Journal of Econometrics 48 (1991) 119–134. North-Holland

Bounded-influence estimators for the SURE model

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Received November 1988, final version received February 1990

This paper considers robust estimation of the seemingly unrelated regression equations (SURE) model. Following the infinitesimal approach to robustness, we characterize the class of optimal bounded-influence estimators and we propose two computationally simple estimators. We also present the results of a set of Monte Carlo experiments carried out to study the behavior of these estimators.

1. Introduction

In recent years much research has been devoted to developing regression estimators which are robust, that is, not too sensitive, to violations of some of the underlying statistical assumptions [see, e.g., Krasker (1980), Huber (1981, 1983), Krasker and Welsch (1982), Hampel et al. (1986)]. There have been also some interesting econometric applications of these estimators. For example, Krasker, Kuh, and Welsch (1983) and Small (1986) estimate hedonic price models for housing, Swartz and Welsch (1986) estimate and forecast energy demand, and Thomas (1987) uses a very large data set to estimate Engel curves for food. All these studies show that robust methods can lead to significant differences with respect to ordinary least squares (LS) in terms of point estimates, inference, and forecasts. This is due to the fact that robust methods are much less sensitive than LS to local violations of the model

*I would like to thank Ed Leamer and Arnold Zellner for useful comments on an earlier draft of this paper.

assumptions. All available studies, however, assume that the disturbances have a scalar covariance matrix at the 'central' model, which rules out important practical cases, such as systems of seemingly unrelated regression equations (SURE).

In this paper we consider robust estimation of the SURE model. Following the infinitesimal approach to robustness [Hampel et al. (1986)], we present a class of regular (that is, consistent and asymptotically normal) M-estimators that have a bounded and continuous influence function (IF). All these estimators are therefore qualitatively robust [Hampel (1971)], that is, small perturbations of the assumed statistical model can only have small effects on the distribution of the estimates. This desirable local stability property is not shared by the conventional maximum-likelihood (ML) estimator based on the assumption of normal (Gaussian) disturbances. In the class of estimators with a bounded and continuous IF, the ones proposed in this paper have minimum asymptotic mean square error (MSE) at the Gaussian model, and therefore attain the best trade-off between efficiency and robustness.

The rest of this paper is organized as follows. Section 2 examines the robustness properties of the Gaussian ML estimator. Section 3 characterizes the class of optimal bounded-influence estimators. Section 4 presents some Monte Carlo results. Section 5 contains the conclusions.

2. Robustness properties of the Gaussian ML estimator

Consider a system of q regression equations of the form

$$y_{in} = x_{in}'\beta_i + u_{in}, \qquad i = 1,\ldots,q, \quad n = 1,\ldots,N,$$

where β_i is a $k_i \times 1$ vector of possibly unknown parameters and x_{in} is a $k_i \times 1$ vector of exogenous regressors. The disturbances $\{u_{in}\}$ are serially independent with zero mean and finite variance, but are contemporaneously correlated across regression equations. The system can be rewritten as

$$y_n = x_n'\beta_0 + u_n, \qquad n = 1,\ldots,N, \tag{1}$$

where y_n is a $q \times 1$ vector, $\beta_0 = (\beta_1',\ldots,\beta_q')'$ is a $k \times 1$ vector of parameters with $k = \sum_{i=1}^{q} k_i$,

$$x_n = \begin{bmatrix} x_{1n} & & \\ & \ddots & \\ & & x_{qn} \end{bmatrix}$$

is a $k \times q$ matrix of exogenous regressors, and $\{u_n\}$ are $q \times 1$ independently and identically distributed random vectors with zero mean and finite variance Σ_0, a symmetric, positive definite (p.d.) $q \times q$ matrix. Let Θ and $\theta_0 = (\beta_0', \sigma_0')' \in \Theta$ denote, respectively, the parameter space and the vector of parameters to be estimated. The parameter of interest is β_0, whereas $\sigma_0 = \text{vec}\, \Sigma_0$ is a nuisance parameter.[1]

The parameter θ_0 can be estimated using various methods, including single-equation LS or, if efficiency gains are sought, Zellner's (1962) feasible GLS procedure and the method of ML. In the ML case, it is common to assume that the distribution of the disturbances in (1) is q-variate Gaussian. If this model is correctly specified and possesses a finite, positive definite (p.d.) Fisher information matrix, then the Gaussian ML estimator $\hat{\theta} = (\hat{\beta}', \hat{\sigma}')'$ is consistent and asymptotically efficient. Further, $\hat{\beta}$ is asymptotically independent of $\hat{\sigma}$ and is asymptotically equivalent to Zellner's estimator.

