of vertical inclination $\varphi$ and the inclination $\delta$ of in XY plane (figure1).

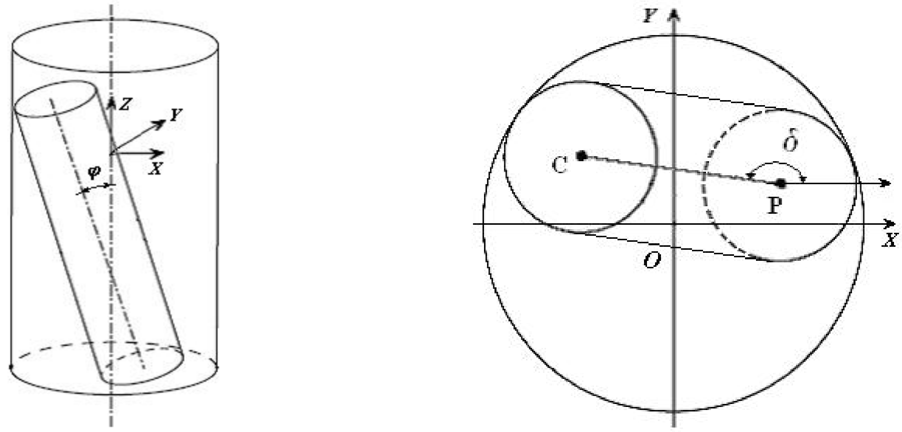


Figure 1 : Geometric parameters $\varphi$ and $\delta$

In particular, it has been analyzed an assembly with two pin-hole connections (figure 2). Concerning the possible errors, always within the design range, all three translations have been allowed, two in the connection plane and one in vertical direction, and it has been also allowed only one rotation around a axis which is perpendicular to the connection plane. The other two rotations have been neglected.

