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MODELLING OF HYDRODYNAMIC JOURNAL BEARING IN SPATIAL MULTIBODY SYSTEMS

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ABSTRACT

The model of a three dimensional journal bearing with hydrodynamic lubrication is herein presented. This model is suitable for embodiment into the equations of spatial multibody systems. Both rotational and squeeze effects together with tilting effect have been taken into account. Moreover a simplified model of friction has been also reported. The proposed methodology has been applied to an example concerning an unbalanced rotor supported by two journal bearings.

INTRODUCTION

Accurate analysis of rotating machinery is useful to improve performance and reliability. In many practical applications the modelling of a journal bearing as ideal revolute joint may lead to neglect some interesting dynamic effects [1]. Some authors proposed to describe such a bearing using bushing elements [2]. Often these models use linearized stiffness/damping which are not suitable for transient analysis or for heavy/variable loads. On the other hand there are many contributions about dry joints with clearances [3-9]. These concern contact/impact algorithm or penalty method approach in order to describe the interaction between bearing surfaces. Taking into account also lubrication effects leads to very complex models. One of them has been proposed in [10] and it is based on an hybrid model including both contact and squeeze effect. The model does not include effects due to rotational speed of the possibility of tilting (it can be used only in planar mechanisms with low rotational speed).

The detailed and comprehensive models coming from the theory of lubrication [11-15] are too complex to be included into equations of motion of multibody systems because they require lengthy computations and very accurate numerical procedure in order to obtain correct solution.

For this reason the authors developed a model which, although it contains some assumptions and simplifications, allows to simulate a three dimensional journal bearing with both effects of squeeze and sliding speed and also the possibility to exert reaction moments by tilting. The effect of friction in bearing has been also estimated using semi empirical formulas.

The standard approach to describe bearing with ideal revolute joint leads to 5 scalar constraint equations (permitting 1 d.o.f. which is the relative rotation along hinge axis). The proposed model replaces these equations with two forces and three torques, thus the system increases the number of degrees of freedom. The ideal joint imposes kinematic constraints (on position), the proposed model introduces instead force constraints.

This kind of force constraints often avoid structures to be kinematically overconstrained. Let us consider a shaft which rotates supported by two bearings. When modelling the bearings with revolute joints the system will be overconstrained (one joint will be overabundant) and the computation of reaction forces from Lagrange multipliers will produce wrong results. Using forces instead of position constraints one obtains correct results from computation and the system is not overconstrained.

NOMENCLATURE

B bearing axial length
f friction coefficient
$\{F_e\}$ external generalized forces
$[J]$ inertia matrix w.r.t. center of mass principal axes
h fluid film thickness
$[M]$ mass matrix
M_x, M_y, M_z bearing reaction torques along x, y and z axis, respectively

p fluid film pressure ($p=p(\theta, z)$)
p_m fluid film pressure mean value
$\{q\}$ generalized coordinates
r_0 journal radius
S Sommerfeld number
x_i displacement of inner shaft center (for z=0) along x-axis w.r.t outer ring axis
x_j displacement of inner shaft center (for z=B) along x-axis w.r.t outer ring axis
y_i displacement of inner shaft center (for z=0) along y-axis w.r.t outer ring axis
y_j displacement of inner shaft center (for z=B) along y-axis w.r.t outer ring axis
W_x, W_y bearing reaction forces along x and y axis, respectively
δ bearing clearance
φ relative clearance
$\{\psi\}$ vector of constraint equations
μ lubricant viscosity
$\{\tau\}$ applied torque
ω_0 bearing angular speed (it is the angular speed of the inner shaft w.r.t. the angular speed of the outer shaft).

