An integrated tool for design, shape modelling and performance analysis of 3D cam

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Abstract: In this paper an easy procedure to integrate three dimensional cam profile synthesis and Computer Aided Design is presented. The aim of the proposed method is to combine an accurate kinematic analysis with shape modelling using CAD programmes. In many cases the manufacturing process and cutting approximation cannot be neglected and affect the final drawing. For specific needs, as spatial cams, an appropriate and more accurate synthesis is required. On the other hand, the only geometrical design misses all the advantages of modelling, for example the correct calculation of mass properties, interferences, and the possibility to perform dynamical and stress analysis. The authors develop an integrated tool which interacts with both CAD software and algebraic solver in order to draw an accurate cam model and to compute performances as regards errors on position, acceleration, jerk and jounce. This methodology can help the designers in the definition of digital mock-up.

Keywords: CAD modelling; 3D barrel cam design.

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1 INDUSTRIAL BACKGROUND

The increasing trend of the application of project data management and concurrent engineering strategies requires new approaches of design methodologies. For this reason, the CAX (Computer Aided Everything) systems, and in particular the CAD/CAM systems allow the increase of data integration between the design and manufacturing phases.

The design of cams, in particular three dimensional ones, plays an important role in improving the performance of machinery, especially in high speed components or a high precision manipulator.

These cams can be found in several mechanical devices. Apart from indexing mechanisms (Tsay et al., 2002) they are included in much packaging and typographical machinery, and also in many road vehicle gearboxes (Pennestrì et al. (2004). In Figure 1 a 3D turret cam for automotive applications is presented.

In practical application three dimensional cam investigations are often less accurate than the two dimensional ones. In many cases the manufacturing of these cams is just performed using the envelope principle (Rubino, 2003). It means that the profile is generated by a milling machine whose cutter is driven with a prescribed law in order to reproduce the relative motion between cam and pin. This approach neglects the understanding of generated surfaces' mathematical properties and it does not care about the differences between the expected and the actual performances. In fact the only enveloping techniques allow the position law to be reproduced quite properly losing instead velocity, acceleration, jerk and jounces laws information. Moreover the designer does not have the instruments for comparing different motion law performances which are, especially in high speed applications, very important. In order to improve cam

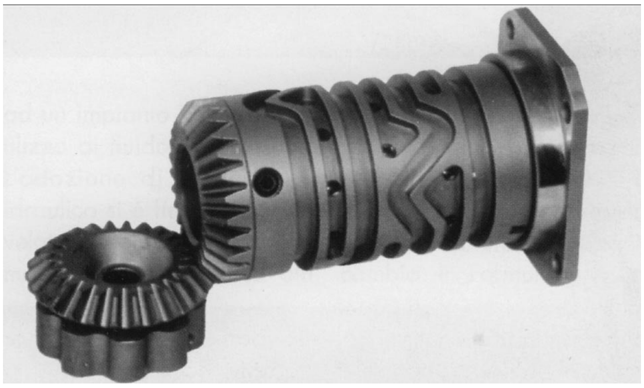


Figure 1 *An automotive barrel cam with several grooves*

performances a more accurate study has to be made before machining them.

Many outstanding authors' specialised books (Norton, 2002; Pennestrì, 2002) face only manufacturing problems about two-dimensional cams. Many geometrical and technological difficulties arise when a 3D cam has to be designed, so there is the need to support engineers. Moreover there are very few commercial CAD/CAM applications which offer tools to synthesise 3D cam profiles, but they are only drawing instruments and they lack accurate analysis utilities.

Studies in literature (Pennestrì et al., 2004) show that best performances in actuating processes can be achieved only choosing the appropriate motion law and the differences between them cannot be neglected. Moreover, just to give an example, during last year the car market increased requests for robotised gearboxes with three dimensional cams by about 150%. The trend is to improve reliability, drive comfort reduce time spent shifting gears. The first robotised gearboxes were equipped with three-dimensional turret cams with very inaccurate profiles: users' disappointment and low performances impose changes.

2 THE PROPOSED METHODOLOGY

As shown in the previous section, in many practical industrial applications there is the need for improved design strategies. Neither specialised books, nor commercial software can solve this problem at present. In order to fill this gap, the authors developed a procedure and implemented a numerical code that integrates a geometrical synthesis (Chakraborty and Dhande, 1977; Chen, 1982; Gonzales-Palacioos and Angeles, 1993; Neklutin, 1969; Rothbart, 1956) of the cam active profile, the simulation of manufacturing process and performance analysis within a CAD programme which can help the designer to compare different solutions before machining it.

