Maximizing the Number of Broadcast Operations in Static Random Geometric Ad-Hoc Networks*

Tiziana Calamoneri[1], Andrea Clementi[2], Emanuele G. Fusco[1], and Riccardo Silvestri[1]

[1] Dipartimento di Informatica
Università di Roma "La Sapienza"
{calamoneri,fusco,silver}@di.uniroma1.it
[2] Dipartimento di Matematica
Università Tor Vergata di Roma
clementi@mat.uniroma2.it

Abstract. We consider static ad-hoc wireless networks where nodes have the same initial battery charge and they may dynamically change their transmission range at every time slot. When a node v transmits with range $r(v)$, its battery charge is decreased by $\beta \times r(v)^2$ where $\beta > 0$ is a fixed constant.

The goal is to provide a range assignment schedule that maximizes the number of broadcast operations from a given source (this number is denoted as the *length* of the schedule). This maximization problem, denoted as MAX LIFETIME, is known to be NP-hard and the best algorithm yields worst-case approximation ratio $\Theta(\log n)$, where n is the number of nodes of the network [5].

We consider *random geometric* instances formed by selecting n points independently and uniformly at random from a square of side length $\sqrt{n}$ in the Euclidean plane.

We first present an efficient algorithm that constructs a range assignment schedule having length, with high probability, not smaller than $1/12$ of the optimum.

We then design an efficient distributed version of the above algorithm where nodes initially know n and their own position only. The resulting schedule guarantees the same approximation ratio achieved by the centralized version thus obtaining the first distributed algorithm having *provably-good* performance for this problem.

1 Introduction

Range assignments in ad-hoc networks. In static ad-hoc radio networks (in short, ad-hoc networks), nodes have the ability to vary their transmission ranges (and, thus, their energy consumption) in order to provide good network connectivity and low energy consumption at the same time. More precisely, the transmission ranges determine a (directed) communication graph over the set V of nodes. Indeed, a node v, with range r, can transmit to another node w if and only if w belongs to the *disk* centered in v and of radius r. The transmission range of a node depends, in turn, on the energy power

* Research partially supported by the European Union under the Project IP-FP6-015964 AEOLUS.

E. Tovar, P. Tsigas, and H. Fouchal (Eds.): OPODIS 2007, LNCS 4878, pp. 247–259, 2007.

supplied to the node. In particular, the power P_v required by a node v to correctly transmit data to another station w must satisfy the inequality (see [22])

$$\frac{P_v}{\mathtt{dist}(v,w)^2} \geq \eta \tag{1}$$

where $\mathtt{dist}(v,w)$ is the Euclidean distance between v and w while η is a constant that, wlog, can be fixed to 1.

In several previous theoretical works [1,9,16,21], it is assumed that nodes can arbitrarily vary their transmission range over the set $\{\mathtt{dist}(v,w) \mid v,w \in V\}$ However, in some network models (like sensor networks), the adopted technology allows to have only few possible transmission range values. For this reason, we will assume that nodes have the ability to choose their transmission range from a finite set $\Gamma = \{0, r_1, r_2 \ldots, r_k\}$ (with $0 < r_1 < r_2 < \ldots < r_k$) that depends on the particular adopted technology (see [7,8,22]). Further technical constraints on Γ will be given and discussed in Subsection 1.1.

A fundamental class of problems, underlying any phase of a dynamic resource allocation algorithm in ad-hoc wireless networks, is the one known as *range assignment problems*. In these problems the goal is to find a transmission range assignment $r : V \rightarrow \Gamma$ such that (1) the corresponding communication graph satisfies a given connectivity property Π, and (2) the overall energy power $\mathsf{cost}(r) = \sum r(v)^2$ required to deploy the assignment is minimized (see for example [16,21]). Clearly, the maximal range value r_k in Γ must be sufficiently large to guarantee that at least one feasible solution exists.

Several research works [1,9,16] have been devoted to the case where Π is defined as follows: *Given specific source $s \in V$, the transmission graph has to contain a directed spanning tree rooted at s (a broadcast tree from s)*. The relevance of this problem (denoted as MIN ENERGY BROADCAST) is due to the fact that any communication graph satisfying the above property allows the source to perform a *broadcast* operation. Broadcast is a task initiated by the source that wants to transmits a message to all nodes. This task constitutes a basic and thus fundamental operation in real life multi-hop radio networks [2,3,16]. As for the worst-case complexity, MIN ENERGY BROADCAST is known to be NP-hard [9] (even when $|\Gamma| = 3$ and r_1 is a small positive constant) and a series of constant-factor approximation algorithms are available in [1,4,9,18]. The best known approximation factor is close to 4 and it is given in [6]. In [5], a more general version of MIN ENERGY BROADCAST is given where not uniform *node efficiency* is considered. In this version, a function $e : V \rightarrow R^+$ is given and the energy cost, required to transmit from node v to w, is $d(v,w)^2/e(v)$. This non-symmetric version of MIN ENERGY BROADCAST seems to be harder: the best known algorithm yields approximation ratio $\Theta(\log n)$ [5].