JOURNAL BEARING MODEL

Let us consider a journal bearing (Figure 1) and a reference frame placed on the outer shaft with the origin coincident to its centre. Assume that the axis of the inner shaft is parallel to reference frame z axis (absence of tilting). The Reynolds' differential equation describing the lubricant motion for a two dimensional flow are [14]:

$$\frac{1}{r_0}\frac{\partial}{\partial\theta}\left(h^3\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\mu\left(\omega_0\frac{\partial h}{\partial\theta}+2\frac{\partial h}{\partial t}\right) \tag{1}$$

where h can be computed using (in this case $x_i = x_j$ and $y_i = y_j$):

$$h=\delta-x_i\cos\theta-y_i\sin\theta \tag{2}$$

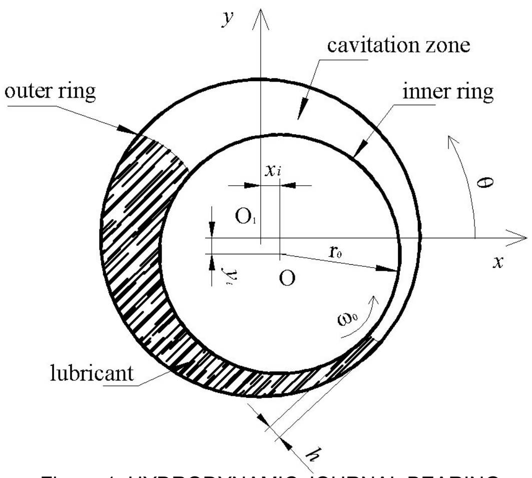

Figure 1. HYDRODYNAMIC JOURNAL BEARING

For bearings with $B/2r_0<1$ (many journal bearings satisfy this geometrical condition) we neglect the contribution of $h^3\partial p/\partial\theta$ and we obtain an approximate solution of the pressure field as:

$$p(\theta,z)=\frac{3\mu z(z-B)}{h^3}\left[\omega_0\left(x_i\sin\theta-y_i\cos\theta\right)-2\left(\dot{x}_i\cos\theta-\dot{y}_i\sin\theta\right)\right] \tag{3}$$

The first term of (3) inside square brackets is the contribution due to relative sliding speed (rotational speed) between inner and outer ring. The second term is the contribution of squeeze effect and depends on the relative velocity between inner and outer ring in the radial direction.

Let us now consider tilting. In this case the axis of the inner shaft is not parallel to z axis and the fluid film thickness can be computed by means of the following expression:

$$h=\delta-\left[x_i-\left(x_i-x_j\right)\frac{z}{B}\right]\cos\theta-\left[y_i-\left(y_i-y_j\right)\frac{z}{B}\right]\sin\theta \tag{4}$$

This function is dependent on the variable z, so it makes the (1) difficult to be integrated. In order to have a simpler solution we substitute (4) into the (1) and obtain:

$$p(\theta,z)=3\mu z(z-B)\frac{\omega_0\left(x\sin\theta-y\cos\theta\right)-2\left(\dot{x}\cos\theta-\dot{y}\sin\theta\right)}{\left(\delta-x\cos\theta-y\cos\theta\right)^3} \tag{5}$$

where (see Figure 2):

$$x=x_i-\left(x_i-x_j\right)\frac{z}{B} \qquad y=y_i-\left(y_i-y_j\right)\frac{z}{B}$$

$$\dot{x}=\dot{x}_i-\left(\dot{x}_i-\dot{x}_j\right)\frac{z}{B} \qquad \dot{y}=\dot{y}_i-\left(\dot{y}_i-\dot{y}_j\right)\frac{z}{B}$$

For a generic journal bearing we can estimate the pressure profile as depicted in Figure 3.

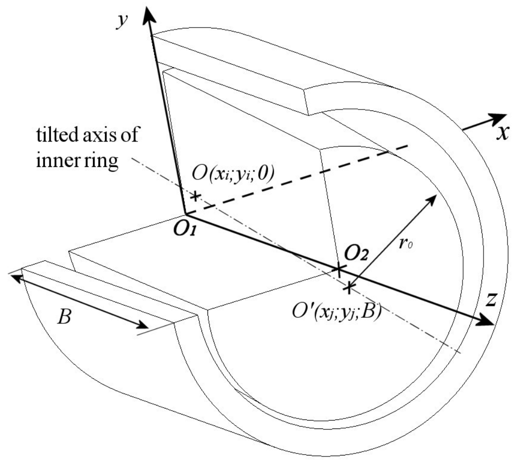

Figure 2. TILTED BEARING AXIS

Thus, the present model of journal bearing requires the position and velocity of two end centre points (O and O') and by means of pressure field in (5) computes the following components of force and torque along x and y axes:

