

# TOLERANCE ALLOCATION IN FLEXIBLE ASSEMBLIES: A PRACTICAL CASE

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## ABSTRACT

Flexible mechanical parts can be assembled having the use of their elasticity together with their geometrical shape. In this case the choose of dimensional and geometrical tolerances is difficult and neglecting elasticity effects may lead to produce a lot of rejected conformal components. Flexible assemblies can be found in many mechanical application.

In this paper the authors discuss a study of the influence of dimensional errors on the functioning of a flexible assembly. The methodology has been applied to a plastic earphone set of a mobile phone which has to be mounted on a metallic hub. In this case the flexibility of the part is necessary to strictly connect the two parts. On the other hand, excessive dimensions may lead to high internal stresses and damage by fatigue phenomena. The aim is to propose a methodology to improve and optimize the allocation of tolerances in systems which can be assembled with deformation. This approach is based on a Computer Aided parametric model in order to simulate dimensional errors, build up Finite Element Model and perform assembly analysis.

**Keywords:** flexible assembly, tolerance allocation, Computer Aided Design.

**Topic:** “Tolerancias y control de calidad geométrica en el producto industrial”.

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## 1. Introduction

In many industrial applications the manufacturing geometrical and dimensional tolerances play an important role for minimize the rejection of imperfect parts. This chose can be made using classical theory of tolerance propagation [1-5] in assemblies (i.e. stack-up formulas, six sigma method, kinematic approaches, maximum material condition, envelope principle and international standard prescriptions). When the investigated assembly is flexible, i.e. the elastic deformation of some parts cannot be neglected, the problem of allocation requires a careful study [6-7].

Designers, according to Global Dimensioning and Tolerancing standards, often assume these bodies as rigid and neglect elasticity, so they apply standard methods. International standards such as ISO 10579 provides only poor information about the dimensional check after manufacturing.

Flexible assemblies can be found in many mechanical application. One of them is the large amount of compliant mechanisms (especially those in micro-electro-mechanical-systems, MEMS) where the kinematic connecting joints are changed into flexibility parts of the links. Many plastic assemblies can also be considered as flexible systems (there are a lot of them in computers, monitors, mobile phones, household appliances, etc.).

The main problem in their study is to consider the flexibility during the assembly because the part can deform in order to be assembled. The limit of this deformation is due to permanent deformation (yielding) or to fatigue consideration. In many cases the deformation is necessary to the correct functioning (for example a locking plastic pin which can be inserted and resists thanks to its flexibility) so the part has to be designed for it.

In all this case the limits of acceptance of a component are wider than those of rigid assemblies. It depends on the possibility to have a positive assembling solution with a deformation under control. Although this deformation can occur also in common metal parts where it can be ignored, it cannot be neglected in all the cases where the parts can deform considerably (large displacement fields).

The investigated assembly is an earphone set of a mobile phones. Experience shows that this device undergoes to frequent failures located at the hub due to low-cycle fatigue stress (Figure 1). This problem impose to improve the design of this earphone set hub an in particular to study in depth the errors that can be allowed during manufacturing.