The MAX LIFETIME **problem.** The above power assignment problems do not consider important ad-hoc network scenarios where nodes are equipped with batteries of limited charge and the goal is to maximize the number of broadcast operations. This important (*maximization*) range assignment problem has been first analytically studied in [5] and it is the subject of our paper.

The goal is to maximize the *lifetime* of the network while having, at any *time period* t, a broadcast tree from a given source. Formally, each node is initially equipped with a battery charge [1] $B > 0$ that, at every time period t, is reduced by amount $\beta \times r_t(v)^2$ where $r_t(v)$ denotes the range assigned to node v during t and $\beta > 0$ is a fixed constant depending of the adopted technology. In this paper, we assume $\beta = 1$, however, all our results can be easily extended to any $\beta > 0$. A *range assignment schedule* is a set of functions $\{r_t : V \to \Gamma, t = 1, \ldots, m\}$. A range assignment schedule is said to be *feasible* if, at any time period t, r_t yields a broadcast tree from s and, for any $v \in V$, it holds that

$$\sum_{t=1}^{m} r_t(v)^2 \leq B$$

Then, the MAX LIFETIME problem is to find a feasible range assignment schedule of maximal length m.

In [5], MAX LIFETIME is shown to be NP-hard. In the same paper, by means of a rather involved reduction to MIN ENERGY BROADCAST with non uniform node efficiency, a polynomial time algorithm is provided yielding approximation ratio $\Theta(\log n)$. This positive result also holds when the initial node battery charges are not uniform.

A static version of MAX LIFETIME has been studied in [20]: the broadcast tree is fixed during the entire schedule and the quality of solutions returned by the MST-based algorithm is investigated. Such results and techniques are clearly not useful for our (dynamic) MAX LIFETIME problem.

Several other problems concerning network lifetime have been studied in the literature [7,8,20]. Their definitions vary depending on the particular node technology (i.e. fixed or adjustable node power) and on the required connectivity or covering property. However, both results and techniques (mostly of them being experimental) are not related to ours.

Our results. To the best of our knowledge, previous *analytical* results on MIN ENERGY BROADCAST and MAX LIFETIME concern worst-case instances only. Some *experimental* studies on MIN ENERGY BROADCAST have been done on *random geometric instances* [10,18]. Such input distributions turn out to be very important in the study of range assignment problems. On one hand, they represent the most natural random instance family where greedy heuristics (such as the MST-based one - see [16]) have a bad behaviour [18]. On the other hand, random geometric distributions is a first good way to model well-spread networks located on flat 2-dimensional regions [7,8,16,20].

We study MAX LIFETIME in random geometric instances of arbitrary size: set V is formed by n nodes selected uniformly and independently at random from the 2-dimensional square of side length $\lfloor\sqrt{n}\rfloor$. Such instances will be simply denoted as *random sets*. Notice that the maximal Euclidean distance among two nodes in random sets is $\sqrt{2n}$, so the maximal range value r_k can be assumed to be not larger than $\sqrt{2n}$.

A natural and important open question is thus to establish whether efficiently-constructible range assignment schedules exist for MAX LIFETIME having *provably-good* length on random sets. Moreover, the design of efficient *distributed* implementations of such schedules is of particular relevance in ad-hoc networks.

[1] So we here assume that, at the very beginning, all nodes are in the same energy situation.

To this aim, we first provide an upper bound on the length of optimal (i.e. maximal) range assignment schedules for *any* finite set V in the 2-dimensional plane. So this upper bound holds in the *worst-case*. Then, we present an efficient algorithm that, on any instance (V, s), returns a feasible schedule. Furthermore, when V is a random set, we prove the schedule length is, with high probability[2] (in short, *w.h.p.*), not smaller than $1/12$ of the optimum. The algorithm is centralized and works in $O(n^2 + nT)$ time where T is the number of broadcast operations yielded by the schedule.