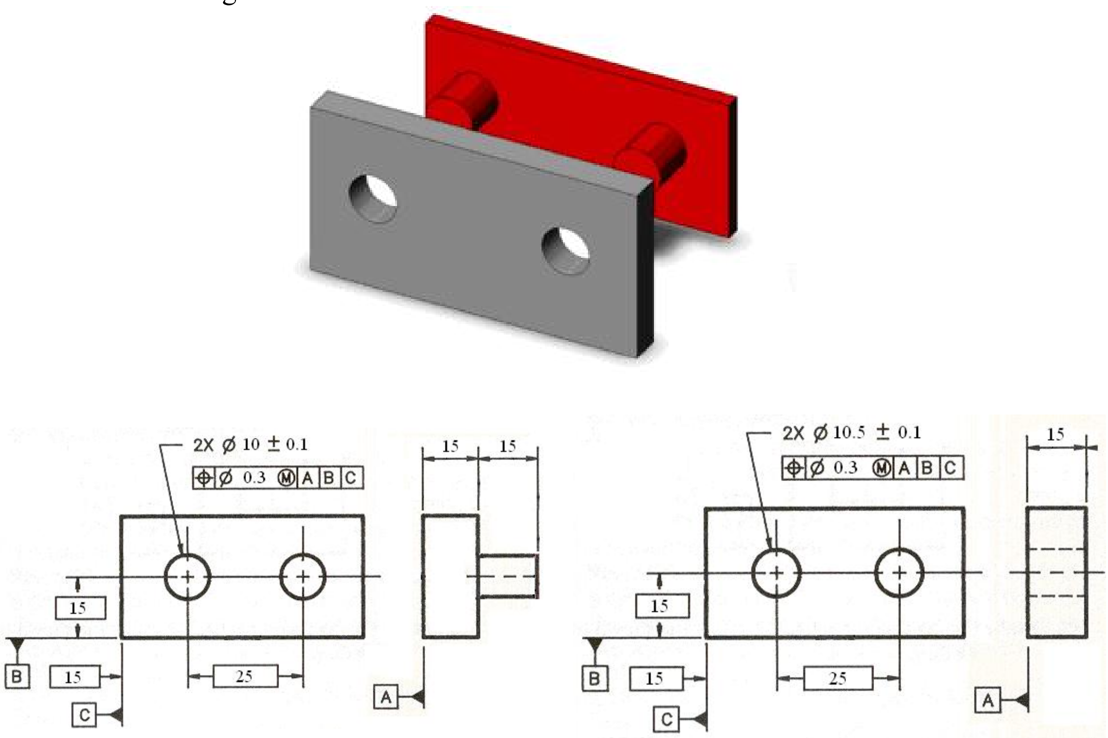


Figure 2 : Examined assembly and applied tolerances

Moreover a population of possible assemblies has been generated, respecting imposed tolerances; for all these components the possibility to be assembled has been verified.

Four possible configurations have been allowed for the pin-hole connection. As shown in figure 3 the possible cases are: a) there is no contact with the lateral surface of pin and hole, b) there is only one point of contact at one end of the pin, c) there are two points of contact (one at each end) d) there is a line contact along one entire length of the pin.

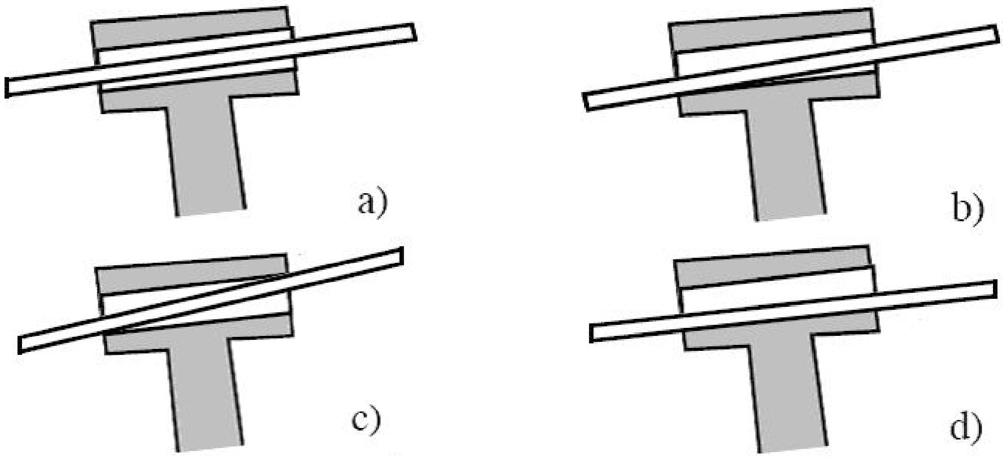

Figure 3 : Possible configurations allowed for pin-hole connection

Subsequently all rejected assemblies have been analyzed taking into account their deformability, in particular way only the pin has been considered deformable, assuming the hole as a rigid body. Then the percentage of consequently mountable assemblies has been assessed, in accordance with levels of stress and deformations which does not reduce resistance and life of components.

## 2. The geometric model and the implemented method.

In this paragraph it will be described the geometric model with all the assumptions and simplifications chosen for the examined problem [1]. First of all, the projections of base and head circles of pins on XY plane have been assumed perfectly circular (figure 4).