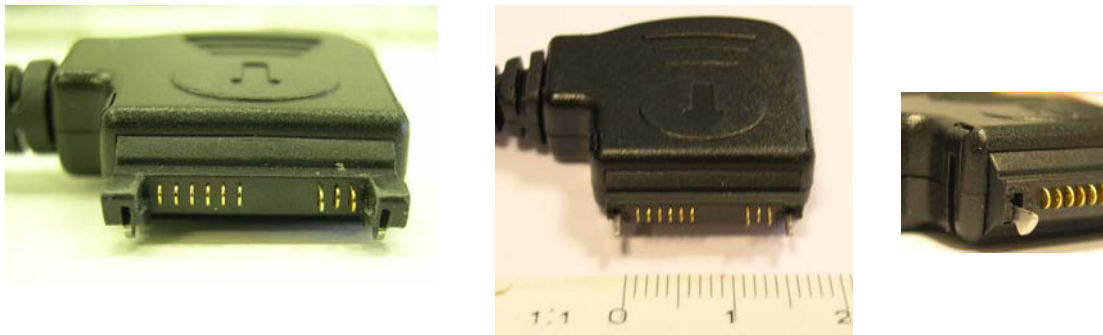
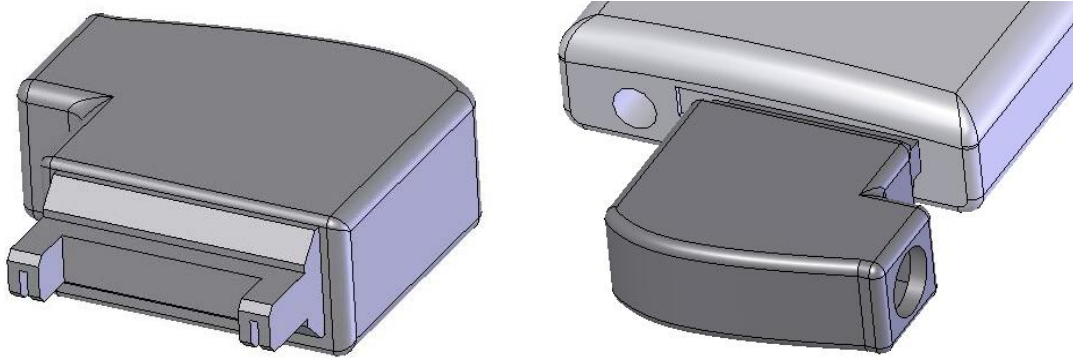


Figure 1: Picture of the earphone set hub before (on the left) and after (on the right) failure

## 2. Description of the assembly

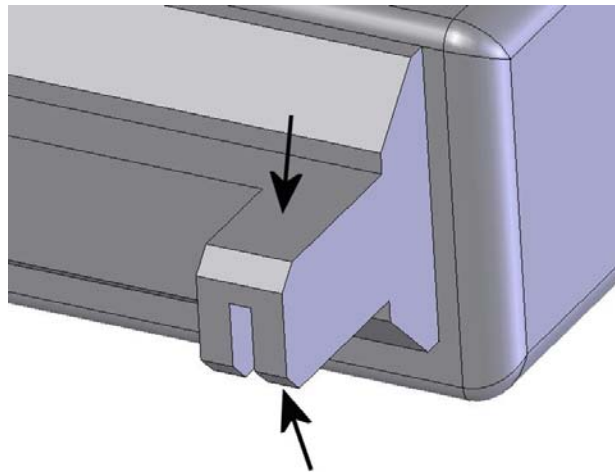
The studied assembly is depicted in a three dimensional CAD reconstruction in Figure 2. It consists of two parts: the mobile phone lower part and the earphone set connecting hub. The correct installation requires a force to push the two square pins of the hub into the housing of the phone. This operation causes a compression of the two pins and the assembly is kept fixed thanks to the friction force between the compressed faces of the pins and the faces on the housing. During normal operation, the hub and the housing undergo to cyclical stresses (during

mounting and dismounting). Moreover the hub is completely made up of plastic material while the housing has also a metallic case to reinforce the structure. Experience shows that many failures occur at hub pin. These can be attributed to fatigue process which weaken the resistance every assembling-disassembling process. The stresses in the hub can increase if the distance between pins causes them to bend in order to fully insert into the housing. This type of error (tolerance between pin relative distance) can be accepted if the structural stress is below the fatigue limit for the considered material. In this case the localization of the pin can also exceed the value suggested by the common rigid assembly approaches.



*Figure 2: CAD Model of the assembly: the earphone set on the left and the complete assembly on the right*

In Figure 3 a detail of one of the pin is depicted and the surfaces which assure the reaction forces during the assembly are shown.



*Figure 3: Detail of one of earphone pin, with the two surfaces compressed in the ideal fitting*

### 3. Flexible assembly

Let first consider the assembly of ideal parts without manufacturing errors. In this case the functional requirement is that the hub, when fitted in, can disengage the household only when is pull away with a minimum force  $F_d$ . This force is balanced by friction acting on both top and bottom surfaces in Figure 3. Assuming a static friction coefficient  $\mu_0$  the normal force  $F_n$  can be computed easily:

$$F_n = \frac{F_d}{\mu_0} \quad (1)$$

This force is generated by the reaction to the compression of both pins during the fitting in the housing.

