THE PROBLEM OF QUANTUM-LIKE REPRESENTATION IN ECONOMY COGNITIVE SCIENCE, AND GENETICS

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We outline our programme to create quantum-like representations in economy, cognitive science, psychology, genetics,....

The basis of the *quantum-like paradigm* consists in understanding that the mathematical apparatus of quantum mechanics and especially quantum probability is not rigidly coupled with *quantum physics* but can have a wider class of applications.

Recall that differential and integral calculus were developed to serve classical Newtonian mechanics. However nowadays nobody is surprised that these tools are widely used everywhere – in engineering, biology, economy, ... In the same way, although the mathematical apparatus of quantum mechanics was developed to describe phenomena in the microworld, it could be applied to the solution of various problems outside physics.

One of the interesting open problems is to apply quantum probability e.g. to cognitive science or to financial markets. One of the main distinguishing features of quantum probability is the use of complex probability

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amplitudes. In the abstract approach such amplitudes are represented by normalized vectors in a complex Hilbert space while the so called mixed states are represented by density matrices. Probabilities (which are compared with experimental relative frequencies) are given by Born's rule:

$$P_\psi(a = \alpha) = |\langle\psi, e_\alpha\rangle|^2,$$

where the observable a is represented by a self-adjoint operator $\widehat{a}$ and e_α is its eigenvector corresponding to the eigenvalue $\alpha : \widehat{a}e_\alpha = \alpha e_\alpha$ (Only the case of operators with purely discrete and nondegenerate spectra is considered here).

During the past 70 years the development of quantum mechanics has been characterized by intensive debates on the origin of quantum randomness and in particular on possibilities to reduce it to the classical ensemble randomness. For example, von Neumann was convinced that quantum randomness is irreducible, but Einstein had the opposite view to this problem: for him the discovery of quantum mechanics was merely a discovery of a special mathematical formalism (quantum formalism) for description of a special incomplete representation of information about microsystems.

This debate is directly related to the problem of creation of quantum-like representations outside of quantum physics. In the majority of applications, e.g., in economy or biology, the conventional models are based on macroscopic variables. For example, contemporary neurophysiology is based on a model with a neuron as the basic unit of information processing.

According to the Copenhagen interpretation of QM a pure quantum state (wave function) describes an *individual quantum system* not an ensamble of systems in the sense of classical probability. As a consequence of such an "individual interpretation" a concrete physical system can be prepared in a physical superposition of pure states.

In the majority of textbooks on QM we can read about e.g. an atom in superposition of different energy states, or an electron in superposition of spin-up and spin-down states, in the famous two slit experiment a photon is in a superposition of passing through both slits.

An attempt to apply the mathematical formalism of QM outside of the microworld in combination with the Copenhagen interpretation would create visible difficulties: it is not easy to imagine a macroscopic system e.g. in economics which is in a real, physical, superposition of two states. The authors of this paper are well aware about macroscopic quantum systems as well as of the attempts to use the Copenhagen interpretation even in this case – e.g. by Legget in superconductivity, by Zeilinger in the two slit experiments for macroscopic systems, by De Martini in experiments with "macroscopic Schrödinger cats" etc. ....

It is well known that such attempts to proceed with the Copenhagen interpretation for macroscopic quantum systems does not provide a clear physical picture of the phenomena. One of the possibilities is to use De Broglie's wave length for characterization of the wave features of a macroscopic system. Since it is very small for a large system, it is always possible to say that, although a macroscopic system has wave features, they are hardly observable.

This kind of compromise is hardly satisfactory as a solution to a conceptual problem.

Moreover, as already pointed out by Pauli in the early times of QM, any attempt to interpret the wave function as a physical wave clashes against the fact that, for most interesting physically systems, these wave functions are defined in a multi–dimensional mathematical space. Thus the supporters of the *wave–particle duality* face the paradox of believing in *a physical wave in a non physical space.*

As a consequence of the above mentioned difficulties with the interpretation of macroscopic quantum systems, a popular attitude today is to proceed beyond conventional models (e.g. in biology) which operate with states of macroscopic systems.

For example, in cognitive science a group of researchers (e.g., Penrose and Hameroff) developed the reductionist approach to the brain functioning. They moved beyond the conventional neuronal paradigm of cognitive science and tried to reduce processing of information in the brain to quantum micro processes – on the level of quantum particles composing the brain. Penrose repeated many times that a neuron (as a macroscopic system) could not be in a physical superposition of two states: firing and nonfiring.