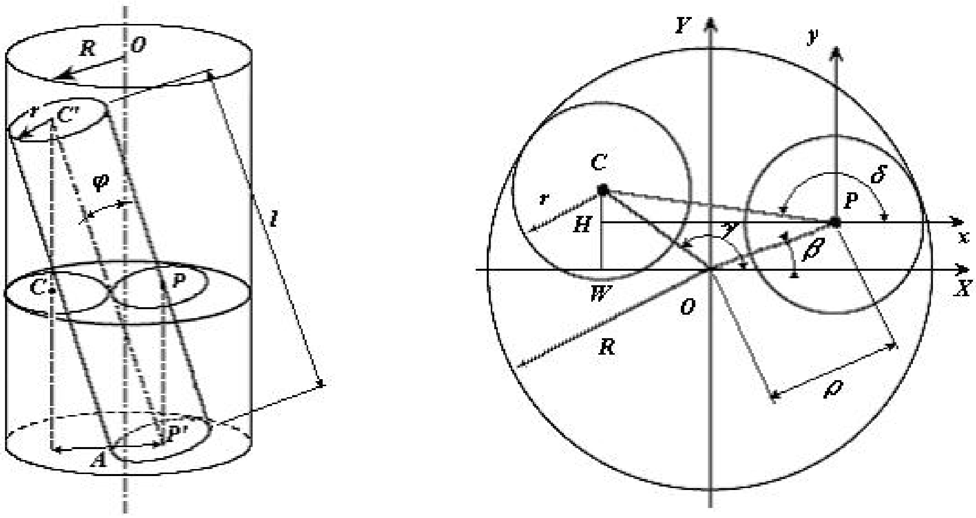

Figure 4 : Geometric model of pin-hole connections

For every couple of parts to assemble, the equations linking the geometric characteristics of the pin with the maximum possible angle for a correct assembly have been written. For this reason, two different vector loops (in figure 5) have been considered [2] [3].

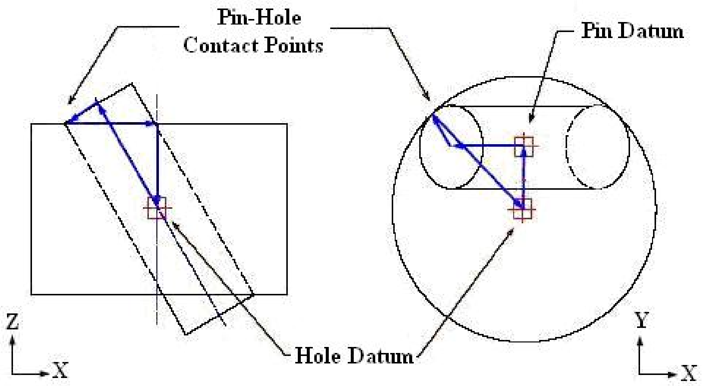


Figure 5 : Vector loops

Let us consider figure 4: it is possible to write the expression of the maximum inclination angle $\varphi_{\max}$, so as to carry the upper pin circumference in contact with the cylindrical surface of the hole.

The expression of $\varphi_{\max}$ is given by:

$$\varphi_{\max} = \arcsin\left(\frac{A}{l}\right)$$

where *l* is the length of the pin and $CP = A = \sqrt{\left(X_P - X_C\right)^2 + \left(Y_P - Y_C\right)^2}$.

Moreover the angle $\delta$ :

$$\delta = 180 - \arcsin\left(\frac{CW - \rho\sin\beta}{CP}\right)$$

where $CW = (R-r)\sin\gamma$ and $CP = A = \sqrt{\left(X_P - X_C\right)^2 + \left(Y_P - Y_C\right)^2}$

In the same way, the expressions for $\delta$ with respect to the relative position of the centres *C* and *P* inside the hole can be deduced. Moreover, $\gamma$ is the angle that describe the position of the pin upper circle measured from the hole centre.

For an assigned position of the pin (pin radius $r$ = 8 mm, hole radius $R$ = 20 mm, $\rho$ =12 mm, $\beta$ =30°), the graphs representing the angles $\varphi$ and $\delta$ are shown in figure 6.