In Section 4, we modify our centralized algorithm in order to design a distributed protocol for MAX LIFETIME on random sets. The protocol assumes that every node initially knows n and its Euclidean position only. This assumption is reasonable in *static* ad-hoc networks since node position can be either stored in the set-up phase or it can locally computed by every node by using GPS systems. This operation is not too expensive in terms of energy consumption since it is performed *once and for all* in the set-up phase.

We then show that the resulting scheduling is equivalent to that yielded by the centralized version and, hence, it achieves w.h.p. a constant approximation ratio as well. We thus get the first distributed protocol for MAX LIFETIME having provably good performance.

The protocol performs, somewhat in parallel, two tasks: (1) It constructs a broadcast communication subgraph starting from the source and (2) transmits the source message along this subgraph to all nodes. We emphasize that all node costs due to both the above tasks are taken into account: whenever a node transmits any message with range r, its battery charge is decreased by r^2.

Our analysis thus evaluates the number of broadcast operations achieved by our protocol. This suffices for bounding the approximation ratio. However, we also analyze the *amortized completion time* of single broadcast operations produced by our protocol. To this aim, we consider the *synchronous* model of communication [2,3,12,11,14] and take care of the MAC layer too: in fact, we also consider time delays due to avoid *collisions*.

Node communications thus work in synchronous *time-slots* and the *amortized completion time* of a protocol, yielding T broadcast operations, is the overall number of elapsed time slots divided by T.

It turns out that our protocol has amortized completion time

$$O\left(\frac{r_2 n\sqrt{n}}{T} + r_2^2 + \frac{\sqrt{n}}{r_2}\right)$$

Since our protocol w.h.p. returns an almost maximal number T of broadcast operations, we can point out some interesting facts.

Assume that $r_2 \in \Gamma$ is close to the *connectivity threshold* of *random geometric graphs* [15,19,23,24], i.e., $r_2 = \Theta(\sqrt{\log n})$ (this setting is relevant in our random input - see Subsection 1.1). Then, the worst scenario for our protocol is when the initial battery charge B is very small so that T is as well small, say $T = O(1)$. In fact, we get an amortized completion time $O(n\sqrt{n \log n})$ that is a factor $\sqrt{n \log n}$ larger than the best-known distributed broadcasting time [15], i.e., $O(n)$.

[2] Here and in the sequel the term *with high probability* means that the event holds with probability at least $1 - \frac{1}{n^c}$ for some constant $c > 0$.

However, those optimal-time distributed protocols [15] do not care about *node energy* costs and, thus, about the lifetime of the network. Our protocol, instead, somewhat trades global network lifetime with completion time of each single broadcast operation. This fact clearly arises whenever B is large enough to allow $T = \Omega(\sqrt{n})$ number of broadcast operations: in this case, we get $O(n\sqrt{\log n})$ amortized completion time, thus very close to the best-known distributed broadcasting completion time.

1.1 Preliminaries

A *random set* V is formed by n nodes selected uniformly and independently at random from the square Q of side length $\lfloor\sqrt{n}\rfloor$. The source node s can be any node in V. The length of a maximal feasible range assignment schedule (in short, schedule) for an input (V, s) is denoted as $\mathtt{opt}(V, s)$.

Given a set V of n nodes in the 2-dimensional Euclidean space and a positive real r, the *disk graph* $G(V, r)$ is the symmetric graph where two nodes in V are linked if $d(v, w) \leq r$. When V is a random set, the resulting disk graph distribution is known as *geometric random graphs* that are the subject of several important studies related to wireless networking [15,19,23,24]. In particular, it is known that, for sufficiently large n, a random geometric graph $G(V, r)$ is w.h.p. connected if and only if $r \geq \mu\sqrt{\log n}$, where $\mu = 1 + \epsilon$ for any constant $\epsilon > 0$ [19,23,24] . The value $\mathtt{CT}(n) = \mu\sqrt{\log n}$ is known as the *connectivity threshold* of random geometric graphs.

Assumptions on range set Γ. As for set

$$\Gamma = \{0, r_1, r_2 \ldots, r_k\}, \quad \text{with } 0 < r_1 < r_2 < \ldots < r_k \leq \sqrt{2n}$$

we make the following assumptions that are motivated by our choice of studying random sets.

The first positive value in Γ, i.e. r_1, is assumed to be $1 \leq r_1 < \mathtt{CT}(n)$. Observe that if $r_1 \geq \mathtt{CT}(n)$ then MAX LIFETIME on random sets admits a trivial schedule which is, w.h.p., a constant factor approximation: indeed the source must transmit at every time period with range at least r_1 and so all other nodes can transmit with the same range at every time period.