The idea is to choose an ideal follower motion law and to deduce the mathematical equations of profile 3D geometry by means of an analytical procedure. The cam will then be automatically modelled using CAD features as they appear in a manufacturing process, simulating the enveloping system discussed in the first section. The difference between the ideal profile and the actual one will then be assessed.

All these steps have to be embodied in a unique tool to offer a competitive design instrument and have a complete overview of the problem. The results coming from this approach can be used successfully to perform other kinds of analysis such as those regarding dynamics and stress-wear prevention which play an important role (Tsay et al., 2002).

The proposed method is divided into three steps. The first is a synthesis of the profile according to the rules of conjugate surfaces and kinematics. The second is the manufacturing process choice and the build-up of the LISP-code in order to generate automatically the CAD features to model the cam (using a procedural approach), and the third is the comparison between the ideal profile with the modelled profile in order to evaluate the theoretical errors of position, velocity, acceleration, jerk and jounces. The executive scheme is shown in Figure 2: each step will be discussed in the following sections.

3 SYNTHESIS OF THREE DIMENSIONAL CAM PROFILE

In this section a method to synthesise a cam profile in a three dimensional cam application is presented (Chakraborty and Dhande, 1977).

Let the follower have a cylindrical surface as an active profile with the axis of the roller perpendicular to the follower arm axis, which is assumed to be the direction of motion of the roller. In this case, referring to Figures 3 and 4, the radius vector and the unit normal vector at any point **P** on follower surface are given by

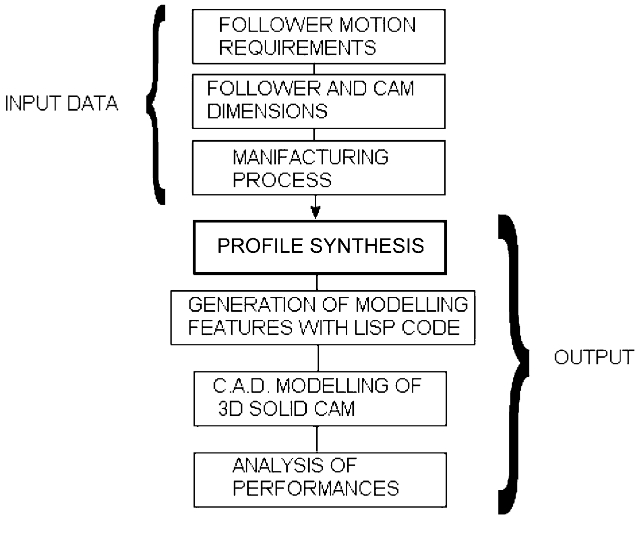


Figure 2 *Procedure scheme*

$$\begin{aligned}\mathbf{R}^{(P)} &= \{\, r\cdot\cos(\vartheta) \quad r\cdot\sin(\vartheta)+s_1 \quad -\delta \,\}^T \\ \mathbf{e}^{(P)} &= \{\, \cos(\vartheta) \quad \sin(\vartheta) \quad 0 \,\}^T\end{aligned} \quad . \tag{1}$$

The synthesis starts from a given law of motion required to the follower, that can be expressed in terms of the displacement variable s (see Figure 3)

$$s_1 = s_1(\phi). \tag{2}$$

For the following expressions we use the subscript '1' referring to the follower, and the subscript '2' referring to the cam.

Let Σ_1 and Σ_2 be the two surfaces coming into contact; both of them can be expressed in parametric form using the general formula

$$\Sigma = S(\vartheta,\delta) = \left\{\, x(\vartheta,\delta) \quad y(\vartheta,\delta) \quad z(\vartheta,\delta) \,\right\}. \tag{3}$$