Assuming  $F_d = 5 \text{ N}$  and  $\mu_0 = 0.5$  (metal-plastic), we get  $F_n = 10 \text{ N}$ . In order to understand the stress field generated to this force a finite elements model can be built. For this model a constant self equilibrated pressure has been applied to both the contact surfaces. In Figure 4 a preliminary result is depicted. As it is clear the influence of compression is a local effect (according to the Saint Venant theory) and the maximum stress concentration can be found in the middle of the pins. For any other value of the fitting force it is sufficient to multiply the stress value according to a proportional coefficient (we still assume to deal with linear elastic deformation).

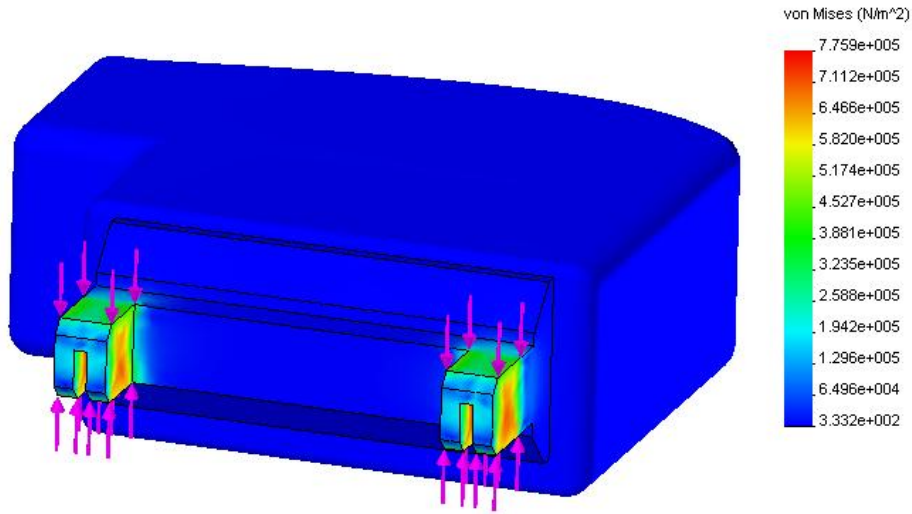


Figure 4: Stress contour of the ideal fitting with the minimum compression force

Let now examine the effect of increasing of 0.1 mm of the distance between the two pins. In this case we can solve a more complex finite element model searching the convergence imposing the shrink fit between the housing and the pin lateral surfaces. This condition can be written as constraint equation on displacement of pin and housing nodes:

$$\vec{P}_{pin} \wedge \vec{n} = \vec{P}_{hous} \wedge \vec{n} \quad (2)$$

where  $\vec{P}_{pin}$  and  $\vec{P}_{hous}$  are the vectors of displacement of pin and housing contact surface nodes, respectively and  $\vec{n}$  is the normal vector of the same surface.

What we get in this case can be observed in Figure 5.

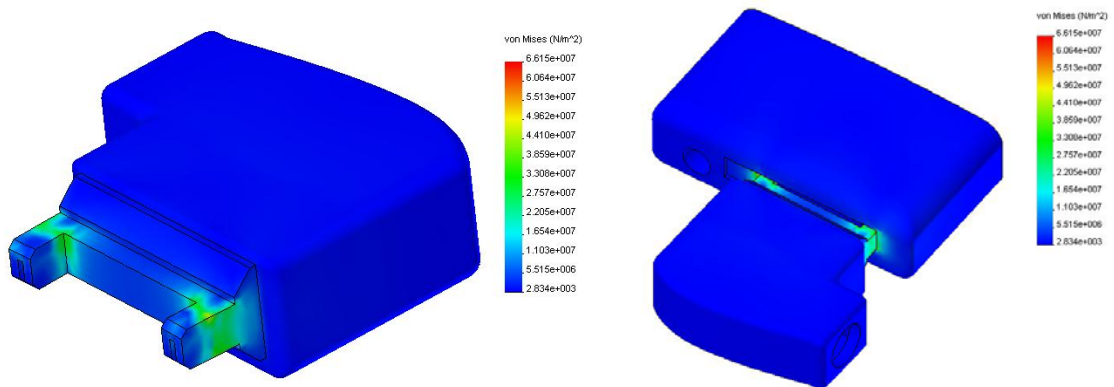


Figure 5: FEM results of an increasing of distance between two pins: the shrink fit causes bending