All other values in Γ can be arbitrarily fixed in input provided that all of them are *not smaller* than $\mathtt{CT}(n)$ and at least one of them is larger than $2\sqrt{2}c\sqrt{\log n}$, where $c > \mu$ is a small constant that will be defined in Lemma 2. Informally speaking, we require that *at least* one value in Γ is a bit larger than the connectivity threshold. This is reasonable and relevant in energy problems related to random geometric wireless networks since this value is the *minimal* one achieving w.h.p. global connectivity. Further discussion on such assumptions can be found in Section 5.

2 The Upper Bound

In this section, we provide an upper bound on the length of any feasible range assignment schedule for a set V.

Consider the disk graph $G(V, r_1)$ and let k_1 be the size of the connected component C_s of G containing source s.

Lemma 1. *Given a set V and a source $s \in V$, it holds that* $\mathsf{opt}(V,s) \leq \frac{B}{r_1^2}$. *Furthermore, if $k_1 < n$ then*

$$\mathsf{opt}(V,s) \leq \min\left\{\frac{B}{r_1^2}, \frac{B}{r_2^4}(k_1 r_2^2 + r_1^2 - k_1 r_1^2)\right\}$$

Proof. Since the source must transmit with range at least r_1 at any time period, the first upper bound follows easily.
If $k_1 < n$ then consider any feasible range assignment schedule $\mathcal{S}$. Let l_1 and l_2 be the number of time periods where the source transmits with range r_1 and *at least* r_2, respectively. It must hold that

$$l_1 r_1^2 + l_2 r_2^2 \leq B$$

Since $k_1 < n$ then, in each of the l_1 time periods of $\mathcal{S}$, there is at least one node in C_s but s having radius at least r_2. This yields

$$l_1 r_2^2 \leq (k_1 - 1)B$$

By simple calculations, from the above two inequalities, we derive an upper bound on the number of time periods of $\mathcal{S}$, i.e.

$$l_1 + l_2 \leq \min\left\{\frac{B}{r_1^2}r, \frac{B}{r_2^4}(k_1 r_2^2 + r_1^2 - k_1 r_1^2)\right\}$$

□

Notice that if V is a random set then, since $r_1 < \mathtt{CT}(n)$, it holds w.h.p. $k_1 < n$.

3 The Algorithm

In this section we present a simple and efficient algorithm for MAX LIFETIME and then we analyze its performance. For the sake of simplicity, in this extended abstract we restrict ourselves to the case $r_2 \geq c\sqrt{\log n}$. Nevertheless, it is easy to extend all our results to the more general assumption described in Section 1.1.

In order to prove the approximation ratio achieved by the schedule returned by our algorithm, we will use the following result that is a simple consequence of Lemma 1 in [17].

Lemma 2. *Constants $c > 0$ and $\gamma > 0$ exist such that the following holds. Given a random set $V \subseteq Q$ of n nodes, consider the partition of Q into square* cells *of side length ℓ where $c\sqrt{\log n} \leq \ell \leq \sqrt{n}$. Then, w.h.p., every cell contains at least $\gamma\ell^2$ nodes. The constants can be set as $c = 12$ and $\gamma = 5/6$.*

Theorem 1. *Let $V \subseteq Q$ be a random set of n nodes and $s \in V$ be any source node. Then, w.h.p., the range assignment schedule returned by* BS *is feasible and it has length at least* $\beta\mathsf{opt}(V,s)$*, where $\beta = 1/12$.*

Proof. From Lemma 2, every cell contains w.h.p. a Pivot (transmitting with range r_2) at every time period. At every time period, there is a Pivot in W_s. This implies that, at any time period, the set of Pivots w.h.p. forms a strongly-connected subgraph that

Algorithm 1. BS (Broadcast Schedule)

1: Input: Set $V \subseteq Q$ of n nodes; a source $s \in V$; a battery charge $B > 0$; the range set $\Gamma = \{0, r_1, r_2 \ldots, r_k\}$
2: Partition Q into square *cells* of side length $r_2/(2\sqrt{2})$; For any cell Q_j, let V_j be the set of nodes in Q_j; construct an arbitrary ordering in V_j
3: Let C_s be the connected component in $G(V, r_1)$ that contains s
4: **if** $|C_s| \leq r_2^2$ **then**
5:     $W_s \leftarrow C_s$
6: **else**
7:     W_s is defined as any connected subgraph of C_s such that it contains s and $|W_s| = r_2^2$
8: **end if**
9: Construct an arbitrary ordering of W_s
10: **for** any time period $t = 1, \ldots,$ **do**
11:     **if** node with index $t \bmod |W_s|$ in W_s has remaining battery charge at least r_2^2 **then**
12:         it is selected as *Pivot* and range r_2 is assigned to it
13:     **else**
14:         The algorithm stops
15:     **end if**
16:     **for** any cell Q_j **do**
17:         **if** node with index $t \bmod |V_j|$ in Q_j has remaining battery charge at least r_2^2 **then**
18:             it is selected as *Pivot* and range r_2 is assigned to it
19:         **else**
20:             The algorithm stops
21:         **end if**
22:     **end for**
23:     All nodes in W_s not selected in lines 11 and 17 have radius r_1
24:     All nodes in $V \setminus W_s$ not selected in line 17 have range 0
25: **end for**