To investigate the robustness properties of the Gaussian ML estimator, let $\hat{\theta}_{N-1}$ denote the estimate corresponding to a sample of size $N-1$, and consider adding to the sample an additional observation $z = (y, (\text{vec}\, x)')' \in \mathcal{Y}$, where $\mathcal{Y}$ denotes the sample space. Let $\hat{\theta}_{N,z}$ denote the resulting estimate. The rescaled difference $N(\hat{\theta}_{N,z} - \hat{\theta}_{N-1})$ can be shown to converge in probability to a finite limit. This limit, viewed as a function of z, is called the influence function (IF) of $\hat{\theta}$ [Hampel (1974)] and is a measure of the asymptotic bias of $\hat{\theta}$ when the distribution of the observations is subject to an infinitesimal amount of contamination at a given point in the sample space. The IF therefore provides a description of the local robustness properties of an estimator. It follows from standard results [see, e.g., Serfling (1980)] that the IF of $\hat{\theta}$ (and of any other estimator asymptotically equivalent to $\hat{\theta}$), evaluated at the Gaussian model $\{F_\theta\}$, is given by

$$IF(z, \hat{\beta}, F_\theta) = J_\beta(\theta)^{-1} xVr,$$

$$IF(z, \hat{\sigma}, F_\theta) = \tfrac{1}{2} J_\sigma(\theta)^{-1} [V \otimes V'] \text{vec}(rr' - I_q),$$

where V is a finite, p.d. matrix such that $VV' = \Sigma^{-1}$, $r = V'(y - x'\beta)$ is the vector of standardized disturbances, $J_\beta(\theta)$ and $J_\sigma(\theta)$ are the diagonal blocks of the Fisher information matrix, and $\otimes$ denotes the Kronecker product. Notice that $IF(z, \hat{\beta}, F_\theta)$ and $IF(z, \hat{\sigma}, F_\theta)$ are both unbounded functions of z.

[1]'Vec' denotes the operator that stacks the columns of a matrix in a single column vector. Strictly speaking, we should consider the $q(q+1)/2$-vector of distinct elements of the matrix Σ_0, but this would only complicate the notation.

This reflects the fact that one large disturbance or one gross-error in x are sufficient to completely spoil the estimates. Also notice that while the influence of a single disturbance on $\hat{\beta}$ is linear, its effect on $\hat{\sigma}$ is quadratic.

3. Bounded-influence estimation

An M-estimator of θ_0 is a root of an implicit equation of the form

$$\sum_{n=1}^{N} \eta_N(z_n, \theta) = 0, \tag{2}$$

where the vector function $\eta_N(\cdot, \theta)$, defined on $\mathscr{Z} \times \Theta$, is called the score function associated with the estimator. Clearly, we obtain the ML estimator when $\eta_N(\cdot, \theta)$ is equal to the likelihood score.

As is well known, the efficiency and robustness properties of an M-estimator are closely related to the properties of its IF [see, e.g., Serfling (1980) and Huber (1981)]. Given a parametric model, an M-estimator is efficient if and only if its IF is a nonsingular linear transformation of the likelihood score. On the other hand, an M-estimator is qualitatively robust in the sense of Hampel (1971) if and only if its IF is bounded and continuous. Qualitative robustness is a desirable property, because it ensures that small departures from the assumed statistical model can only have small effects on the distribution of an estimator. The Gaussian ML estimator is clearly not qualitatively robust. A natural quantitative measure of the robustness of an M-estimator is given by the sup-norm of its IF, called the estimator's sensitivity. This measure provides an upper bound on the asymptotic bias that may arise under small departures from the model assumptions. An estimator with a bounded IF or, equivalently, a finite sensitivity is called a bounded-influence estimator.

In this paper we allow for separate sensitivity bounds for the estimators of β_0 and σ_0, and we consider the class of bounded-influence estimators $\bar{\theta} = (\bar{\beta}', \bar{\sigma}')'$ for which

$$\sup_{z \in \mathscr{Z}} \| IF(z, \bar{\beta}, F_\theta) \|_{B_1} \leq \gamma_1, \tag{4}$$

$$\sup_{z \in \mathscr{Z}} \| IF(z, \bar{\sigma}, F_\theta) \|_{B_2} \leq \gamma_2, \tag{5}$$

where (B_1, B_2) are p.d. matrices, (γ_1, γ_2) are finite constants, and $\|x\|_B =$

$(x'Bx)^{1/2}$ denotes the norm of the vector x in the metric of the p.d. matrix B. An estimator in this class is called optimal if it has minimum asymptotic mean square error (MSE) at the assumed Gaussian model $\{F_\theta\}$.

An optimal bounded-influence estimator $\tilde{\theta}$ can be characterized by its score function. Peracchi (1987) shows that the optimal score has a relatively simple form when the MSE criterion is defined in the metric of a block-diagonal matrix with subblocks equal to (B_1, B_2). In particular, the symmetry of the Gaussian error distribution implies that the first k components of the optimal score $\eta(z,\theta)$ are given by

$$\eta_1(z,\theta) = w_1(z,\theta)xVr, \tag{6}$$

where $w_1(z,\theta)$ is a scalar weight function defined by

$$w_1(z,\theta) = \min\{1, \gamma_1/\|A_1 xVr\|_{B_1}\},$$

and A_1 is a p.d. matrix root of the equation

$$\mathrm{E}\min\{1, \gamma_1/\|A_1 xVr\|_{B_1}\}xVrr'V'x' - A_1^{-1} = 0,$$

with E denoting expectations taken with respect to the Gaussian model. It can be shown that this equation has a solution only if

$$\gamma_1 \geq (\operatorname{trace} B_1)\big/\left[\mathrm{E}\|xVr\|_{B_1}\right].$$