$$W_x = -\int_0^B \int_0^{2\pi} p(z,\theta) r_0 \cos\theta d\theta dz \quad (6)$$

$$W_y = -\int_0^B \int_0^{2\pi} p(z,\theta) r_0 \sin\theta d\theta dz \quad (7)$$

$$M_x = -\int_0^B \int_0^{2\pi} p(z,\theta) r_0 z \sin\theta d\theta dz \quad (8)$$

$$M_y = \int_0^B \int_0^{2\pi} p(z,\theta) r_0 z \cos\theta d\theta dz \quad (9)$$

In order to take into account cavitation, the integrals (6-9) have to be evaluated only when pressure p has a positive value. Pressure computed as in (5) is a 2π-periodic function hence the interval in which it has positive values is equal to π. Alternatively, the computation of the integrals (6-9) can be performed using abs(p) instead of p and then dividing the results by 2.

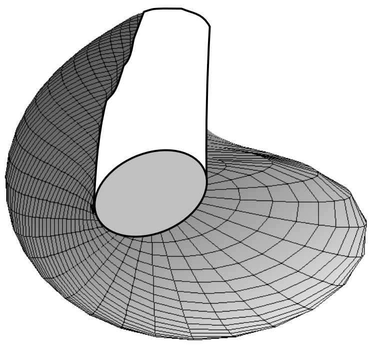

Figure 3. AN EXAMPLE OF A PRESSURE FIELD AROUND A 3D BEARING ACCORDING TO EQUATION (5).

FRICTION IN BEARING

In many applications it is necessary to compute the resistant torque due to friction. Although it is possible to compute its value using complex integrals [12] [14], practice shows that simplified empirical formulas can estimate friction effect with acceptable accuracy.

Let first introduce an important bearing parameter, the Sommerfeld number which can be defined as:

$$S = \frac{p_m \varphi^2}{\mu \omega_0} \quad (10)$$

where the mean pressure p_m and the relative clearance φ can be computed as:

$$p_m = \frac{\sqrt{W_x^2 + W_y^2}}{2 r_0 B} \qquad \varphi = \frac{\delta}{2 r_0}$$

Starting from (10) the friction coefficient f can be computed using:

$$f = \frac{3\varphi}{S} \quad \text{for} \quad S<1$$

$$f = \frac{3\varphi}{\sqrt{S}} \quad \text{for} \quad S>1$$

As it is clear, we can have two behaviours: the first one (when $S<1$) happens when the bearing is rotating at high speed; the second (when $S>1$) happens when the bearing is under heavy load and rotates with low speed.

The friction torque can be computed with the following equation:

$$M_z = f r_0 \sqrt{W_x^2 + W_y^2} \quad (11)$$

It is meaningful to observe that if $S<1$, then the friction torque does not depend on the applied loads, but only on the rotational speed, i.e.:

$$M_z = \frac{6 r_0^2 B \mu \omega_0}{\varphi} \quad \text{for} \quad S<1$$

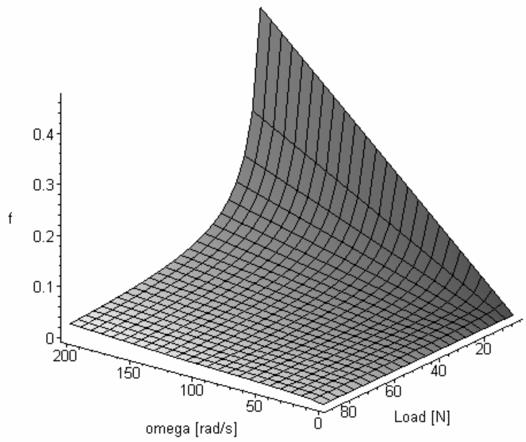

Figure 4. FRICTION COEFFICIENT FOR A BEARING WITH PROPERTY REPORTED IN TABLE 2.

Moreover the friction model in (11) can be also included as an external torque in a mechanical system with bearing modelled as an ideal joint. The advantage is to have a friction coefficient depending on rotational speed, load, lubricant and bearing geometrical properties. In Figure 4 the coefficient of friction for a bearing with geometrical and lubricant properties reported in Table 2, is plotted as a function of load and rotation speed ω. For a lubricated bearing, a realistic value of this coefficient is about 0.02÷0.03.