covers all nodes in V and s is connected to one of such Pivots. Moreover, BS assigns, to every node, an energy power which is never larger than the current battery charge of the node.

We now evaluate the length T of the scheduling produced by BS, so T is the last time period of the BS's run on input (V, s). Let w be any node in $V \setminus W_s$ then, from Lemma 2, in its cell there are w.h.p. at least $(\gamma r_2^2)/8$ nodes. So, w spends at most energy

$$\left(\frac{8T}{\gamma r_2^2}\right) r_2^2 \tag{2}$$

From (2), T can be any value such that

$$T \leq \frac{\gamma B}{8} \tag{3}$$

During the schedule, every node v in W_s will have range r_1 or r_2. Let $|W_s| = k$, then the energy spent by v is at most

$$\left(\frac{T}{k} + \frac{8T}{\gamma r_2^2}\right) r_2^2 + T r_1^2 \tag{4}$$

Notice that in (4) we have considered the fact that a node in W_s can have range r_2 because it has been selected as Pivot of its cell (Line 17) or as Pivot of W_s (Line 11). Now, two cases may arise.

- If $k \geq \left(\frac{r_2}{r_1}\right)^2$, since $r_1 \geq 1$, from (4) the amount of spent energy is at most $Tr_1^2\,(2 + 8/\gamma)$. So, T can be any value such that

$$T \leq \frac{B}{r_1^2\,(2 + 8/\gamma)} \tag{5}$$

Observe that every value T that satisfies 5, it also satisfies Eq. 3. So T can assume value $\frac{B}{r_1^2(2+8/\gamma)}$ and, from Lemma 1, we have that

$$T \geq \frac{\mathsf{opt}(V, s)}{2 + 8/\gamma}$$

- If $k < \left(\frac{r_2}{r_1}\right)^2$, according to the definition of W_s, we have $k = k_1$. From (4) and some simple calculations, the energy spent by $v \in W_s$ is at most

$$T\,\frac{r_2^4 + k_1 r_1^2 r_2^2 + (8/\gamma)k_1 r_2^2}{r_2^2 k_1 + r_1^2 - k_1 r_1^2}$$

where we used the fact that $r_1^2 - k_1 r_1^2 \leq 0$. Observe also that since $k_1 < \left(\frac{r_2}{r_1}\right)^2$ and $r_1 \geq 1$, we get

$$k_1 r_1^2 r_2^2 + (8/\gamma)k_1 r_2^2 \leq r_2^4 \left(1 + \frac{8}{\gamma r_1^2}\right) \leq r_2^4 \left(1 + \frac{8}{\gamma r_1^2}\right)$$

It thus follows that the energy spent by v is at most

$$T\,\frac{r_2^4 + k_1 r_1^2 r_2^2 + (8/\gamma)k_1 r_2^2}{r_2^2 k_1 + r_1^2 - k_1 r_1^2} \leq T\frac{r_2^4(2 + 8/\gamma)}{r_2^2 k_1 + r_1^2 - k_1 r_1^2}$$

It follows that T can be any value such that

$$T \leq \frac{r_2^2 k_1 + r_1^2 - k_1 r_1^2}{r_2^4(2 + 8/\gamma)} B \tag{6}$$

Finally, by combining (3), (6), and Lemma 1, we get again

$$T \geq \frac{\mathsf{opt}(V, s)}{2 + 8/\gamma}$$

So, the Theorem is proved for $\beta = 1/(2 + 8/\gamma) > 1/12$. □

4 The Distributed Version

In this section, we present a distributed version of BS. As mentioned in the Introduction, we adopt the synchronous model of node communication: the protocol acts in homogeneous *time slots*.

The resulting protocol is non spontaneous and assumes that every node v knows the number n of nodes, its own position (w.r.t. an absolute coordinate system) and, clearly, Γ.