The remaining q^2 components of the optimal score are given by

$$\eta_2(z,\theta) = w_2(z,\theta)\operatorname{vec}(rr' - I_q), \tag{7}$$

where $w_2(z,\theta)$ is a scalar weight function defined by

$$w_2(z,\theta) = \min\{1, \gamma_2/\|A_2 \operatorname{vec}(rr' - I_q)\|_{B_2}\},$$

and A_2 is a p.d. matrix root of the equation

$$\mathrm{E}_\Phi \min\{1, \gamma_2/\|A_2 \operatorname{vec}(rr' - I_q)\|B_2\}$$

$$\times \operatorname{vec}(rr' - I_q)\operatorname{vec}(rr' - I_q)' - A_2^{-1} = 0,$$

with expectations taken with respect to the multivariate standard normal distribution. It can be shown that this equation has a solution only if

$$\gamma_2 \geq (\text{trace}\, B_2)\Big/\left[\mathrm{E}_{\Phi}\|\text{vec}(rr' - I_q)\|_{B_2}\right].$$

Since the optimal score function $\eta(\cdot,\theta)$ is bounded and continuous, $\tilde{\theta}$ is qualitatively robust. If $(\gamma_1,\gamma_2) \to \infty$, then $\tilde{\theta}$ reduces to the Gaussian ML estimator of θ_0. Notice that if γ_1 or γ_2 are finite, system estimation is required even when each equation contains exactly the same regressors and there are no cross-equations nor covariance restrictions.

The optimal estimator $\tilde{\theta}$ can be interpreted as a weighted ML estimator. Geometrically, the likelihood score for one observation is shrunk so as to satisfy the robustness constraints (3) and (4). The weight applied to the likelihood score depends on the choice of the matrices (B_1, B_2). For example, when B_1 is equal to the identity matrix we obtain the SURE analogue of the Hampel–Krasker estimator of regression [Hampel (1978), Krasker (1980)]. When $B_1 = AV(\tilde{\beta}, F_\theta)^{-1}$, we obtain the analogue of the regression estimator of Krasker and Welsch (1982).

Computation of $\tilde{\theta}$ for a given sample can be expensive because eq. (2) must be solved numerically, and each iteration requires solving two implicit matrix equations for A_1 and A_2. Following the suggestion of Peracchi (1987), the arbitrariness of the choice of B_1 and B_2 can be exploited to simplify the computation. In particular, B_1 and B_2 can be chosen such that

$$w_1(z,\theta) = \min\{1, \gamma_1/\|xVr\|\}, \tag{8}$$

$$w_2(z,\theta) = \min\{1, \gamma_2/\|\text{vec}(rr' - I_q)\|\}, \tag{9}$$

where $\|\cdot\|$ denotes the Euclidean norm. The resulting estimator, denoted by BI_1, is not invariant under a reparameterization of the model. An invariant estimator, denoted by BI_2, can be obtained by choosing B_1 such that

$$w_1(z,\theta) = \min\{1, \gamma_1/\|xVr\|_{J_\beta(\theta)^{-1}}\}, \tag{10}$$

where $J_\beta(\theta) = \mathrm{E}x\Sigma^{-1}x'$.

In this paper we shall also consider a simple generalization of Huber's M-estimator of regression. The score function of this estimator has the same form as (6) and (7), with

$$w_1(z,\theta) = \min\{1, \gamma_1/\|r\|\}, \tag{11}$$

and $w_2(z,\theta)$ given by (9). This estimator has a bounded *IF* only if the regressors take values in a bounded set, but should have good robustness properties when disturbances have a thick-tail distribution.

Let $\bar{\theta} = (\bar{\beta}, \bar{\sigma})$ denote any of the BI_1, BI_2, and Huber-type estimators. For a sample of size N, estimates of β_0 and Σ_0 can be obtained by a simple iteratively reweighted LS algorithm, with the $(i+1)$th iteration given by

$$\bar{\beta}_{(i+1)} = \left[\sum_{n=1}^{N} w_{1n}^{(i)} x_n \bar{\Sigma}_{(i)}^{-1} x_n'\right]^{-1} \sum_{n=1}^{N} w_{1n}^{(i)} x_n \bar{\Sigma}_{(i)}^{-1} y_n,$$

$$\Sigma_{(i+1)} = \left[\sum_{n=1}^{N} w_{2n}^{(i)}\right]^{-1} \sum_{n=1}^{N} w_{2n}^{(i)}\left(y_n - x_n'\bar{\beta}_{(i)}\right)\left(y_n - x_n'\bar{\beta}_{(i)}\right)',$$

where $w_{1n}^{(i)}$ is of the form (8), (10), or (11) and $w_{2n}^{(i)}$ is of the form (9), with β, Σ, and V replaced by the corresponding values obtained from the ith iteration. The algorithm can be started at the single-equation LS estimate of β_0. Starting at some robust estimates is however preferable, and is essential if the algorithm is iterated only a few times.