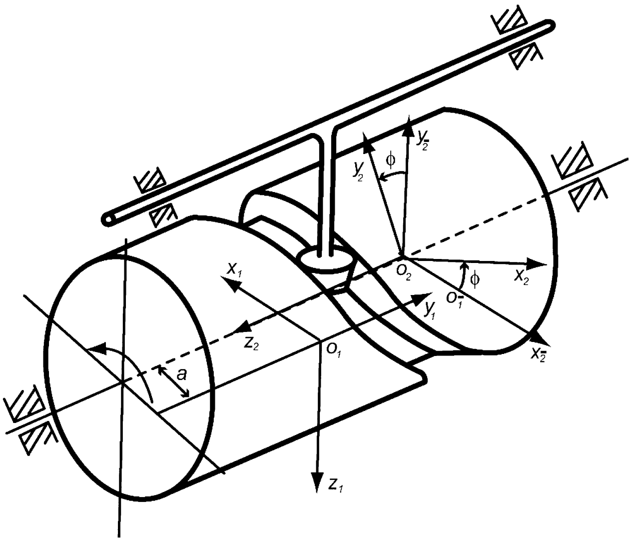


Figure 3 *Cam nomenclature*

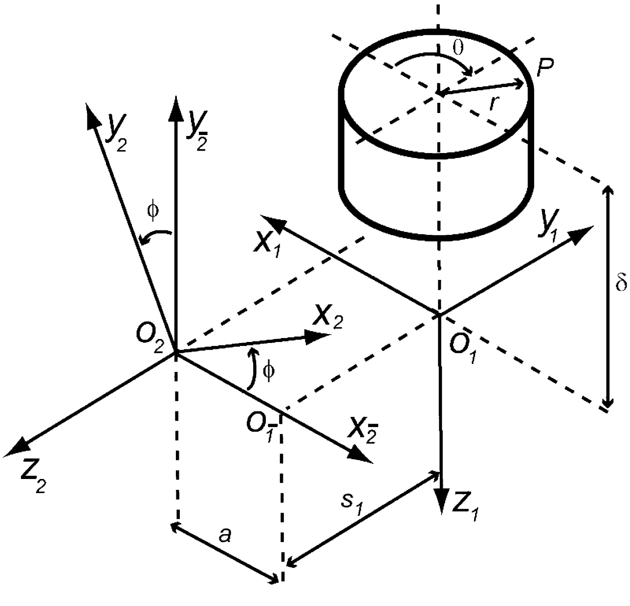


Figure 4 *Follower nomenclature*

The tangent vectors of each surface can be expressed using

$$\begin{aligned}\mathbf{t}_\vartheta &= \frac{\partial S}{\partial \vartheta} \\ \mathbf{t}_\delta &= \frac{\partial S}{\partial \delta}\end{aligned} \quad . \tag{4}$$

The normal vector can be computed using

$$\mathbf{n} = \frac{\mathbf{t}_\vartheta \wedge \mathbf{t}_\delta}{\|\mathbf{t}_\vartheta \wedge \mathbf{t}_\delta\|}. \tag{5}$$

Let us assume $\mathbf{v}_{12}$ as the relative velocity vector. During the contact between the two surfaces, this vector has to lay on the common tangent plane. This condition can be derived imposing that the relative velocity vector is perpendicular to the common normal vector, and so we can impose the dot product between them to be zero:

$$\mathbf{n}_1 \cdot \mathbf{v}_{12} = 0. \tag{6}$$

Thus the condition (6) represents the relationship between the parameters ϑ and δ along the contact curve, and so the parametric representation of the contact line.

Using Equation (6) in (3) one obtains the equation of the contact surface in parametric form. To express this surface in the reference frame of Σ_2 a change of coordinates from Σ_1 to Σ_2 by means of the transformation matrix $\mathbf{M}_{12}$ is needed:

$$\begin{Bmatrix} \mathbf{R}_2^c \\ 1 \end{Bmatrix} = \mathbf{M}_{12} \begin{Bmatrix} \mathbf{R}_1^c \\ 1 \end{Bmatrix}. \tag{7}$$

For the specific case, the equation of the cylindrical follower and its normal vector are expressed by (1).