In what follows, the eccentricity of source s in W_s (i.e. the maximal distance between s and a node in W_s) is denoted as $h(W_s)$ and the t-th message sent by the source is denoted as m_t. We assume that m_t contains the value of time period t.

Protocol: DBS (Distributed Broadcast Schedule)

Preprocessing: /* Construction of $W_s \subseteq C_s$ such that $h(W_s) \leq r_2^2$. */

- *One-to-All.* Starting from s, use round robin among nodes and range transmission r_1 to inform all nodes in C_s that are at most within r_2^2 hops from s: such nodes will form W_s. The one-to-all operation induces a spanning tree `Tree` of W_s rooted at s.
- *All-to-One.* By a simple bottom-up process on `Tree` and using round robin on each level, s collects all node labels and the structure of `Tree`.
- *Initialization.* Every node sets a local counter $\texttt{counter} = -1$. Furthermore, each node has a local array P of length $(\gamma/8)r_2^2$ where it will store the ordered list of the first $(\gamma/8)r_2^2$ labels belonging to its own cell. This array is initially empty.

Let us observe that at the end of the Preprocessing phase, source s has full knowledge of W_s.

Broadcast operations:

- For $t = 0, 1, \ldots$ /* time periods */
  Execute Procedure BROADCAST(m_t)

Procedure BROADCAST(m_t)

Nodes in W_s only:

- Source s selects the $(t \bmod \min\{|W_s|, r_2^2\})$-th node in W_s as *Pivot* (range r_2 will be assigned to it);
  s transmits, with range r_1, $\langle m_t, P\rangle$ where P is the path in `Tree` from s to the Pivot.
- When a node in W_s receives $\langle m_t, P\rangle$, it checks whether its label is the first in P. If this is the case, it transmits, with range r_1, $\langle m_t, P'\rangle$ where P' is the residual path to the Pivot.
- When the selected Pivot p of W_s receives $\langle m_t, P = (p)\rangle$, it transmits, with range r_2, $\langle m_t, i\rangle$ where i is the index of its cell.

All nodes:

- If $(t \leq (\gamma/8)r_2^2)$ then
  - When a node v receives, for the first time w.r.t. time period t, $\langle m_t, i\rangle$ from the Pivot of a neighbor cell i, it becomes active.

    - An active node, at every time slot, increments `counter` by one and checks whether its label is equal to the value of its `counter`. If this is the case, it becomes the Pivot of its cell and transmits, with range r_2, $\langle m_t, i\rangle$ where i is the index of its cell.
    - When an active node in cell i receives $\langle m_t, i\rangle$, it (so the Pivot as well) records in $P[t]$ the current value of counter c, i.e. the label of the Pivot, and becomes inactive.
- else (i.e. $(t > (\gamma/8)r_2^2)$)
    - When a node v receives, for the first time w.r.t. time period t, $\langle m_t, i\rangle$ from the Pivot of a neighbor cell i, it checks if its label is equal to $P[t \bmod (\gamma/8)r_2^2]$. If this is the case, it becomes the Pivot of its cell and transmits, with range r_2, $\langle m_t, j\rangle$ where j is the index of its cell.

The above protocol has the following properties that are a key-ingredient in the performance analysis.

Fact 2. *Even though they initially do not known each other, all nodes in the same cell are activated (and disactivated) at the same time slot, so their local counters share the same value at every time slot. Furthermore, after the first $(\gamma/8)r_2^2$ broadcast operations, all nodes in the same cell know the set P of Pivots of that cell.*

More precisely, if $l_0 < l_1 < l_2 < \cdots$ are the labels of the nodes in a cell, then, during the first $(\gamma/8)r_2^2$ broadcast operations (i.e.time periods), the Pivot of the cell at time period t will be node having label l_t.

Lemma 3. *Given a random set $V \subseteq Q$ and any source $s \in V$, if the length of the broadcast schedule yielded by* BS *is T, then the length of the broadcast schedule yielded by* DBS *is at least $T - 2$.*

Proof. Notice that, the only difference in terms of power consumption between BS and DBS lies in the Preprocessing phase required by the latter. In that phase, at most two messages with range r_1 are sent by a node to discover W_s. Indeed, thanks to Fact 2, the `if` branch of the Broadcast procedure for nodes in V spends *time* instead of *power* in order to discover the set of Pivots of each cell. Hence, in the worst case, the distributed version performs two broadcasts less than the centralized algorithm. □

Corollary 1. *Let $V \subseteq Q$ be a random set of n nodes and $s \in V$ be any source node. Then, w.h.p., the range assignment schedule returned by* DBS *is feasible and it has a length at least $\beta\mathsf{opt}(V, s) - 2$ where $\beta = 1/12$.*

Proof. Direct consequence of Theorem 1 and Lemma 3. □

We now evaluate message and time complexity of DBS.