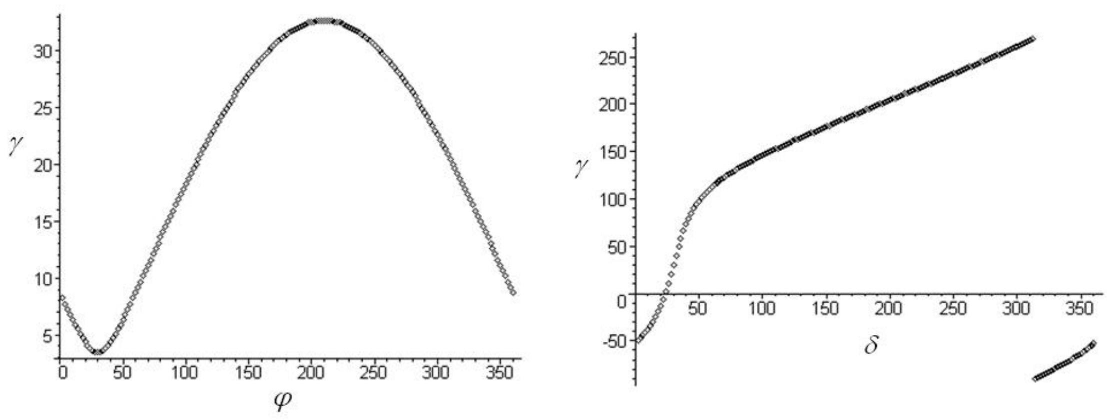


Figure 6 : Angles $\varphi$ e $\delta$

Therefore, it's easy to determine the possibility to assemble or not the two parts comparing the angle $\varphi_{\max}$, previously found, with the real inclination of the object examined. Logically, if this real angle were greater than the maximum value calculated the assembly would not be possible and the parts would be discarded or opportunely modified.

About the studied case, there are two pins that must be inserted in the respective holes (the method can be extended in the same way to connections with more than two pins). The relative positions and distances between the two pins centre are known, assigning tolerances of localization for both of them. So, the base circle of the first pin has been placed, in several locations, inside the hole verifying its assembly; subsequently, knowing the position of the second pin, it has been verified if its base circle falls inside of the respective hole and, then, if its inclination allows a correct assembly, in the same way as previously exposed. Logically if at least a configuration exists for which all the previous steps are verified, then the two parts can be correctly assembled, otherwise these two parts can not be assembled together, considered as rigid bodies.

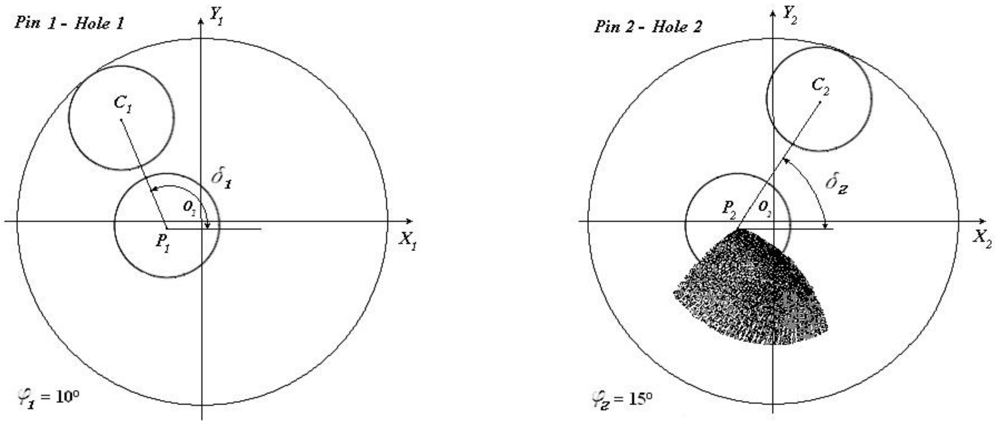

Figure 7: Possible localizations of the pin inside the hole

One of the result of, the described procedure for an assembly with two holes (distance between holes $=\overline{O_1O_2}$ ) is illustrated in figure 7. The dark region represents the zone in which it is possible to locate the 2nd pin centre inside the hole (it's shown only on the second hole because, for sake of simplicity, in this case the errors of localization of the pins and the assembly rotation are summed to be equal zero).

All the parts rejected can be examined again introducing a possible deformation [4] [5] of the pins. The new procedure consists in a gradually reduction of the inclinations of the two pins and, by means of the previously exposed procedure, in search of the starting angle $\varphi$ from which it is possible to assemble correctly the two parts. If it is found an entire zone that allows a correct assembly, it can be chosen, as preferential position of the pin, the location that minimized the energy of deformation stored in the bodies, or in the system.