NUMERICAL REMARKS

Let now focus on the implementation of the model into a multibody system (Figure 5). The constant inputs are the geometrical properties of the bearing (i.e. B, r_0 and δ) and the lubricant viscosity. The variable inputs are the relative angular speed between inner and outer ring, the position and the velocity of inner ring centre points (O and O') w.r.t the outer ring centre points (O_1 and O_2, see Figure 2).

Every integration step the expressions (6) (7) (8) (9) and (11) are evaluated. The p function quick goes to infinity when h is near to 0 and it cannot be integrated or the result may be inaccurate. A smart check can be performed on h value. If h is lower than a threshold value h_{min}, then the fluid film thickness is not sufficient to avoid the direct contact between inner and outer surface and the friction coefficient arises to high values. In this case a contact or hybrid algorithm [11] can be successfully applied. The designer often tries to keep the h value higher than h_{min} in order to ensure the lubricant effect and reduce the friction torque. If during the simulation h reaches the threshold value, it is better to change the bearing dimensions or the lubricant.

Since the evaluation of the two dimensional integrals in (6) (7) (8) and (9) is complex, another simplification can be introduced. If the bearing rotates with high angular speed then the term $2(\dot{x}\cos\theta - \dot{y}\sin\theta)$, describing squeeze effect in (5), is neglected.

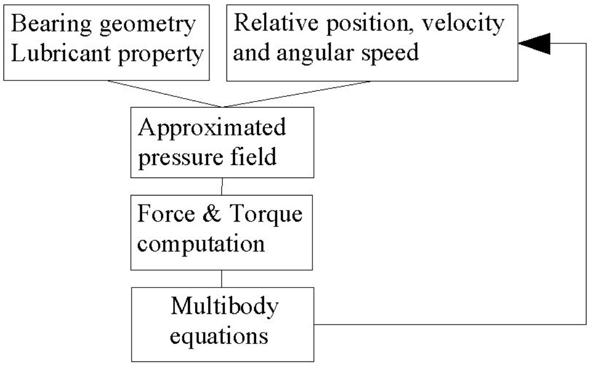

Figure 5. COMPUTATIONAL ALGORITHM TO COMPUTE JOURNAL BEARING FORCES AND TORQUES.

In case of constant rotational speed and slightly variable load, the friction coefficient can be assumed constant instead of updating its value at each integration step.

The effect of tilting can be neglected when the bearing is axially long. In this case the pressure field as estimated in (3) can be used in the integrals (6-9) instead of the more complex expression in (5). In this case the integral w.r.t variable z has a symbolic solution.

AN EXAMPLE

Let us now test the proposed model on a simple example. We consider a rotor with two journal bearings. Moreover, we assume that the body rotates at constant speed and is unbalanced (i.e. its centre of mass does not belong to rotation axis (see Figure 6)).

The geometrical and mass properties of the rotating body are summarized in Table 1.

Table 1: ROTOR PROPERTIES

Mass	1.0 kg
Principal Inertia Moments	[0.01; 0.01; 0.3] kg m^2
Eccentricity	0.001 m
Bearing 1 distance from G (d_1)	0.2 m
Bearing 2 distance from G (d_2)	0.1 m
Rotational velocity	100 rad/s

The bearing and lubricant properties are summarized in Table 2.

Table 2: BEARING AND LUBRICANT PROPERTIES

Bearing radius r_0	0.02 m
Bearing length B	0.02 m
Clearance δ	0.0003 m
Lubricant viscosity	0.0242 kg/ms