The relative velocity is

$$\mathbf{v}_{12} = \omega_2 \left\{ -\delta \quad \frac{ds_1}{d\phi_2} \quad [a - r\cdot\cos(\vartheta)] \right\}^T. \tag{8}$$

The parametric expression of contact line can be derived imposing the contact condition (6) and so one gets:

$$\frac{ds_1}{d\phi}\sin(\vartheta) - \delta\cdot\cos(\vartheta) = 0. \tag{9}$$

Using (10) as transformation matrix $\mathbf{M}_{12}$, the equation of the parametric surface in the reference frame of the cylindrical cam is obtained, as well as the shape of the groove (11), always in a parametric representation.

$$\mathbf{M}_{12} = \begin{bmatrix} -\cos(\phi_2) & 0 & -\sin(\phi_2) & a\cdot\cos(\phi_2) \\ \sin(\phi_2) & 0 & -\cos(\phi_2) & -a\cdot\sin(\phi_2) \\ 0 & -1 & 0 & -s_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$\mathbf{R}_2 = \begin{Bmatrix} -r\cdot\cos(\phi_2)\cdot\cos(\vartheta) + \sin(\phi_2)\cdot\delta + a\cdot\cos(\phi_2) \\ r\cdot\sin(\phi_2)\cdot\cos(\vartheta) + \cos(\phi_2)\cdot\delta - a\cdot\sin(\phi_2) \\ -s_1 - r\cdot\sin(\vartheta) \end{Bmatrix}. \tag{11}$$

4 MANUFACTURING PROCESSES AND CAD MODELLING TOOL

For a given value of delta, Equation (11) expresses the intersection of the surface of the cam with a cylinder coaxial with the cam itself. So the cam can be modelled starting from a cylinder and then subtracting a solid portion obtained extruding a section of the follower using Equation (11) with a fixed value of delta as the path curve. CAD software uses interpolation in order to describe an analytical path. This approximation is often imprecise and may lead to wrong profile manufacturing. Thus the exact profile expressed by (11) could be only approximated by the spline software algorithm. The kind of approximation depends on the method of manufacture. The simplest type of Numerical Control (NC) Machine (Norton, 2002) moves the tool (or workpiece) to a specified x, y, z location and then drives the tool (like a drill) down through the workpiece to make a hole. This process is repeated as many times as necessary to create the entire profile. The NC is merely a computerised version of the old manual method of cam milling, which is often called plunge-cutting to refer to plunging the spinning milling cutter down through the workpiece. This is not the best way to machine a cam because it leaves 'scallops' on the surface due to the fact that the machinist can only plunge at a discrete number of positions around the cam. In a CNC (Continuous Numerical Control) machine (Norton, 2002), the tool is in constant contact with the workpiece, always cutting, while the computer controls the movement of the workpiece from position to position as stored in its memory. The cam displacement functions (i.e. those expressed in (11)) have to be discretised or sampled at some angular increment. Since the machine has information about x, y, z location only about a few points, it has to figure out how to get from one point to another while cutting. The most common method used is linear interpolation (L1). In this case the machine computer calculates the straight line between each pair of data points and then drives the cutter or workpiece along this path. An improvement can be made in this interpolation by fitting a cubic spline curve to the cam coordinate data (McKinley and Levine, 2000; Nievergelt, 1993).

The CAD modelling of the cam reproduces the manufacturing process. For the CN process, the cam is built starting from a solid cylinder and then subtracting as many cylinders (of the same dimension as the cutter) as there are sampling points. In this case the cam surface presents several scallops which modify its performance.

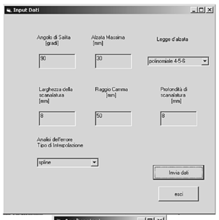
For a CNC process the cam is built starting from the same solid cylinder and then subtracting solids generated from the sweep of a rectangular sketch along cubic spline curves. In this case there are no scallops, but the cam profile is still approximated.

All the information about the geometrical feature to be modelled are coded in an LISP programme. This can be easily imported in a common CAD software such as Autodesk–Mechanical Desktop (Figure 5). The entire procedure (from the profile synthesis, to the generation of the solid model, to the performance analyses) is completely automatic and it is implemented in a VISUAL BASIC procedure written by the authors themselves.

5 INTERPOLATION ERROR ANALYSIS

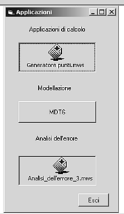
The accuracy of the interpolation and performance of the cam depends on the manufacturing process (Pennestrì et al., 2004). For a high speed cam the profile accuracy is very important. Mechanical errors or

Input dialog window: here the user can define the properties of the follower motion, the geometrical shape of the follower and the manufacturing process to simulate and analyse.