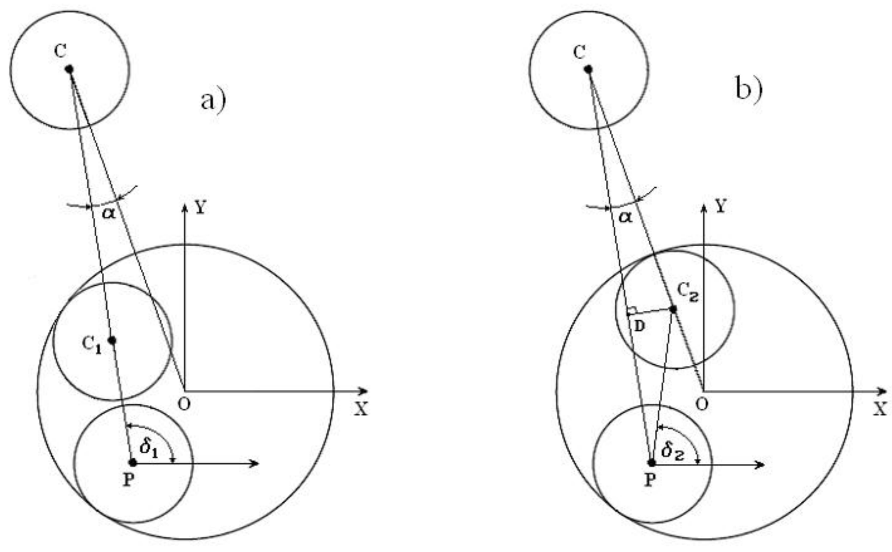

Figure 8 : Corrections on deformed pin geometry

After the correct relative positions between parts has been computed, it is possible to make small adjustments on deformed geometry of the pins that, otherwise, would be like in figure 8a. In order to shift from this configuration to that shown in figure 8b, physically more acceptable, it's important to consider that the contact forces between hole and pin head circumference act along the normal at the contact point, and this normal passes also through the centre of the hole. Therefore, as in figure 8, the configuration from the A position, in which the angle $\varphi$ was only reduced in order to permit the assembly, changes to the B position in which the pin presents two different deformations, $d_1 = \overline{CC_2}$ and $d_2 = \overline{CD} = d_1 \sin\alpha$, where $\alpha = |\delta_1 - \delta_2|$.

As concern the stresses of the members, the pin is simply considered like an embedded beam with two deformations applied to its extremity equal to $d_1$ and $d_2$. So it is possible to calculate the maximum bending moment at the end of the pin:

$$M_{f1} = \frac{3EI}{l^2} d_1 \qquad M_{f2} = \frac{3EI}{l^2} d_2$$

and in order to obtain the values of stress at the pins end:

$$\sigma_f = \frac{M_f}{W_f} = \frac{32 M_f}{\pi d^3}$$

where $d = 2r$ and $M_f = \sqrt{{M_{f1}}^2 + {M_{f2}}^2}$.

Choosing a stress limit for the resistance of the assembly, many pieces can further be saved and not discarded. At the end of the entire analysis we can have three different situations: a) two parts can be assembled, b) can be assembled with deformations (with acceptable levels of stresses), c) or could be not assembled.

## 3. Results

With the method previously exposed, some tests have been done on parts with particular values of the errors. For the first part of procedure with no deformation allowed, the verify has been simple because there are only geometric considerations applied to rigid bodies, controllable during the production process.

On the other hand, in order to verify the correctness the second part of the method, concerning flexible bodies analysis, the results obtained for every assembly have been confronted with finite elements simulations in which the external pin surface and the internal hole surface have been constrained with a contact pair (fitting).

Starting from an interference position, through an iterative process, the parts reach a correct position with appropriate deformations.

At a first time, it has been verified the correctness of the positioning algorithm and the procedure of the position correction of the pin head. As in figure 9, that shows only one of the two pins, it can be observed a real angle of 64.10° in agreement with the calculated angle of 64.16° ($\delta_1 = 60°$, $\delta_2 = 64.10°$, $\alpha = 4.10°$).