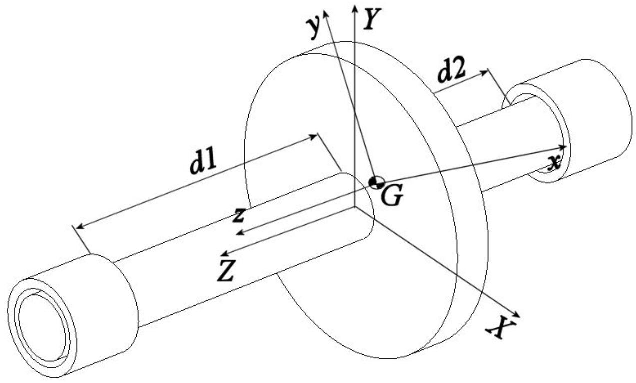

Figure 6. UNBALANCED ROTOR BETWEEN TWO JOURNAL BEARINGS

The motion of the rotor can be described using 7 generalized coordinates (3 for the translation and 4 Euler's parameters to describe the attitude [16]): $\{q\}=\{x\ y\ z\ e_0\ e_1\ e_2\ e_3\}^T$. The equations of motion can be written using differential-algebraic equations (DAE) system of index 3 [17]:

$$\begin{cases} [M]\{\ddot{q}\}+\left[\Psi_q\right]^T\{\lambda\}=\{F_e\} \\ \{\Psi\}=0 \end{cases} \tag{12}$$

where:

- $[M]$ is the mass matrix and it can be computed as:

$$[M]=\begin{bmatrix} \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} & [0_{3\times4}] \\ [0_{4\times3}] & 4[G]^T[J][G] \end{bmatrix} \quad \text{with} \quad [J]=\begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix}$$

$$\text{and} \quad [G]=\begin{bmatrix} -e_1 & e_0 & e_3 & -e_2 \\ -e_2 & -e_3 & e_0 & e_1 \\ -e_3 & e_2 & -e_1 & e_0 \end{bmatrix};$$

- $\left[\Psi_q\right]$ is the Jacobian matrix of constraint vector $\{\Psi\}$. There are only two constraints applied to the rotor:

  $\Psi_1 \equiv z = 0$ z translation constraint

  $\Psi_2 \equiv e_0^2+e_1^2+e_2^2+e_3^2-1=0$ normalization of Euler's param.
- $\{\lambda\}$ is the vector of Lagrange's multipliers
- $\{F_e\}$ is the vector of applied load (parameters with subscript A and B are referred to Bearing 1 and 2, respectively):

$$\{F_e\}=\begin{Bmatrix} W_{xA}+W_{xB} \\ W_{yA}+W_{yB} \\ 0 \\ 8\left[\dot{G}\right]^T[J]\left[\dot{G}\right]\{e_0 \quad e_1 \quad e_2 \quad e_3\}^T+2[E]^T\{\tau\} \end{Bmatrix} \quad \text{with}$$

$$[E]=\begin{bmatrix} -e_1 & e_0 & -e_3 & e_2 \\ -e_2 & e_3 & e_0 & -e_1 \\ -e_3 & -e_2 & e_1 & e_0 \end{bmatrix} \text{ and } \{\tau\}=\begin{Bmatrix} M_{xA}+M_{xB}-W_{yA}d_1+W_{yB}d_2 \\ M_{yA}+M_{yB}+W_{xA}d_1-W_{xB}d_2 \\ M_{zA}+M_{zB} \end{Bmatrix}$$

Integrating the DAE system (12) we can simulate the dynamic behaviour of the rotor. The centre of mass trajectory in *x-y* plane is monitored. In Figure 7 the trajectories for the ideal revolute joints and for the case of hydrodynamic journal bearing joints are compared. It is clear that the motion of the rotor is more complex. There is a rotational motion due to eccentricity and a local circular motion due to the presence of the lubricant fluid film. In Figure 8 and 9 the time histories of centre of mass *X* and *Y* position are plotted and compared to those of rotor supported with ideal revolute joints.