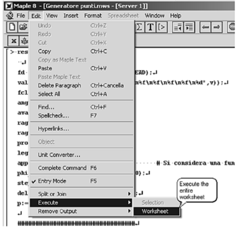


Angolo di Salita [gradi]: 90
Alzata Massima [mm]: 30
Legge d'alzata: polinomiale 4-5-6
Larghezza della scanalatura [mm]: 8
Raggio Camma [mm]: 50
Profondità di scanalatura [mm]: 8
Analisi dell'errore Tipo di Interpolazione: spline

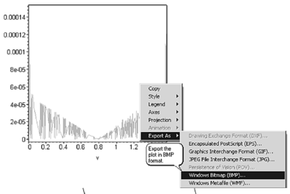
Applications menu: the user can choose to generate the profile analytically (by means of Maple V), numerically and to build a LISP code to be written by a commercial CAD software (in this case this software is Mechanical Desktop)



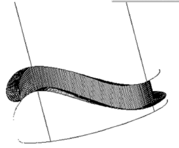
The Maple V automatic procedure allows to compute and plot results



An example of performance analysis: error in velocities



The automatic simulation of manufacturing process to model the cam surface starting from a solid cylinder.



The rendered solid model of the three-dimensional cam mechanism with the prescribed law of motion.

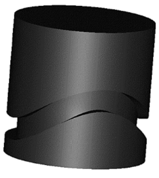


Figure 5 *Developed tool scheme and snapshots*

wrong interpolation may lead to errors in the position of the follower. Moreover the discontinuities in profile (and also of its derivative) causes discontinuities in velocities, accelerations, high or (theoretically) infinite jerks and jounces. In many mechanical applications these problems have to be avoided in order to increase the performance, reliability and the life of the cam (Neklutin, 1969; Norton, 2002; Pennestrì, 2002; Rothbart, 1956). We started our synthesis imposing a displacement law of the follower, then we obtained the exact (in closed form) expression for the cam surface. The approximation arises when this surface has to be cut by the milling machine. In this case the exact function is interpolated by discrete cut (in NC machinery) or linear or cubic curve (in CNC machinery).

In order to have an assessment of the errors, the error function is defined as:

$$\varepsilon = |f_{ideal} - f_{actual}| \tag{12}$$

where ε is the mechanical error, f_{ideal} is the parameter (position, velocity, acceleration, jerk, jounce) computed in the theoretical synthesis, and f_{actual} is the same parameter computed with the approximation of manufacturing process.

6 AN EXAMPLE

In this section an example of the explained procedure is described and some details of the developed automatic software are discussed.

The example refers to the design of a gearbox turret cam which the University of Tor Vergata performs in cooperation with an Italian industrial partner. Only the design of one groove will be investigated for sake of brevity. The turret cam (see Figure 1) is a device which can be mounted on the gearbox and it moves the sleeves during shifting phases. Moreover, the followers are the pins connected to their sleeves. More details about this industrial solution can be found in Tsay et al. (2002).

We are assuming the design of a three dimensional cam starting from a solid cylinder of 35 mm of radius. The follower is another cylinder with a radius of 7.5 mm. Let us assume the motion law of the follower to be a polynomial 3–4–5 (Chakraborty and Dhande, 1977):

$$s_1 = s_1(\phi_2) =$$

$$\frac{8.5\left[10\left(\frac{\phi_2}{6.28/13}\right)^3 - 15\left(\frac{\phi_2}{6.28/13}\right)^4 + 6\left(\frac{\phi_2}{6.28/13}\right)^5\right]}{1000}. \tag{13}$$

A graphical representation is shown in Figure 6. Isolating the variable ϑ in the contact condition (6), one gets:

$$\vartheta = \arctan\left(\frac{\delta}{2.262\phi_2^2 - 9.365\phi_2^3 + 9.693\phi_2^4}\right), \tag{14}$$

so the profile surface, according to Equation (11), has the following mathematical expression.