A simple modification of the argument in Maronna and Yohai (1981) can be used to establish consistency and asymptotic normality of $\bar{\theta}$ under general conditions. The asymptotic variance matrix of $\bar{\theta}$ can be estimated consistently by $P_N^{-1} Q_N P_N^{-1}$, where $P_N = N^{-1}\sum_{n=1}^{N}(\partial/\partial\theta)\eta(z_n, \bar{\theta}_N)$ and $Q_N = N^{-1}\sum_{n=1}^{N}\eta(z_n, \bar{\theta}_N)\eta(z_n, \bar{\theta}_N)'$. If the distribution of the disturbances in (1) is symmetric about zero, the asymptotic variance matrix of $\bar{\theta}$ is block-diagonal with respect to $\bar{\beta}$ and $\bar{\sigma}$, which implies that $\bar{\beta}$ and $\bar{\sigma}$ are asymptotically independent.

The asymptotic normality of $\bar{\theta}$ and the asymptotic independence of $\bar{\beta}$ and $\bar{\sigma}$ lead to simple Wald-type tests of hypothesis concerning the regression parameters. Score tests can also be constructed, based on the average score evaluated at the restricted estimates. The asymptotic normality of $\bar{\theta}$ implies that the Wald- and score-test statistics have an asymptotic χ^2 distribution under the null hypothesis. Since $\bar{\theta}$ is a bounded-influence estimator all these tests are robust, that is, their level and power are relatively insensitive to small deviations from the assumed statistical model [Peracchi (1987)]. This property is not shared by tests based on the Gaussian ML or the Huber-type estimators.

The difference between $\bar{\theta}$ and the Gaussian ML estimator of θ_0 can be used as the basis for specification tests of the type proposed, among others, by Hausman (1978). In the case of bounded influence estimators, such specification tests are likely to be quite powerful because, while $\bar{\theta}$ is only slightly less efficient then $\hat{\theta}_{ML}$ at the assumed model, the difference between $\bar{\theta}$ and $\hat{\theta}_{ML}$ can be large when the model is misspecified.

Finally, in the case of bounded-influence estimators, the weights $\{w_{1n}\}$ and $\{w_{2n}\}$ summarize all the information on the influence of a particular observation and can therefore be used as effective diagnostics for outliers and influential observations. Since the weights are jointly computed with the estimates, no further calculation is required.

4. Monte Carlo results

In this section we report the results of a set of Monte Carlo experiments, carried out in order to study the behavior of the various estimators under small departures from the assumed Gaussian model (1). The departures considered include nonnormal disturbances and gross-errors in the data.

4.1. Basic design of the experiments

The 'central' model consists of two simple regression equations:

$$y_{in} = \beta_{i1} + \beta_{i2} x_{in} + u_{in}, \qquad i = 1, 2,$$

where $\beta_{i1} = \beta_{i2} = 1$. The regressors in each equation have been randomly drawn from a $U(-\xi, \xi)$ distribution, and experiments have been carried out with different values of ξ. The 2×1 disturbance vector has been generated as $u_n = Cr_n$, where C is an upper triangular matrix and r_n is a vector with uncorrelated components. Different choices of C have been considered to allow for different correlation across the equations.

The estimators considered include the single-equation LS estimator, Zellner's feasible GLS estimator, the Gaussian ML estimator, the Huber-type estimator, and the BI_1 and BI_2 estimators. The starting values for bounded-influence estimation are given by the Huber-type estimates. The sensitivity bounds for bounded-influence and Huber-type estimators have been chosen so as to attain an average weight of approximately 95% at the central model. This choice results in an asymptotic relative efficiency of about 95% at the Gaussian model.

Each Monte Carlo experiment consists of 1000 replications, and each set of experiments has been carried out for samples of size 25 and 50.[2]

[2]All programs are written in GAUSS, Version 1.49b. Computations have been carried out on an IBM PS/2 Model 80.

Table 1

Measures of tail length.

Distributions	Tail length
Uniform	0.569
$N(0,1)$	1.00
$CN(0.05,3)$	1.24
$CN(0.10,3)$	1.53
$CN(0.05,5)$	1.73
$CN(0.10,5)$	2.50
$CN(0.05,10)$	3.43
$CN(0.10,10)$	4.93
Slash	7.85