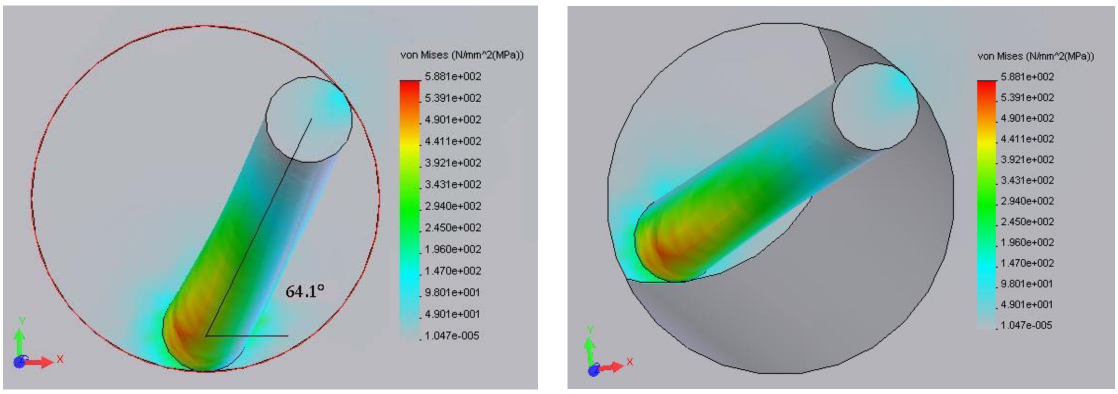


Figure 9 : Right position of deformed pin

In this phase, the pin and hole dimensions have been chosen very different in order to make the results more easily to be observed.

In figure 10, positions of both pins and relative stresses levels are shown. For all tests carried out, the coordinates of the centres of the upper and lower pin circumferences inside the holes have a maximum percentage error of the 3÷4%.

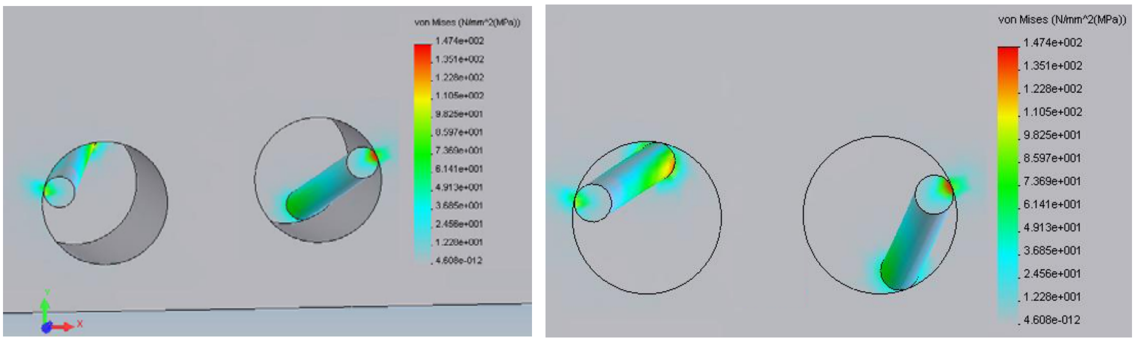

Figure 10 : Positioning of both pins

As concern the studied case, the model has been also verified in its actual dimensions. In figure 11, pin and hole stresses are shown: the maximum stress is at the base of the pin, as previously assumed, with a peak values in agreement with the theoretical ones.