As was already mentioned, the majority of attempts to apply the mathematical formalism of quantum mechanics outside physics were based on the reduction of the processes under consideration to some underlying quantum processes in the microworld. This reductionist approach was heavily based on the following argument: since everything in this world is composed of quantum particles, any kind of process might be (at least in principle) reduced to a quantum processes.

The unification dream is in principle correct, and it has played an important role in the development of natural sciences, in this spirit any attempt to apply quantum mechanics to e.g. cognitive science should be welcome. However, it is very difficult (if even possible at all) to establish a natural correspondence between conventional macroscopic models and underlying quantum models. There is a huge difference in scales of parameters in those models. Moreover even in quantum physics the *correspondence principle* is vaguely formulated and not totally justified and, on the other hand, even in classical physics, the unification dream is far from being accomplished in spite of the important successes of statistical mechanics in the reduction of thermodynamics to to mechanics. For example structures such as crystals, which are relatively simple with respect to biological structures, at the moment have not been deduced from first principles neither in classical nor in quantum physics.

We point out that it is possible to escape the above mentioned difficulties by rejection of the Copenhagen interpretation and association of a pure quantum state (wave function) not with an individual quantum system, but with an ensemble of identically prepared systems.

Such an interpretation is called the *statistical interpretation* of QM. It has been originally proposed by many authors among which Einstein, Popper, Margenau, De Broglie, Bohm, Ballentine, ... but only with the development of quantum probability it could overcome the traditional critiques which prevented, for over 50 years, the majority of physicists to accept this apparently natural interpretation. The main objection to it, to which the above mentioned authors never gave a satisfactory answer, was that the statistical interpretation is contradicted by the experimental data.

Concerning this objection the main point of quantum probability is that the experimental data contradict the use of the Kolmogorov model of probability and not the statistical interpretation. If one keeps to the statistical interpretation, then one can assume that the quantum probabilistic description need not be based on irreducible quantum randomness [1].

The quantum probabilistic calculus can be used for incomplete description of statistical data.[a]

One could not even exclude that in some cases a Kolmogorov model

[a]Here we should distinguish between the theoretical view of quantum probability as the study of *all* non Kolmogorovian models and the more restricted point of view that wants to limit the investigations to the original quantum model. Moreover quantum probability proves that the appeal to an *irreducible quantum randomness* is not necessary, but it does not prove that it is wrong. This is a personal belief that cannot be scientifically proved or disproved and it is a fact that a number of scientists, including renown pioneers of quantum probability like Hudson and Belavkin, still adhere to irreducible randomness.

can be found beyond the quantum probabilistic description. The crucial point is that the role of the presence of a "hidden Kolmogorovian model" is negligible if one has no access to data described by the latter (typically unobservable joint probabilities). In such cases the only reasonable possibility is to use the quantum probabilistic description or different non Kolmogorovian models.

Thus we propose to test the approach based on the accepting of Einstein's viewpoint: incompleteness of quantum mechanics. The natural question which is typically asked as first reaction to our proposal is the following:

What about the known no-go theorems?

We will not enter here into a debate on the complicated problem of the validity of no-go theorems (for this we refer to ([3])). In fact, the main problem of the no-go ideology is that it is directed against all possible prequantum models (the so called hidden variable models). [b] Supporters of no-go activity formulate new theorems excluding various classes of models with hidden variables, but one could never be sure that a natural model which does not contradict any known no-go theorem would be finally found. In particular, Accardi[2] pointed out to the possibility to produce non classical, i.e. non Kolmogorovian, statistics by using *classical adaptive local dynamical systems*, see e.g. Ohya[4], for modelling of the process of measurement. Such models are known as chameleon models: this animal adapts his color to color of surface. Chameleon realism differs essentially from Einstein realism – association of values of quantum observables directly with states of systems.

Einstein realism does not take into account the dynamics of the process of interaction of a system with the measurement device. In fact chameleon realism matches well with the ideas of the father of the Copenhagen interpretation N. Bohr who permanently pointed out that the whole experimental arrangement should be taken into account[5]. One might speculate that Bohr would prefer chameleon realism to such rather strange things as nonlocal realism or "quantum nonlocality".