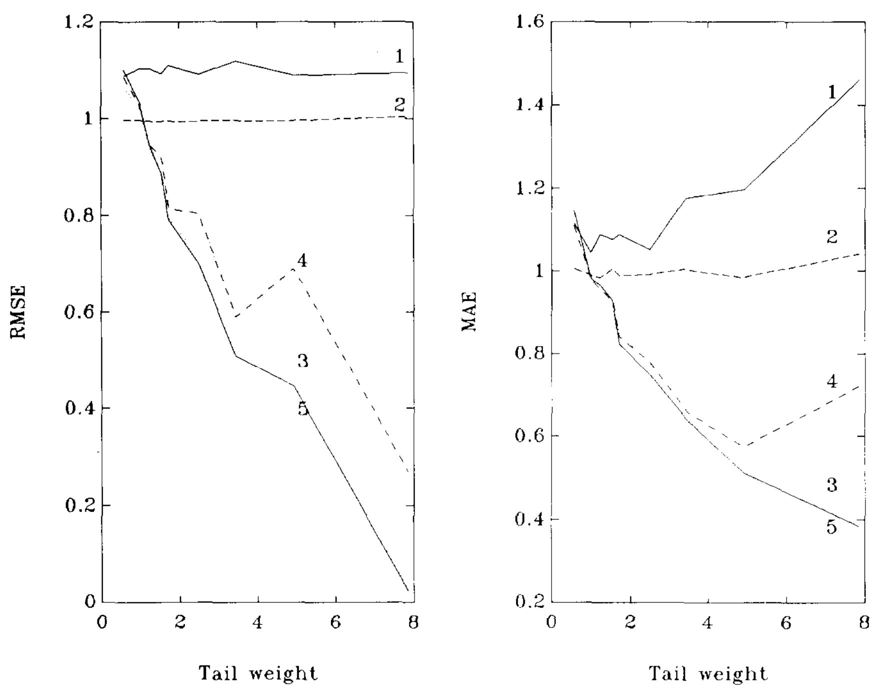

Fig. 1. Efficiency of ML relative to the other estimators ($N = 25$, population $R^2 = 0.50$). 1: OLS, 2: Zellner's GLS, 3: Huber, 4: BI_1, 5: BI_2.

To avoid bothering the reader with an excess of tables, results are presented in graphical form. Detailed numerical tables are available from the author upon request. For simplicity we only present results for the case when $\xi = \sqrt{3}$ and CC' is a matrix with ones on the main diagonal and off-diagonal elements equal to 0.5. For the Gaussian model, this corresponds to a population R^2 of 0.5 for each equation.

4.2. Nonnormal disturbances

First we examine the behavior of the various estimators under small departures of the error distribution from normality. The distributions that we consider are ordered in table 1 by the index of tail length suggested by Rosenberger and Gasko (1983), namely,

$$\tau(F) = \frac{F^{-1}(0.99) - F^{-1}(0.5)}{\Phi^{-1}(0.99) - \Phi^{-1}(0.5)} \Big/ \frac{F^{-1}(0.75) - F^{-1}(0.5)}{\Phi^{-1}(0.75) - \Phi^{-1}(0.5)}.$$

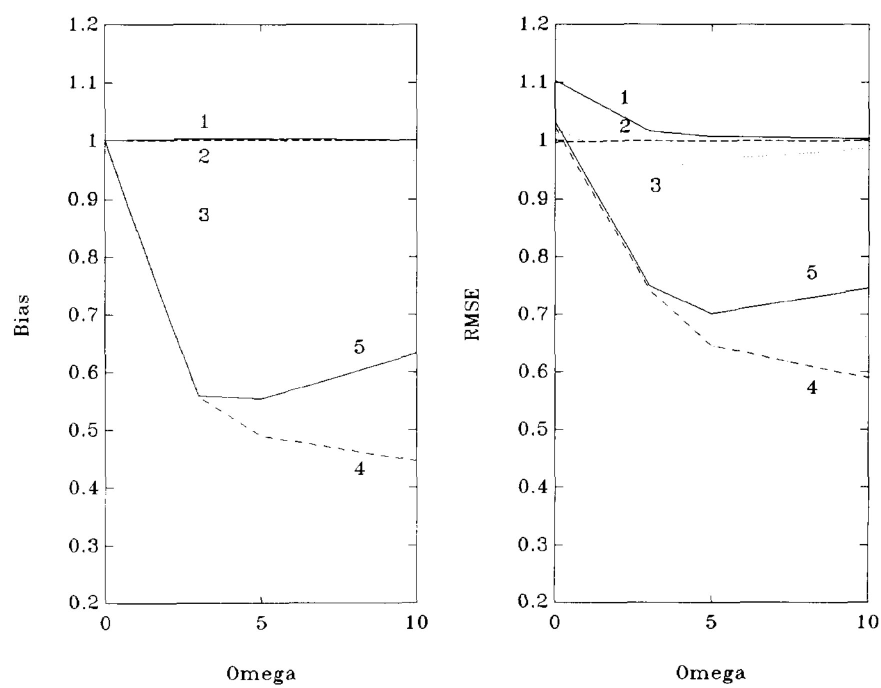

Fig. 2. Bias and efficiency of ML relative to the other estimators (gross-error model: $\delta = 0$, $\pi = 0.05$, $N = 25$). 1: OLS, 2: Zellner's GLS, 3: Huber, 4: BI_1, 5: BI_2.

$CN(\pi, \sigma)$ denotes a contaminated normal distribution with contamination proportion equal to π and variance of the contaminating distribution equal to σ^2. $CN(0.05, 5)$ and $CN(0.10, 5)$ have about the same tail weight as Student's t_2 and t_3 distributions, respectively. The 'slash' is the distribution of the ratio of two independent $N(0,1)$ and $U(0,1)$ random variables. The tail behavior of this distribution is less extreme than the Cauchy, for which $\tau(F) = 9.22$. We also include the uniform to allow for distributions with thin tails. Because all distributions are symmetric, no bias arises and the relevant issue is the precision of the various estimators.