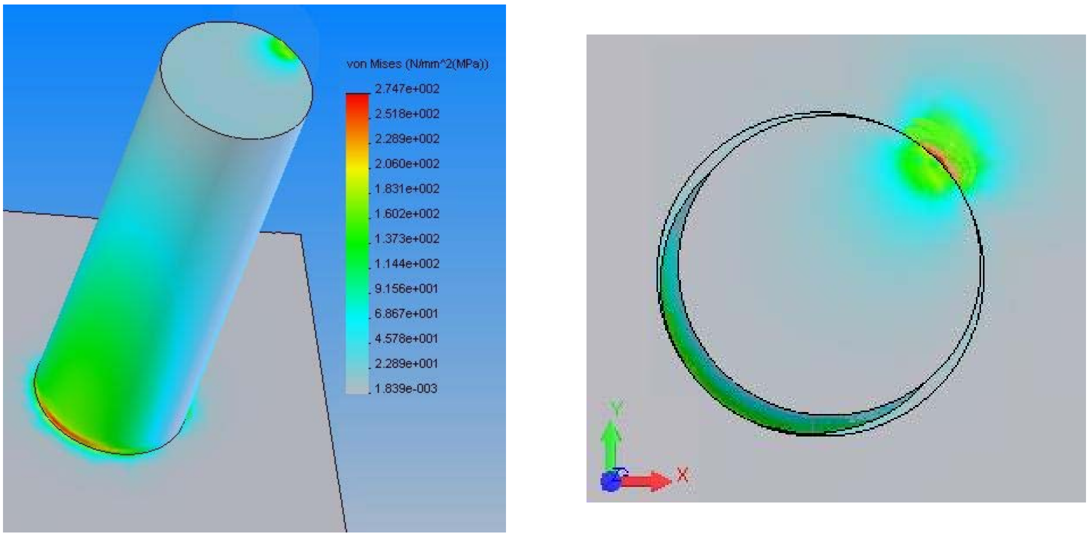

Figure 11 : Pin stresse level

In particular, in figure 11 the stresses level of the pin exceeds the prescribed $\sigma_{amm}$, hence it could not be assembled without damage. The stress field at the head of the pin belongs only to contact phenomena, it's absolutely a local perturbation and the peak level is lower than the stress at the pin's base.

With the finite element analyses, the obtained results validate the method here proposed. The agreement between the two sets of results has given full validity to the proposed methodology. The tests have been carried out on pins and holes of different dimensions (figure 9) in order to show and examine, in a better way, the geometric variables sensitivity and the relative parts position.

## 4. Statistics Analysis with the Montecarlo method

With Montecarlo method, it is possible to inquire the percentage of rejects during production knowing every single tolerances applied to the mechanism and their relative statistics distributions [6].

A Montecarlo simulation method consists of a generation of random values for the positions and dimensions of every single part, in accordance with their respective probability distributions, and an analysis of the possible assembled configurations thanks to the equations previously shown [7]. Repeating this procedure for a large number of assemblies, it can be made an histogram with the dimensions variations from the ideal values (in our case the positions between holes and pins). The figure 12 shows graphically how the process works.

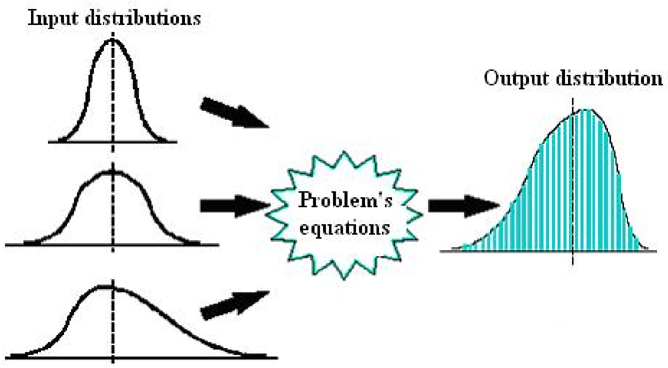


Figure 12 : Graphical representation of Montecarlo method

The obtained results can then be compared with the design limits in order to predict the total number of rejects (assemblies that fall outside the design limits). The distribution commonly used is the normal or Gaussian distribution in order to describe the variations of the parts on which the tolerances are applied. The number of considered assemblies depends on the output result precision. Reliable results are obtained with a large number of assemblies.

Applying this method to the studied assembly, the distributions in Figure 13 are obtained for the assembly errors.

