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Concentration around a sphere for a singularly perturbed Schrödinger equation.

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The behaviour as $h \to 0$ of positive radially symmetric solutions $u = u_h$ to $-h^2\Delta u + V(x)u = |u|^{p-2}u$ is studied. Here $x \in \mathbf{R}^n$ with $n \geq 3$, $2 < p < 2n/(n-2)$, and the potential $V(x) = V(|x|) \in C^1(\mathbf{R}^n, \mathbf{R})$ is radially symmetric and such that $\inf_{x\in\mathbf{R}} V(x) > 0$. Under additional assumptions on V, which basically require that $V = V(r)$ is sufficiently large in some interval $r \in [r_1, r_2]$, it is shown that in the semiclassical limit $h \to 0$ (along a subsequence) the functions u_h will concentrate on a sphere of positive radius. Thus the potential barrier prevents concentration at the origin, as is normally found in these kinds of problems.

Reviewed by *Markus Kunze*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.