Fig. 1 shows the efficiency of the Gaussian ML estimator relative to the other estimators. For brevity, we only present results for the slope parameter in the first equation and samples of size 25. In terms of root mean square error (RMSE), the single-equation LS estimator is always dominated by Zellner's and Gaussian ML estimators. These two estimators behave very similarly. They are more efficient than robust estimators for thin-tailed distributions, but they loose their efficiency very quickly as the tail length increases. The BI_2 and Huber estimators behave very similarly and are both

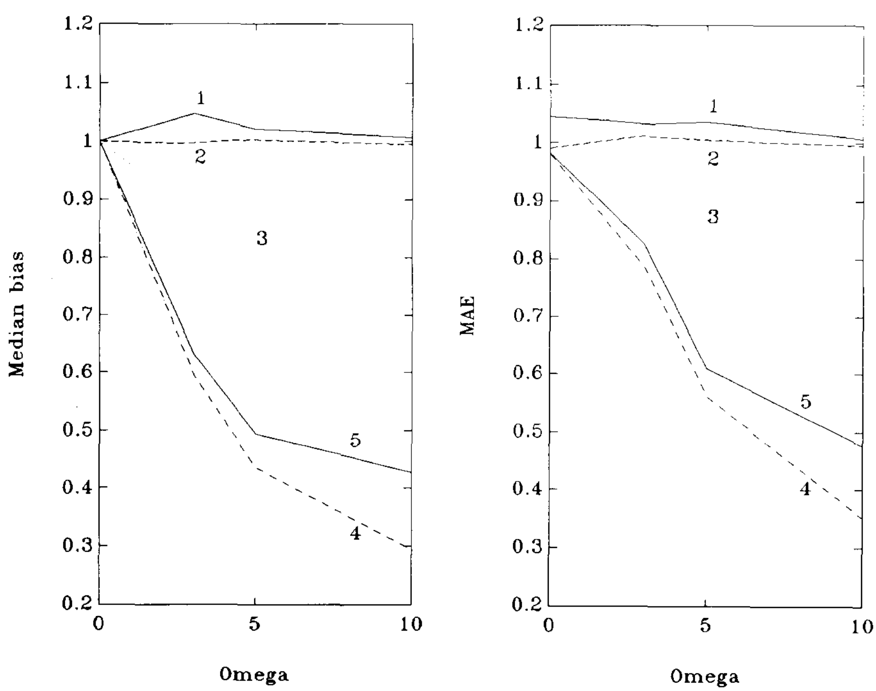

Fig. 2 (continued)

more efficient than the BI_1 estimator for heavy-tailed distributions. In the case of distributions with long tails, the median absolute error (MAE) provides a better measure of an estimator's variability around the true parameter value. The results in terms of MAE agree with the ones in terms of RMSE, except that the decline in the efficiency of Zellner's and Gaussian ML estimators relative to more robust estimators is less pronounced.

Very similar results have been obtained for the slope parameter in the second equation, the intercept parameters in both equations, different choices of the variance of the regressors and different contemporaneous correlation across the two equations.

4.3. Gross-errors

Next we report some results that illustrate the robustness properties of the various estimators under contamination by gross-errors. Observations are

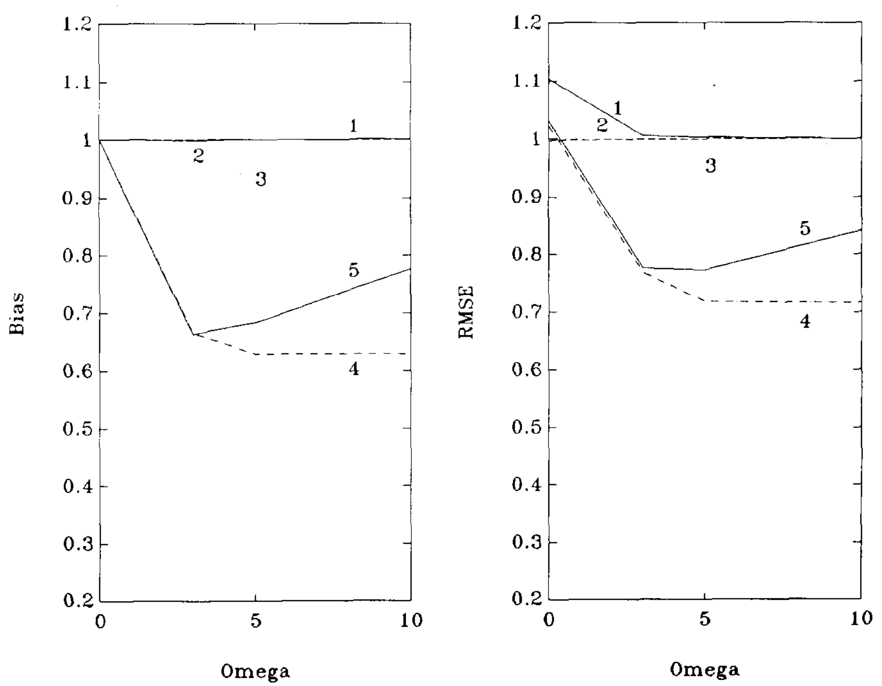

Fig. 3. Bias and efficiency of ML relative to the other estimators (gross-error model: $\delta = 0$, $\pi = 0.10$, $N = 25$). 1: OLS, 2: Zellner's GLS, 3: Huber, 4: BI_1, 5: BI_2.