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[b]We do not agree with Bell's attempt to couple the so called "quantum nonlocality" with the problem of completeness of quantum mechanics. It has now been experimentally proved that "quantum nonlocality" is an absurd alternative to incompleteness. Unfortunately, in spite of the mathematical and experimental evidence, nowadays quantum nonlocality has become extremely popular in quantum information theory. Moreover, this idea diffuses outside quantum physics: it became fashionable to refer to quantum nonlocality in cognitive and social sciences and even in parapsychology.

We share Einstein's views only partially. We keep to the statistical interpretation of the quantum state and consequently incompleteness of QM, but we agree with Bohr in considering the values of some quantum observables as responses to interactions with apparata rather than objective properties of quantum systems.

We know that by using e.g. adaptive dynamical system one can obtain a local realistic model for quantum measurements. We could summarize our proposal as follows:

" Be not afraid to consider the quantum description as an incomplete one. Look for applications of quantum probability outside quantum physics! Use adaptive dynamical systems to describe interactions among "conventional variables" (e.g. neuronal states) which produce a quantum-like behavior".

As a comment the use of the notion quantum-like(QL) behavior, cf. Khrennikov [6, 7], we think that it would be useful to preserve the term "quantum" for quantum physics while, in other models which are still based on quantum or, more generally non Kolmogorovian, probabilistic description we should use the term "quantum-like". In particular, in this way we can distinguish our approach from a purely reductionist one. For example, the quantum brain model is a reductionist model of the brain functioning, but the quantum-like brain model is a model in which the wave function provides a (incomplete) probabilistic representation of information produced by the neurons[8] and not a model for the actual physical state of them.

The QL modelling immediately meets one complex problem: the creation of QL-representations (in complex Hilbert space) of classical probabilistic data. For example, looking for a QL model of image recognition, see e.g. Fichtner et al[9], it would be natural to represent an image by a wave function. Image processing may be modelled by using e.g. (in the simplest case) Schrödinger's equation. But we should solve the problem of initial conditions:

How does the brain represents the initial image by the wave function?

If one considers the brain as a kind of probabilistic machine, then this problem can be formulated as the *inverse Born problem:*

To construct a complex probability amplitude on the basis of probabilities.

An attempt to solve this problem was done in a series of works of Krennikov [6, 7]. There was created so called QL-representation algorithm. *Improvement of this algorithm, its generalization as well creation of new QL-*

representation algorithms is an important problem in the realization of the QL paradigm.

We now couple the QL paradigm with another important probabilistic paradigm. Nonclassical statistical data are not covered completely by the conventional quantum model. As one of the authors (Luigi Accardi) [2] pointed out the main distinguishing feature of quantum probability is its *non-Kolmogorovainity.*

It was emphasized that in the same way as in geometry (where starting with Gauss, Lobachevsky, Riemann, ..., various non-Euclidean geometries were developed and widely applied e.g. in relativity theory) in probability theory various non-Kolmogorov models may be developed to serve applications. The QM probabilistic model was one of the first non-Kolmogorovian models which had important applications. Thus one may expect development of other types of probabilistic models which would be neither Kolmogorovian nor quantum.

One of such models was presented – it is the model with so called hyperbolic interference [7]. It is based on representation of probabilities by amplitudes taking values in the algebra of so called hyperbolic numbers. This example motivates extension of the QL paradigm by attempting to develop and apply models in which probability amplitudes take values in various commutative and even noncommutative algebras. The corresponding generalizations of Born's rule should be presented, analogues of the QL-representation algorithms should be created. These are interesting and complex problems!

It is reasonable to extend our definition of a QL model by considering a multiplicity of probabilistic models differing from the conventional Kolmogorov model which is a purely mathematical construction expressing one possible probabilistic description of data. We would like to present a "physical definition" of a QL model coupled to the process of measurement. The main idea behind the QL paradigm is the possibility to represent incomplete probabilistic data about some class of systems. We formalize this idea in the following way.

Postulate H. (Weak Form of the Heisenberg Uncertainty Principle). There exist observables which cannot be measured simultaneously with arbitrary precision.

Definition. (QL Model). Any probabilistic model describing data obtained from observations satisfying the Weak Form of the Heisenberg Uncertainty Principle is called quantum-like.

To create a QL model, one should first find at least two observables which cannot be measured simultaneously. Corresponding probabilistic data should be incorporated into a model, typically by using some algebraic structure, e.g. complex or hyperbolic Hilbert space.

We are now analyzing several different families of empirical data in order to realize concretely the programme outlined in the present paper. These developments will be discussed elsewhere.

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