$$\Sigma =$$

$$\left\{\begin{array}{l} 3.5\dfrac{\cos(\phi_2)}{\sqrt{1+\dfrac{\delta^2}{2.262\phi_2^2 - 9.365\phi_2^3 + 9.693\phi_2^4}}} + \sin(\phi_2)\cdot\delta \\ 3.5\dfrac{\sin(\phi_2)}{\sqrt{1+\dfrac{\delta^2}{2.262\phi_2^2 - 9.365\phi_2^3 + 9.693\phi_2^4}}} + \cos(\phi_2)\cdot\delta \\ -0.754\phi_2^3 + 2.341\phi_2^4 - 1.938\phi_2^5 \\ -3.5\dfrac{\dfrac{\delta}{(2.262\phi_2^2 - 9.365\phi_2^3 + 9.693\phi_2^4)}}{\sqrt{1+\dfrac{\delta^2}{2.262\phi_2^2 - 9.365\phi_2^3 + 9.693\phi_2^4}}} \end{array}\right\}.$$

From the active profile surface the cutting path can be computed choosing a value for δ. In this investigation the value of the outer radius of the solid cylinder has been chosen. For guided cutting this path has been replicated to obtain another path translated of 7.5 mm in axial direction of the cam. The sketch to sweep is the rectangular section of the follower, and so a groove of the same dimension as the follower has been produced. A CNC with cubic interpolation has been assumed as the manufacturing process. After cutting the cam looks like those in Figure 7.

The position, velocity, acceleration, jerk and jounce error between the analytical profile and the cubic spline approximation are shown in Figure 8. The results show that the mechanical error increases at the start and at the end of the profile, and this suggests an increase in the sampling points around those intervals. In the case of NC cutting, the errors are greater than CNC cutting,

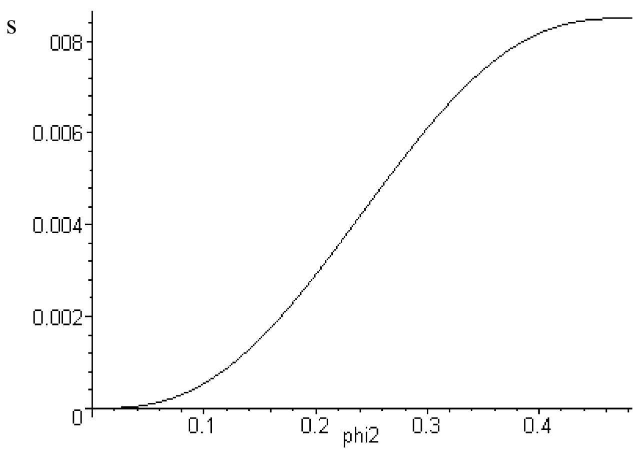


Figure 6 *Follower displacement vs cam angle*

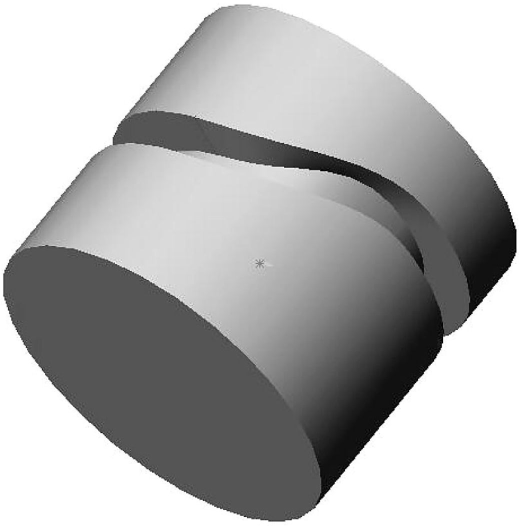


Figure 7 *Complete modelling of cam with synthesised profile*

and in many cases they reach the value of (theoretically) infinite. Actually the elasticity of the milling machine makes the profile smoother, thus avoiding the discontinuity of the derivative of motion law.

7 CONCLUSIONS

The proposed example shows how quickly a three dimensional barrel cam can be modelled, simulating the manufacturing process, starting from a given law of motion. The presented Equations (9) and (11) can be successfully used to design a great variety of cam profiles acting on a cylindrical follower. Moreover, the expression of relative velocity (8) can also be used to perform efficiency and wear analyses to appreciate other cam performances (Pennestrì et al., 2004). The Visual Basic procedure developed by the authors links the kinematic