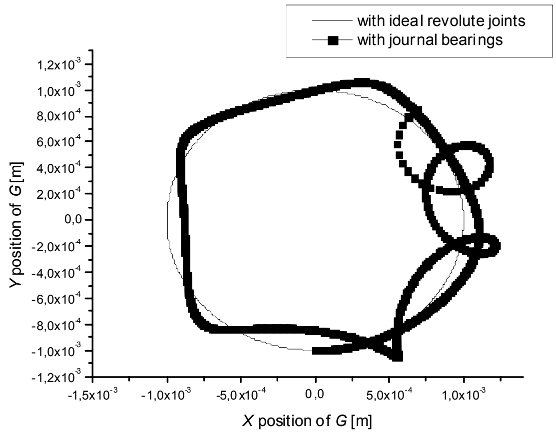

Figure 7. COMPARISON BETWEEN ROTOR CENTRE OF MASS TRAJECTORIES

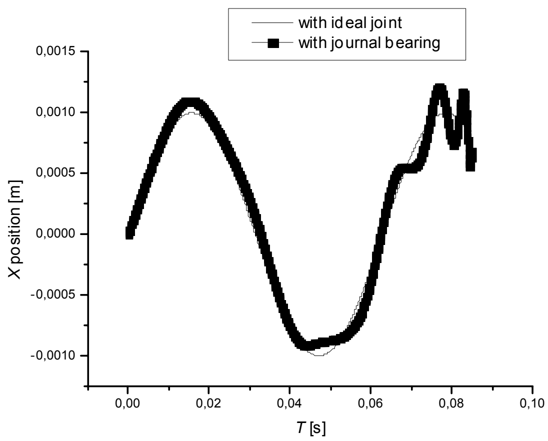

Figure 8. ROTOR CENTRE OF MASS TRAJECTORIES IN *X* DIRECTION

Also the trajectories of the two pin centers are plotted in order to evaluate their orbits inside the journals. Figure 10 shows the trajectory of left bearing in Figure 6 centre in *x-y* plane.

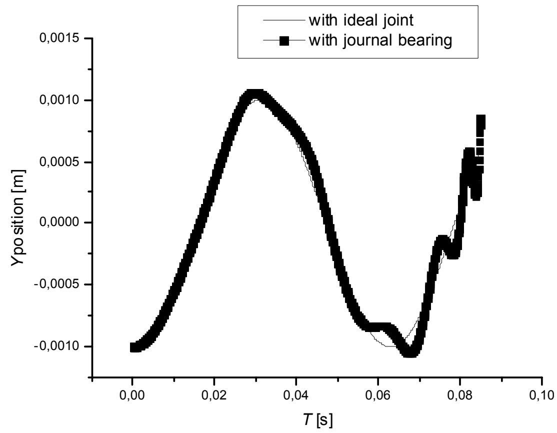

Figure 9. ROTOR CENTRE OF MASS TRAJECTORIES IN *Y* DIRECTION

In case of ideal joint this trajectory will be just a point located in (0; 0). The lubricant fluid film effect together with joint clearance causes the bearing to describe an orbital path.

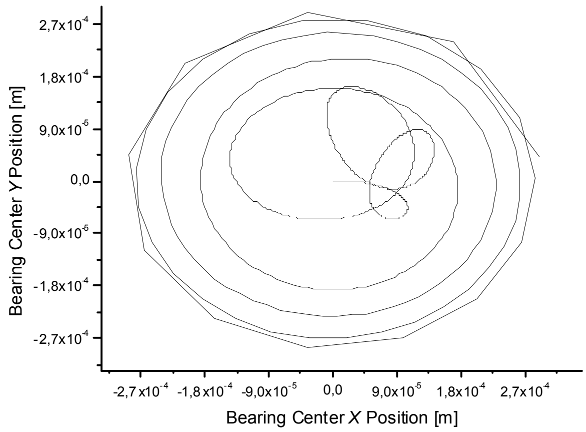

Figure 10. BEARING 1 END A CENTRE TRAJECTORY

CONCLUSIONS

An hydrodynamic bearing model has been discussed in this paper. Starting from fluid film equations the expressions concerning bearing reaction forces and torques have been deduced. These equations, expressed by (6) (7) (8) (9) and (11) can be included into multibody codes. They allow to take into account rotational speed, squeeze and tilting effects. With this approach the bearing is described by force constraints instead of kinematic constraints as well as ideal joints. This allows to avoid kinematically overconstrained models which are difficult to manage. On the other hand, this model requires accurate time integration procedures. In the proposed example the authors used the FORTRAN subroutine Radau5 by Hairer and Wanner [18]. The proposed bearing equations are completed with a simplified friction model in order to estimate the power losses as function of load and rotational speed.

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