We can observe that stress concentration in this second analysis occurs in a different region of the hub (compare Figure 4 to Figure 5). Moreover the stress values due to normal fitting are less than those computed in case of an increment of distance between pins. It can be explained considering that in the second case the pins undergo to bending (instead of simple compression of the first case), so the structure is more stressed. Similar results can be obtained considering a decreasing of the distance between the two pins.

Now we can figure out the sum of two stress field and build up a correlation matrix. It can be observed that in general, contact problems are non linear. In this case, several structural solutions demonstrate that just for small errors we can approximate the stress fields as a linear function of the geometry overlapping error, and this result simplify the procedure.

#### 4. Tolerance allocation optimization

In the previous section we have assessed the effect of the two different causes on the stress which the structure undergoes to. Now we propose a methodology to improve the tolerance allocation for the assembly. Let now introduce the stress limit of the structure. This threshold can be computed taking into account the fatigue cycles during earphone life. Although plastic material properties change with the temperature, according to ASTM experimental tests we can assume as a valid threshold the value of  $S = 13.8$  MPa which is a common value for PC/ABS plastics<sup>2</sup>. We can combine the stresses obtained in the two different analysis types in order to obtain the global stress field as function of two dimensional error (length and height of the pin) computed in two reference points (in the middle of the pin and at one edge of the pin) which can be considered as “control points”.

In Figure 6 the plot of both functions are depicted. This plot shows that the error on pin height causes a structural stress lower than the error on the distance between two pins. Sectioning the surface plot on Figure 6 with planes orthogonal to stress axis we can obtain for a certain value of stress the admissible combination of two errors. These sections can be used as design charts in order to allocate tolerances taking into account the possible deformation and the stress threshold due to fatigue consideration.

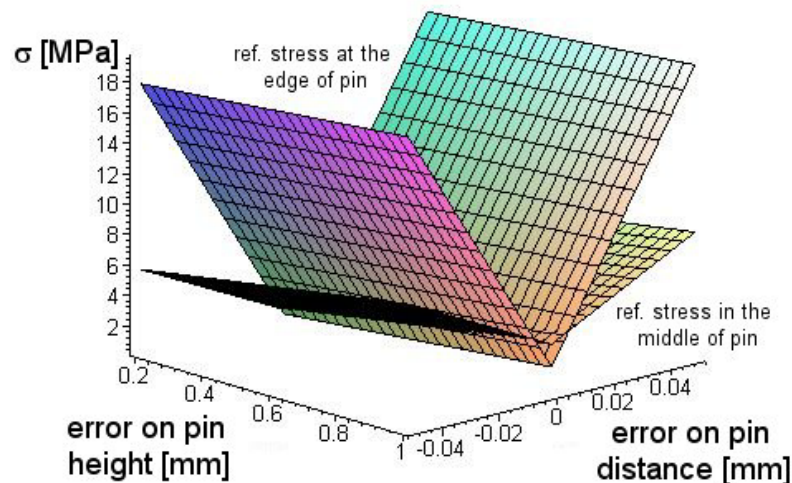


Figure 6: Global stress field as function of dimensional errors

<sup>2</sup> The threshold value of fatigue stress can be scaled according to different working condition. During these tests the temperature was 25° and the speed was very low (almost static tests). Choosing different threshold value does not affect the methodology.

## 5. Conclusions

The proposed methodology is based on a computer aided simulation approach. CAD modelling and finite element method have revealed to be a powerful instrument to improve the design of these flexible assembly. Moreover the developed analysis model can manage both interferences and contact problems. The output results of the study have been summarized into design charts which can be easily used to reduce the rejection of parts without required properties.

The proposed practical application is only one example of the methodology which can be extended to a large amount of industrial application. Another important advantage is the capability of including the presented approach to other typical methods for allocating tolerances such as kinematic approach both in two dimensions and in three dimensions.

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