now generated from a simple errors-in-variables model, where measurement errors occur with small probability. More precisely, observations are generated as follows:

$$y_{in}^* = \beta_{i1} + \beta_{i2} x_{in}^* + u_{in},$$

$$y_{in} = y_{in}^* + \nu_{in},$$

$$x_{in} = x_{in}^* + w_{in}, \qquad i = 1,2,$$

where $\beta_{i1} = \beta_{i2} = 1$, the regressors are uniformly distributed on $[-\sqrt{3}, \sqrt{3}]$, and the regression disturbances $\{u_{in}\}$ are serially uncorrelated and normally distributed with mean zero, unit variance, and $\text{cov}(u_{1n}, u_{2n}) = 0.5$. The measurement errors $\{\nu_{in}\}$ are equal to zero with probability $1 - \pi_\nu$, and with probability π_ν are randomly drawn from a $N(0, \delta^2)$ distribution. Similarly,

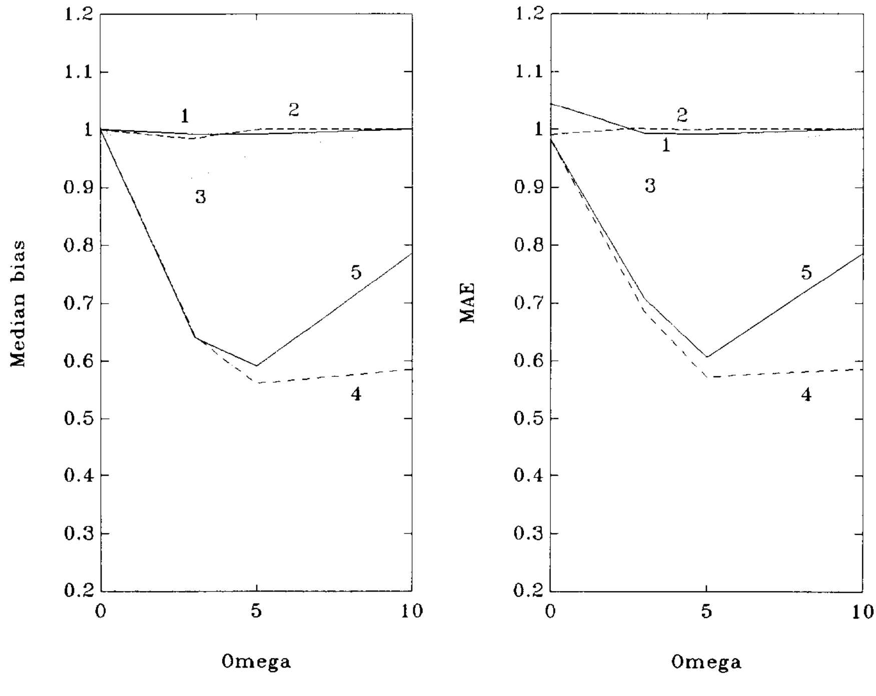

Fig. 3 (continued)

the measurement errors $\{w_{in}\}$ are equal to zero with probability $1-\pi_w$, and with probability π_w are randomly drawn from a $N(0,\omega^2)$ distribution. For simplicity, we set $\pi_v=\pi_w=\pi$. To study the effects of changes in the amount and type of contamination, we considered different choices of π, δ, and ω. When $\pi=0$, we obtain model (1) with Gaussian disturbances. When $\pi=1$, we obtain the classical errors-in-variables model. If the regressors are measured with errors (that is, $\pi>0$), all estimators are biased. We focus attention on the relative behavior of the various estimators in terms of their bias and precision when measurement errors occur with small probability. Bias is measured by the difference between the (Monte Carlo) mean or median and the true parameter value, and precision is measured by the RMSE and the MAE. For brevity, we only report results for the slope parameter in the first equation and samples of size 25.

Fig. 2 refers to the case when $\pi=0.05$ and only the regressors are measured with error, that is, $\omega>0$ and $\delta=0$. The bias and the imprecision