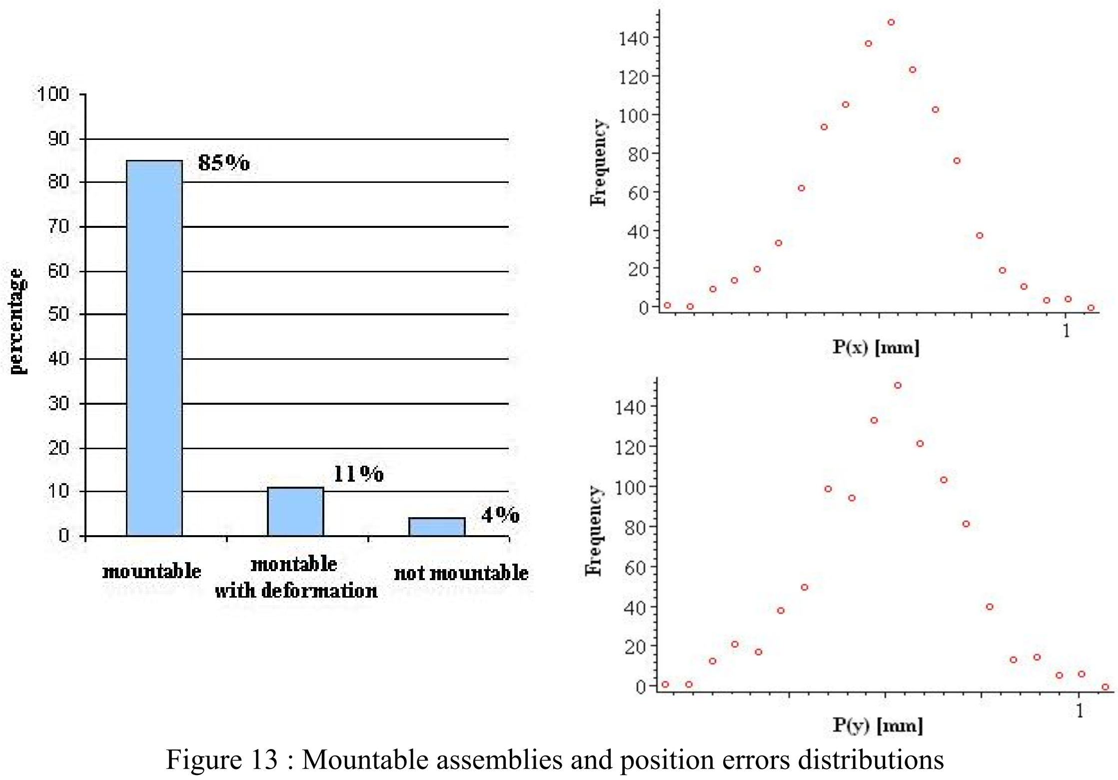


Figure 13 : Mountable assemblies and position errors distributions

In fact, Figure 13 shows the results obtained at the end of a simulation with 1000 assemblies; starting from a 85% of mountable assemblies, considering the bodies deformations, it is possible to accept an ulterior 11% of assemblies, with only 4% of rejects. Considering all mountable assemblies, the position errors distributions in direction *x* and *y* between the two plates are also represented in figure.

We have inquired, in last analysis, the effect of the dimension of the population on the correctness of the simulation. Using every time greater sample (until a million of mechanisms), we have noticed that the variation on the value of the previewed rejects was less than 1%. For this reason, the used sample (of 1.000 assemblies) has been defined enough representative, considering also the short computational time.

## 5. Conclusions

In this work, a methodology has been introduced in order to inquire the relations and influences of the mechanical errors on the functionality of assemblies whose connections are of pin-hole type. This approach has been validated in order to supply a useful design help for the tolerances allocation in order to keep under control the number of rejects. This number has been also calculate considering possible deformations of the assemblies; in fact, many parts, otherwise rejected, have been saved and possible to assemble in accordance with original design.

The proposed procedure is completely generic and can be applied to a large kind of complex assemblies or even mechanisms.

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