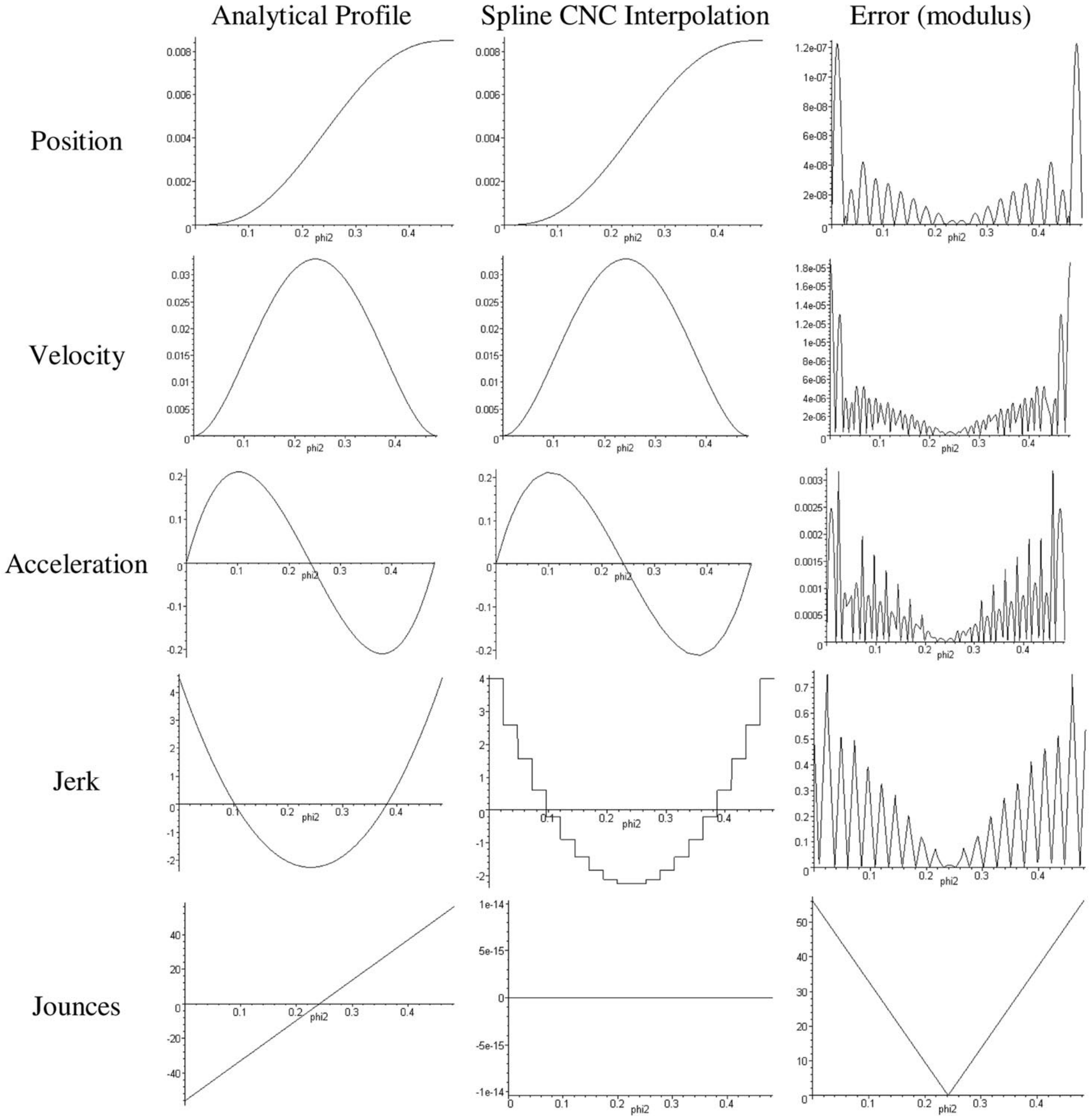


Figure 8 *Comparison between analytical profile and spline CNC interpolation*

analysis to the CAD programme. In fact the generation of the groove can be performed as a repetition of common features of subtraction of a cylinder from a base solid cam. The design problem is faced starting from synthesis and completed with the analysis. This methodology fills the current gap in cam manufacturing giving the designer an integrated method to understand the actual profile mathematics which cannot be neglected. Moreover, during the actual manufacturing, some interference or undercutting errors may occur, and so they can be avoided simulating the milling procedure using CAD/CAM. The error introduced by the approximation of the manufacturing machines has also been studied and a numerical example of a practical case has been presented. The proposed methodology is completely general and the developed tool can be applied to a large variety of cam systems. The only requirement is a CAD parametric programme working with features.

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