Lemma 4. *The overall number of node transmissions (i.e. the message complexity) of* DBS *is $O(|W_s| + T \cdot ((n/r_2^2) + r_2^2))$, where T is the length of the schedule.*

Sketch of the proof. Observe that in the Preprocessing phase only nodes in C_s can exchange messages. In particular, s and all nodes within r_2^2 hops from s send only one message; all other nodes within 1 and $r_2^2 - 1$ hops from s send two messages. So, the message complexity of the Preprocessing phase is $\Theta(|W_s|)$. Thanks to Fact 2, during each broadcast, exactly one message per cell is sent, so globally $O(8n/r_2^2)$ messages are exchanged; to this number of messages, we have to add those sent by the nodes of path P in W_s: this value is bounded by r_2^2. □

Theorem 3. *The overall number of time slots required by* DBS *to perform T broadcast operations is w.h.p.*

$$O(r_2 n\sqrt{n} + T \cdot (r_2^2 + \sqrt{n}/r_2))$$

Sketch of the proof. For a single broadcast operation performed by DBS, we define the *delay* of a cell as the number of time slots from its activation time and the selection of its Pivot. Observe that the sum of delays introduced by a cell during the first $(\gamma/8)r_2^2$ broadcasts is at most n. Then, the delay of any cell becomes 0 for all broadcasts after the first $(\gamma/8)r_2^2$ ones. Moreover, a broadcast can pass over at most $O(\sqrt{n}/r_2)$ cells. By assuming that a maximal length path (this length being $\Theta(\sqrt{n}/r_2)$) together with maximal cell delay can be found in each of the first $\min\{(\gamma/8)r_2^2, T\}$ broadcasts, we can bound the maximal overall delay with

$$O(r_2 n\sqrt{n}) \tag{7}$$

In the Preprocessing phase, DBS uses round robin to avoid collisions. During the All-to-One phase, each node needs to collect all messages from its children before sending a message to its parent in `Tree`. Hence, the whole phase is completed in

$$O(nr_2^2) \tag{8}$$

time slots as the height of `Tree` is bounded by r_2^2.

Finally, the number of time slots required by every broadcast without delays and Preprocessing time is

$$O(r_2^2 + \sqrt{n}/r_2) \tag{9}$$

since r_2^2 is the upper bound on $h(W_s)$ and the length of any path on the broadcast tree outside W_s is $O(\sqrt{n}/r_2)$.

By combining (7), (8), and (9), we get the theorem bound without considering collisions among cell Pivots. In order to avoid such collisions, we further organize DBS into iterative phases: in every phase, only cells with not colliding Pivot transmissions are active. Since the number of cells that can interfere with a given cell is constant, this further scheduling will increase the overall time of DBS by a constant factor only. This iterative process can be efficiently performed in a distributed way since every node knows n and its position, so it knows its cell. □

From Theorem 3, the amortized completion time of a single broadcast operation performed by DBS is

$$O\left(\frac{r_2 n\sqrt{n}}{T} + r_2^2 + \frac{\sqrt{n}}{r_2}\right)$$

5 Open Problems

In this paper, we have studied the MAX LIFETIME problem on random sets. Further interesting future studies should address other basic operations than broadcasting: for instance, the gossiping operation which is known to be NP-hard too [5]. A more technical problem, left open by our research, is the study of MAX LIFETIME when Γ contains more than one *positive* values *smaller* than the connectivity threshold $\mathtt{CT}(n)$ of random geometric graphs. This case seems to be very hard since it concerns the size and the structure of the connected components of such random graphs *under* the threshold connectivity [19,23].