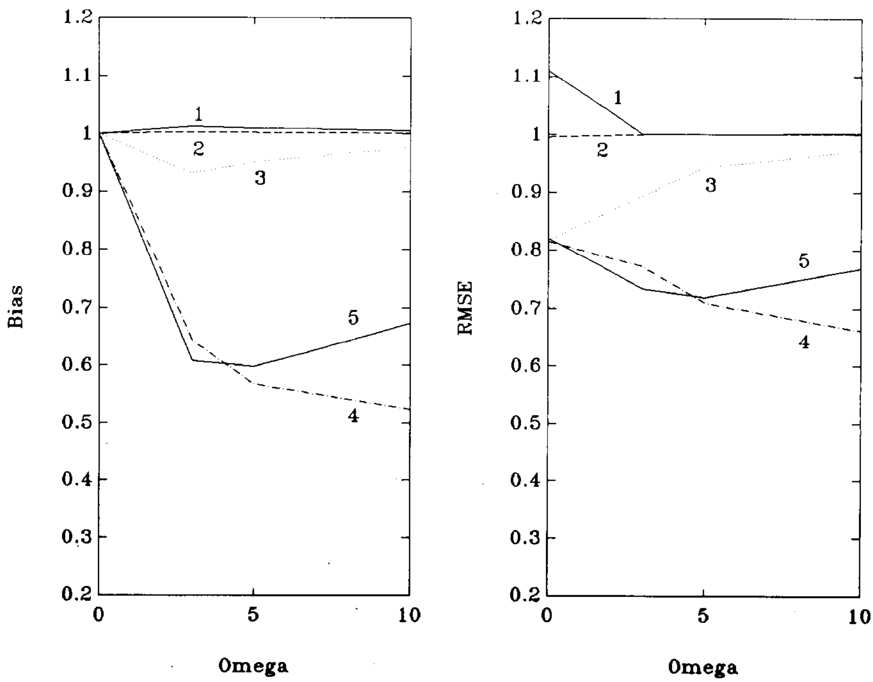

Fig. 4. Bias and efficiency of ML relative to the other estimators (gross-error model: $\delta=5$, $\pi=0.05$, $N=25$). 1: OLS, 2: Zellner's GLS, 3: Huber, 4: BI_1, 5: BI_2.

increase with ω for all estimators, but this increase is very moderate for the two bounded-influence estimators, and especially for the BI_1 estimator. Notice that the Huber-type and the Gaussian ML estimators now behave very similarly in terms of both the bias and the imprecision, particularly for large value of ω. Also notice how the superiority of bounded-influence estimators is even clearer if the median bias and the MAE are considered.

Our justification for bounded-influence estimators is that they offer protection against small departures from the assumed statistical model. We now consider the effects of increasing the 'degree of misspecification' of the model, by either increasing the measurement error probability π or by introducing measurement errors in the dependent variable as well. When the measurement error probability π is increased from 0.05 to 0.10 (fig. 3), our general conclusions do not change, but the superiority of bounded-influence estimators is somewhat reduced. This can also be seen in fig. 4, where the measurement error probability is kept at 0.05, but measurement errors can occur for both the dependent variables and the regressors.

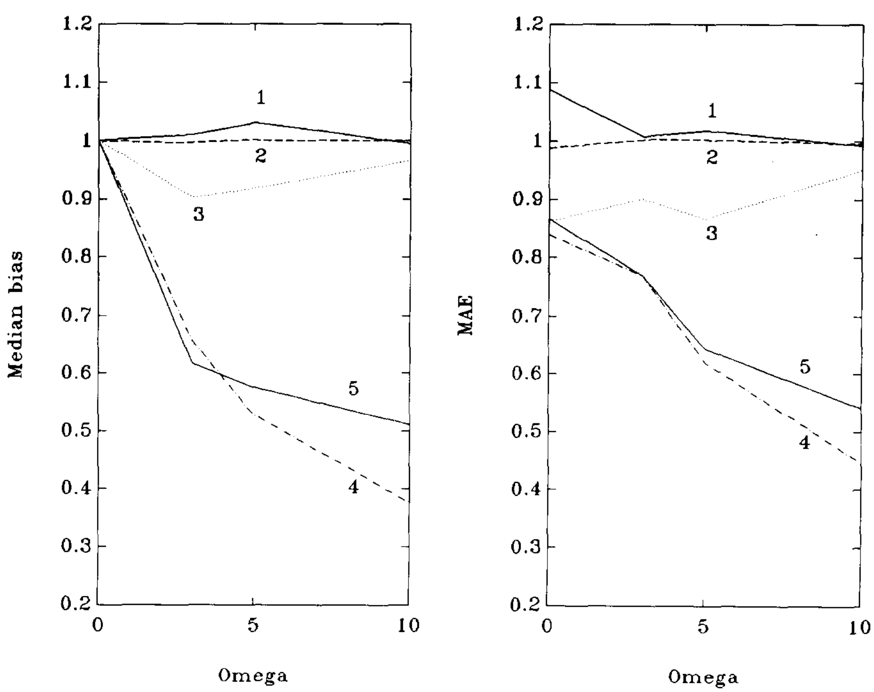

Fig. 4 (continued)

5. Conclusions

The derivation of optimal bounded-influence estimators relies on asymptotic arguments. The Monte Carlo results presented in this paper indicate that these estimators maintain good efficiency and robustness properties even in small samples. This provides further evidence in favor of using optimal bounded-influence estimators when an investigator seeks protection against small departures from the model assumptions, while retaining high efficiency at the central model. The price that must be paid, namely some efficiency loss if the assumed model is exactly correct, seems rather small compared with the potentially large gains in terms of reduced bias and increased precision of the estimates of the parameters of interest. Further, one is free to choose the efficiency loss that he/she is willing to tolerate.

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