References

1. Ambuehl, C.: An optimal bound for the MST algorithm to compute energy efficient broadcast trees in wireless networks. In: Caires, L., Italiano, G.F., Monteiro, L., Palamidessi, C., Yung, M. (eds.) ICALP 2005. LNCS, vol. 3580, pp. 1139–1150. Springer, Heidelberg (2005)
2. Bar-Yehuda, R., Goldreich, O., Itai, A.: On the time-complexity of broadcast in multi-hop radio networks: An exponential gap between determinism and randomization. JCSS 45, 104–126 (1992)
3. Bar-Yehuda, R., Israeli, A., Itai, A.: Multiple communication in multi-hop radio networks. SICOMP 22(4), 875–887 (1993)
4. Calinescu, G., Li, X.Y., Frieder, O., Wan, P.J.: Minimum-Energy Broadcast Routing in Static Ad Hoc Wireless Networks. In: Proc. of *20th IEEE INFOCOM*, pp. 1162–1171 (April 2001)
5. Calinescu, G., Kapoor, S., Olshevsky, A., Zelikovsky, A.: Network lifetime and power assignment in ad hoc wireless networks. In: Di Battista, G., Zwick, U. (eds.) ESA 2003. LNCS, vol. 2832, pp. 114–126. Springer, Heidelberg (2003)
6. Caragiannis, I., Flammini, M., Moscardelli, L.: An exponential improvement on the MST heuristic for the minimum energy broadcast problem. In: Proc. of ICALP 2007. LNCS, vol. 4596, pp. 447–458. Springer, Heidelberg (2007)
7. Cardei, M., Du, D.-Z.: Improving wireless sensor network lifetime through power organization. Wireless Networks 11, 333–340 (2005)
8. Cardei, M., Wu, J., Lu, M.: Improving network lifetime using sensors with adjustable sensing ranges. Int. J. Sensor Networks 1(1/2), 41–49 (2006)
9. Clementi, A., Crescenzi, P., Penna, P., Rossi, G., Vocca, P.: On the Complexity of Computing Minimum Energy Consumption Broadcast Subgraphs. In: Ferreira, A., Reichel, H. (eds.) STACS 2001. LNCS, vol. 2010, pp. 121–131. Springer, Heidelberg (2001), http://www.dia.unisa.it/~penna
10. Clementi, A., Huiban, G., Penna, P., Rossi, G., Verhoeven, Y.C.: On the Approximation Ratio of the MST-based Heuristic for the Energy-Efficient Broadcast Problem in Static Ad-Hoc Radio Networks. In: Proc. of IPDPS 2003, p. 222 (2003)
11. Clementi, A.E.F., Monti, A., Silvestri, R.: Distributed broadcast in radio networks of unknown topology. Theor. Comput. Sci. 302(1-3), 337–364 (2003)
12. Chrobak, M., Gasieniec, L., Rytter, W.: Fast broadcasting and gossiping in radio networks. J. Algorithms 43(2), 177–189 (2002)
13. Chu, W., Colbourn, C.J., Syrotiuk, V.R.: The effects of synchronization on topology transparent scheduling. Wireless Networks 12, 681–690 (2006)
14. Czumaj, A., Rytter, W.: Broadcasting algorithms in radio networks with unknown topology. J. Algorithms 60(2), 115–143 (2006)

15. Dessmark, A., Pelc, A.: Broadcasting in geometric radio networks. Journal of Discrete Algorithms (2006)
16. Ephremides, A., Nguyen, G.D., Wieselthier, J.E.: On the Construction of Energy-Efficient Broadcast and Multicast Trees in Wireless Networks. In: Proc. of 19th IEEE INFOCOM, pp. 585–594 (2000)
17. Flaxman, A.D., Frieze, A.M., Vera, J.C.: On the average case performance of some greedy approximation algorithms for the uncapacitated facility location problem. In: Proc. of the 37-th ACM STOC 2005, pp. 441–449 (2005)
18. Flammini, M., Navarra, A., Perennes, S.: The Real Approximation Factor of the MST Heuristic for the Minimum Energy Broadcast. In: Nikoletseas, S.E. (ed.) WEA 2005. LNCS, vol. 3503, pp. 22–31. Springer, Heidelberg (2005)
19. Gupta, P., Kumar, P.R.: Critical power for asymptotic connectivity in wireless networks. In: Stochastic Analysis, Control, Optimization and Applications, Birkhauser, Boston, MA, pp. 547–566 (1999)
20. Kang, I., Poovendran, R.: Maximizing network lifetime of wireless broadcast ad hoc networks. J. of ACM Mobile Networks and Applications (6) (2005)
21. Kirousis, L.M., Kranakis, E., Krizanc, D., Pelc, A.: Power Consumption in Packet Radio Networks. Theoretical Computer Science 243, 289–305 (2000)
22. Pahlavan, K., Levesque, A.: Wireless Information Networks. Wiley-Interscience, Chichester (1995)
23. Penrose, M.: Random Geometric Graphs. Oxford University Press, Oxford (2003)
24. Santi, P., Blough, D.M.: The Critical Transmitting Range for Connectivity in Sparse Wireless Ad Hoc Networks. IEEE Trans. on Mobile Computing 2, 25–39 (2003)